

Harald Dimmelmeier

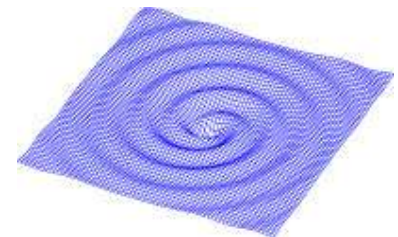
Nonlinear Effects in Pulsations of Rotating Neutron Stars

Presented work in collaboration with

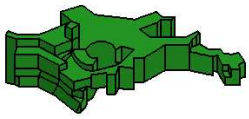
Nick Stergioulas (Aristotle University Thessaloniki)

Toni Font (Universidad de Valencia)

Dimmelmeier, Stergioulas, and Font,
Mon. Not. R. Astron. Soc., 2006, submitted
(astro-ph/0511394)



DFG SFB Transregio 7 “Gravitational Wave Astronomy”

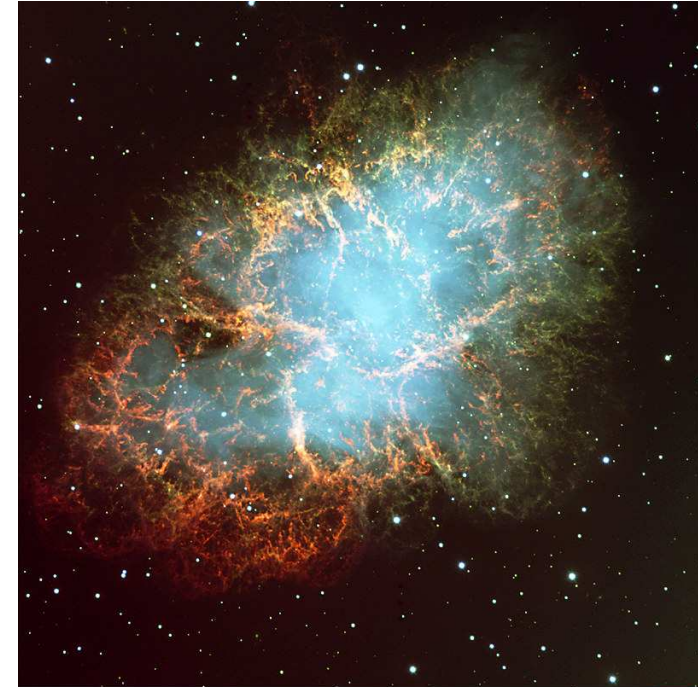


Pulsations of Rotating Neutron Stars

Excitation mechanisms for pulsations:

- Rotational supernova **core collapse**.
- **Accretion-induced collapse**.
- **Core quakes** due to strong phase transitions in EoS.
- Formation of **hypermassive neutron star** (from binary neutron star merger).

Potential source of detectable HF gravitational waves!



The Crab Nebula in Taurus (VLT KUEYEN + FORS2)

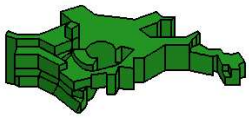
ESO PR Photo 40f/99 (17 November 1999)

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Aim: Use wave signal to determine **neutron star structure** and constrain **high density EoS**.

For nonrotating neutron star models:
Theory of asteroseismology already exists
(e.g. Andersson and Kokkotas, 1998;
Benhar, Ferrari, Gualtieri, 2004).



Previous Work

In spherical symmetry or slow rotation limit:
Use **perturbative methods** to determine mode frequencies.

Possible since few years:
Study pulsations of rotating neutron stars
with **fully nonlinear evolution codes**.

- Several studies in Cowling approximation
(e.g. Font et al., 2001; Yoshida et al., 2002;
Stergioulas, Apostolatos, Font, 2004)

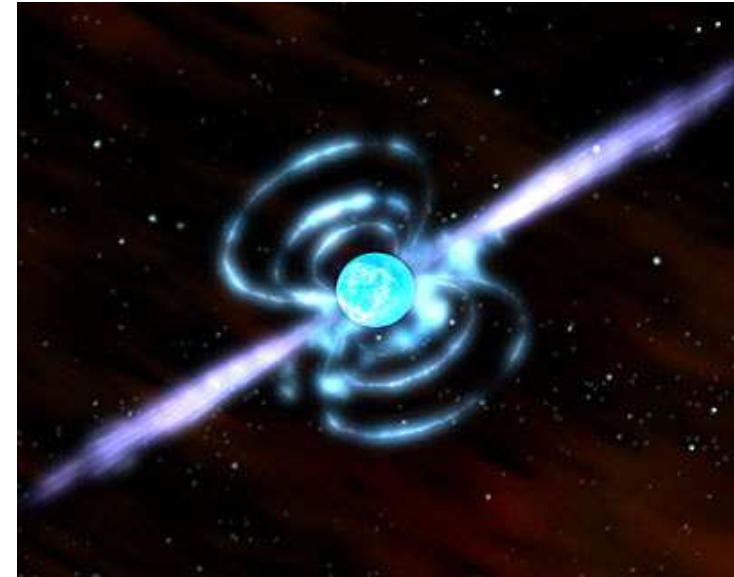
Result: **Frequencies in Cowling wrong by up to 100%!**

Hope: Use Cowling simulations to establish **empirical relation for correct mode frequencies**.

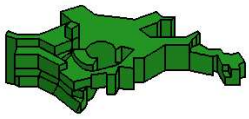
- Full evolution using **Cactus with GR-Hydro** (Font et al., 2002).

Very limited set of models and modes
(computationally expensive 3d code, no high resolution)

Review article: Nick Stergioulas, “Rotating Stars in Relativity”,
Living Rev. Relativity, 3, 2003, <http://www.livingreviews.org/lrr-2003-3>.



(used by permission of The University of Texas McDonald Observatory)



New Nonlinear Simulations

In nonlinear codes:

Excite pulsations with small amplitude perturbations, **use FFT** to extract mode frequencies.

General advantages of nonlinear codes:

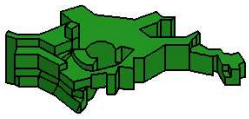
- Can also **study nonlinear phenomena** (mass-shedding-induced damping, instabilities, mode coupling, avoided crossing).
- Can obtain information about **relative mode strengths**.

We use **CoCoNuT** code:

- **Full spacetime evolution** (no Cowling approximation).
- **Conformal flatness approximation** for 3 + 1 metric equations (CFC – excellent and tested approximation for rotating neutron stars).
- **HRSC methods** for hydrodynamic equations and **spectral methods** for metric equations (Mariage des Maillages; Dimmelmeier et al., 2004).
- **Spherical polar coordinates**, equidistant grid inside star (160×60 grid points).
- **Axisymmetry** and equatorial symmetry (computationally fast).



(www.madlantern.com)



Models and Linear Mode Frequencies

We simulated 4 sequences of equilibrium models (same as in Stergioulas, Apostolatos, Font, 2004):

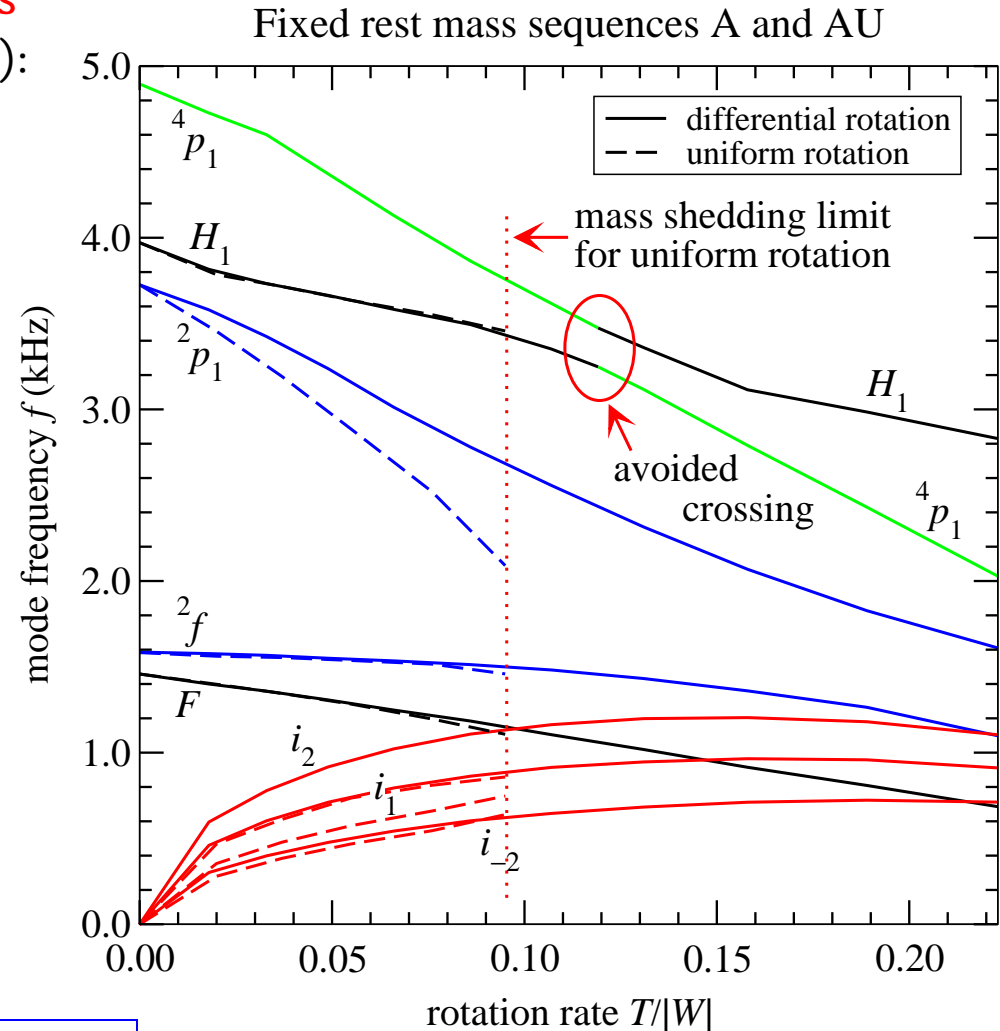
- A: Fixed rest mass, differential rotation.
- AU: Fixed rest mass, uniform rotation.
- B: Fixed central density, differential rotation.
- BU: Fixed central density, uniform rotation.

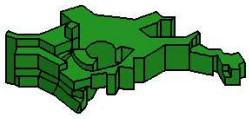
Example:

Dependence of mode frequencies on rotation (F and H_1 -mode, 2f and 2p_1 -mode, 4p_1 -mode).

Obvious avoided crossing between H_1 and 4p_1 -mode!

Observe also low-frequency inertial modes.





Comparison to Fully General Relativistic Results

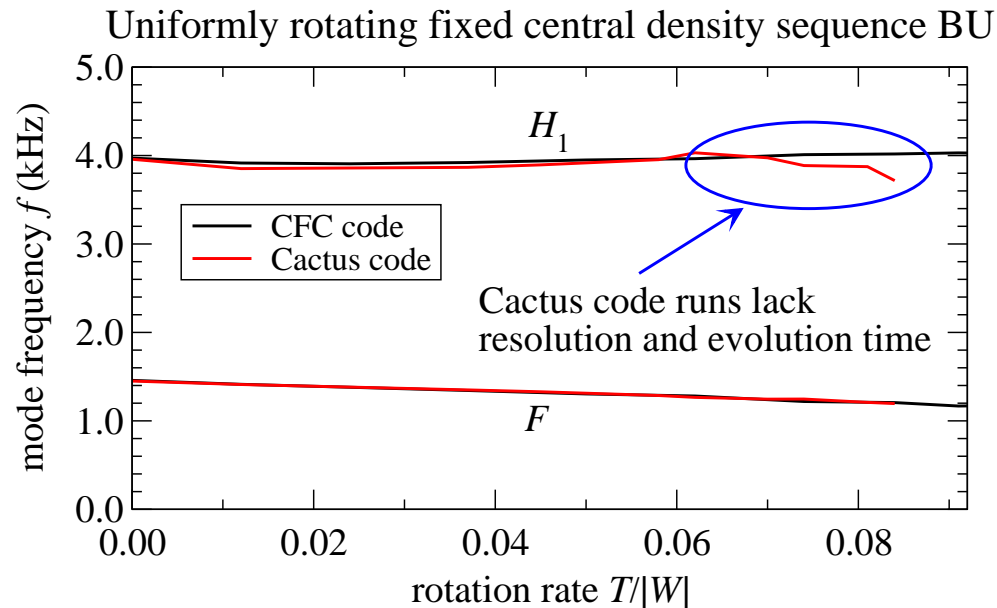
Sequence BU was already **simulated with Cactus GR-Hydro code** (Font et al., 2002).

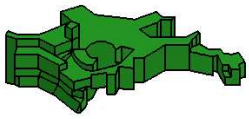
Comparison with our results:

- **Excellent agreement** for fundamental F -mode.
- **Good agreement** for first overtone H_1 .

Close to mass-shedding limit:

Cactus code runs probably have **insufficient resolution or boundary problems**.





Comparison to Results in Cowling Approximation

For F and 2p_1 -mode:

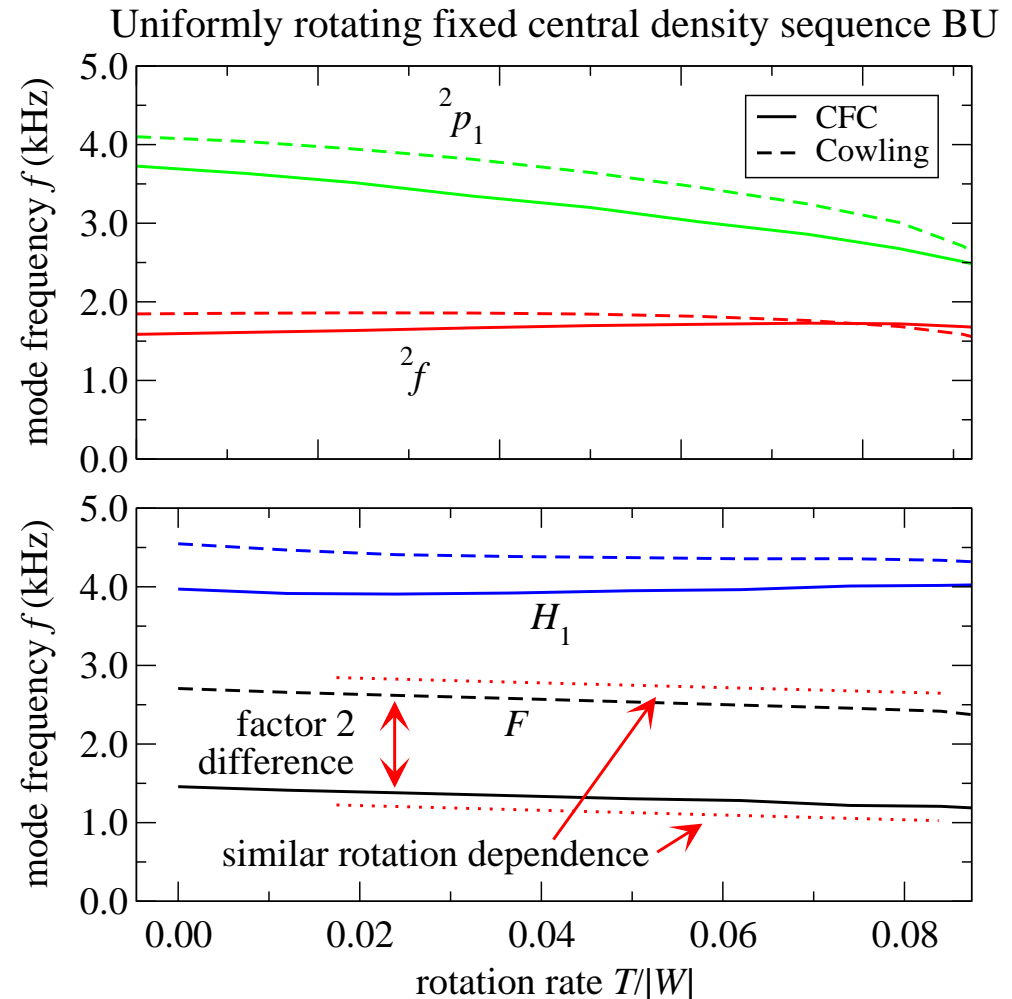
Cowling simulation **reproduce correct rotation dependence** reasonably well.

But: **F -mode frequencies too high**
by factor ~ 2 in Cowling (~ 1 kHz).

For H_1 and 2f -mode:

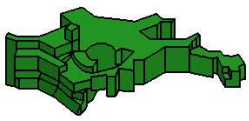
Cowling simulations yield **frequencies similar to correct ones**.

But: **Rotation trend not clearly reproduced**
(dependence is small anyway).



Idea in Font et al., 2001; Stergioulas, Apostolatos, Font, 2004:

Use results from Cowling simulations and empirical relation to obtain correct frequencies!



Failure of Empirical Relations

We have checked these empirical relations for

- F -mode frequency in sequence BU (Font et al., 2001)

Relation yields accuracy of better than 2%!

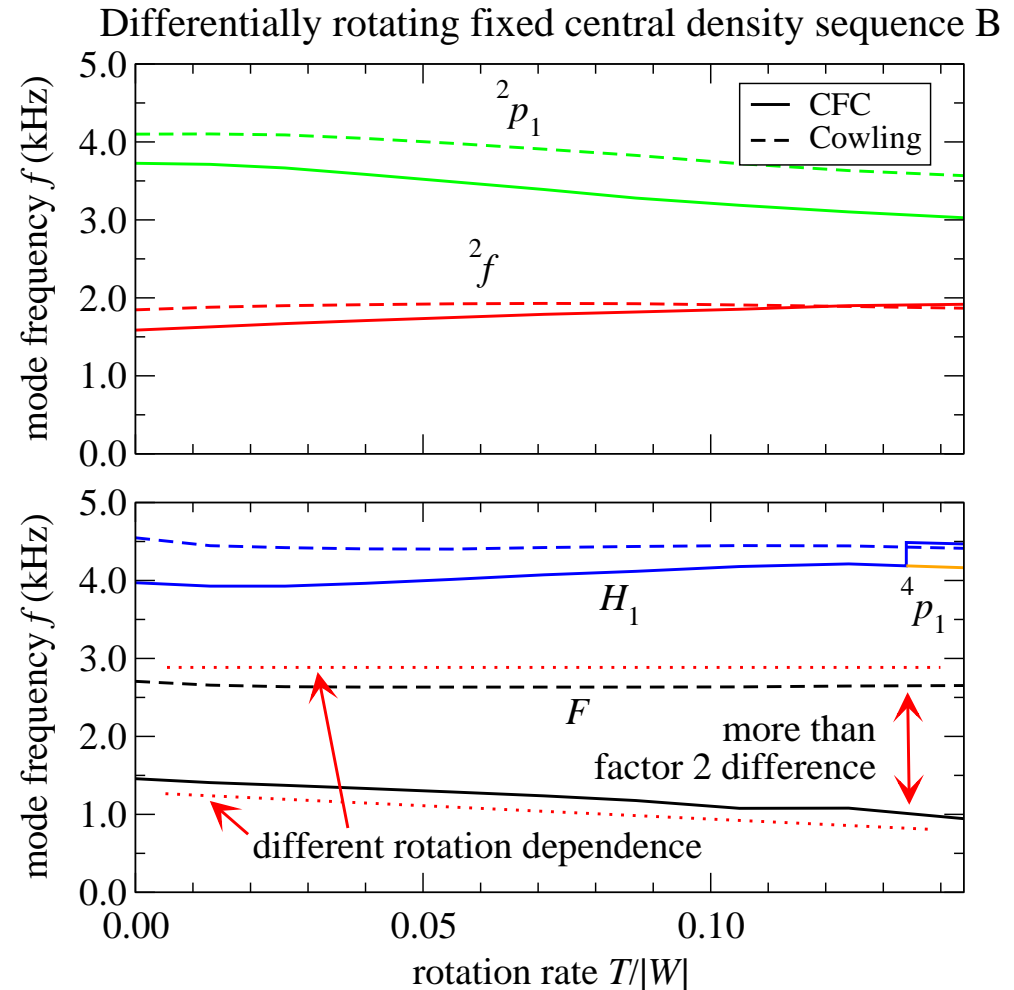
- F and 2f -mode frequency in sequence A (Stergioulas, Apostolatos, Font, 2004; using information about compactness of models from Yoshida and Eriguchi, 2001).

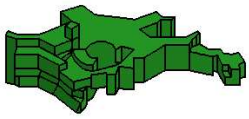
Predicted uncertainty of these relations:
Few percent.

For most rapidly rotating model of sequence A:

Difference of $\sim 30\%$ for F and 2f -mode!

Bottomline: Such relations must be used with caution!





Eigenfunction Recycling

As initial perturbation to excite pulsations:

Use **analytic “trial” eigenfunctions**

(for various parity, $l = 0, 2, 4$).

These differ from exact eigenfunction.

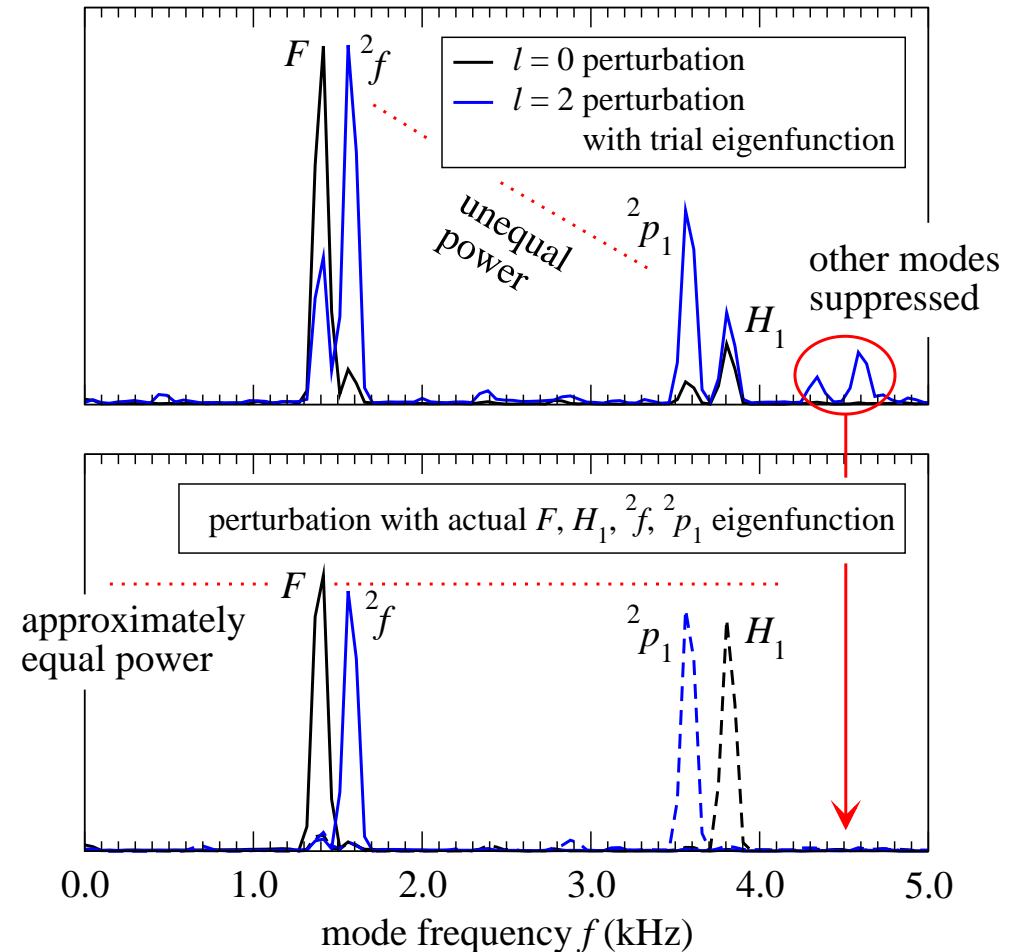
⇒ They **also excite unwanted modes**
(e.g. higher order harmonics).

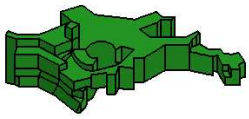
New approach: **Use eigenfunction “recycling”!**

- **Extract eigenfunction** of selected mode
(at specific frequency).
- Use extracted eigenfunction as **initial perturbation** in second run (“recycling run”).
- Convenient check:
Compare eigenfunctions from original run and recycling run.

Can select single mode for excitation and efficiently suppress all other modes!

With appropriate choice of perturbation amplitude: Get **constant peak height in PSD**.





Nonlinear Harmonics

Genuine nonlinear effect:

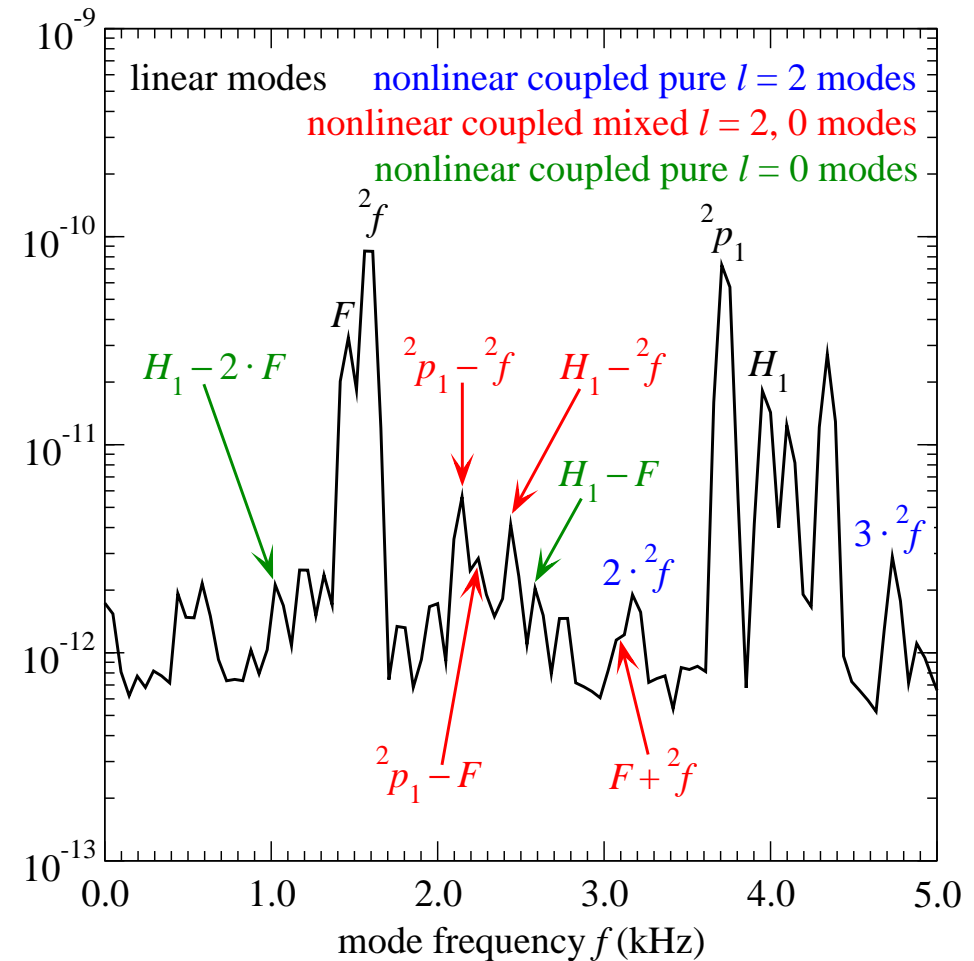
Nonlinear harmonics of linear pulsation modes

(sums and differences of linear modes, including self-couplings).

Recently: Such nonlinear harmonics **observed in oscillating tori around Kerr black holes** (Zanotti et al., 2005).

Example: Nonrotating star, excited by $l = 2$ trial eigenfunction.

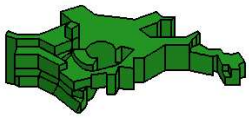
Observe also coupling of $l = 0$ and $l = 2$ modes!



In linear approximation for nonrotating star: Modes of different l are **orthogonal to each other**.

- Two effects:
- Approximate nature of $l = 2$ eigenfunction: **$l = 0$ modes are also excited**.
 - Nonlinear effects **couple all linear modes** even with different l .

Criterion to distinguish linear modes and harmonics: **Scaling with perturbation amplitude**.



Nonlinear 3-Mode Couplings

Rotation effects modes differently.

⇒ When neutron star is rotating:
Presence of nonlinear harmonics opens
possibility for 3-mode couplings.

Modes interact when frequencies cross!

Example: $H_1 - F$ -mode and 2p_1 -mode.

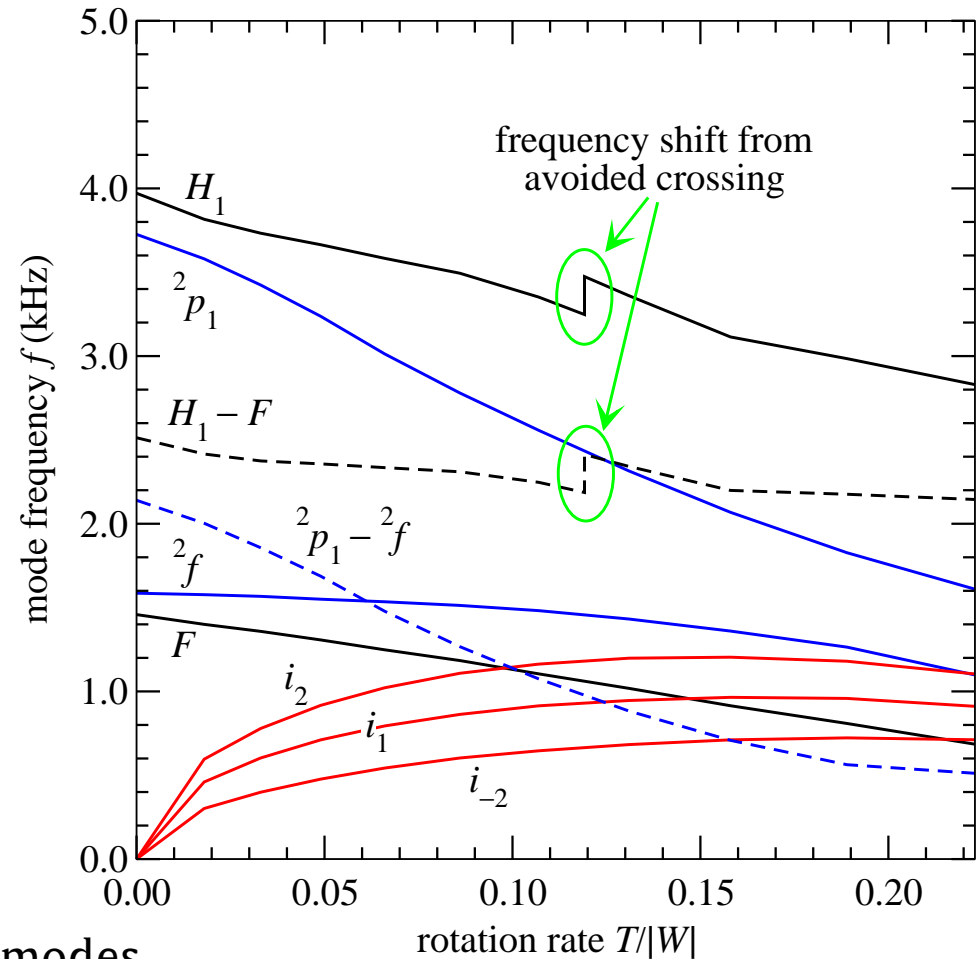
Other possibility:
Mode crossing with some inertial mode.

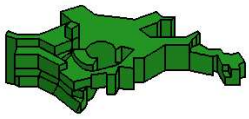
This could potentially lead to **resonance effects**
or even **parametric instabilities.**

⇒ Possibly significant **energy transfer** between modes.

Most interesting case of energy transfer for **gravitational waves**:
Strong oscillating, weakly radiating mode → weakly oscillating, strongly radiating mode
(suggested by Clark, 1979; Eardly, 1983).

We only observe if **necessary conditions are fulfilled**; we perform no further studies.



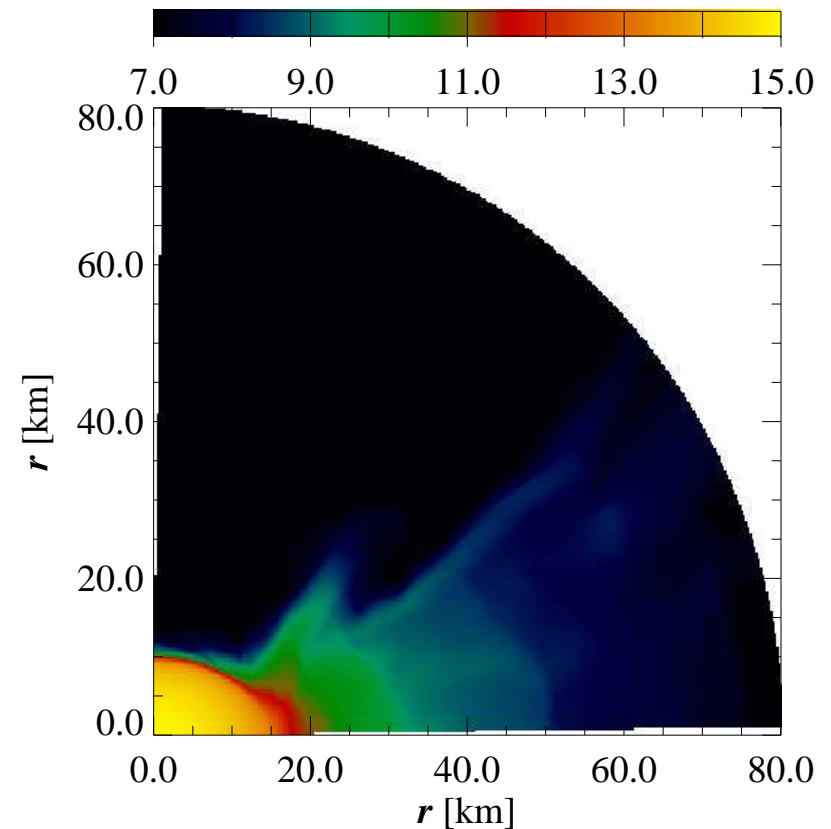
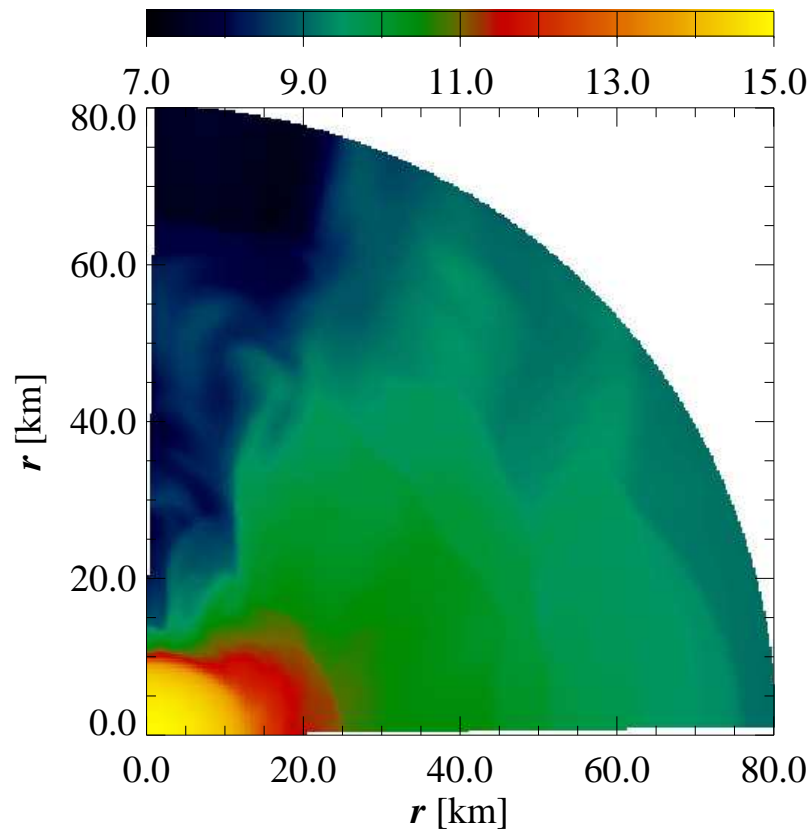


Mass-Shedding for Rapidly Rotating Neutron Stars

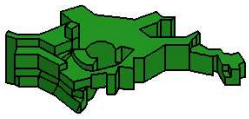
For small but finite pulsation amplitudes:

Mass-shedding occurs in (almost) maximally rotating neutron stars.

- In Cowling approximation (left): Mass-shedding **creates extended high-entropy envelope**.
- With coupled spacetime evolution (right): Mass-shedding **strongly suppressed**.



We do not know exact reason for this observed mechanism!



Mass-Shedding-Induced Damping

Mass-shedding-induced damping is another **striking nonlinear effect**.

At each pulsation, star **ejects mass into envelope**.
⇒ Pulsation energy is lost, **pulsation damped**.

If **damping time scale** is comparable to **growth time scale** of unstable modes (e.g. CFS-instability driven f or r -modes):

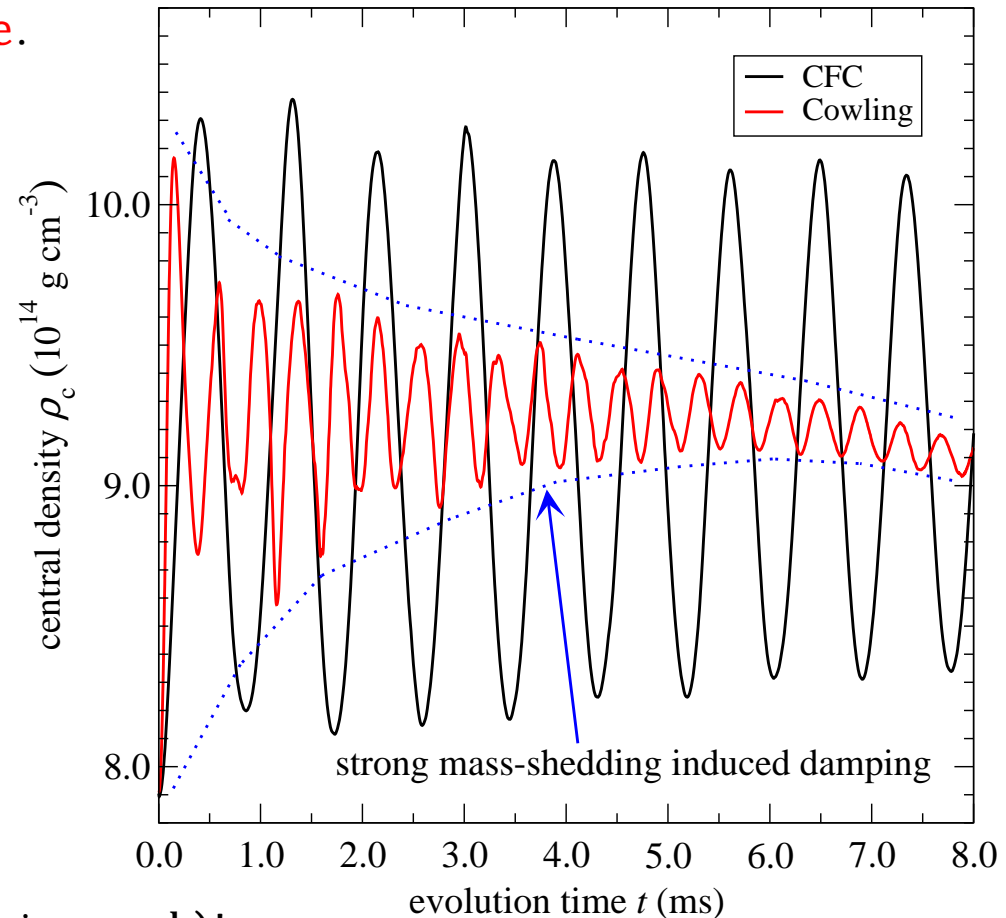
Damping mechanism could **limit mode growth**.

Example:

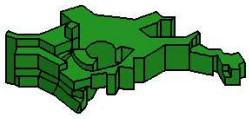
$l = m = 2$ f -mode becomes unstable only near mass-shedding limit.

But with coupled spacetime evolution:

Much longer damping time scale (not detectable in graph)!



This could efficiently reduce this damping effect for unstable modes!



Gravitational Wave Power Spectrum

We also extract **gravitational waves emitted by pulsations**.

We use **Newtonian quadrupole formula**
(in time-integrated form).

Example:

Gravitational wave **power spectrum**
for slowly rotating model from sequence A
(using eigenfunction recycling technique).

As expected:

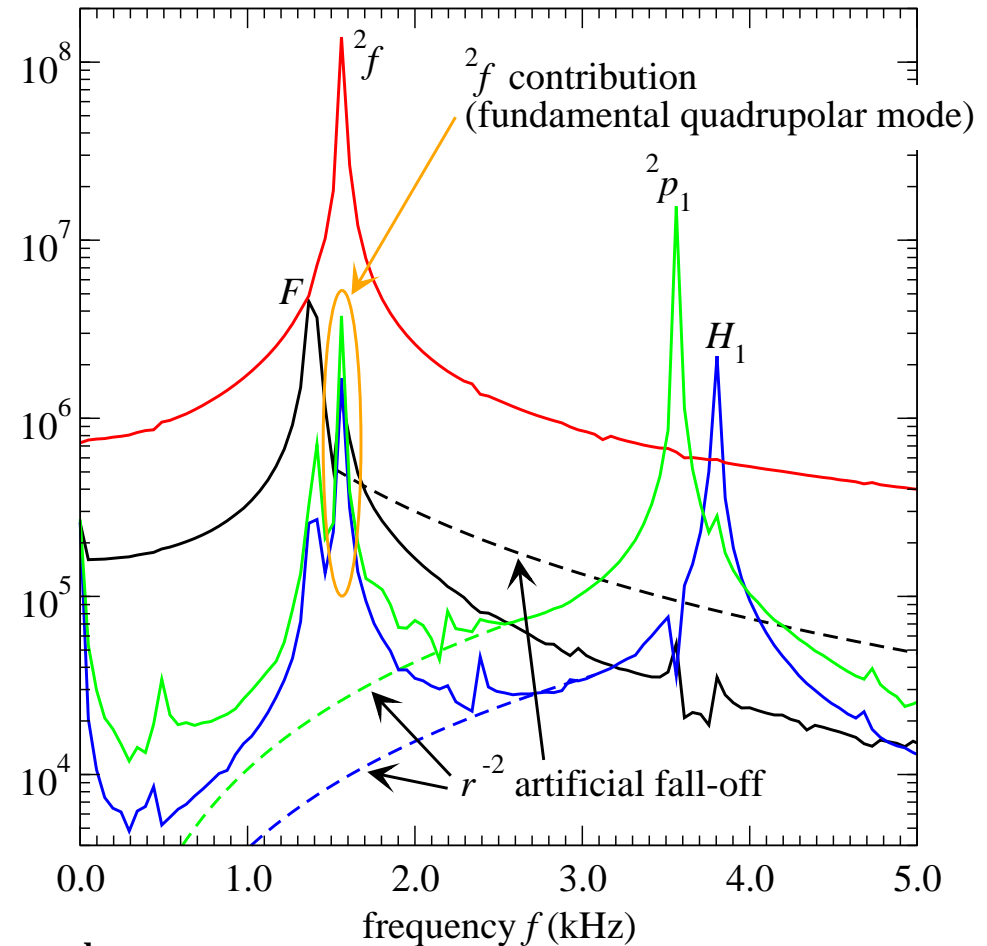
Quadrupolar 2f -mode is **strong emitter**.

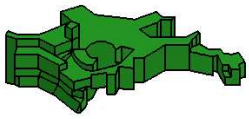
Observation:

Small leaking of energy into 2f -mode from other modes:

Creates **strong contribution in signal**.

⇒ **Suppress that contribution** in signal processing.





Detectability of Gravitational Waves

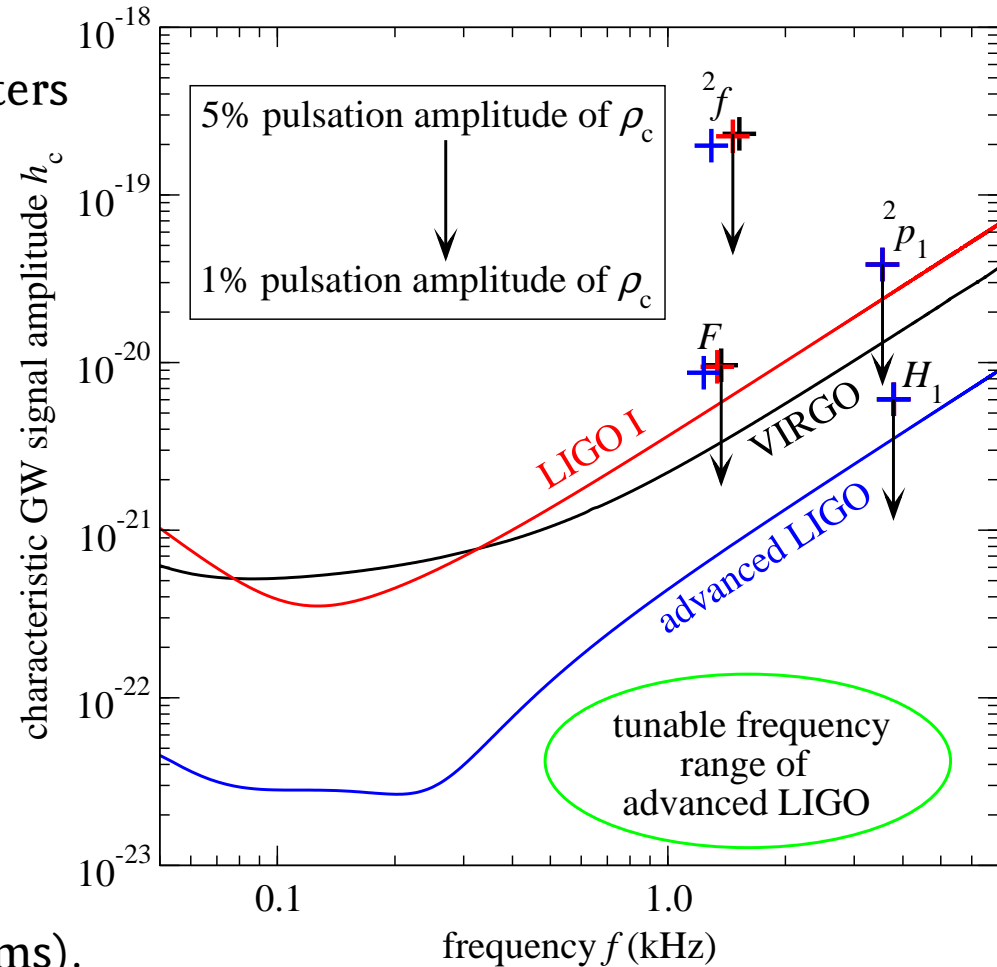
Estimate **detectability prospects** by interferometers (VIRGO, LIGO I, advanced LIGO):

Compute (slightly detector dependent)

- characteristic **signal amplitude**,
- characteristic **signal frequency**,
- **signal-to-noise ratio**.

Exploit linear scaling properties:

- Initial **perturbation amplitude**.
- **Pulsation amplitude** during evolution.
- Gravitational wave **signal amplitude**.
- Square root of **signal duration time** (here 20 ms).



Can construct relation between detectability and required pulsation amplitude.

Location of signals along ascending high frequency slope of detector:
Dependence of mode frequency on rotation is crucial for detection.