Harald Dimmelmeier

Gravitational Waves from Core Collapse Supernovae: New Simulations and their Practical Application

Work done at the MPA Garching
in collaboration with E. Müller and J.A. Font-Roda.

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http://www.mpa-garching.mpg.de/rel_hydro/

Gravitational Waves from Core Collapse Supernovæ

Problem with observing a core collapse supernova:

We only see optical light emission (light curve) of the explosion
(hours after collapse – envelope optically thick).

But: There are two direct means of observation of the central core collapse:

- Neutrinos; signal decreases with R^{-2} (seconds after the collapse – only for galactic supernovae).
- Gravitational waves; signal decreases with R^{-1}
(coherent motion of central massive core – synchronous with collapse – possibly extragalactic).

Some of the new gravitational wave detectors are already taking data
(LIGO, VIRGO, GEO600, TAMA300, ACIGA).

Challenge: The signal is very complex! Signal analysis is like search for a needle in a haystack.
⇒ "Numerical Relativity Simulations are badly needed!" (David Shoemaker – LIGO Collaboration)

Our contribution to this quest:

The first *relativistic* simulation of rotational core collapse to a neutron star.

Physical Model

Physical model of a core collapse supernova:

- Massive star of $\gtrsim 8M_\odot$ develops a (rotating) iron core ($M_{\mathrm{core}} \approx 1.5M_\odot$).
- When core exceeds a critical mass, it collapses ($T_{\mathrm{collapse}} \approx 100$ ms).
- At supernuclear density, neutron star forms (EoS of matter stiffens $\Rightarrow$ bounce).
- Shock wave propagates through stellar envelope and disrupts rest of the star (visible explosion).

During the various evolution stages, core collapse involves many areas of physics:

> Gravitational physics (GR!), stellar evolution, particle and nuclear physics, neutrino transport, hydrodynamics, element nucleosynthesis, radiation physics, interaction of the ejecta with interstellar medium, ...
>
> ... in multi-dimensions (rotation)!

$\Rightarrow$ Numerical simulations are very complicated, many approximations necessary.

So far no nonspherical consistent simulations including all known physics (too complicated)!

And not even all the physics is known: Supernuclear EoS, rotation rate and profile of iron core, ...

Signal waveform will reveal new physics!

Assumptions about the Model

To reduce the complexity of the problem, we assume

- axisymmetry and equatorial symmetry,
- simplified ideal fluid equation of state, $P(\rho, \epsilon) = P_{\mathrm{poly}} + P_{\mathrm{th}}$ (neglect complicated microphysics),
- rotating polytropes in equilibrium as initial models,
- constrained system of the Einstein equations (Wilson's CFC approximation).

Goals

The main goals of our simulations are to

- extend research on Newtonian rotational core collapse by Zwerger and Müller to GR,
- obtain more realistic waveforms as "wave templates" for interferometer data analysis,
- have a 2D GR hydro code for comparison with future simulations in other formulations.

How do GR effects change collapse dynamics?
What influence does that have on gravitational wave signals?
What is the role of rotation?

Relativistic Field Equations – ADM Metric

Einstein field equations of general relativity + Bianchi identity
↓
Divergence equations for the energy momentum tensor (equations of motion)

$$G^{\mu\nu} = 8\pi T^{\mu\nu} \quad \longrightarrow \quad \nabla_\nu T^{\mu\nu} = 0,$$

with Einstein tensor $G^{\mu\nu}$ (spacetime curvature) and energy momentum tensor $T^{\mu\nu}$ (matter).

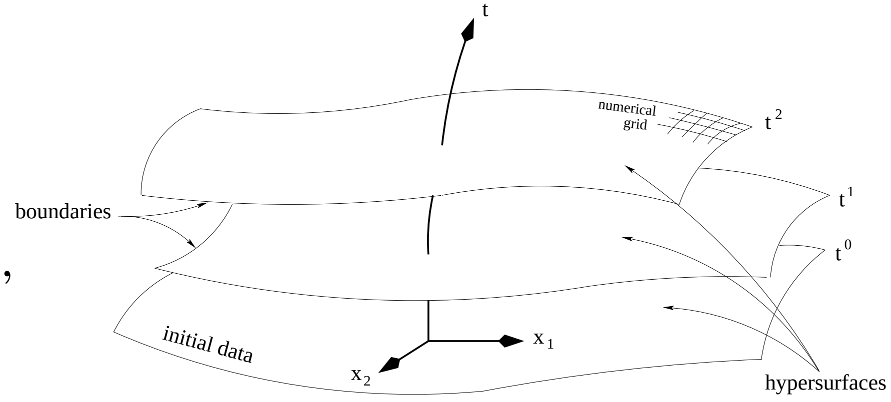
We want to do numerical physics. ⇒ Choose a suitable spacetime slicing.

We use the ADM $\{3+1\}$ formalism.
Split spacetime into a foliation of 3D hypersurfaces.

⇒ This defines Cauchy problem:
Evolve initial data with given boundary conditions.

ADM metric: $ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$,
with lapse α, shift vector β^i and three-metric γ_{ij}.

The metric has 10 independent components.
Use gauge freedom for adaption to specific situations.



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Conservation Equations

Define a set of conserved hydrodynamic quantities:

$$D = \rho W, \qquad S_i = \rho h W^2 v_i, \qquad \tau = \rho h W^2 - P - D,$$

with density ρ, pressure P, internal energy ϵ, enthalpy h, 3-velocity v_i, Lorentz factor W.

Relativistic equations of motion for an ideal fluid

$\downarrow$

System of hyperbolic conservation equations

$$\frac{1}{\sqrt{-g}}\left(\frac{\partial\sqrt{\gamma}\rho W}{\partial t} + \frac{\partial\sqrt{-g}\rho W\hat{v}^i}{\partial x^i}\right) = 0,$$

$$\frac{1}{\sqrt{-g}}\left(\frac{\partial\sqrt{\gamma}\rho h W^2 v^j}{\partial t} + \frac{\partial\sqrt{-g}(\rho h W^2 v_j\hat{v}^i + P\delta^i_j)}{\partial x^i}\right) = T^{\mu\nu}\left(\frac{\partial g_{\nu j}}{\partial x^\mu} - \Gamma^\delta_{\mu\nu} g_{\delta j}\right),$$

$$\frac{1}{\sqrt{-g}}\left(\frac{\partial\sqrt{\gamma}(\rho h W^2 - P - \rho W)}{\partial t} + \frac{\partial\sqrt{-g}((\rho h W^2 - \rho W - P)\hat{v}^i + P v^i)}{\partial x^i}\right) = \alpha\left(T^{\mu 0}\frac{\partial \ln\alpha}{\partial x^\mu} - T^{\mu\nu}\Gamma^0_{\mu\nu}\right),$$

with the Christoffel symbols $\Gamma^\lambda_{\mu\nu}$, and $g = \det g_{\mu\nu}$, $\gamma = \det\gamma_{ij}$, $\hat{v}^i = v^i - \beta^i/\alpha$.

These equations are the GR extension of the hydro equations in Newtonian gravity.

High-Resolution Shock-Capturing Methods

For solving the hydro equations, we exploit their hyperbolic and conservative form:

$$\frac{1}{\sqrt{-g}}\left[\frac{\partial\sqrt{\gamma}F^0}{\partial x^0}+\frac{\partial\sqrt{-g}F^i}{\partial x^i}\right]=S,$$

with the vectors of conserved quantities F^0, fluxes F^i and sources S.

Modern recipe for solving such equations: High-resolution shock-capturing (HRSC) methods.

⇒ Use analytic solution of (approximate) Riemann problems
(time evolution of piecewise constant initial states).

This method guarantees

- convergence to physical solution of the problem,
- correct propagation velocities of discontinuities. and
- sharp resolution of discontinuities.

ADM Metric Equations

In the ADM metric:

Einstein field equations for the spacetime metric
↓
Set of evolution and constraint equations

$$
\begin{aligned}
\partial_t \gamma_{ij} &= -2\alpha K_{ij} + \nabla_i \beta_j + \nabla_j \beta_i, && \text{three-metric evolution,} \\
\partial_t K_{ij} &= -\nabla_i \nabla_j \alpha + \alpha (R_{ij} + K K_i j - 2 K_{im} K^m_j) + \beta^m \nabla_m K_{ij} + && \text{extrinsic curvature evolution,} \\
& \quad + K_{im} \nabla_j \beta^m + K_{jm} \nabla_i \beta^m - 8\pi T_{ij}, && \\
0 &= R + K^2 - K_{ij} K^{ij} - 16\pi \alpha^2 T^{00}, && \text{Hamiltonian constraint,} \\
0 &= \nabla_i (K^{ij} - \gamma^{ij} K) - 8\pi S^j, && \text{momentum constraint,}
\end{aligned}
$$

with Riemann scalar R and extrinsic curvature K_{ij}.

Note mathematical similarity with the Maxwell equations!

These equation for the metric have been the workhorse of numerical relativity for decades.

Problems with the ADM equations – Conformal Flatness Approach

But: These equations are often numerically unstable (especially in 2D: axis!).

⇒ Many attempts to reformulate these equations (numerical application mostly experimental).

Thus, the quest for the "Holy Grail of Numerical Relativity", a code which

- evolves an arbitrary spacetime,
- has no symmetry restrictions
- avoids/handles singularities,
- can deal with black holes,
- maintains high accuracy, and
- runs indefinitely long,

is still a formidable and unattained task.

However, one can approximate the full equations in various ways (poor man's grail):
Newtonian approximation – Special relativity – Post-Newtonian approximation.

Our approach (Wilson's conformal flatness condition – CFC):
Approximate the exact three-metric by a conformally flat one, $\gamma_{ij} = \phi^4 \hat{\gamma}_{ij}$.

Advantages:	Tradeoffs:
• Hydro and metric equations are much simpler.	• No emission of gravitational waves
• Discretized equations are numerically stable.	• (need indirect methods for wave extraction).
• CFC is exact in spherical symmetry.	• Deviation from exact metric hard to estimate.

CFC Metric Equations

In the CFC approximation:

ADM equations for exact metric
↓
System of five coupled elliptic equations for CFC metric

$$\hat{\Delta}\phi = -2\pi\phi^5\left(\rho W^2 - P + \frac{K_{ij}K^{ij}}{16\pi}\right),$$

$$\hat{\Delta}\alpha\phi = 2\pi\alpha\phi^5\left(\rho h(3W^2 - 2) + 5P + \frac{7K_{ij}K^{ij}}{16\pi}\right),$$

$$\hat{\Delta}\beta^i = 16\pi\alpha\phi^4 S^i + 2K^{ij}\hat{\nabla}_j\left(\frac{\alpha}{\phi^6}\right) - \frac{1}{3}\hat{\nabla}^i\hat{\nabla}_k\beta^k,$$

where $\hat{\nabla}$ and $\hat{\Delta}$ are the flat space Nabla and Laplace operators, respectively.

Note that these equation exhibit no *explicit* time dependence!

We solve this system iteratively (Newton–Raphson scheme).

Gravitational Waves

The CFC results in abandoning the two polarizations of gravitational radiation.
⇒ System cannot emit gravitational waves. But we want to extract gravitational waves!

Gravitational wave emission is calculated via standard quadrupole formula:

$$h_{ij} = \frac{2}{R}\ddot{Q}_{ij},$$

with mass quadrupole moment

$$Q_{ij} = \int dV\,\rho\left(x_i x_j - \frac{1}{3}\delta_{ij} r^2\right).$$

In supernova core collapse:
Evolution dynamics are not influenced by gravitational radiation ($E_{\mathrm{gw}} \sim 10^{-6} E_{\mathrm{tot}}$).
⇒ This method of wave extraction is justified.

Rotating Neutron Stars

We have dynamically evolved rapidly rotating neutron stars models.

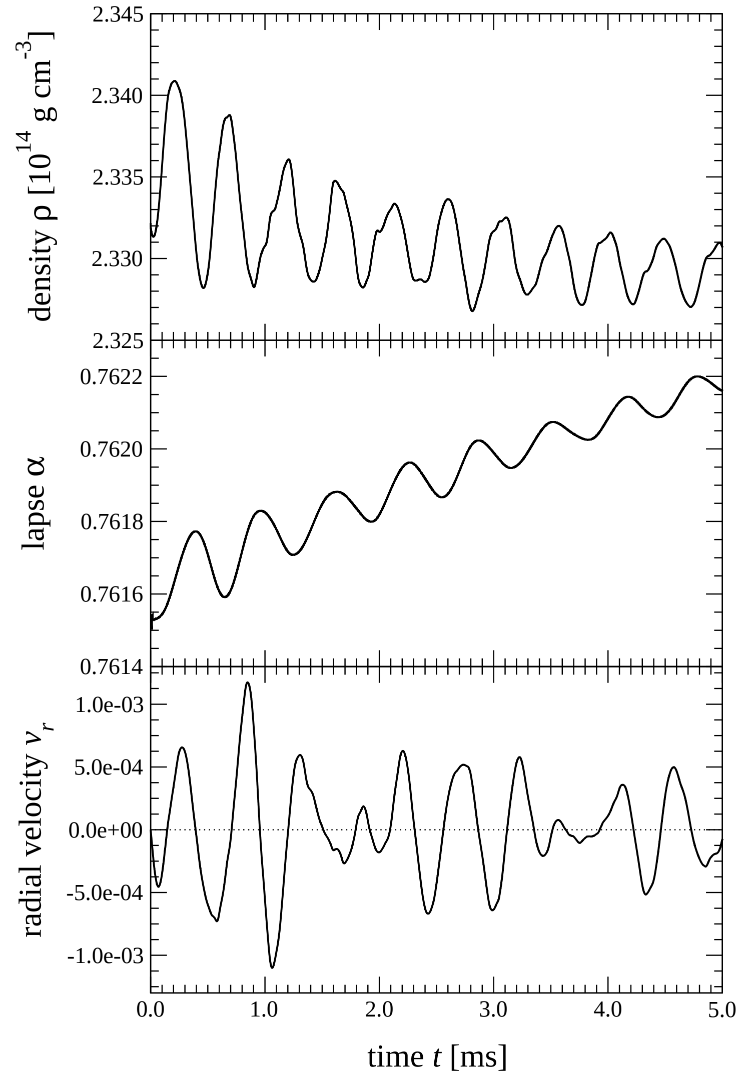
The code can keep the density and rotation profile stable and accurate over many rotation periods!

By exciting oscillation modes in the radial and angular direction, the code can be used to study pulsations.

This stability test demostrates that the CFC approximation is justified even for

- strongly gravitating and
- rapidly rotating

systems like neutron stars rotating at breakup speed.



Spherical Core Collapse

In spherical symmetry the CFC is exact.

We have compared

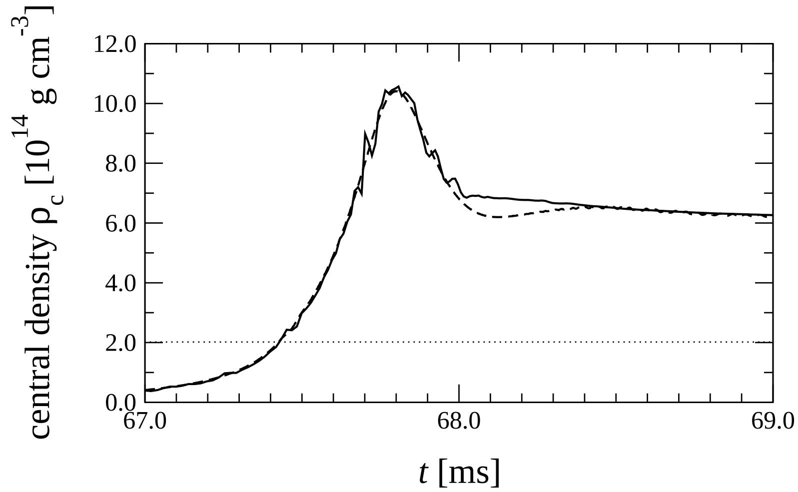
- our Eulerian code to
- a Lagrangian finite difference code (with artificial viscosity)

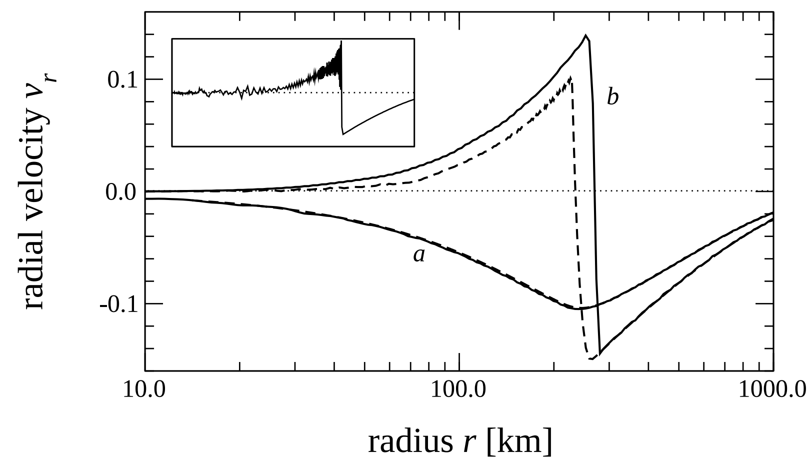
in core collapse to a neutron star or a black hole.

We find excellent agreement in various quantities.

The shock front after the bounce is resolved very clearly and confined to about 2 gridpoints!

Note the superior radial resolution of the comoving coordinates in the Lagrangian code.





Rotational Core Collapse Simulations – Models

Initial models for the iron core:
Rotating $\gamma = 4/3$ polytropes with central densities $\rho = 10^{10}$ gm cm^{-3} and $R_{\mathrm{core}} \approx 1\,500$ km.

The collapse is initiated by a change in the EoS (polytropic index is lowered).

Parameters specify

- rotation profile (from uniform to extremely differential rotation),
- rotation rate $\beta = E_{\mathrm{rot}}/|E_{\mathrm{pot}}|$ (no rotation up to the mass shedding limit), and
- polytropic index during collapse (speed of contraction) and at $\rho > \rho_{\mathrm{nuc}}$ (hardness of bounce).

We have performed a parameter study of 26 models (initial rotation profile is not well known).

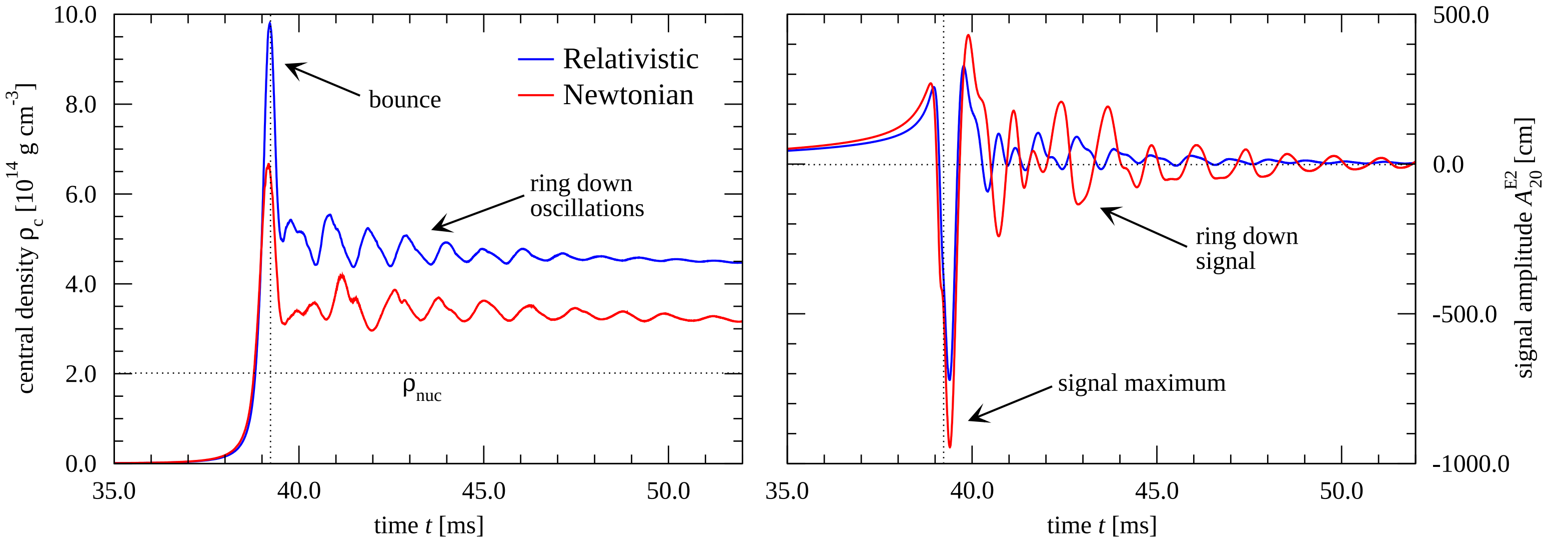
Goal: Identification and quantification of relativistic effects.

Influence of relativistic gravity is visible in many aspects. $\Rightarrow$ Quantitative and *qualitative* changes.

Example: Select three particular models and compare them with Newtonian simulations.

Relativistic Effects

Model A: Slow, almost uniform rotation, fast collapse ($\approx$ 40 ms), soft supernuclear EoS.



- Deep dive into the potential, high supernuclear central densities.
- Regular single bounce, subsequent ring down.
- GR simulation: Higher central density and signal frequency, but *lower* signal amplitude.

Compactness of the Core

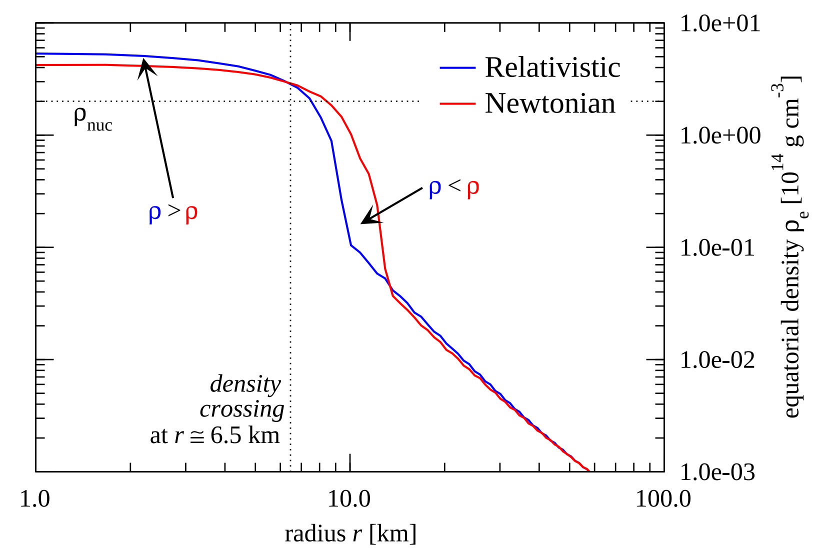
Problem: Why is signal amplitude in GR simulations often smaller than in Newtonian simulations (despite higher central densities, collapse und rotation velocities, accelerations)?

Explanation: GW signal is determined by accelation of *extended* mass distribution:

$$A_{20}^{\mathrm{E2}} = \ddot{Q} \propto \frac{d^2}{dt^2} \int dV \rho \boxed{r^2}.$$

$\uparrow$

weight factor!



- Dynamics of *entire* inner core are important.
- Outer parts can contribute more to signal than interior.

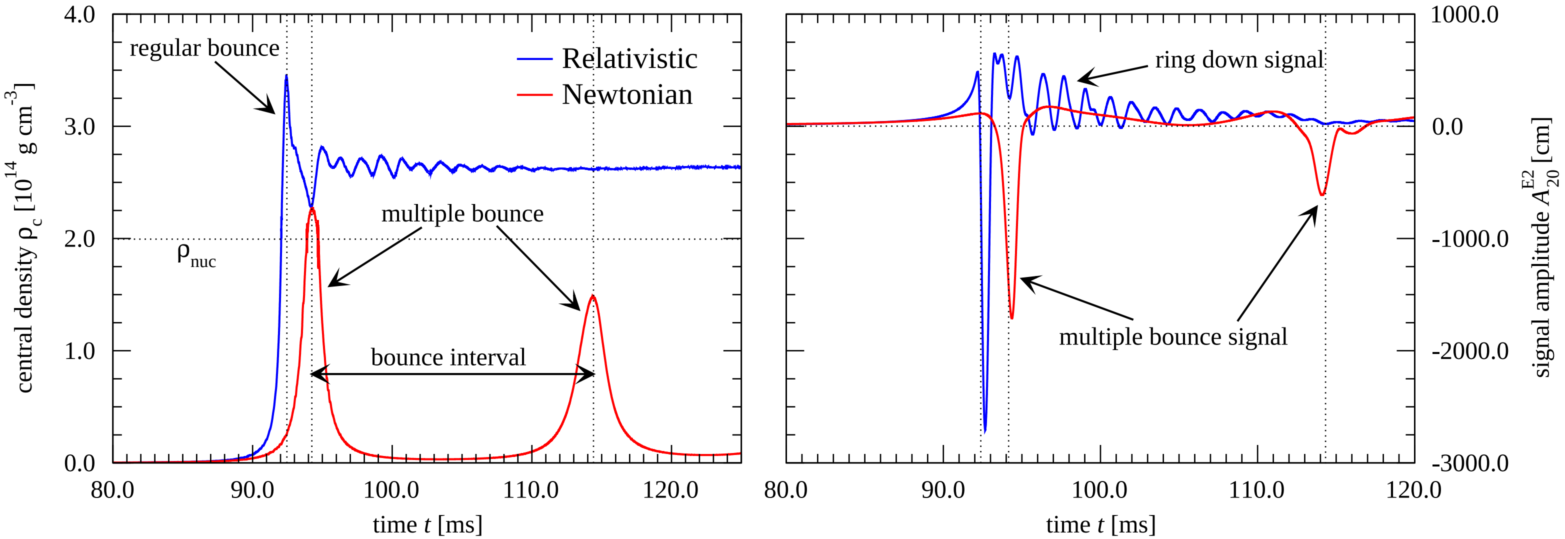
Density profiles cross inside the proto-neutron star ("density crossing")!

The higher central densities in GR simulationen do *not always* translate into higher signal amplitudes in the gravitation waves!

Nevertheless: Relativistic effects increase the signal frequencies.

Relativistic Effects

Model B: Slow, almost uniform rotation, slow collapse (≈ 90 ms).



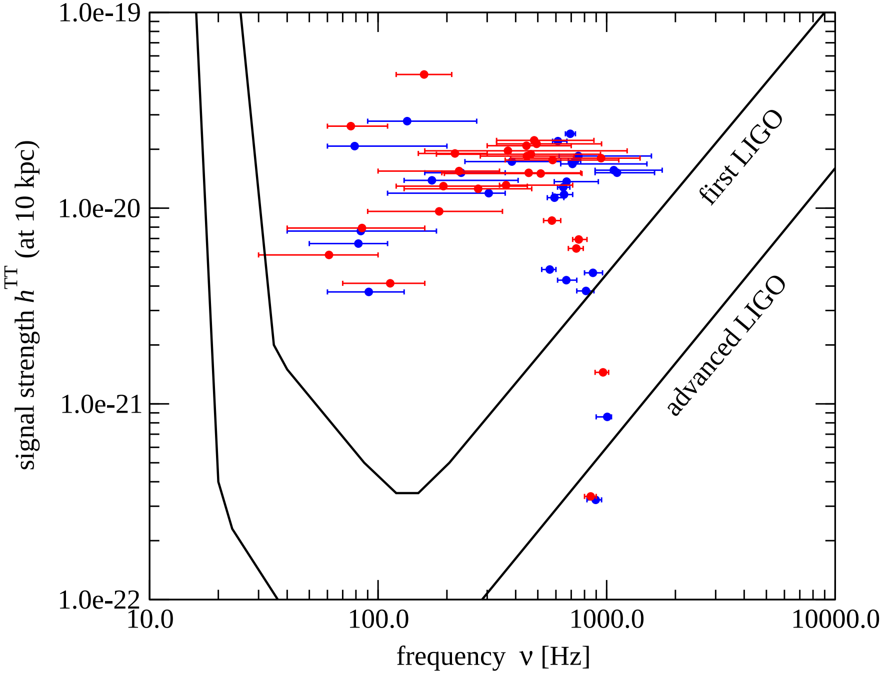
- Rotation increases strongly during collapse (conservation of angular momentum!).
- Newtonian: Nuclear density is hardly reached, multiple centrifugal bounce with re-expansion.
- GR: Nuclear density is easily reached, regular single bounce.
- Relativistic simulations show multiple bounces only for a few extreme models.

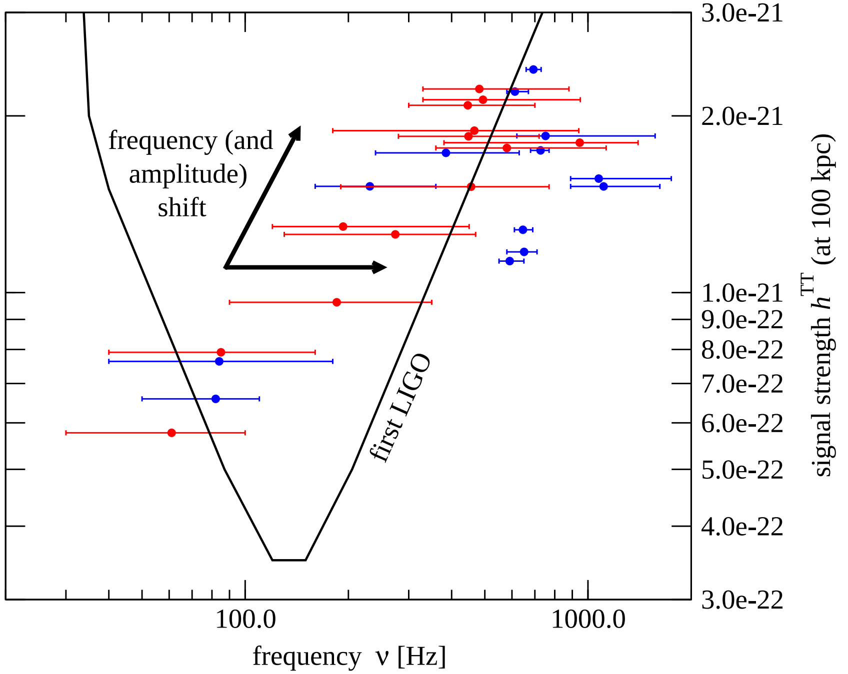
Strong *qualitative* difference in the collapse dynamics and thus in the signal form.

Many models exhibit this behavior. ⇒ Important consequences for a possible detection!

Gravitational Wave Signals

Influence of relativistic effects on the signals: Investigate amplitude–frequency diagram.





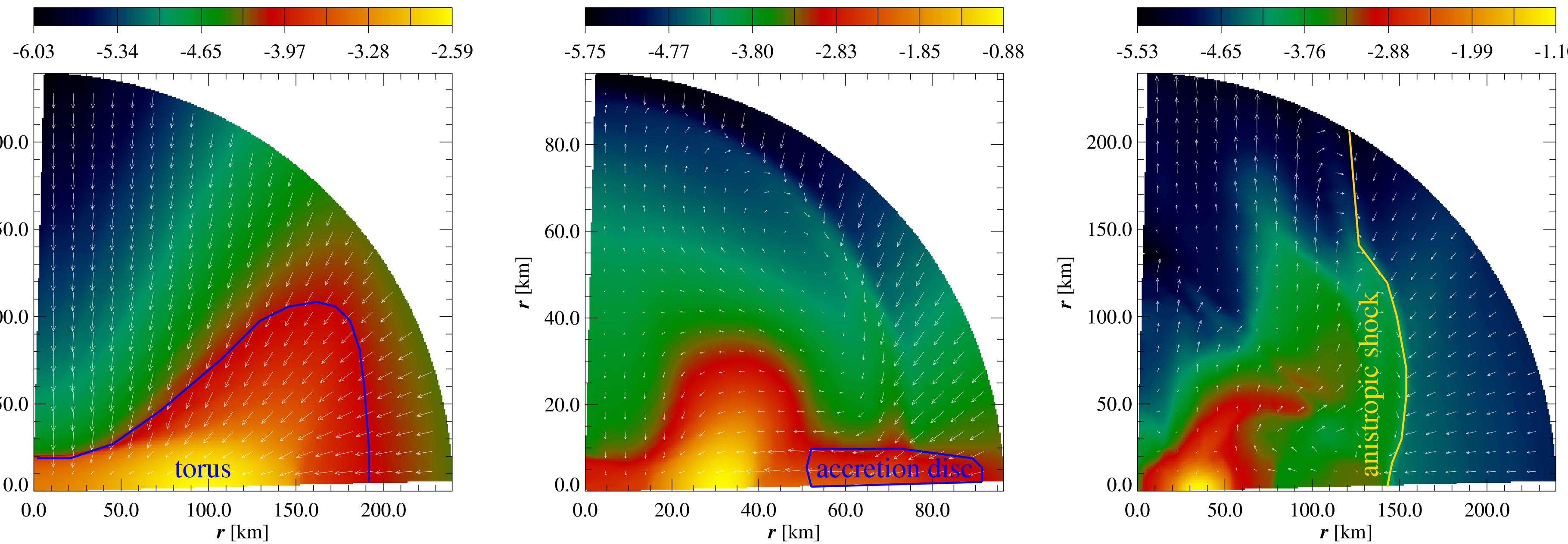
- **Spread of the 26 models does not change much. ⇒ Signal of a galactic supernova detectable.**
- **On average: Amplitude ⟶, Frequency ↗.**

  ⇒ Best case: GR effects shift model parallel to the high frequency sensitivity threshold.

Otherwise: Signal could fall out of the sensitivity window!

Relativistic Effects

Model C: Fast and extremly differential rotation, rapid collapse (≈ 30 ms).



- Initial model already has a toroidal density distribution.
- During contraction, the torus becomes more pronounced.
- Proto-neutron star is surrounded by a disc-like structure, which is accreted.
- After bounce, a strongly anisotropic shock front forms.

Summary

Our simulations show:

- Central densities are significantly higher than in Newtonian simulations.
- Many previous multiple bounce models collapse to supernuclear densities in relativity.
- On average, the signal amplitude does not change, but the signal frequency increases; we still have $h^{\mathrm{TT}} \approx 10^{-23} \cdot 10\ \mathrm{Mpc}/R$ for axisymmetric supernova core collapse.
- Relativistic effects increase rotation rate; many models could develop triaxial instabilities.

Applications – Future Projects

These are the first gravitational wave templates obtained by simulations of rotational supernova core collapse in *general relativity*.

- These templates supplement and replace previous results; we will make them publicly available.
- Our simulations are an important step towards three-dimensional simulations.
- We plan to increase the accuracy of the CFC approximation (CFC Plus).
- Detectors start their measurements in 2002; now we wait for the next galactic supernova...

Validity of the CFC

Again: The CFC is sufficiently accurate for

- not very nonspherical matter distributions
  (fulfilled very good in core collapse – compare to rotating dust disks, Schäfer and Kley), and
- if the energy of gravitational wave emission can be neglected
  (no significant gravitational radiation backreaction on the dynamics – $E_{\mathrm{gw}} \lesssim 10^{-7} E_{\mathrm{tot}}$!).

Facts and results from accuracy tests for the CFC approximation:

- CFC makes no explicit assumptions about the time-dependence of spacetime.
- CFC metric solves the ADM constraints.
- Evolution equations for γ_{ij} are only slightly violated.
- Evolution equations for K_{ij} are violated stronger
  (K_{ij} are a particular combination of metric components – they are never used in our approach).
- We can maintain long-term stability for rotating neutron stars.
- Even for strongly deformed rotating neutron stars, CFC is a fair approximation.