Bayesian Radio Interferometric Calibration and Imaging

RESOLVE

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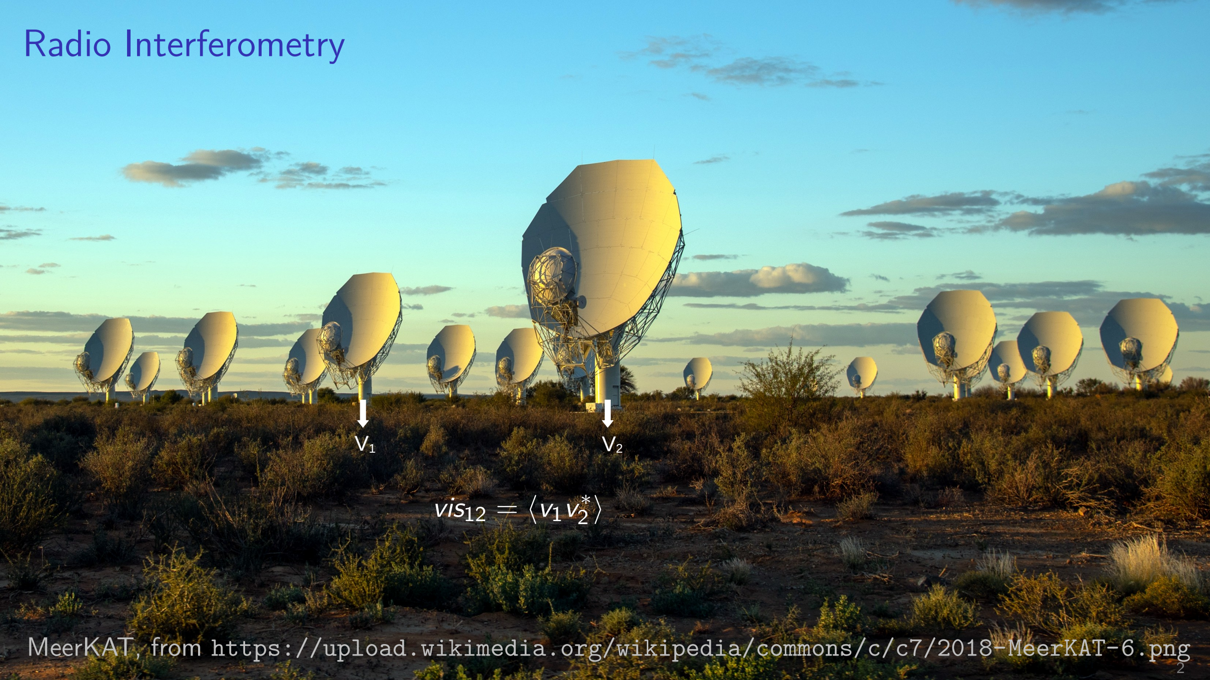
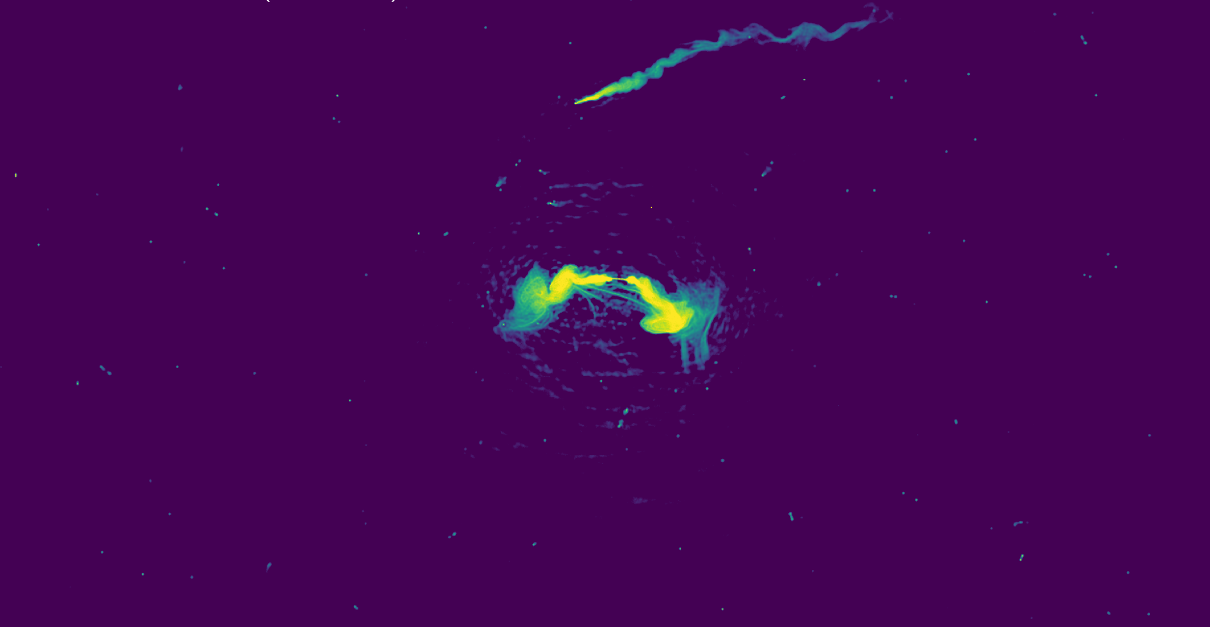
Radio Interferometry



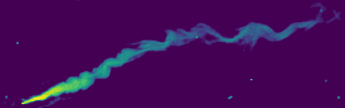
$$vis_{12} = \langle v_1 v_2^* \rangle$$

MeerKAT, from https://upload.wikimedia.org/wikipedia/commons/c/c7/2018-MeerKAT-6.png

ESO 137 @ 1 GHz (MeerKAT)



ESO 137 @ 1 GHz (MeerKAT)



Measurement equation

$$vis_{pqt} = R(I, G) + n = \int \frac{dldm}{n(l, m)} I(l, m) G_p(t, l.m) G_q(t, l, m) e^{-i2\pi(ul + vm - w(n-1))} + n$$

- Sky brightness I
- Antenna gain G
- Visibilities: fourier modes of the sky

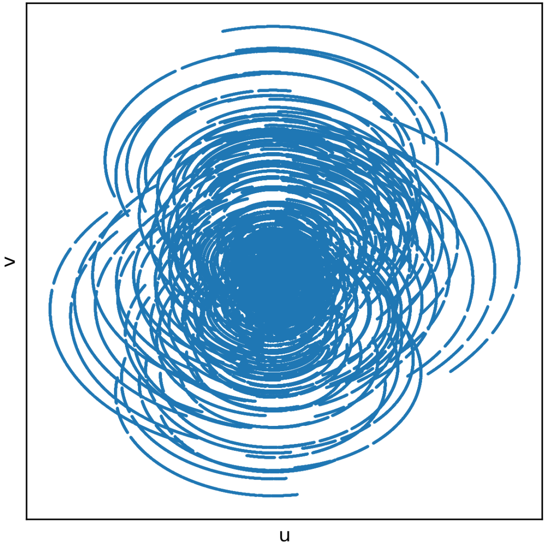
Radio interferometric imaging

Measurement equation

$$vis_{pqt} = R(I, G) + n$$
$$= \int \frac{d\boldsymbol{l}}{n(\boldsymbol{l})} G_p(\boldsymbol{l}, t) G_q(\boldsymbol{l}, t) I(\boldsymbol{l}) e^{-i2\pi \boldsymbol{lk}} + n$$

- Sky brightness I
- Antenna gain G
- Visibilities: fourier modes of the sky

- R is not invertible
- G is unknown
- Imaging is an inverse problem



Inverse Problem – Bayes' Theorem

Bayes' theorem

$$\mathcal{P}(s|d) = \frac{\mathcal{P}(d|s)\mathcal{P}(s)}{\mathcal{P}(d)}$$

- Data: d (e.g. visibilities)
- Signal: s (e.g. sky brightness I, antenna gain G)

Inverse Problem – Bayes' Theorem

Bayes' theorem

$$\mathcal{P}(I, G|vis) = \frac{\mathcal{P}(vis|I, G)\mathcal{P}(I, G)}{\mathcal{P}(vis)}$$

- Data: d (e.g. visibilities)
- Signal: s (e.g. sky brightness I, antenna gain G)

- How to choose the prior $\mathcal{P}(I, G)$?
- How to construct the likelihood $\mathcal{P}(vis|I, G)$?
- How to compute the posterior $\mathcal{P}(I, G|vis)$?

Bayesian Imaging – Software

NIFTy

- `https://gitlab.mpcdf.mpg.de/ift/nifty`
- Prior Models
- Inference Algorithms
- New summary paper[a]

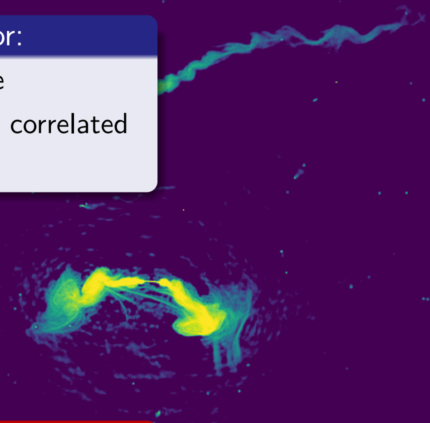
[a] G. Edenhofer et al. "Re-Envisioning Numerical Information Field Theory (NIFTy.re): A Library for Gaussian Processes and Variational Inference". In: (2024). arXiv: 2402.16683 [astro-ph.IM].

RESOLVE

- `https://gitlab.mpcdf.mpg.de/ift/resolve`
- Handling radio interferometric data
- Measurement equation

Physics to be encoded in the prior:

- Sky brightness positive definite
- Diffuse emission: nearby pixels correlated
- Flexible prior



Idea

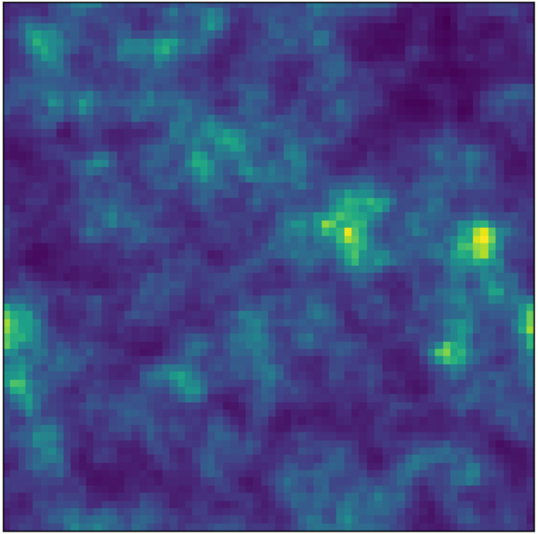
Generative prior model

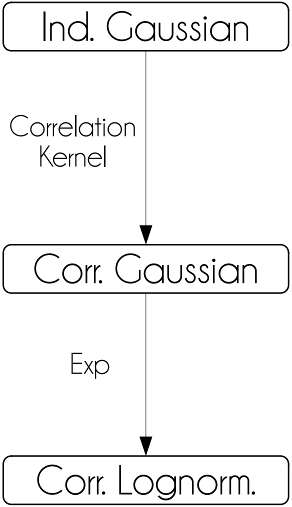
Prior as Generative Model

Prior as a standardized generative model:

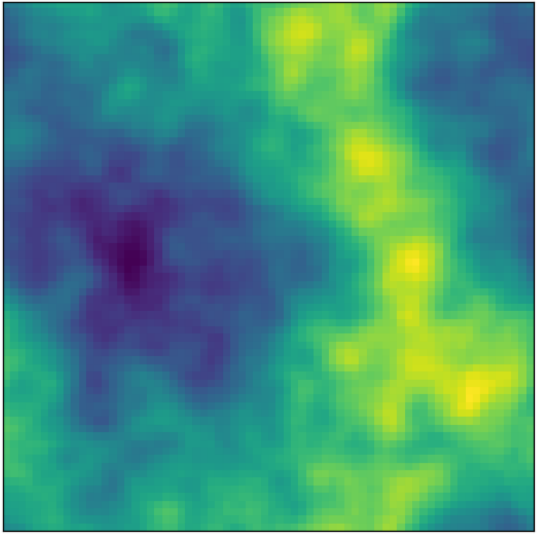
- Latent parameters ξ
- $\mathcal{P}(\xi) = \mathcal{G}(0, 1)$
- $\xi \mapsto s$

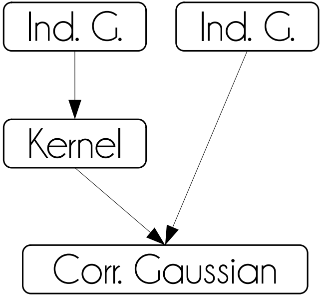
Prior as Generative Model

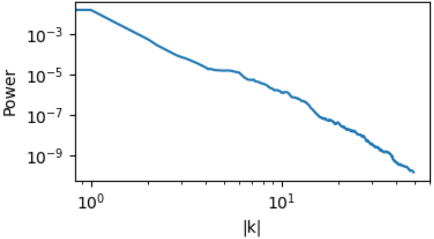




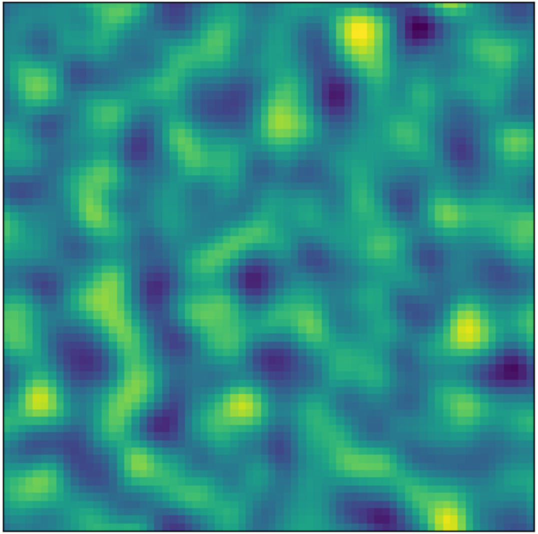
Prior as Generative Model

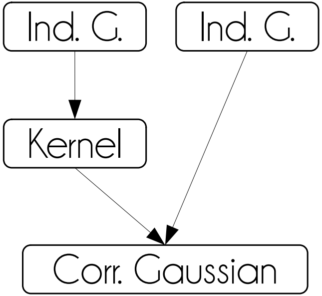


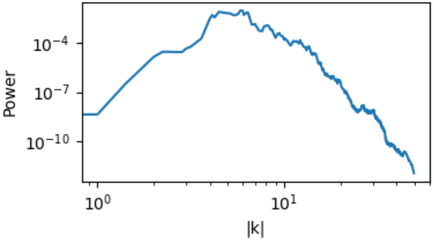




Prior as Generative Model







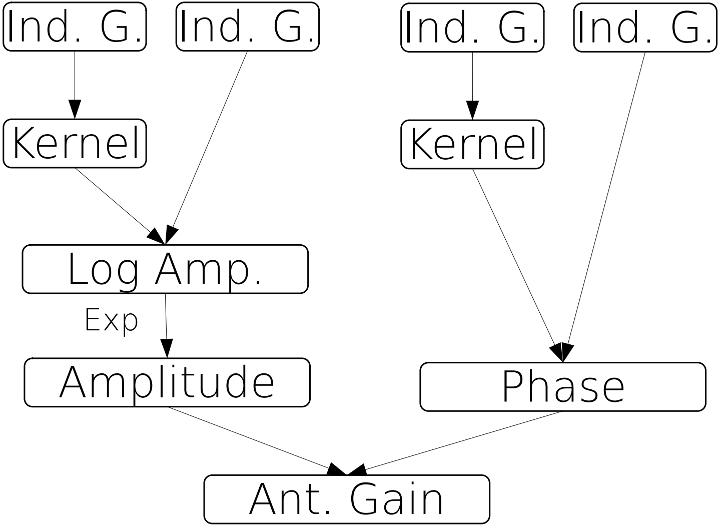
Prior as Generative Model

- Antenna gain Model:

$$G_p(t, l, m) = \alpha(t, l, m)e^{i\phi(t,l,m)}$$

- amplitude α: sensitivity of antenna
- phase ϕ: delay of radio wave

Prior as Generative Model



$$\mathcal{P}(s|d) = \frac{\mathcal{P}(d|s)\mathcal{P}(s)}{\mathcal{P}(d)}$$

$$\Downarrow$$

$$\mathcal{P}(\xi|d) = \frac{\mathcal{P}(d|s(\xi))\mathcal{P}(\xi)}{\mathcal{P}(d)}$$

How to construct the likelihood $\mathcal{P}(I, G|vis)$

Measurement equation:

$$vis = R(I, G) + n = \int \frac{d\boldsymbol{l}}{n(\boldsymbol{l})} G_p(\boldsymbol{l}, t) G_q(\boldsymbol{l}, t) I(\boldsymbol{l}) e^{-i2\pi \boldsymbol{lk}} + n$$

- Sky brightness I
- Antenna gain G
- Gaussian distributed noise $n \curvearrowleft \mathcal{G}(0, N)$
- Likelihood:

$$\mathcal{P}(vis|I, G) = \mathcal{G}(vis - R(I, G), N)$$

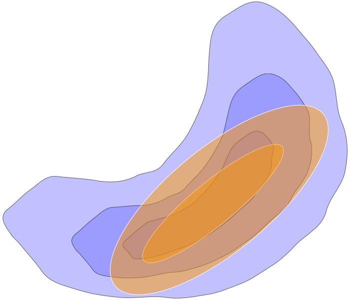
Inference

$$\mathcal{P}(\xi|d) = \frac{\mathcal{P}(d|s(\xi))\mathcal{P}(\xi)}{\mathcal{P}(d)} \tag{1}$$

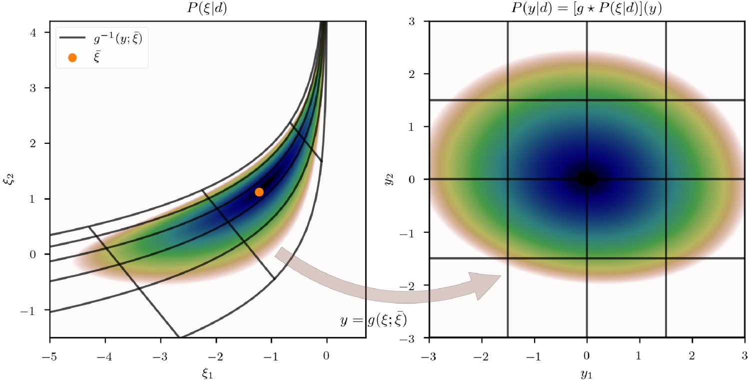
How to obtain the posterior:

- Very low dimensions: Compute directly
- Medium dimensions: Sampling techniques, e.g. HMC
- High dimensions: Variational Inference

GeoVI – Geometric Variational Inference[2]



- Coordinate transformation in latent space
- Approximately transform the Posterior into a Gaussian
- Also linear scaling with number of dimensions



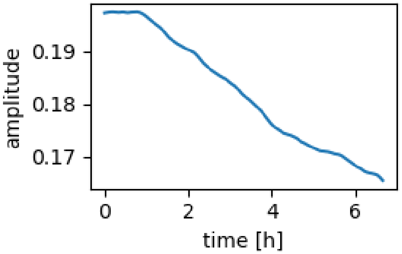
[2] P. Frank, R. Leike, and T. A. Enßlin. “Geometric Variational Inference”. In: *Entropy* 23.7 (2021).

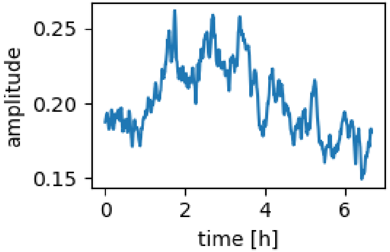
Bayesian Imaging and Calibration

Assumptions:

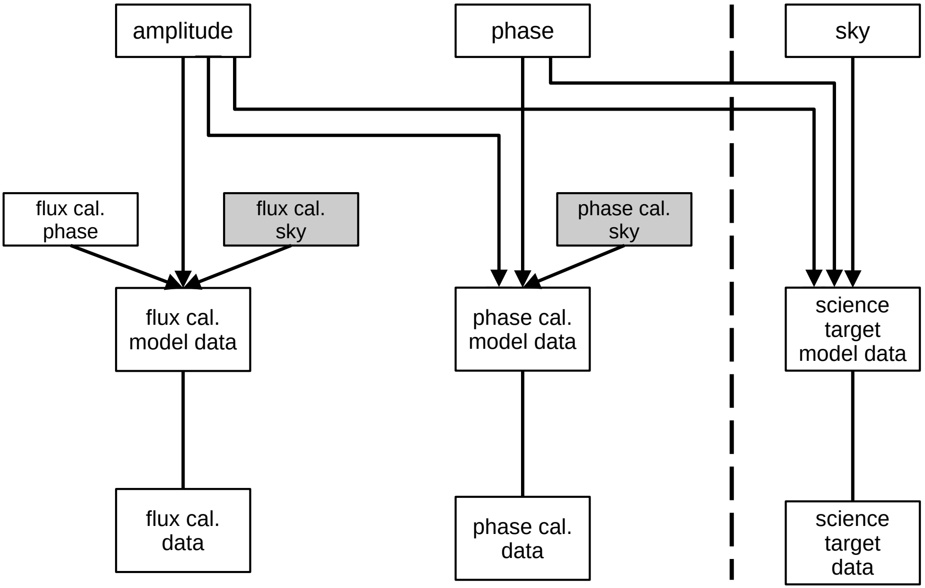
- I, G smooth functions
- Self adaptive degree of smoothness
- Positivity of sky brightness

Example: Joint direction dependent calibration and imaging

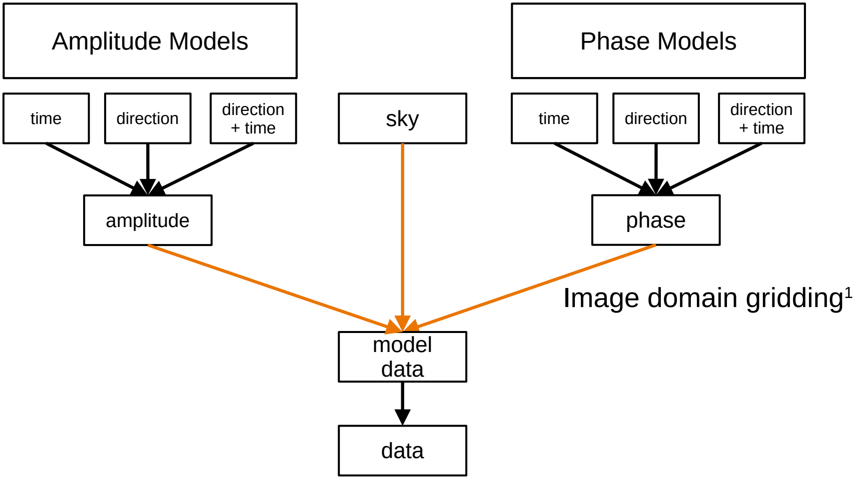




Direction Independent Calibration and Imaging Forward Model

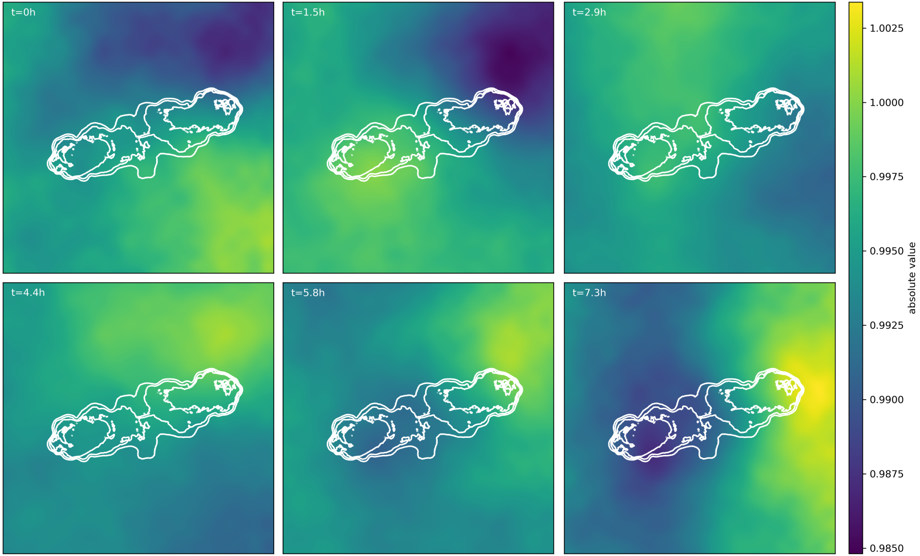


Direction Dependen Calibration and Imaging Forward Model

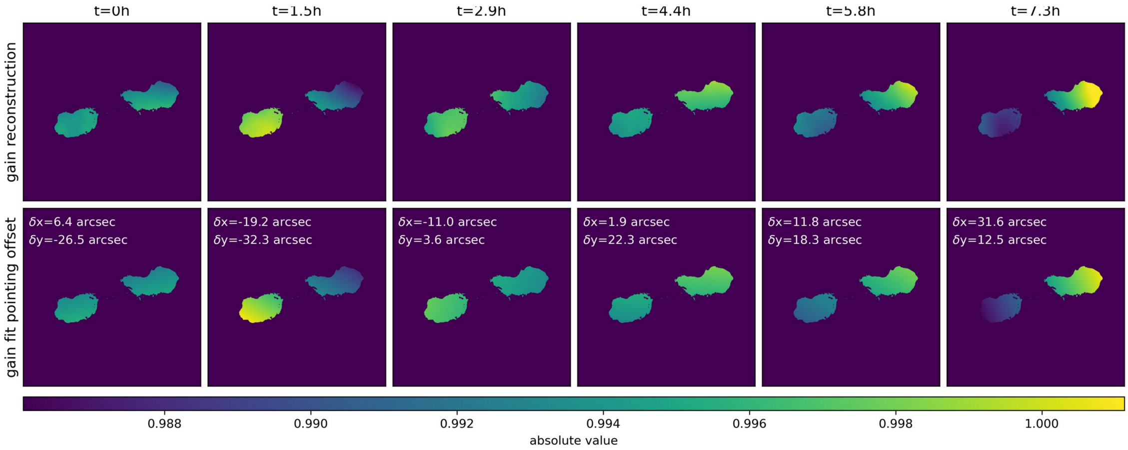


[3]S. van der Tol, B. Veenboer, and A. R. Offringa. "Image Domain Gridding: a fast method for convolutional resampling of visibilities". In: *A&A* 616, A27 (Aug. 2018), A27.

Direction and Time Dependent Calibration



Direction and Time Dependent Calibration – Pointing Erros



Comparison

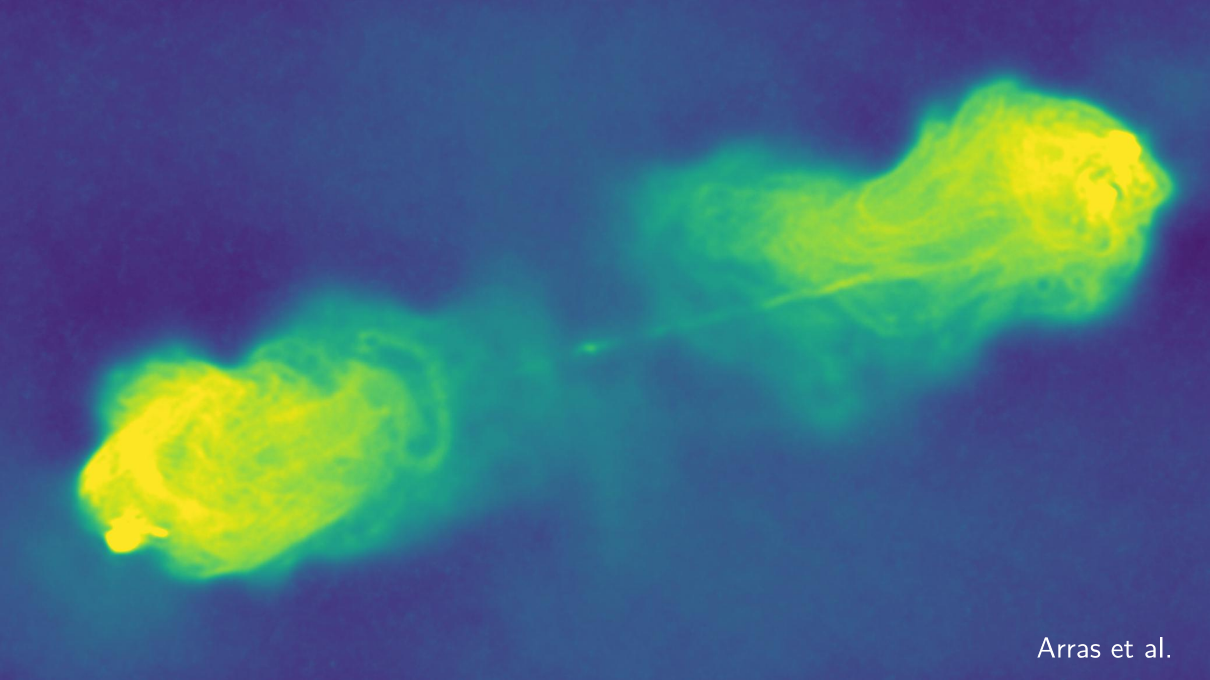
Comparison on VLA 2.05 GHz Data of Cygnus A:

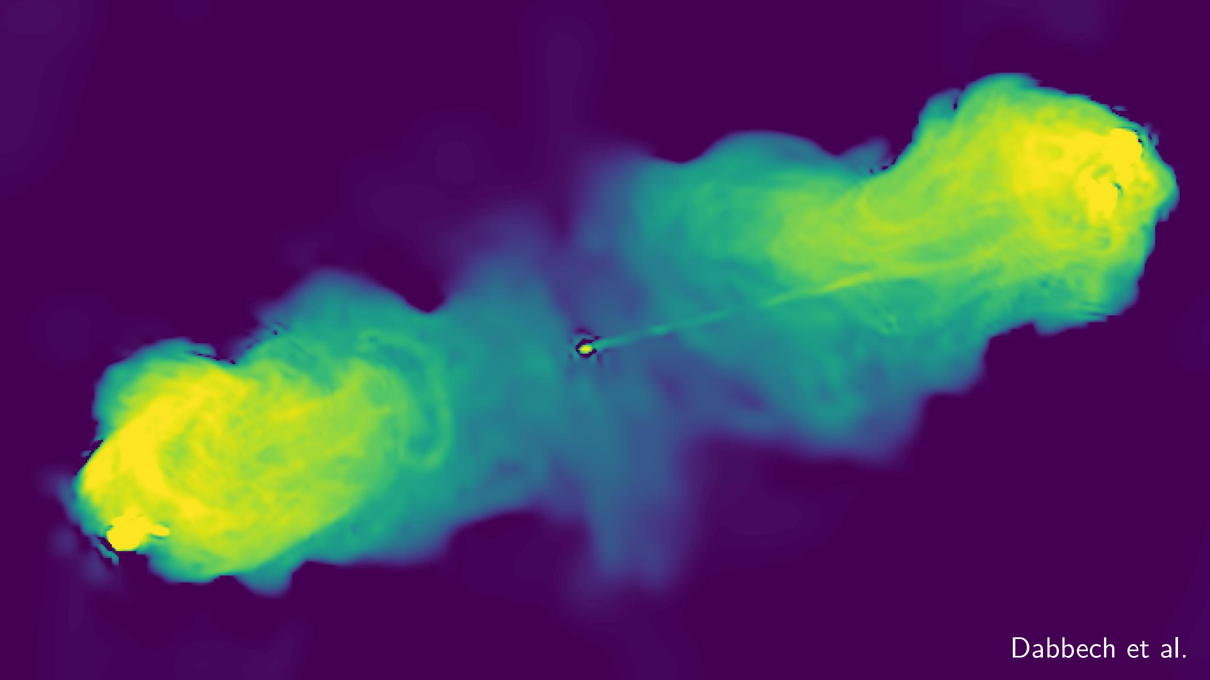
- Roth et al. 2023[4]: `resolve` with direction dependent calibration
- Arras et al. 2021[5]: `resolve` classic calibration
- Dabbech et al. 2021[6]:
  - Compressed sensing method
  - Joint calibration and imaging via non-convex optimization
  - Calibration includes direction dependents

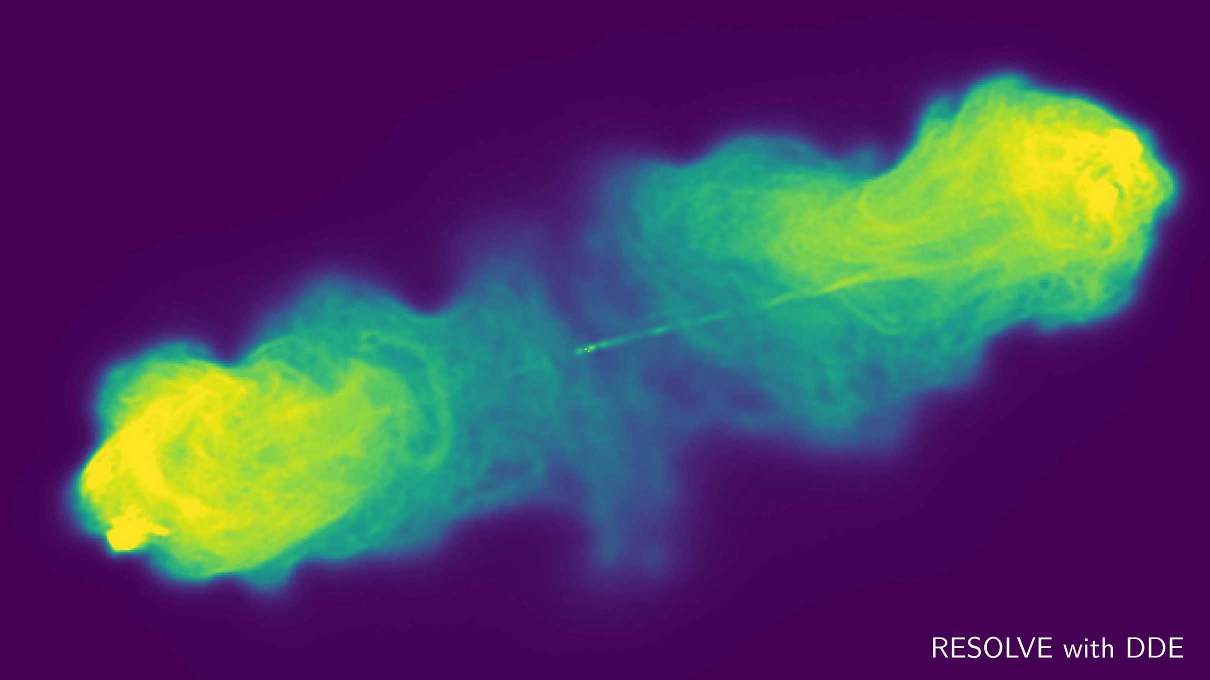
[4] J. Roth et al. "Bayesian radio interferometric imaging with direction-dependent calibration". In: *A&A* 678, A177 (Oct. 2023), A177.

[5] P. Arras et al. "Comparison of classical and Bayesian imaging in radio interferometry. Cygnus A with CLEAN and resolve". In: *A&A* 646, A84 (Feb. 2021), A84.

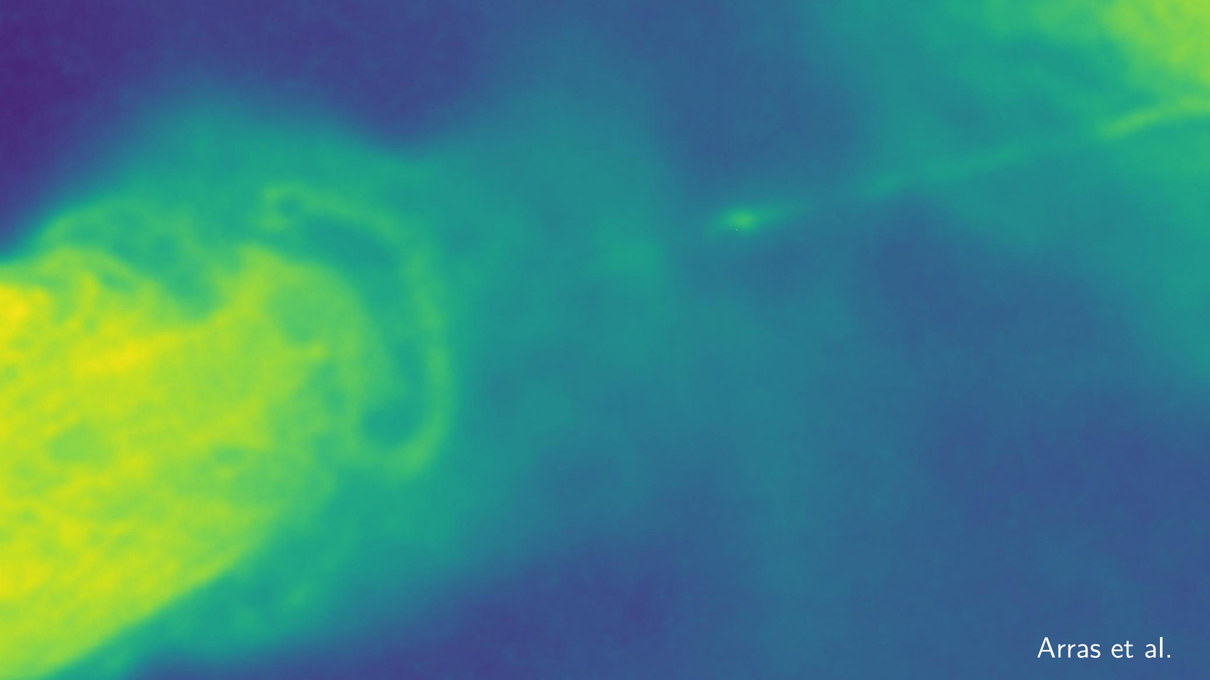
[6] A. Dabbech et al. "Cygnus A jointly calibrated and imaged via non-convex optimization from VLA data". In: *MNRAS* 506.4 (Oct. 2021), pp. 4855–4876.


Arras et al.

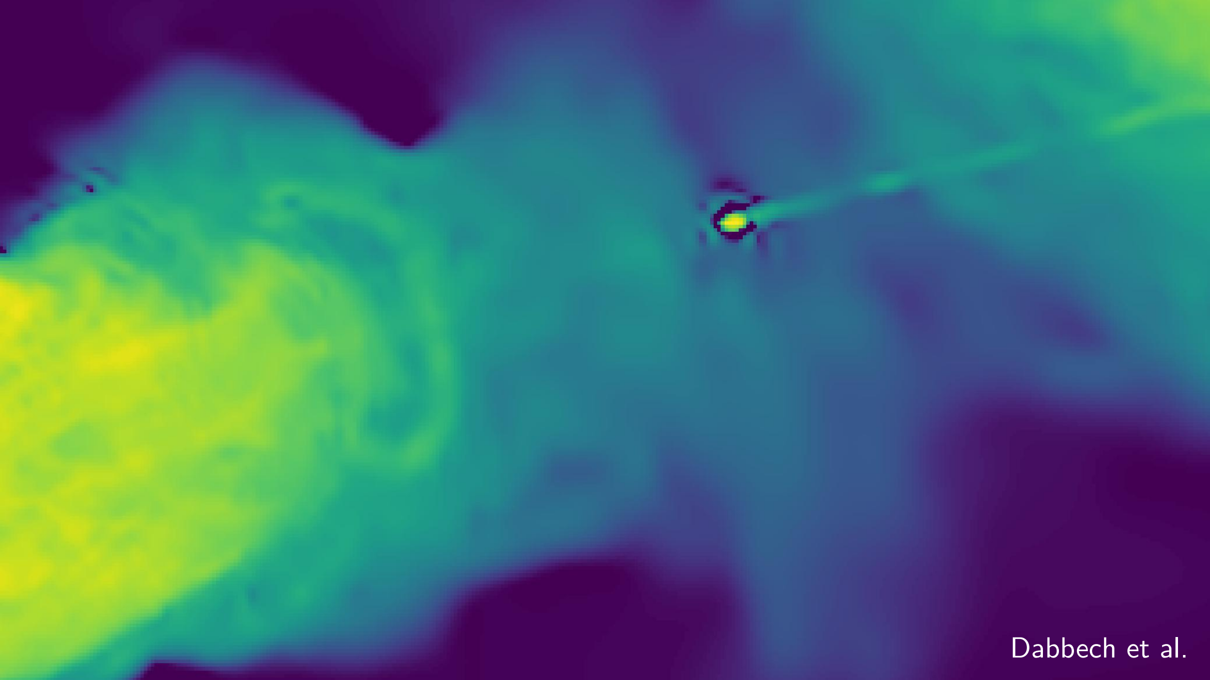

Dabbech et al.



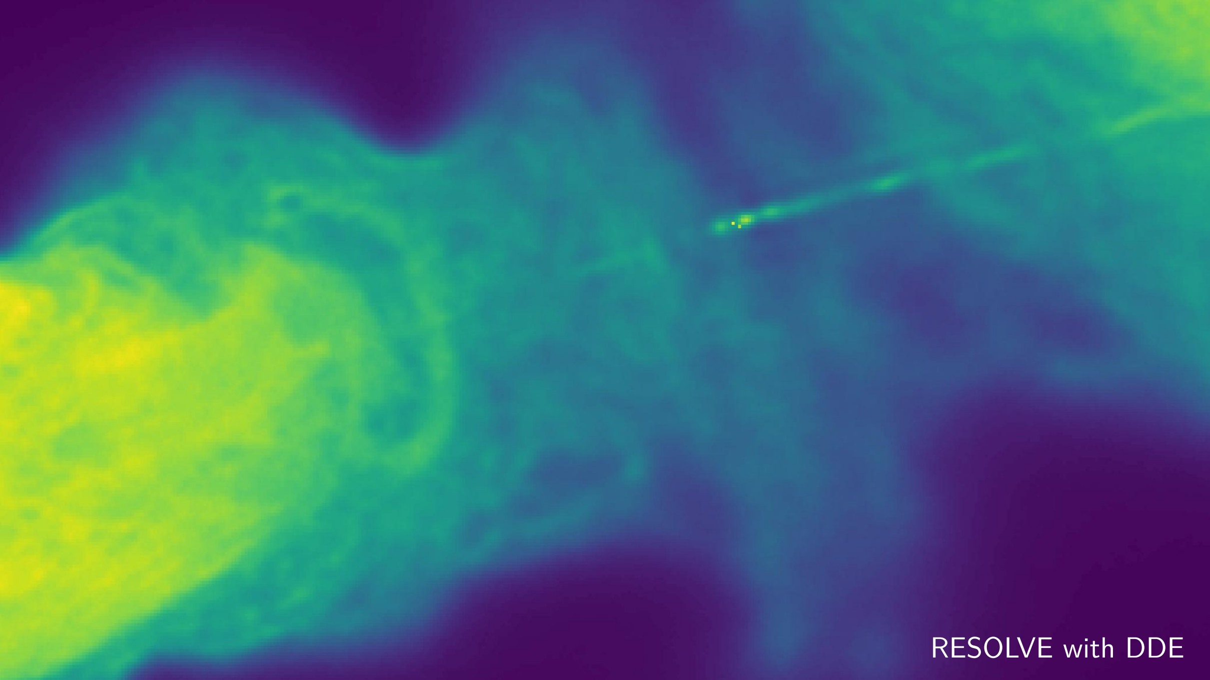
RESOLVE with DDE

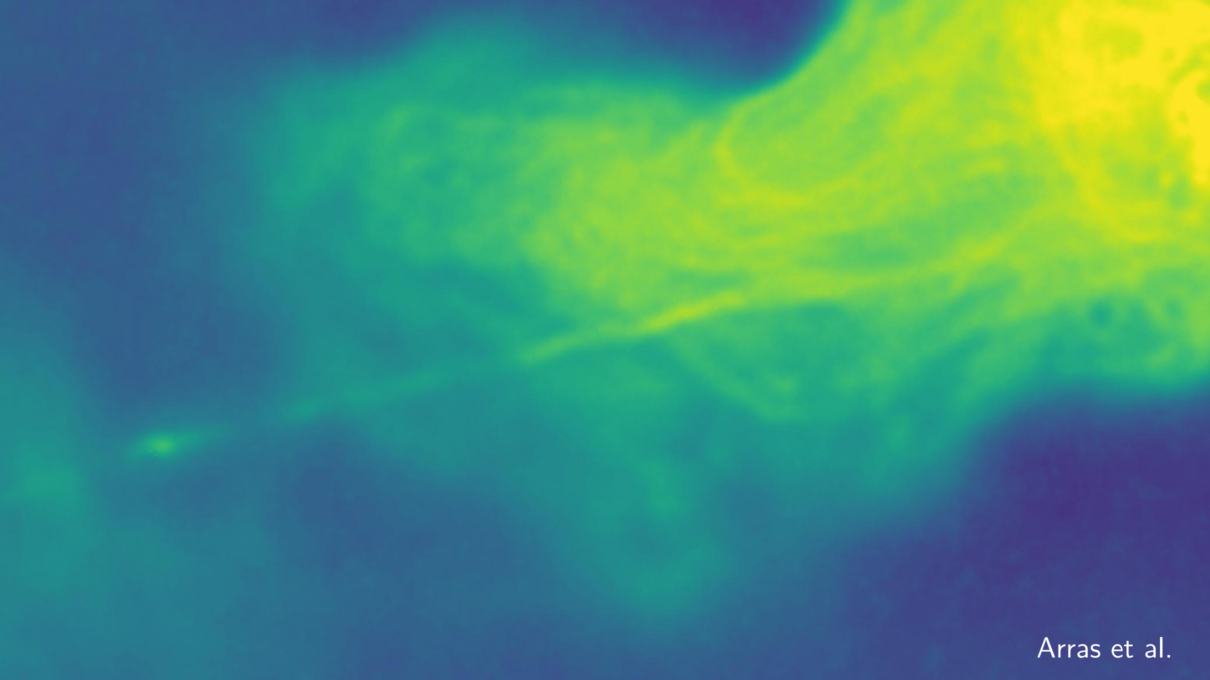


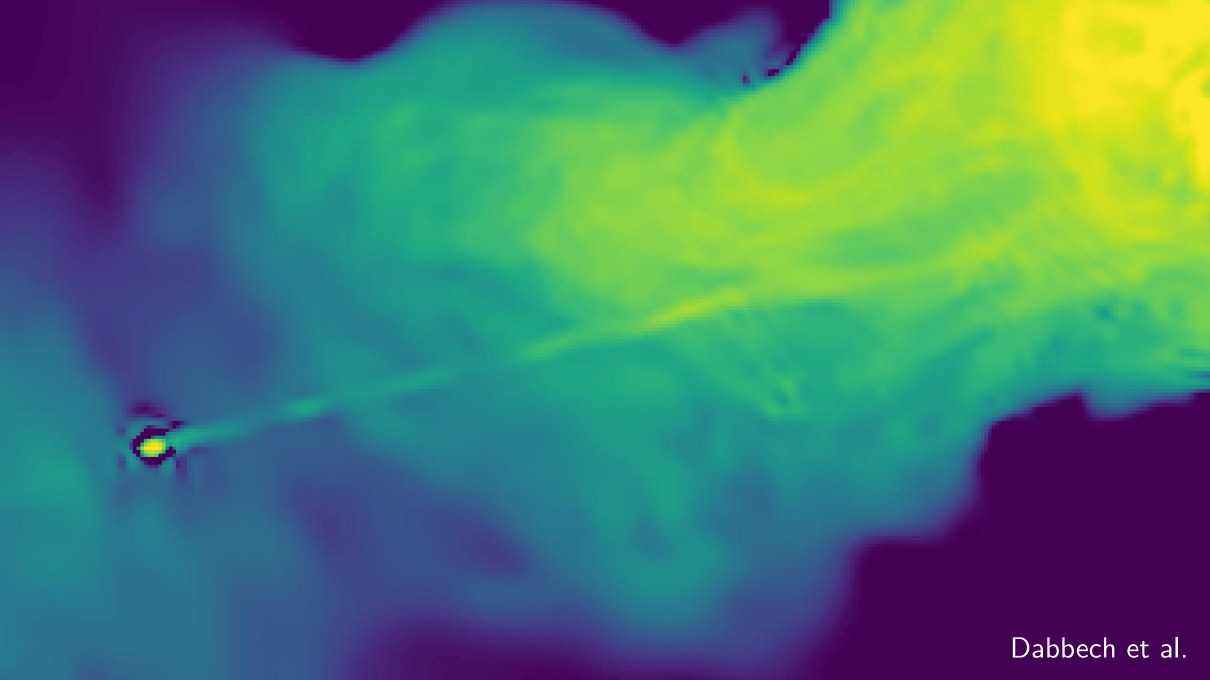
Arras et al.

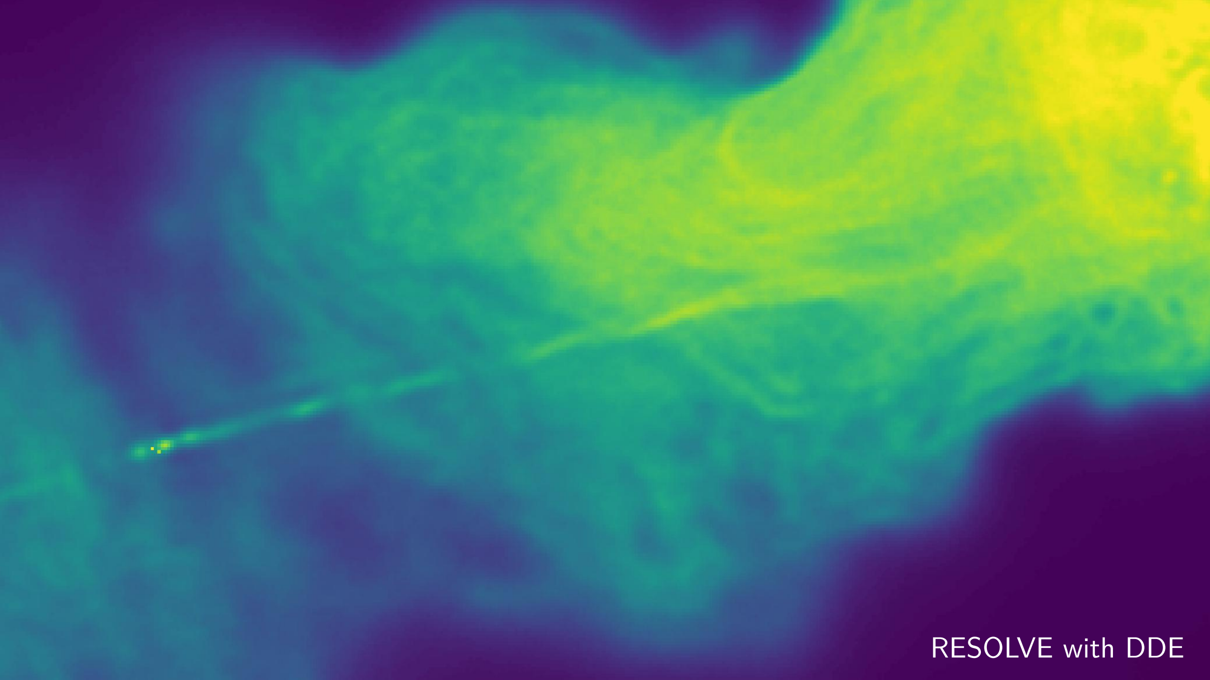


Dabbech et al.

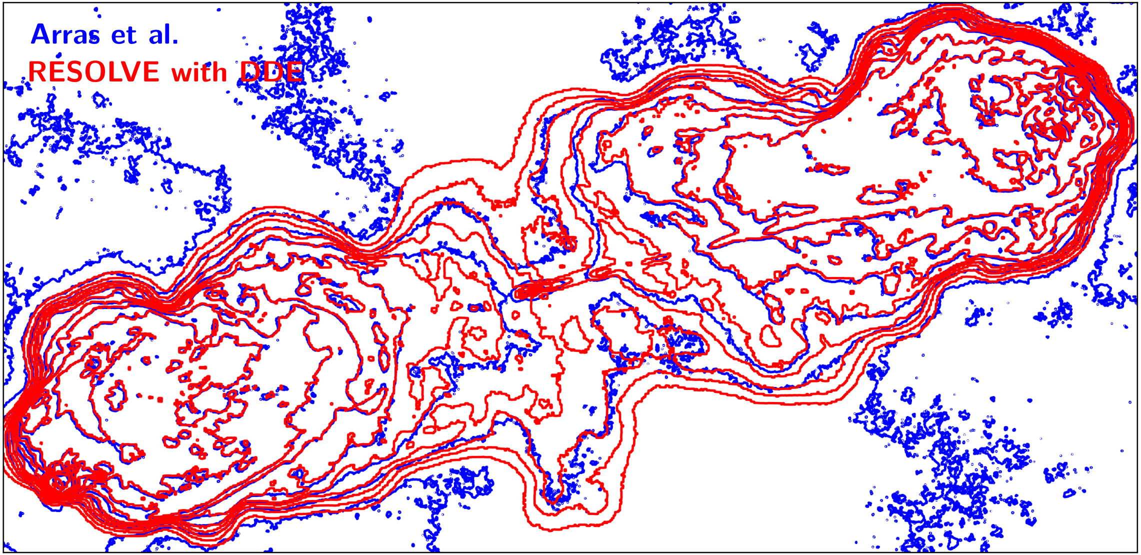

RESOLVE with DDE


Arras et al.


Dabbech et al.

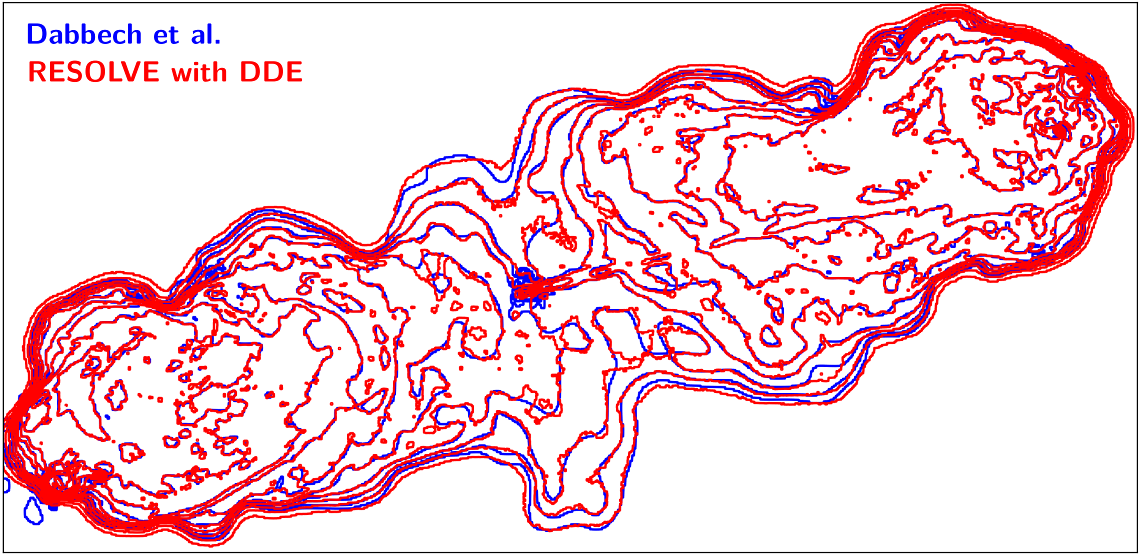

RESOLVE with DDE

Comparison – Flux Contours

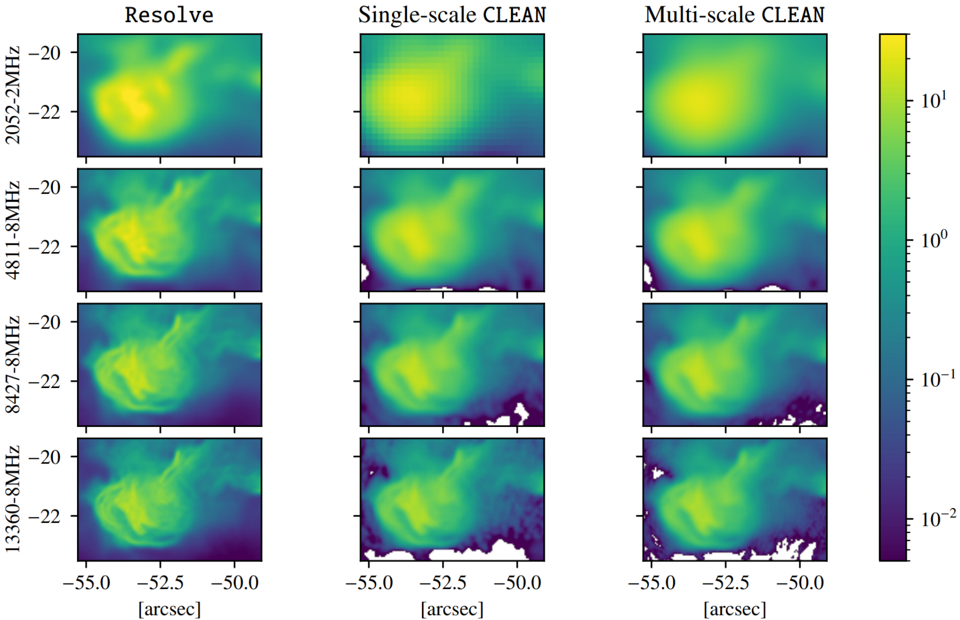


Contours $5 \cdot 10^7$ - $2 \cdot 10^{12}$ **Jy/sr**

Comparison – Flux Contours

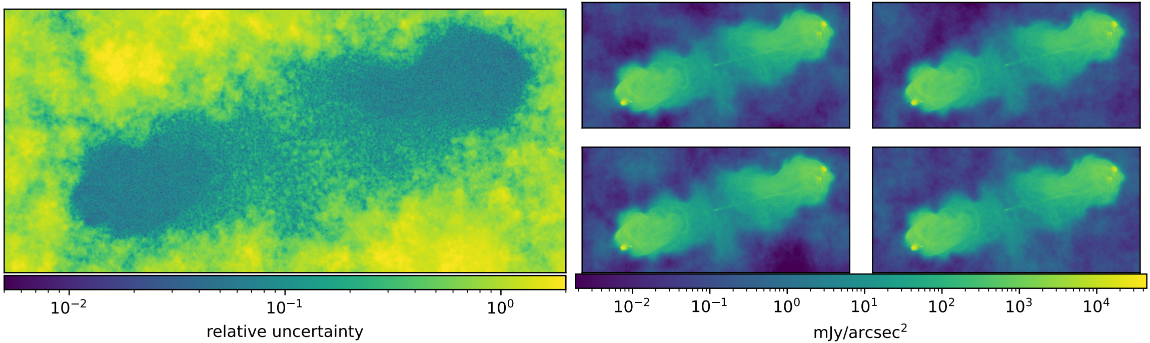


Contours $5 \cdot 10^7$ - $2 \cdot 10^{12}$ **Jy/sr**



Resolve
Single-scale CLEAN
Multi-scale CLEAN
2052-2MHz
4811-8MHz
8427-8MHz
13360-8MHz
−20
−22
−55.0
−52.5
−50.0
[arcsec]
10^1
10^0
10^{-1}
10^{-2}

Uncertanties



Summary

- NIFTy[7]: Prior models and variational inference algorithms
- `resolve`[8]: Bayesian radio interferometric imaging
- Joint calibration and imaging example:
    - Increased resolution
    - Increased dynamic range

[7] https://github.com/NIFTy-PPL/NIFTy
[8] https://gitlab.mpcdf.mpg.de/ift/resolve

[1] G. Edenhofer et al. "Re-Envisioning Numerical Information Field Theory (NIFTy.re): A Library for Gaussian Processes and Variational Inference". In: (2024). arXiv: 2402.16683 [astro-ph.IM].

[2] J. Knollmüller and T. A. Enßlin. "Metric Gaussian Variational Inference". In: (Jan. 30, 2019). arXiv: 1901.11033v3 [stat.ML].

[3] P. Frank, R. Leike, and T. A. Enßlin. "Geometric Variational Inference". In: *Entropy* 23.7 (2021).

[4] S. van der Tol, B. Veenboer, and A. R. Offringa. "Image Domain Gridding: a fast method for convolutional resampling of visibilities". In: *A&A* 616, A27 (Aug. 2018), A27.

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[7] A. Dabbech et al. "Cygnus A jointly calibrated and imaged via non-convex optimization from VLA data". In: *MNRAS* 506.4 (Oct. 2021), pp. 4855–4876.