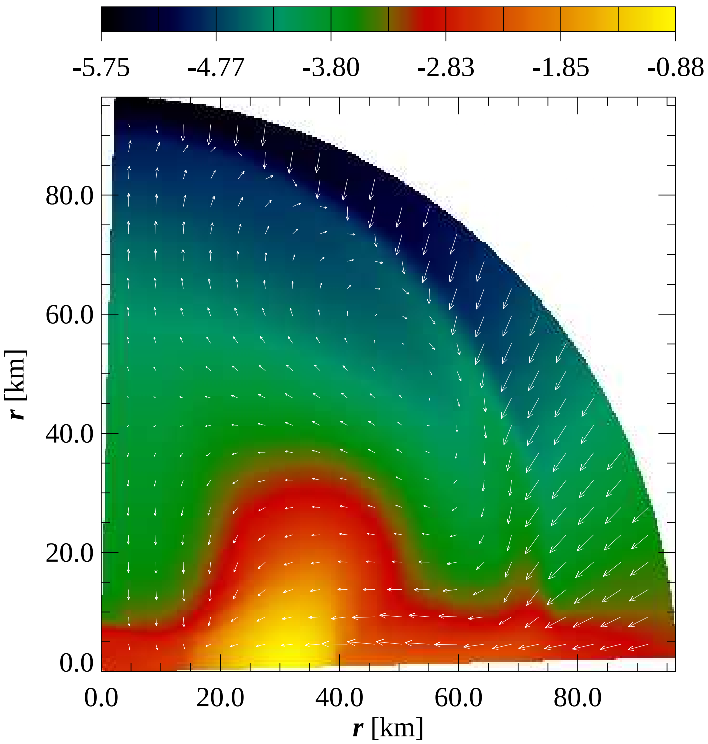


**Harald Dimmelmeier**

# Gravitational Waves from Supernova Core Collapse:

# Current State and New Directions

## A Talk in 20 Questions

Dimmelmeier, Font, Müller, *Astrophys. J. Lett.*, 560, L163–L166 (2001), astro-ph/0103088
Dimmelmeier, Font, Müller, *Astron. Astrophys.*, 388, 917–935 (2002), astro-ph/0204288
Dimmelmeier, Font, Müller, *Astron. Astrophys.*, 393, 523–542 (2002), astro-ph/0204289

`http://www.mpa-garching.mpg.de/rel_hydro/`

# What happens in a Supernova Explosion?

Physical model of a core collapse supernova:

- Massive progenitor star ($M_{\text{progenitor}} \approx 10 - 30 M_\odot$) develops a core of iron group nuclei.
- When core exceeds Chandrasekhar mass ($M_{\text{core}} \approx 1.5 M_\odot$), it collapses ($T_{\text{collapse}} \approx 100$ ms).
- At supernuclear density, EoS of matter stiffens ⇒ bounce, hot proto-neutron star forms.

$\longrightarrow \boxed{\text{Gravitational waves}}$

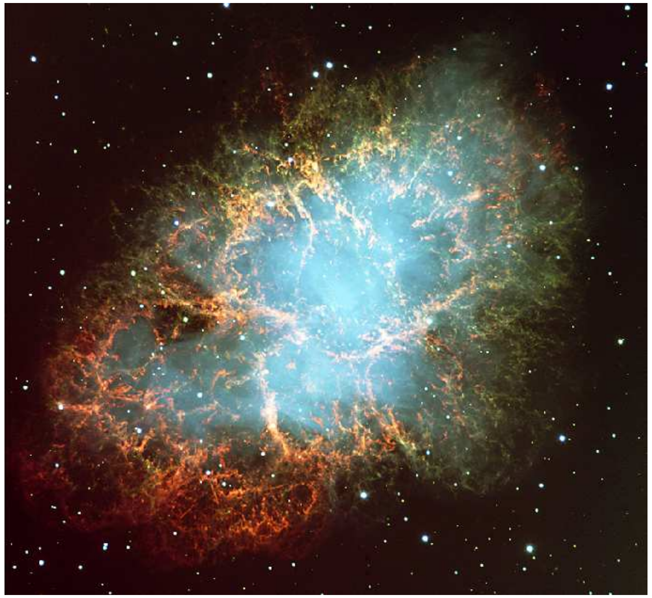
- Hydrodynamic shock propagates outward from sonic sphere, but stalls at $R_{\text{stall}} \approx 300$ km.
- Proto-neutron star cools, collapse energy is released by neutrino emission ($T_\nu \approx 1$ s).
- Neutrinos deposit energy behind stalled shock and revive it (delayed explosion mechanism).

$\longrightarrow \boxed{\text{Gravitational waves}}$

- Shock wave propagates through stellar envelope and disrupts rest of star (visible explosion).
- Neutron star may develop triaxial instabilities due to gravitational wave backreaction.

$\longrightarrow \boxed{\text{Gravitational waves}}$

# Why are Gravitational Waves from Supernovæ so interesting?



Conventional means of observing supernovæ:

- **Optical light** emission:
  Hours after actual collapse;
  emitted from stellar surface;
  no direct information about collapse engine.
- **Neutrinos** from core bounce:
  Directly from engine region; flux decays like $1/r^2$;
  extremely low detectability
  (for SN 1987A: only $\sim 10$ neutrinos detected).

Gravitational waves can directly probe collapse mechanism!
With $1/r$, fall-off behavior is superior to neutrinos.

$$\boxed{\text{Measurement of signal waveform will reveal new physics!}}$$

Gravitational waves will put constraints on rotation states of iron core and neutron star, supernuclear EoS, degree of convection, . . .

# What are the Observational Challenges?

Consider:

- Wave signal from core collapse supernova is close to interferometer detection limit ($h \approx 10^{-20}$ for a Galactic event).
- A burst signal is very complex and short in time (has no chirp signal "trigger" like merger burst signal).

⇒ We need realistic prediction of signal from relativistic numerical simulations!

Our contribution:

*Relativistic* simulations of rotational core collapse to a neutron star in axisymmetry,
and
publicly available gravitational wave signal catalogue from a parameter study.

# What are the Source Simulation Challenges?

During the various evolution stages, simulations of core collapse face many challenges:

- Physical complexity:

  Many and complicated aspects of physics involved.

  Some physics like supernuclear EoS uncertain.

  Initial conditions from stellar evolution (like rotation state of iron core) not well known.

  (Full or approximated) general relativistic gravity needed.

- Numerical difficulties:

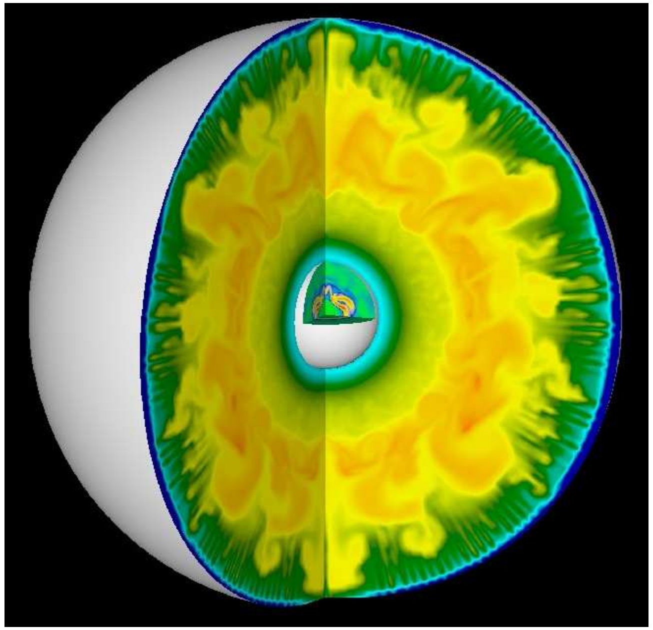
  Many different time and length scales (Comoving coordinates, FMR, AMR).

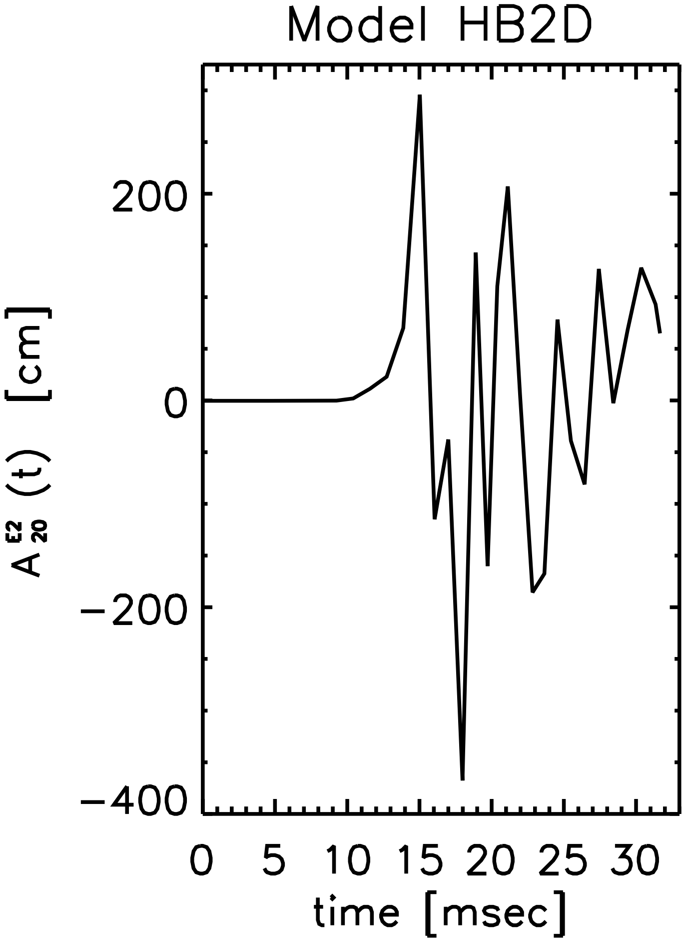
  Multidimensional treatment might be crucial
  (convection in proto-neutron star and neutrino heating region,
  Rayleigh-Taylor instabilities in envelope, rotation, magnetic fields, . . . ).

  Solution of Boltzmann transport equations for consistent treatment of neutrinos.

**Numerical simulations are very complicated, many approximations necessary.**

# Can Convection yield a Detectable Signal?





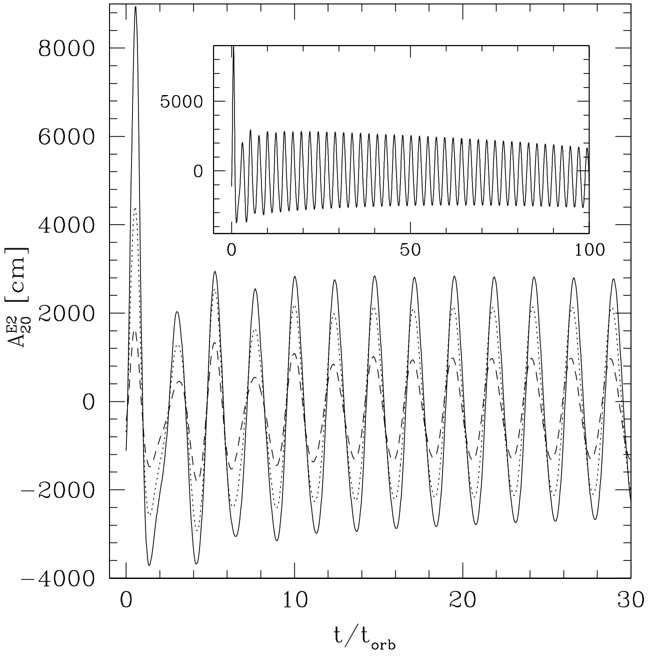
Convection can develop in

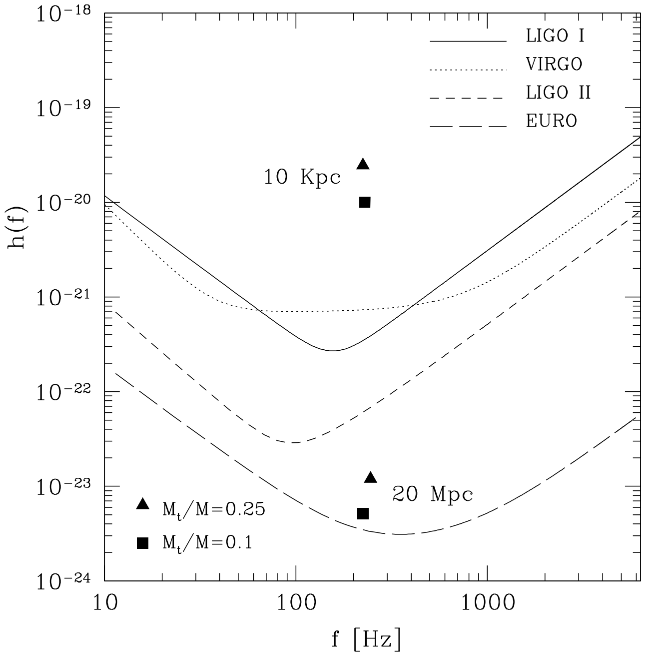
- boiling proto-neutron star,
- reheated post-shock region,
- outer stellar envelope.

This also produces gravitational waves with a signal strength comparable to bounce signal (results from Müller and Janka, 1997).

## What happens if a Black Hole forms?

For large core mass or soft supernuclear EoS, a black hole can form instantaneously of delayed (due to fall-back of matter).





Figures show simulations of oscillating matter torus around a black hole (Zanotti et al., 2003).

Gravitational radiation can be emitted by

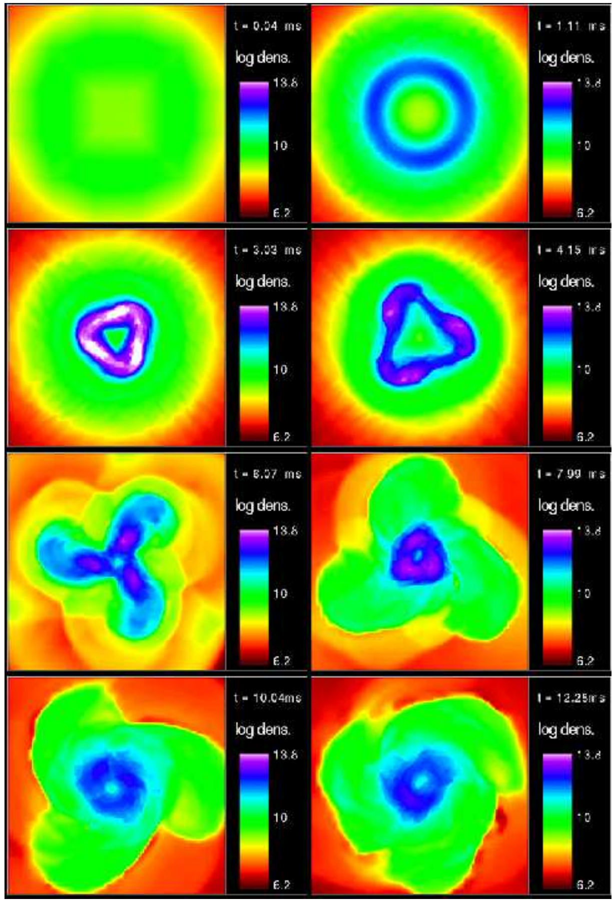
- black-hole quasi-normal mode ringing,
- accretion of matter onto black hole, or
- oscillation of matter surrounding the black hole.

# What Signals can we expect from the Rotating Neutron Star?

Development of triaxial instabilities

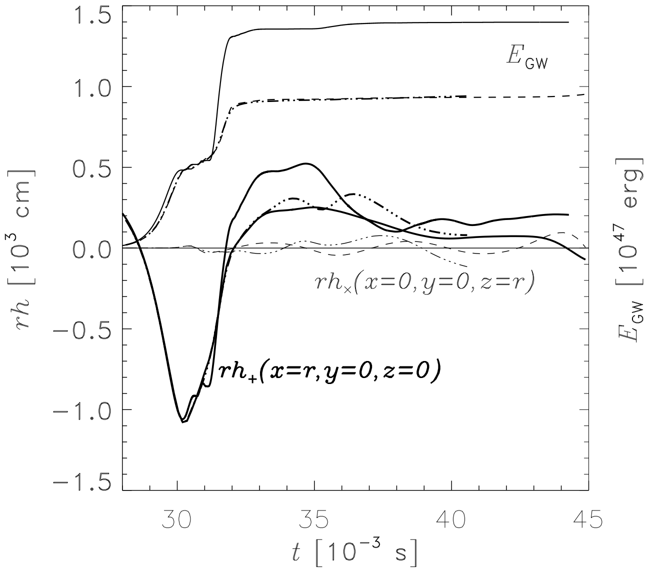
- in proto-neutron stars (dynamical and secular), or
- in old neutron stars (r-mode instability, . . . )

can be another important source of gravitational waves with signal amplitudes comparable to bounce signals.



These processes yield quasi-periodic signals, which reveal information about supernuclear EoS.

Such simulations can only be done in 3d codes (e.g. Newtonian results by Rampp et al., 1998).



# What is the Current State of the Art in Relativistic Core Collapse?

Early attempts to simulate relativistic core collapse we hindered by

- computational limitations, and particularly
- numerical problems (nonconservative hydrodynamics, axis problems, instability of ADM equations).

Major breakthroughs in both aspects. New mathematical and numerical formulations:

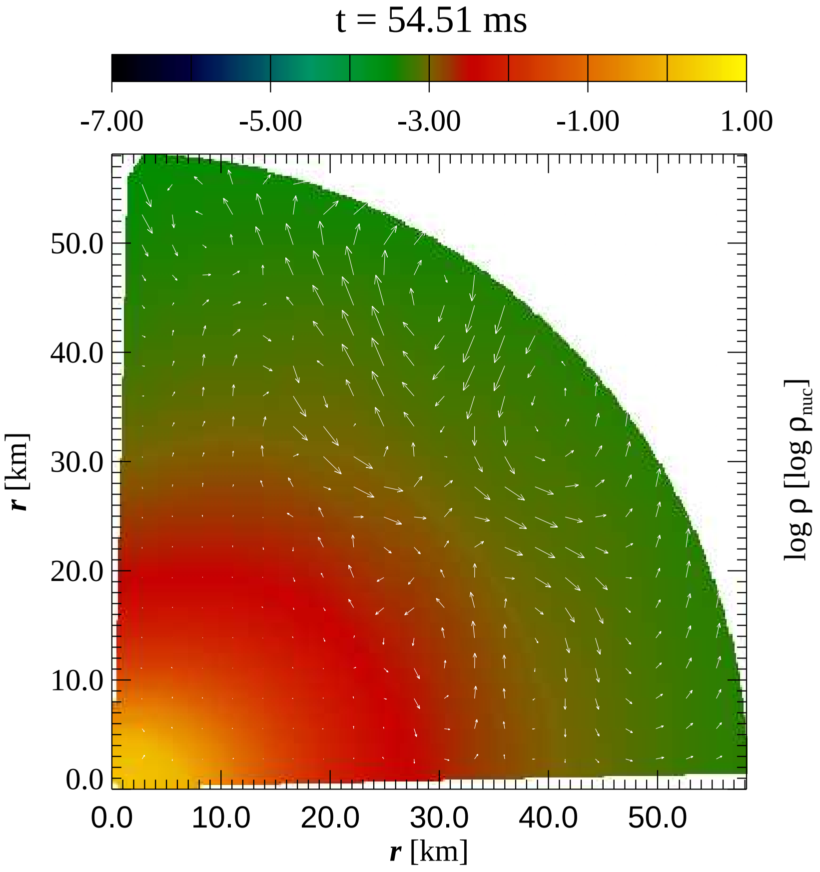
- HRSC schemes exploiting hyperbolic hydrodynamics.
- Reformulation of ADM equations by Baumgarte, Shapiro, Shibata, and Nakamura (BSSN).
- Various approximation approaches of metric equations.

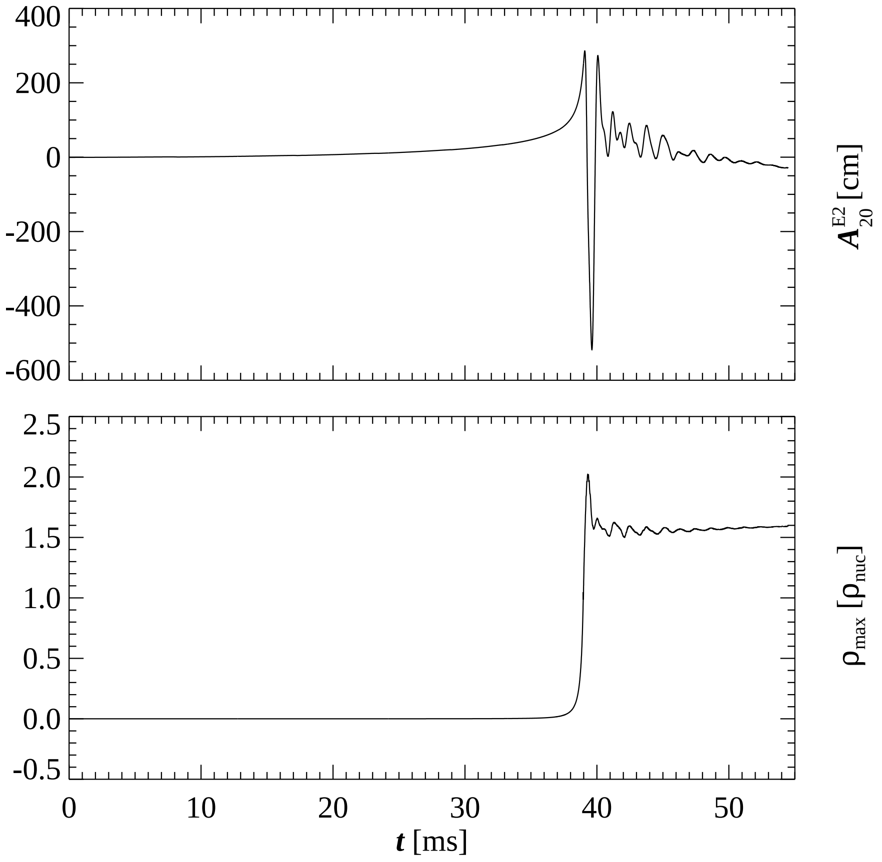
This has given boost to simulations of supernova core collapse, which utilize very different approaches:

- 2d Cartesian Cartoon method with rotation (Shibata).
- 3d Cartesian with BSSN and HRSC or conventional hydrodynamics (AEI Cactus code, Shibata, Baumgarte, Shapiro).
- 3d SPH with spherical gravity (Fryer and Heger).
- 3d hydrodynamics with NewtonPlus gravity (Mezzacappa).

# What is our Contribution to this?

We have developed a relativistic hydro code for simulating rotational core collapse.





## Model Assumptions

# What Simplifications do we make?

To reduce complexity of the problem, we assume

- axisymmetry and equatorial symmetry,
- rotating $\gamma = 4/3$ polytropes in equilibrium as initial models, with $\rho_{\mathrm{c\,ini}} = 10^{10}$ gm cm$^{-3}$, $R_{\mathrm{core}} \approx 1\,500$ km, and various rotation profiles and rates,
- simplified ideal fluid equation of state, $P(\rho, \epsilon) = P_{\mathrm{poly}} + P_{\mathrm{th}}$ (neglect complicated microphysics),
- constrained system of Einstein's equations (assume conformal flatness for spatial 3-metric).

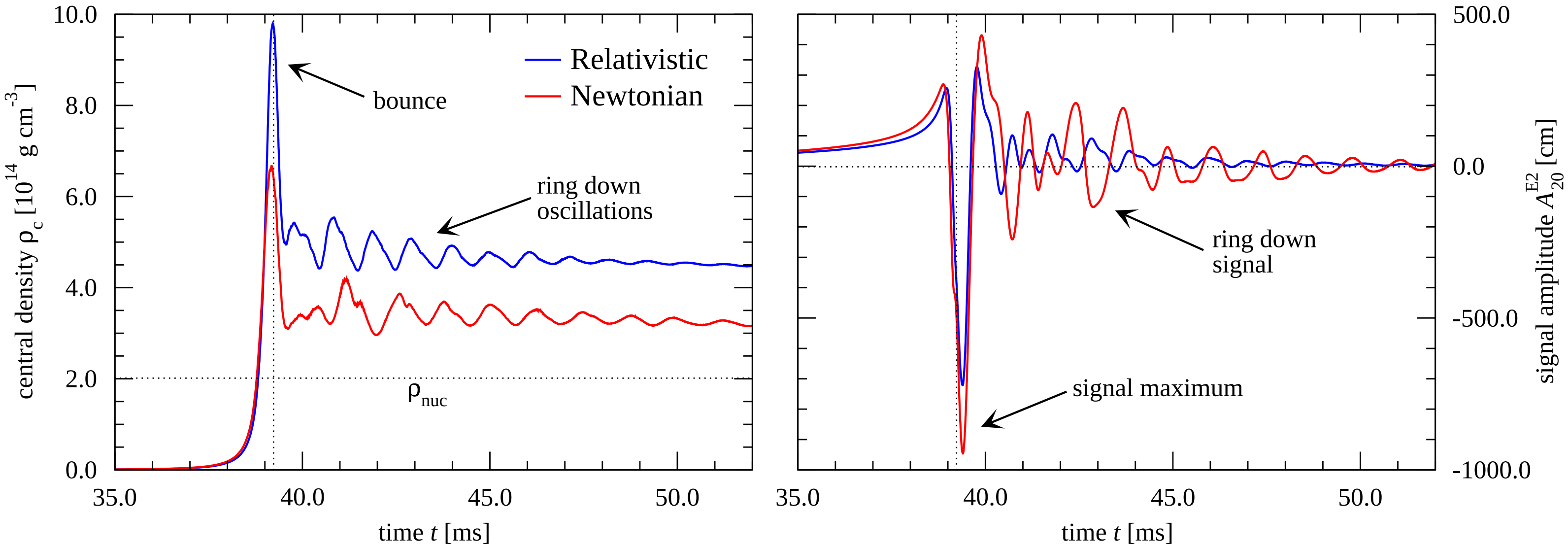
# Which are our Goals?

We have performed a parameter study of 26 models to

- extend research on Newtonian rotational core collapse by Zwerger and Müller to GR,
- obtain more realistic waveforms as "wave templates" for interferometer data analysis, (wave templates are important and actually being used in data analysis: VIRGO data analysis group has used Zwerger's catalogue (Pradier et al., 2000), and already uses our results (Chassande-Mottin, 2002)),
- have a 2d GR hydro code for comparison with future simulations and as a basis for extension.

# Do Higher Densities result in Larger Signal Amplitudes?

**Many models instantaneously acquire a new supernuclear equilibrium state (proto-neutron star).**



- **Deep dive into potential, high supernuclear densities, single bounce, ring down (regular bounce).**
- **GR simulation: Higher central density and signal frequency, but *lower* signal amplitude.**
  **Explanation: GW signal is determined by accelation of *extended* mass distribution:**

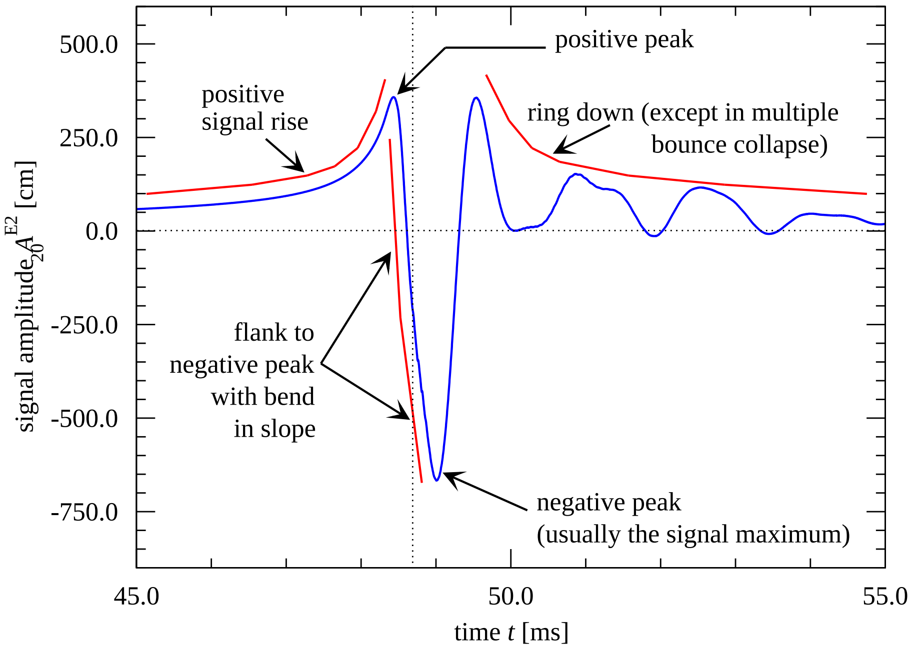
$$A_{20}^{\mathrm{E2}} = \ddot{Q} \propto \frac{d^2}{dt^2} \int dV \rho \boxed{r^2}. \quad \leftarrow$$ **weight factor!**

**In relativistic gravity core is more compact. ⇒ Gravitational waves can have smaller amplitude!**

# What are the Typical Features of a Gravitational Wave Signal?

Most waveform share common features. This is important for filters in data analysis!

Example: Signal from regular core collapse with single bounce.



Goal: Estimate

- robustness of signal, and
- dependence on model parameters.

We present our waveform catalogue at

`http://www.mpa-garching.mpg.de/Hydro/RGRAV/`.

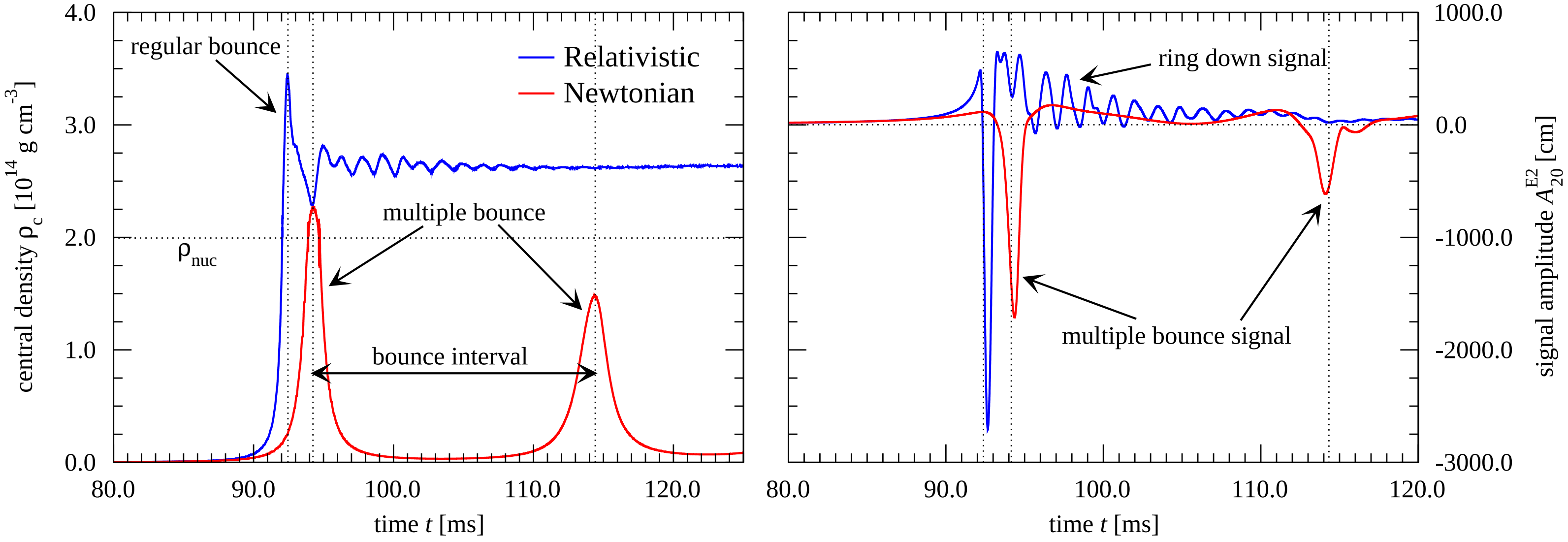
Our offer to data analysis:

Exploration of specific parameter space regions.

Conversely, in a detected signal, these features allow for conclusions about physics of core collapse.

# Can Relativistic Gravity make the Collapse Dynamics change?

**Many Newtonian simulations show multiple core bounces. In relativity, collapse dynamics can change.**

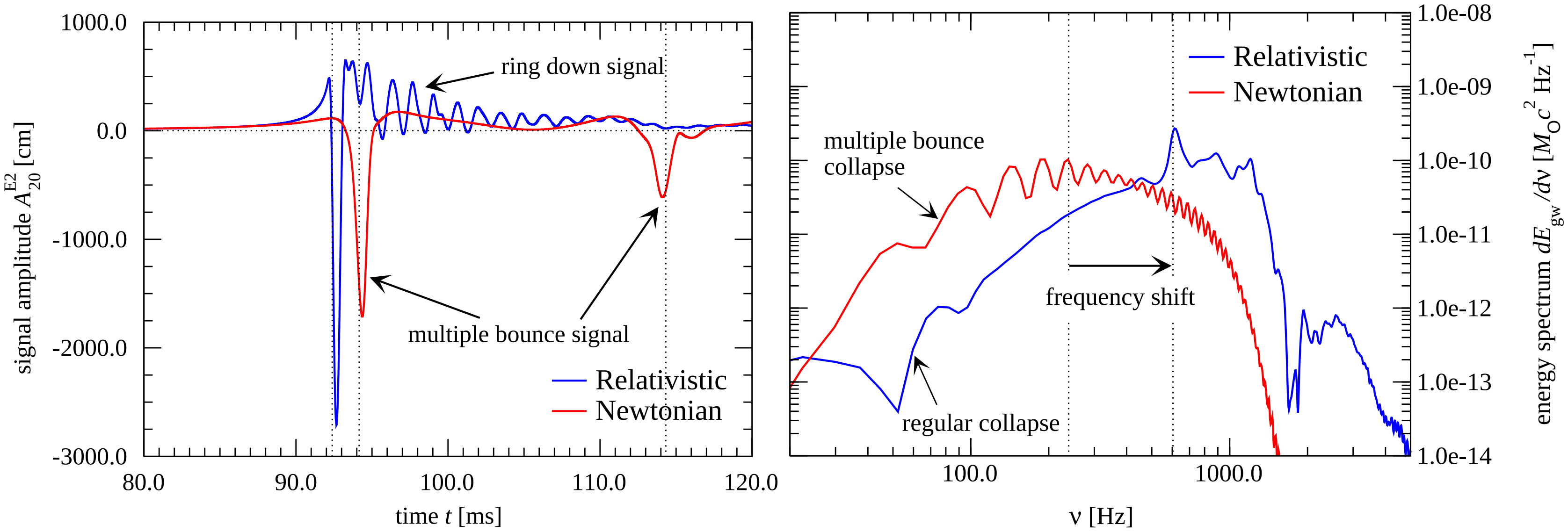


- Rotation increases strongly during collapse (angular momentum conservation!).
- Newtonian: Nuclear density hardly reached, multiple centrifugal bounce with re-expansion.
- GR: Nuclear density easily reached, regular single bounce.
- Relativistic simulations show multiple bounces only for few models.

Strong *qualitative* difference in collapse dynamics and thus in signal form.

# How is this Change of Collapse Dynamics reflected in the Signal?

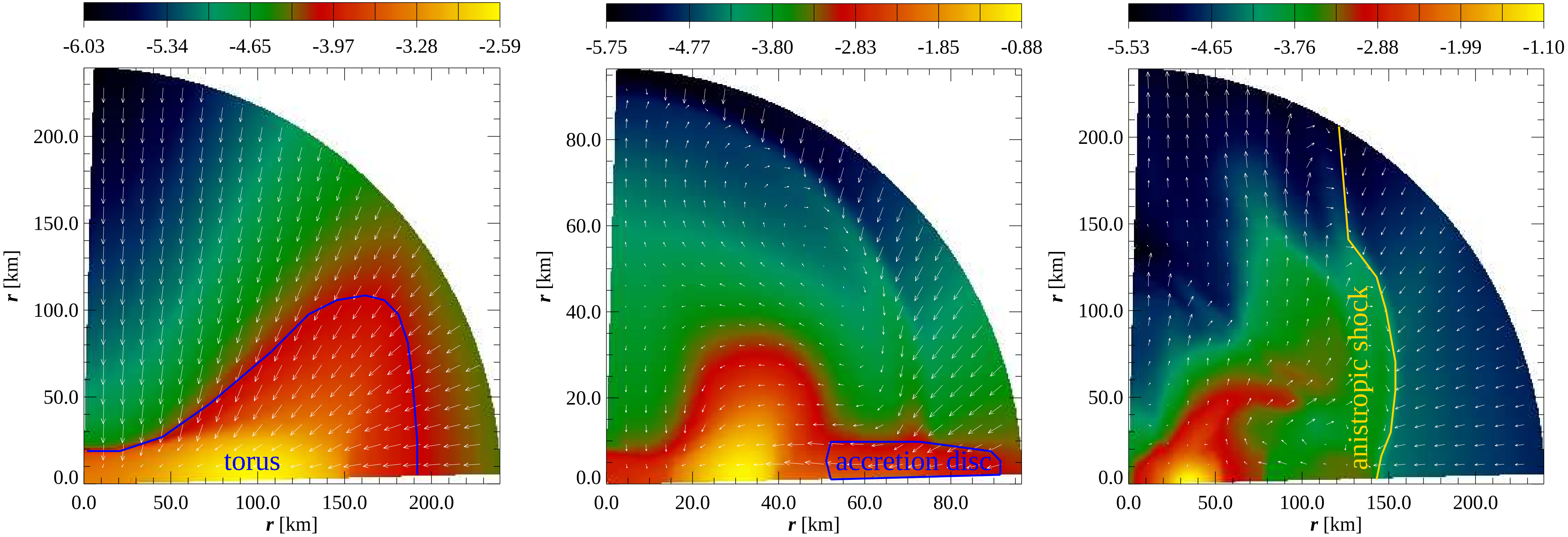
**Change in collapse dynamics is clearly visible in energy spectrum:**



- **Multiple bounce collapse has broad round spectrum which peaks at relatively low frequency.**
- **Regular collapse has steeper spectrum with pronounced peak at higher frequency.**

**⇒ From signal we can infer on collapse dynamics.**

# What Behavior do Rapidly Rotating Models exhibit?

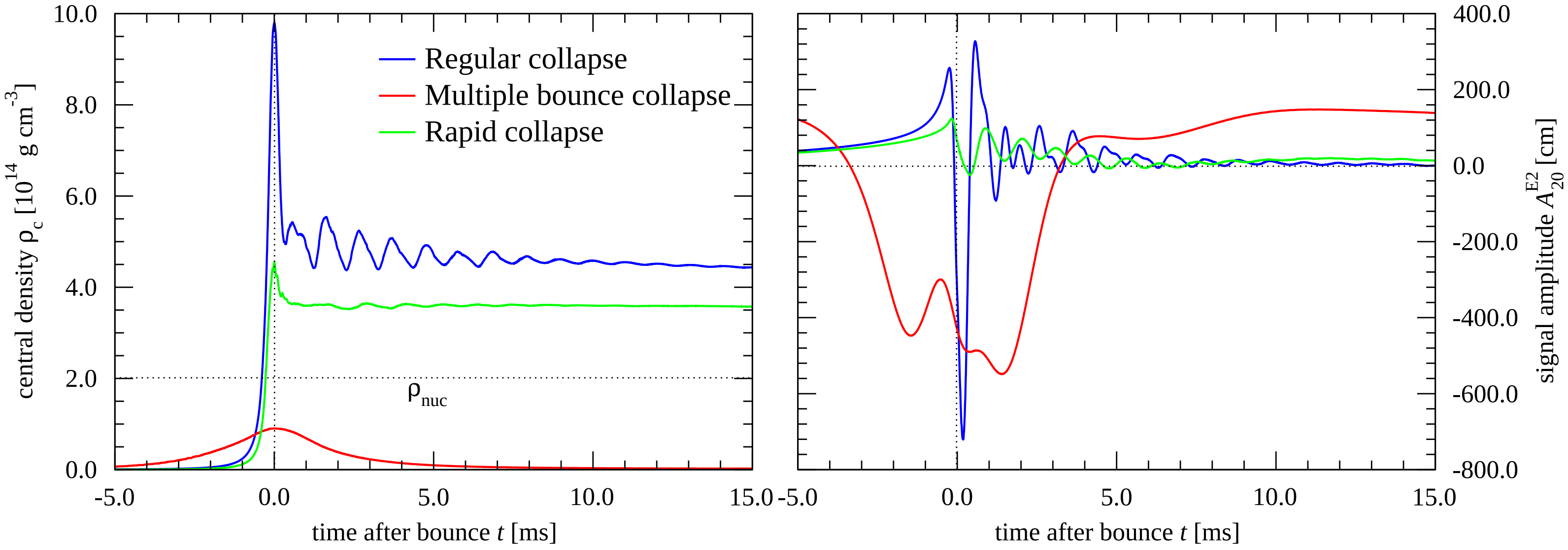


- Initial model has toroidal density shape; torus becomes more pronounced during contraction.
- Proto-neutron star is surrounded by short-lived accretion disc.
- After bounce, a strongly anisotropic shock front forms.
- Bar instability could develop on dynamical timescale; this will produce a characteristic signal (particularly with differential rotation and in GR).

# Can our Models be classified according to the Signal Waveform?

**Just as in Newtonian gravity, in our relativistic simulations we observe**

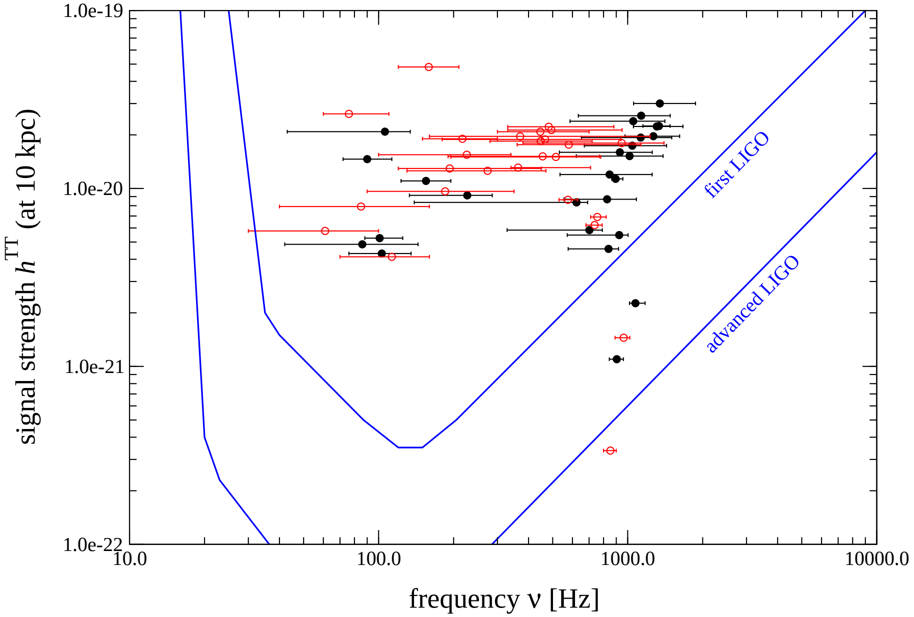
- **three normal collapse types,**
  **regular collapse (signal shows one large negative peak, and clear ring down),**
  **multiple bounce collapse (signal shows distinct multiple large negative peaks, and no ring down),**
  **rapid collapse (signal shows one small positive maximum peak, and low-amplitude ring down),**
- **a separate class of rapidly and differentially rotating models (which form torus).**



**These collapse types can be identified both in density evolution and in signal waveform.**

# Where in the Sensivity Diagram are our Models located?

**Influence of relativistic effects on signals: Investigate amplitude–frequency diagram.**



- **Spread of the 26 models does not change much. ⇒ Signal of a galactic supernova detectable.**
- **On average: Amplitude remains at $h^{TT} \approx 10^{-23} \cdot 10$ Mpc$/R$, frequency increases to $\nu \approx 1000$ Hz.**

**If close to detection threshold: Signal could leave sensitivity window in relativistic gravity!**

# What are the Future Directions of this Research Field?

- Breakthroughs in numerical relativity (new formulations of field equations) and
- increasing computer power (massive parallel computing)

will pave way to more sophisticated simulations.

In next years, we expect following developments:

- Extension of relativistic 2d codes to 3d, including fixed or adaptive mesh refinement.
- Better simulations of black hole formation.
- Inclusion of microphysics/relativistic gravity into relativistic/supernova codes.
- More consistent and realistic rotating initial models.

Increasing detector sensitivity: Consider additional mechanisms for wave generation in core collapse.

Ultimately: Now separate branches of

- numerical relativity (dominated by vacuum solutions), and
- classical astrophysical hydrodynamics (focused on microphysics and explosion mechanism)

will be reunified.

# What are the Future Directions of our Work?

Further developments of our axisymmetric code:

- Extract oscillation frequencies of rotating neutron stars (equilibrium, collapse simulations).
- Include more accurate approximation of spacetime metric (CFC Plus).
- Use spectral methods from LORENE for calculating metric (more accurate and faster).
  Current solver methods:
  - Discretrized multi-dimensional Newton–Raphson iteration (robust, much too slow for 3d).
  - Integral conventional Poisson solver iteration (fast, slow / no convergence in 2d / 3d).
  - Integral Poisson solver iteration using LORENE's spectral methods (fast, intrinsically 3d, rapid convergence).
- Extend code to 3d, and investigate dynamic development of bar instability.
- Add more realistic microphysics (Lattimer–Swesty, neutrinos) and check robustness of signal.

Ongoing or planned other projects:

- Assess quality of CFC approximation by comparison to fully relativistic simulations.
- Use CACTUS code to simulate axisymmetric and fully 3d core collapse.