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Model-Independent Reconstruction of the Expansion History of the Universe from Type Ia Supernovae

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ABSTRACT

Based on the largest homogeneously reduced set of Type Ia supernova luminosity data currently available – the Union2 sample – we reconstruct the expansion history of the Universe in a model-independent approach. Our method tests the geometry of the Universe directly without reverting to any assumptions made on its energy content and thus allows us to constrain Dark Energy models in a straightforward way. This is demonstrated by confronting the expansion history reconstructed from the supernova data to predictions of several Dark Energy models in the framework of the wCDM paradigm. In addition, we test various non-standard cosmologies such as braneworlds, $f(R)$ and kinematical models. This is mainly intended to demonstrate the power of the method. Although statistical rigor is not the aim of our current study, some extreme cosmologies clearly disagree with the reconstructed expansion history. We note that the applicability of the presented method is not restricted to testing cosmological models. It can be a valuable tool for pointing out systematic errors hidden in the supernova data and planning future Type Ia supernova cosmology campaigns.

Key words: supernovae: general – cosmology: cosmological parameters – cosmology: observations – cosmology: theory

1 INTRODUCTION

Type Ia supernovae (SNe Ia) are currently the best (relative) distance indicators out to redshifts of $z \sim 1$ (Tammann 1978; Colgate 1979; Riess, Press & Kirshner 1996; Schmidt et al., 1998; Perlmutter et al., 1999). As such, they have been instrumental in shaping our current picture of the Universe (Leibundgut 2001, 2008). In particular, the notion of an accelerated expansion rate of the Universe was established a decade ago based on SN Ia distance measurements (Riess et al. 1998; Perlmutter et al. 1999).

For lack of a deeper understanding, the cause of this acceleration is commonly parametrized in standard ΛCDM cosmology as a Dark Energy component that currently dominates the energy contents of the Universe (see, e.g., Turner & Huterer 2007, for a recent review). The most plausible explanation for it may be the vacuum energy (or cosmological constant) with a constant equation of state $w = p/\rho = -1$ (Turner & White 1997). In this case, Dark Energy would be an elastic and smooth fluid exerting a repulsive gravity that produces the observed accelerated expansion (Ostriker & Steinhardt 1995; Liddle et al. 1996; Turner & White 1997).

So far, however, all attempts to compute the vacuum energy have led to values that are about 55–120 orders of magnitude off the observed value (Weinberg 1989; Sahni 2002). Other possibilities allow for a Dark Energy equation of state varying with time and avoid the cosmic coincidence and the previously mentioned fine-tuning problems (Zlatev, Wang & Steinhardt 1999; Frieman, Turner & Huterer 2008).

These concepts describe the vacuum energy to be a dynamical, evolving scalar-field slowly rolling toward its lowest energy state (Freese et al. 1987; Frieman et al. 1995;

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Wetterich 1995; Turner & White 1997; Caldwell et al. 1998; Steinhardt 1999). In this context, the simplest parametrization of the Dark Energy equation of state $w(a)$ depending on the cosmic scale factor a reads (Chevallier & Polarski 2001; Linder 2003):

$$w(a) = w_0 + w_\mathrm{a}(1-a), \tag{1}$$

where w_0 is the present-day value of the equation of state and w_a accounts for its time-dependence.

Alternative approaches describe cosmic acceleration as a manifestation of new gravitational physics rather than Dark Energy (see, e.g., Deffayet 2001; Deffayet et al. 2002; Carroll et al. 2004; Nojiri & Odintsov 2006; Amendola et al. 2007). Instead of adding an extra term to the energy-momentum tensor on the right side of Einstein's equations, these modify the geometry terms on their left side in order to reproduce the observations.

Hence, we face the situation that apart from ΛCDM a large variety of cosmological models has been proposed to account for cosmic acceleration. Testing their validity is an important but challenging task. The present-day nearby Universe is fairly well-known and therefore all models under consideration reproduce its characteristics, or contain free parameters that can be tuned to do so. This degeneracy is difficult to break with currently available data.

Usually, Dark Energy models are constrained by starting out from a Friedman cosmology in which the expansion function H is parametrized in terms of the contributions of radiation (r), matter (m), curvature (k), and Dark Energy (de) to the energy density as

$$\begin{aligned} H^2(a) &= H_0^2 E^2(a) \\ &= H_0^2\left[\frac{\Omega_\mathrm{r0}}{a^4}+\frac{\Omega_\mathrm{m0}}{a^3}-\frac{\Omega_{k0}}{a^2}+\Omega_\mathrm{de0}F(a)\right], \end{aligned} \tag{2}$$

where H_0 and $\Omega_{\ldots 0}$ denote the Hubble constant and the present-day density parameters corresponding to the different components, respectively. A possible time-dependence of the Dark Energy equation of state is captured by the function $F(a)$

$$F(a) = \exp\left\{-3\int_1^a \left[1+w(a')\right]\frac{da'}{a'}\right\}. \tag{3}$$

The cosmological parameters occurring in Eq. (2) are determined from fits to observations and Dark Energy models are usually assessed by confronting their predictions to these parameters. The significance of this approach is, however, limited since it automatically assumes a Friedman model. When working with SNe Ia this constraint is unnecessary as they probe the geometry of the Universe directly and no assumptions on the form of the energy-momentum tensor are required to derive the expansion history of the Universe.

Consequently, our analysis follows a recently developed method (Mignone & Bartelmann 2008) to reconstruct the expansion history of the Universe in a model-independent fashion from luminosity distance data. The idea of a model-independent reconstruction extracted straight from the data was already proposed in Starobinsky (1998) and reconstructions of this kind were carried out by Shafieloo et al. (2006); Shafieloo (2007) using SN Ia data, by Fay & Tavakol (2006) adding constraints from measurements of baryon acoustic oscillations (BAO) to the SN Ia data, and by Daly & Djorgovski (2003, 2004) combining SNe Ia luminosity distances with angular-diameter distances from radio galaxies. Seikel & Schwarz (2008, 2009) tested the significance of cosmic expansion directly from SN Ia data in a model-independent way.

The goal of this work is to apply the method of Mignone & Bartelmann (2008) to the most complete SN Ia data set currently available and to constrain some flavors of Dark Energy models. This is intended as a *demonstration* of the general power of SN Ia distance measurements for testing Dark Energy models when analyzed in a model-independent way. A rigorous statistical treatment that would let us rule out specific models is beyond the scope of this paper.

The paper is organized as follows. In Section 2, the essential aspects of the model-independent methodology are reviewed. The application of the method to luminosity-distance measurements is discussed in Section 3. A comparison between SN Ia data and several Dark Energy models is presented in Section 4. In Section 5, the predictions for the expansion history of the Universe of several non-standard cosmologies are confronted with our reconstruction from SN Ia data. Ways of improving the reconstruction with new data that may become available through current surveys are pointed out in Section 6. This shows that our model-independent analysis can be instrumental in planning observational campaigns. Moreover, it can also point to potential systematics in the data that do not generally arise from traditional ways of analysis. This aspect is highlighted in Section 7. Finally, conclusions are drawn and future perspectives are discussed.

2 MODEL-INDEPENDENT METHOD

The minimal assumptions adopted by Mignone & Bartelmann (2008) are that the expansion rate is a reasonably smooth function and that the Universe is topologically simply connected, homogeneous and isotropic, i.e. it is characterized by a Robertson-Walker metric:

$$ds^2 = c^2dt - a^2(t)\left[d\chi^2 + f_k^2(\chi)(d\theta^2+\sin^2\theta d\phi^2)\right] \tag{4}$$

The Robertson-Walker metric allows us to define an angular-diameter distance by

$$D_\mathrm{A}(a) = af_k[\chi(a)], \tag{5}$$

with the comoving angular-diameter distance

$$f_k(\chi) = \begin{cases} \sin\chi & (k=1,\ \text{spherical}) \\ \chi & (k=0,\ \text{flat}) \\ \sinh\chi & (k=-1,\ \text{hyperbolic}) \end{cases} \tag{6}$$

and the comoving distance

$$\chi(a) = \frac{c}{H_0}\int_a^1 \frac{dx}{x^2E(x)}. \tag{7}$$

Through Etherington's relation (Etherington 1933), which holds for any space-time, we can relate the luminosity distance to the angular-diameter distance:

$$D_\mathrm{L}(a) = \frac{1}{a^2}D_\mathrm{A}(a), \tag{8}$$

This allows us to write the former as an integral of the inverse of the expansion rate

$$D_{\rm L}(a) = \frac{c}{H_0}\frac{1}{a}\int_a^1 \frac{dx}{x^2 E(x)} \equiv \frac{c}{H_0}\frac{1}{a}\int_a^1 \frac{dx}{x^2} e(x), \tag{9}$$

with $e(a) \equiv E^{-1}(a)$ and $k = 0$ set in Eq. (6) for simplicity of notation. It is worth noticing that the choice of $k = 0$ does not affect the fundamental method and can be dropped without change of principle if needed. The Hubble length c/H_0, by which the luminosity distance is scaled, shall be dropped in the following for the sake of brevity.

From Eq. (9) the derivative with respect to a is taken

$$D'_{\rm L}(a) = -\frac{1}{a^2}\int_a^1 \frac{dx}{x^2} e(x) - \frac{e(a)}{a^3}. \tag{10}$$

This expression can be transformed to a Volterra integral equation of the second kind for the unknown function $e(a)$,

$$e(a) = -a^3 D'_{\rm L}(a) + \lambda \int_1^a \frac{dx}{x^2} e(x), \tag{11}$$

with the inhomogeneity $f(a) = -a^3 D'_{\rm L}(a)$ and the simple kernel $K(a, x) = x^{-2}$. The general parameter λ will later be fixed to $\lambda = a$. For now it is introduced to make the connection to a class of equations for which solutions are known to exist and to be uniquely described in terms of a Neumann series (see Arfken & Weber 1995):

$$e(a) = \sum_{i=0}^{\infty} \lambda^i e_i(a). \tag{12}$$

A possible choice for the function e_i would be

$$e_0(a) = f(a), \tag{13}$$

$$e_n(a) = \int_1^a K(a, t) e_{n-1}(t) dt, \tag{14}$$

where, for the guess of e_0, the approximation of either the integral or λ to be small has been made in Eq.(11). This approximation is valid in all important cosmological cases and it is subsequently improved until convergence is reached. It essentially means that starting, for instance, from the ΛCDM cosmology observations say that deviations must be small, if they exist at all.

Equation (11) involves the derivative of the luminosity distance with respect to the scale factor a. Observations of SNe Ia provide measurements of the distance modulus, μ_i, and redshifts, z_i (or scale factors $a_i = (1 + z_i)^{-1}$). Thus one could think of taking the derivative of the luminosity distance directly from the data. However, this is an inconvenient procedure since the result would be extremely noisy and the determination of $D'_{\rm L}(a)$ would be unreliable. Therefore, it is necessary to suitably smooth the data in the first place by fitting an adequate function $D_{\rm L}(a)$ to the measurements $D_{\rm L}(a_i)$. The derivative in Eq. (11) can then be approximated by the derivative of $D_{\rm L}(a)$. Thus, the derivative of the fitted data is taken as a representation of the derivative of the real data. The method proposed by Mignone & Bartelmann (2008) achieves this goal via the expansion of the luminosity distance $D_{\rm L}(a)$ into series of (in principle) arbitrarily chosen orthonormal functions $p_j(a)$:

$$D_{\rm L}(a) = \sum_{j=0}^{J} c_j p_j(a). \tag{15}$$

Suitable orthonormal function sets can be constructed by Gram-Schmidt orthonormalisation from any linearly independent function set. The J coefficients c_j are those which minimize the χ^2 statistic function when fitting to the data. Therefore, with this representation of the data, the derivative of the luminosity distance function is simply given by

$$D'_{\rm L}(a) = \sum_{j=0}^{J} c_j p'_j(a). \tag{16}$$

Due to the linearity of Eq. (11), it is possible to solve it for each mode j of the orthonormal function set separately. Thus, the final solution in terms of a Neumann series is

$$e(a) = \sum_{j=0}^{J} c_j e^{(j)}(a), \tag{17}$$

that is, the measured coefficients of the series expansion give the solution for the expansion function.

The number of terms to be included in order to produce an acceptable fit to the data depends on the choice of the orthonormal basis. Due to the limited quality of current observational data, however, it is expected that the coefficients of this expansion can only be determined up to a certain order. Therefore, although the basis of the expansion is arbitrary with ideal data, it is not in practice. A preferred choice would minimize the number of required coefficients. Maturi & Mignone (2009) suggested an optimal basis system derived from principal component decomposition of cosmological observables. The use of this optimal basis would certainly reduce the coefficients needed to obtain an accurate reconstruction and we shall do so in future work. In the current analysis, however, we settle for the arbitrarily chosen basis proposed by Mignone & Bartelmann (2008), which uses the linearly independent set

$$u_j(x) = x^{j/2-1}, \tag{18}$$

orthonormalized via the Gram-Schmidt method. The orthonormalization interval is chosen to be $[a_{\rm min}, 1]$, where $a_{\rm min} = (1 + z_{\rm max})^{-1}$ is the scale factor corresponding to the maximum redshift $z_{\rm max}$ in the supernova sample. This way, an arbitrary set of orthonormal functions $p_j(a)$ is constructed. When projecting the luminosity distance $D_{\rm L}(a)$ onto these basis functions, we can solve for the expansion coefficients. In this case, at least the first five coefficients are different from zero (see Mignone & Bartelmann 2008). However, as shown in Table 1, only the first three can be determined with current data; the fourth and fifth coefficients lose significance. With future space-based telescopes, new and better-quality data will be become available for this kind of analysis. This should procure an improvement on the accuracy of the reconstruction by allowing for determining more coefficients in the expansion.

3 APPLICATION TO SN IA DATA: THE UNION2 SAMPLE

We apply the model-independent method to the largest homogeneously reduced SN Ia sample currently available. The Union2 sample (Amanullah et al. 2010) consists of 557 SNe Ia. It includes the recently extended dataset of distant supernovae observed with the HST (Riess et al. 2007; Amanullah et al. 2010), the data from the SNLS (Astier et al. 2006), ESSENCE (Miknaitis et al. 2007; Wood-Vasey et al. 2007) and SDSS (Holtzman et al.

2008) surveys, several compilations from literature (e.g. Hamuy et al. 1996), and the new data from nearby SNe Ia of Hicken et al. (2009).

In Fig. 1 the Union2 sample is shown, together with the best fit of the luminosity distances to Eq. (15) when using the first three terms in the expansion (dotted line, see Table 1 for values). There is considerable scatter around the fit, mainly introduced by the distant SNe which are more difficult to calibrate and are usually afflicted by host-galaxy extinction and survey selection effects.

In the following two sections, we compare the expansion history derived from the Union2 sample with predictions of various cosmological models. This comparison, however, is not fully consistent. SNe Ia are not standard candles but have to be calibrated as distance indicators applying an empirical calibration. This calibration procedure introduces a dependence between the calibrated measurements that gives rise to finite non-diagonal entries in the covariance matrix for the distance modulus.

Moreover, the parameters of the calibration are determined simultaneously with the cosmological parameters in the SALT2 light curver fitter (Guy et al. 2007) assuming a ΛCDM model. As the Union2 sample was calibrated with SALT2, this applies to the data used in the following and implies that the parameters are model-dependent. This clearly contradicts the intended model-independence of our method and would prevent precise fits to non-ΛCDM cosmologies. Such fits, however, are not our goal here. Apart from that, ΛCDM happens to be an extraordinary good fit to the data and models with more degrees of freedom do not improve the quality of the fit more than statistically expected. That means that the correction parameters will not change significantly if one assumes a different model, as long as it roughly matches ΛCDM. Models that show a $H(z)$ drastically different from ΛCDM could also have different parameters in the calibration, but are not interesting as they would not agree with the data. Additionally, the calibration parameters should be redshift independent and therefore, essentially decoupled from cosmology. Consequently, we argue that the induced covariance is small and the variation of the parameters with cosmology is negligible for our purposes.

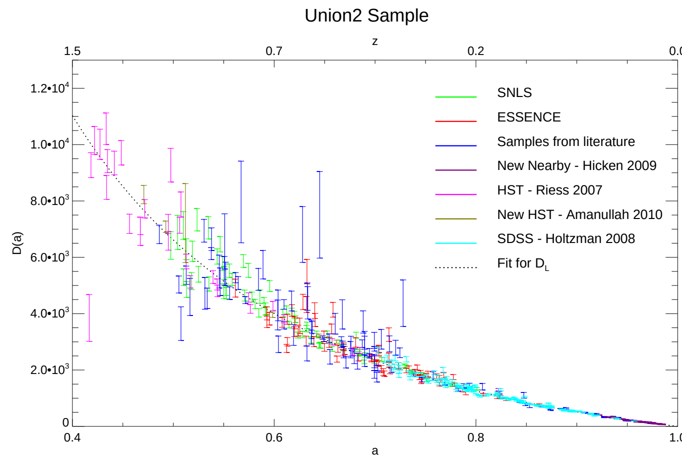

Figure 1. Luminosity distance plotted against scale factor a for the the Union2 sample. The dotted line is the best fit for the luminosity distance obtained by fitting the data to Eq. (15) and using three expansion coefficients.

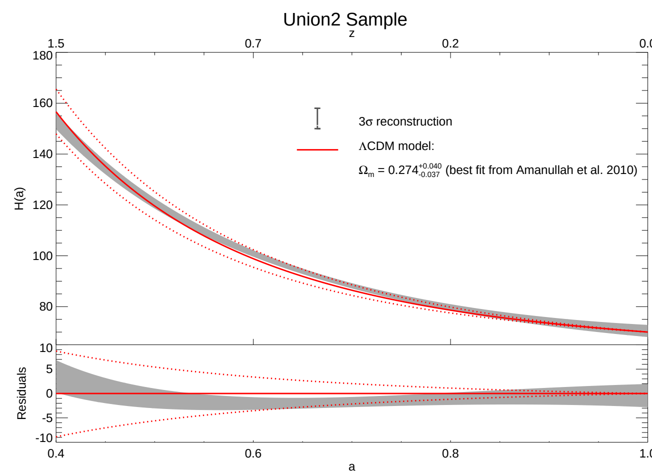

Figure 2. The reconstructed expansion rate, with 3σ errors, extracted from the Union2 sample. The red lines represent a ΛCDM model assuming the best-fit cosmological parameters reported by Amanullah et al. (2010) . It is shown here only for the sake of comparison, this model was not used in our calculations. The bottom panel shows the residuals between the reconstruction and the model. The dotted lines are the limits of the model varying Ω_m with H_0 fixed to 70 km s^{-1} Mpc^{-1}.

4 COMPARISON TO ΛCDM AND DARK ENERGY MODELS

4.1 ΛCDM cosmology

The result of our reconstruction of the expansion rate $H(a)$ from the Union2 sample is compared with a ΛCDM model in Fig. 2. Here, we adopt the values for the today's density parameters from the best fits to the Union2 sample given by Amanullah et al. (2010) – $\Omega_\mathrm{m} = 0.274^{+0.040}_{-0.037}$. The Hubble constant is assumed to be $h = 0.7$ ($H_0 = 100h$ km s^{-1} Mpc^{-1}) and the equation of state to be constant ($w = -1$).

Thanks to the improved quality and size of the Union2 data set, the reconstruction has smaller errors bars than that of Mignone & Bartelmann (2008), who made use of the SNLS data only. The ΛCDM model is in good agreement with the supernova data, although its slope is slightly different. This leads to a deviation at intermediate values of a, but, within the error bars, the ΛCDM model is still consistent with our reconstruction.

4.2 Dark Energy models

Relaxing the assumption of constant equation of state, we analyze a Dark Energy model proposed by Rapetti, Allen & Weller (2005). It is an extension of the parametrization in Eq. (1) proposed by Chevallier & Polarski (2001) and Linder (2003), which assumes a fixed transition redshift ($z_\mathrm{t} = 1$) between the current value of the equation of state and the value at early times, $w_\mathrm{et} = w_0 + w_1$. In contrast, the model discussed by Rapetti et al. (2005) introduces z_t as an extra free parameter so that the equation of state w can be written as

Table 1. The first five coefficients of the reconstructed expansion.

Coefficients	Fitted Values
c_0	2772 ± 14
c_1	-1389 ± 15
c_2	73 ± 11
c_3	12± 5
c_4	-3 ± 1

$$w(a) = \frac{w_{\rm et} z + w_0 z_{\rm t}}{z + z_{\rm t}} = \frac{w_{\rm et}(1-a)a_{\rm t} + w_0(1-a_{\rm t})a}{a(1-2a_{\rm t}) + a_{\rm t}}, \tag{19}$$

where $a_{\rm t}$ is the transition scale factor. From the Friedmann equation, the expansion function in terms of redshift is then given by

$$H(z) = H_0\sqrt{\Omega_{\rm m}(1+z)^3 + \Omega_{\rm de} f(z) + \Omega_k(1+z)^2}, \tag{20}$$

with

$$f(z) = (1+z)^{3(1+w_{\rm et})} {\rm e}^{-3(w_{\rm et}-w_0)g(z;z_{\rm t})}; \tag{21}$$

Here, the function $g(z; z_{\rm t})$ is defined as

$$g(z; z_{\rm t}) = \frac{z_{\rm t}}{z_{\rm t}-1} \ln\left(\frac{z_{\rm t}}{z_{\rm t}+z}\right). \tag{22}$$

Rapetti et al. (2005) constrained the best-fit cosmological parameters for different possibilities within this model, varying the number of free parameters. They made use of SN Ia (the Gold sample by Riess et al. 2004), X-ray galaxy clusters and CMB data for this analysis. In Fig. 3, we compare those possibilities to the model-independent reconstruction of the expansion function extracted from the Union2 sample. This is meant to illustrate that standard cosmologies within the wCDM paradigm with different choices of parameters do not fit the data in all cases. This is not easy to see in other analyses which are model-dependent. Our reconstruction of $H(z)$ can be a useful tool since it allows us to rule out cosmological models based entirely on the data. For example, the two most extreme models shown in Fig. 3 – one with a fixed $z_{\rm t} = 0.35$ (green dashed-dotted line) which splits the SN and the cluster data sets into similarly low and high redshift subsamples, and the one with arbitrary $z_{\rm t}$ (blue dashed line) – are clearly inconsistent with our reconstruction. The other models (marginally) agree with the SN measurements within the error bars. However, they have slightly different slopes and – as for the ΛCDM model – some tension exists at intermediate and small values of a (intermediate and high redshifts).

5 GOING BEYOND ΛCDM

Despite the good agreement with cosmological observations, the ΛCDM model is conceptually problematic, as it leaves the fine tuning and coincidence problems unresolved. A number of alternatives to the ΛCDM parameterization have been suggested in order to address those difficulties. We study some of these alternatives here and compare them to our reconstruction.

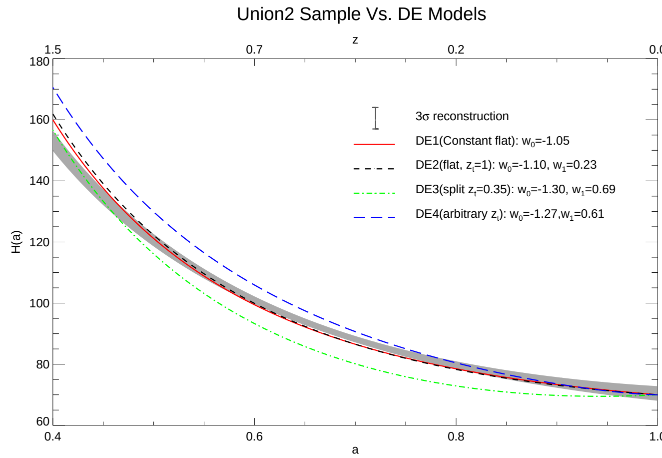

Figure 3. The reconstructed expansion history, with 3σ errors, extracted from Union2 sample and confronted with several Dark Energy models with the best fit cosmological parameters constrained by Rapetti et al. (2005).

5.1 Modified Gravity

The idea of modified gravity differs from dark energy models. It does not describe the late-time acceleration as being caused by some unknown energy component but suggests that General Relativity may be inaccurate at large scales. Such scenarios are likely to diverge in their predictions for $H(z)$ from ΛCDM and therefore they can be constrained from the reconstructed expansion history of the Universe.

5.1.1 The $f(R)$ models

One way of modifying gravity theory is by adding terms to the Ricci scalar (R) in the Einstein-Hilbert Lagrangian (Starobinsky 1980; Carroll et al. 2004). The Palatini formalism provides second order differential field equations that can account for the present cosmic acceleration without any need of Dark Energy (Fay et al. 2007). Several parameterisations of $f(R)$ are possible. For the sake of simplicity, we follow the parametrization of Carvalho et al. (2008) here (for details on other parameterisations see, e.g., Hu & Sawicki 2007; Fay et al. 2007). The general functional $f(R)$ is assumed to have the form

$$f(R) = R - \frac{\beta}{R^n}, \tag{23}$$

where R is the Ricci scalar; n, β, and $\Omega_{\rm m}$, are the parameters of the model. ΛCDM is recovered for $n = 0$ The expansion rate can be written as

$$H^2(z) = H_0^2\left[\frac{3\Omega_{\rm m0}(1+z)^3 + f/H_0^2}{6f'\xi^2}\right], \tag{24}$$

with

$$\xi = 1 + \frac{9}{2}\frac{f''}{f'}\frac{H_0^2\Omega_{\rm m0}(1+z)^3}{Rf''-f'}, \tag{25}$$

where the notation $f' = df/dR$, $f'' = d^2f/dR^2$ has been adopted. By evaluating these equations at $z = 0$ it is possible to obtain a relationship between n, β, and $\Omega_{\rm m}$. Thus, if we specify the values of two of them the other is immediately fixed. In our analysis we treat n and Ω_m as free parameters.

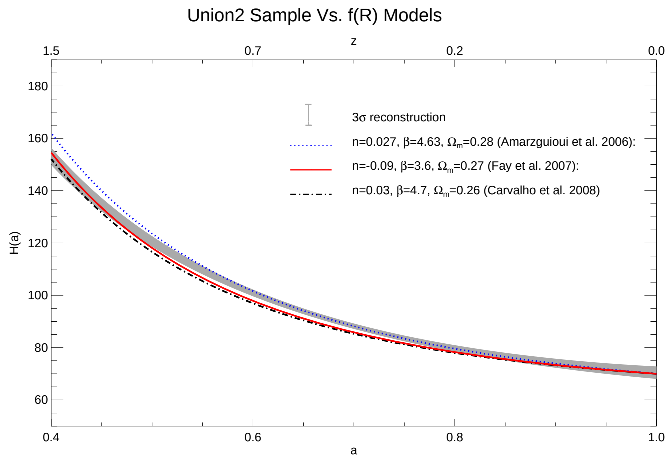

Figure 4. The reconstructed expansion history, with 3σ errors, extracted from Union2 sample compared with several $f(R)$ models. The values for the models are taken from previously reported fits to BAO and CMB data in combination with independent $H(z)$ (black dashed-dotted line) and luminosity distance measurements (red solid – SNLS sample – and blue dotted – Gold sample – lines).

We compare the model-independent expansion history derived from the Union2 sample with different $f(R)$ models in Fig. 4. The adopted values for the model parameters are taken from previously reported fits in the literature. Carvalho et al. (2008) obtained constraints on the $(n, \Omega_{\rm m})$ parameter space from the combination of BAO and CMB data with independent determinations of the Hubble parameter at different redshifts. This model (black dashed-dotted line) has a substantially different slope and diverges from our reconstruction,

In similar studies, other authors have combined SNe data with BAO and CMB. We consider here the numbers obtained by Fay et al. (2007) and Amarzguioui et al. (2006) using the SNLS and Gold samples, respectively. We find these models (red solid and blue dotted lines) closer to our reconstruction although they present deviations at high redshift (for the Gold sample) and at intermediate redshifts (for the SNLS sample).

5.1.2 Brane worlds

Another possibility to modify gravity is to consider higher dimensional models. Braneworld cosmology describes the observable Universe as a three-dimensional brane embedded in higher dimensional spacetime (or "bulk"). All matter particles and fields are trapped on the brane (i.e. conservation of mass-energy holds firm) while gravity is free to propagate in the brane as well as in the bulk.

There are different scenarios within brane-world cosmology (see, e.g., Maartens 2004; Alam & Sahni 2006). We consider here, for example, the Dvali-Gabadadze-Porrati (DGP) model (Dvali, Gabadadze & Porrati 2000). This model allows the extra dimension to be large, and its generalisation to a Friedmann-Robertson-Walker brane produces a self-accelerating solution (Deffayet 2001). The resulting Friedman equation is a modification of the General Relativistic case and reads

$$H^2 - \frac{H}{r_{\rm c}} = \frac{8\pi G\rho}{3}, \tag{26}$$

where the cross-over scale $r_{\rm c}$ is defined as

$$r_{\rm c} = \frac{1}{H_0(1-\Omega_{\rm m})}. \tag{27}$$

The additional term $H/r_{\rm c}$ in Eq. (26) behaves like a Dark Energy component with an effective equation of state that evolves from $w = -1/2$ for $z \gg 1$ to $w = -1$ in the distant future.

For a wCDM model (assuming that Eq. (1) holds) it is possible to obtain an expression for $w(a)$ which mimics the evolution of the DGP model (see Tang, Weller & Zablocki 2006):

$$w(a) = -1 + \frac{\Omega_{\rm m} a^{-3}}{\left[(r_{\rm c}H_0)^{-1} + 2\eta\right]\eta}, \tag{28}$$

with $\eta = \sqrt{\Omega_{\rm m} a^{-3} + 1/2\,(r_{\rm c}H_0)^3}$. At present-day, this tends to

$$w(a=1) = -\frac{1}{1+\Omega_{\rm m}}. \tag{29}$$

This modification allows for a description of the expansion history as well as of the growth of large-scale structure.

One can even go beyond DGP as an isolated theory and consider a phenomenological model which is motivated by the concept of an extra dimension with infinite extent. This is the so called mDGP model (Dvali & Turner 2003; Thomas, Abdalla & Weller 2009). It interpolates between ΛCDM and the DGP model and allows for the presence of an extra dimension through corrections to the Friedman equation by introducing a parameter α. The modified Friedman equation reads

$$H^2 - \frac{H^\alpha}{r_{\rm c}^{2-\alpha}} = \frac{8\pi G\rho}{3}, \tag{30}$$

with the cross-over scale $r_{\rm c}$ defined as

$$r_{\rm c} = (1-\Omega_{\rm m})^{\frac{1}{\alpha-2}} H_0^{-1}. \tag{31}$$

In Fig. 5, the comparison of our model-independent reconstruction with some DGP and mDGP models is presented. The values of the model parameters are taken from previously reported fits to actual cosmological data.

We evaluate two pure DGP models ($\alpha = 1$) adopting the best-fit values for $\Omega_{\rm m}$ reported in Guo et al. (2006) and Liang & Zhu (2011) (green dash-dotted and black dashed lines, respectively). In the first case, the constraints were obtained from the Gold and SNLS samples in combination with BAO. In the second, the best fit values are found by combining cosmology-independent Gamma Ray Burst and SNe Ia data, with BAO, CMB and $H(z)$ measurements. The latter constitutes so far the strongest constraint obtained for the DGP model.

We also consider the results obtained by Thomas et al. (2009) adding weak-lensing data to BAO and SNe (red dotted line). In that particular study, the authors find an upper limit for the α parameter ($\alpha < 0.58$ at 68 per cent confidence level), but are not able to give constraints on the $\Omega_{\rm m}$ parameter. Therefore, and to better understand the effect of changing $\Omega_{\rm m}$ for a given α, we make here use of the best-fit

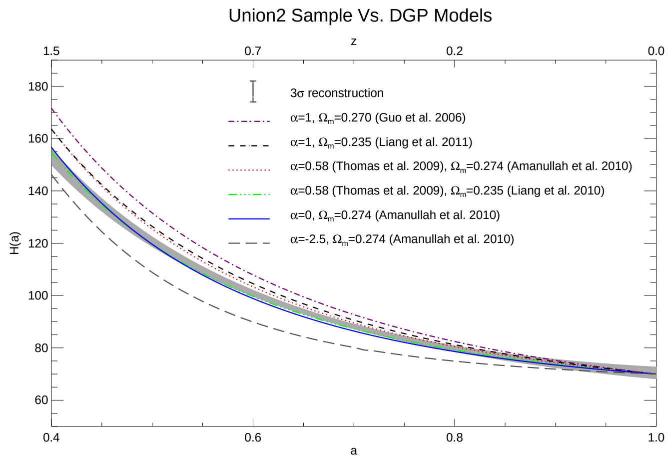

Figure 5. The reconstructed expansion history, with 3σ errors, extracted from Union2 confronted with several DGP/mDGP models. The parameters of the model are taken from previous reported fits to several cosmological data.

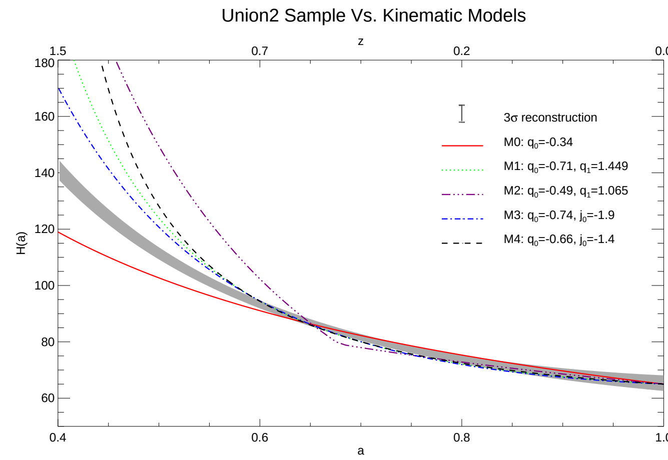

Figure 6. The reconstructed expansion history, with 3σ errors, extracted from the Union2 set compared with several kinematic models with best-fit parameters obtained by Guimarães et al. (2009).

values reported in Liang & Zhu (2011) for the DGP model and in Amanullah et al. (2010) for the ΛCDM model.

For the sake of comparison, a ΛCDM model corresponding to $\alpha = 0$ (blue solid line) and a model with negative α (grey long-dashed line) are also included in Fig. 5.

As it has been found in prior studies, (see, for example, Maartens & Majerotto 2006; Fairbairn & Goobar 2006), a pure DGP cosmology is disfavored by the SNe data. Models with negative values of α also disagree with our reconstruction. However, it is still early to break the degeneracy betweeten ΛCDM and mDGP models with $0 \lesssim \alpha \lesssim 0.50$.

5.2 Kinematic approach

A different way of describing the cosmic expansion history is by means of the so-called kinematic models that do not use quantities from the dynamic description, such as $\Omega_{\rm m}$ or w . Here we discuss the models of Elgarøy & Multamäki (2006) and Guimarães, Cunha & Lima (2009), that are based on different parameterisations of the deceleration parameter, q, and the jerk parameter, j. The deceleration parameter in terms of z is defined as

$$q \equiv -\frac{1}{H^2}\frac{\ddot{a}}{a} = \frac{1}{2}(1+z)\frac{(H(z)^2)'}{H(z)^2} - 1, \quad (32)$$

where the primed quantity denotes the derivative with respect to z. Similarly, the jerk parameter is given by

$$\begin{aligned} j &\equiv -\frac{1}{H^3}\frac{1}{a}\frac{d^3a}{da^3} \\ &= -\left[\frac{1}{2}(1+z)^2\frac{(H^2)''}{H^2} - (1+z)\frac{(H^2)'}{H^2} + 1\right]. \quad (33) \end{aligned}$$

We consider five realizations here: in the first and simplest model, M_0, the deceleration parameter is constant, $q(z) = q_0$. The second model, M_1, is a linear expansion of the deceleration parameter $q(z) = q_0 + q_1 z$. Model M_2 has two phases of constant deceleration, separated by an abrupt transition redshift, i.e. $q(z) = q_0$ for $z \leqslant z_{\rm t}$ and $q(z) = q_1$ for $z > z_{\rm t}$. The fourth model, M_3 is a constant-jerk parametrization, $j(z) = j_0$. The last model, M_4, derives from an expansion of the luminosity distance with free q_0 and j_0 parameters resulting in

$$\begin{aligned} D_L(z) &= \frac{c}{H_0}\left[z + \frac{1}{2}(1-q_0)z^2 \right. \\ &\left. -\frac{1}{6}(1 - q_0 - 3q_0^2 - j_0)z^3\right]. \quad (34) \end{aligned}$$

All these models make specific predictions for the cosmic expansion (see Guimarães et al. 2009, for the corresponding expressions of $H(z)$). They are of particular interest because no assumption on the matter-energy content present in the Universe is made. Guimarães et al. (2009) constrained the different parameters in their models through a Bayesian statistical analysis from the previous Union compilation (Kowalski et al. 2008).

The comparison between the expansion history obtained from Union2 data and the theoretical expansion function calculated for each model is shown in Fig. 6. For the different parameters we adopt the best-fit values obtained by Guimarães et al. (2009). Again, our goal here is to illustrate how the method performs and how families of models, with completely different parameters, can be confronted with the data in order to assess their validity. We note that, for example, model M_2 could be rejected based on current SN Ia data because its shape differs greatly from the reconstructed expansion history. Also M_0 has a different slope which does not even fit the data at low redshifts. Models M_1, M_3 and M_4 agree with the SN data within the error bars up to values of the scale factor $a \sim 0.6$ ($z \sim 0.66$). This is consistent with the transition redshifts $z_{\rm t}$ given in Guimarães et al. (2009). The good agreement may reflect the fact that the models were constructed in a way that mimics ΛCDM at low redshifts. At high redshifts $z > z_{\rm t}$, however, none of the models is consistent with our reconstructed expansion history.

6 IMPROVING THE RECONSTRUCTION WITH MOCK SN IA DATA

More data at various redshifts will be obtained in future SN Ia surveys. In order to test for the effects new data may have on the reconstructed expansion function $H(z)$, we create a mock data sample of nearby SNe Ia with fixed H_0. We generate a sample of 200 mock SNe Ia in a ΛCDM model with $\Omega_{\rm m} = 0.274$, $\Omega_{\rm de} = 0.726$ and $H_0 = 70$ km/s, with an uniform distribution in scale factor from 0.86 to 1. The reason for adding 200 SNe is to approximately equal the number of nearby and distant SNe contained in Union2. We calculate the luminosity distances consistent with equation Eq. (9) and generate random errors with uncertainties 30% smaller than those of Hicken et al. (2009). The random number is drawn from a normal distribution with mean zero and a standard deviation of one.

The result of the model-independent reconstruction based on the Union2 sample including the mock data is shown in Fig. 7. The improvement in the reconstruction is apparent from the significant decrease in the error bars. The reduction of the error at low redshifts was forced by constructing the mock data assuming a fixed H_0. This highlights the importance of a well-measured value of H_0 for cosmological studies. Although SNe Ia are the best *relative* distance indicators out to redshifts $z \gtrsim 1$, they do not provide reliable *absolute* distances. Therefore, the zero point in our $H(z)$ reconstruction is not well-determined. Fixing H_0 from other measurements as a prior in our analysis would be beneficial.

A ΛCDM model resulting from $\Omega_{\rm m} = 0.274^{+0.040}_{-0.037}$ (Amanullah et al. 2010) is shown in Fig. 7 for comparison. This model is consistent with our reconstruction within the error interval. However, a small difference in slope is observed, similar to that for the original Union2 sample (cf. Fig. 2). The slight deviation at high redshifts is probably due to the additional weight put on the mock data which have smaller errors.

This analysis demonstrates the power of our method in testing redshift ranges in which more (and more accurate) SN data would help to significantly improve the constraints on cosmological models. Such considerations may help with the design of future surveys.

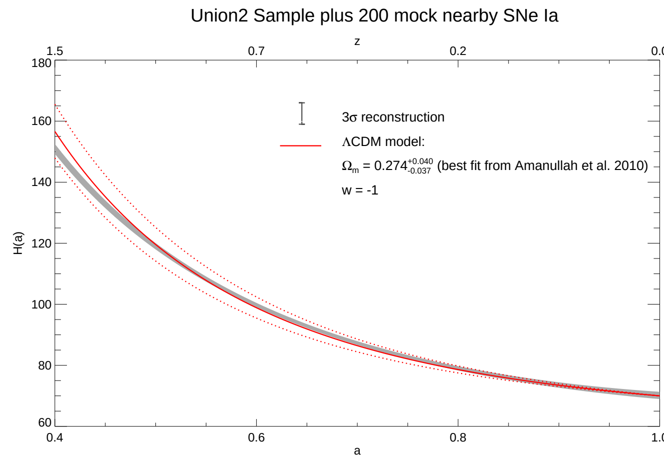

Figure 7. The reconstructed expansion rate with 3σ errors extracted from the Union2 set, plus 200 nearby mock SNe with errors 30% smaller than those of Hicken et al. (2009). Our reconstruction is compared with a ΛCDM model assuming the best-fit cosmological parameters reported by Amanullah et al. (2010).

7 TESTING FOR SYSTEMATICS IN SN IA DATA COMPILATIONS

The method of model-independent reconstruction of the expansion history of the Universe can in principle be used as a tool to study systematics in SN Ia data. Given, for instance, two samples from different surveys, a question to ask is whether the respective cosmological parameters derived from both of them predict an expansion history that is consistent with the direct data analysis.

We demonstrate this on the example of the Union2 data set which is a collection of data from various SN Ia surveys. As noted in Sect. 4.1, there is some tension between the reconstructed expansion history and the predictions of a ΛCDM cosmology at intermediate redshifts, although within the error bars they are consistent. One possibility is that this tension could be due to different characteristics and systematic errors of the sub-samples. In the most extreme case, one sub-sample could be responsible for introducing the divergence from a ΛCDM model. This can be identified by reconstructing the expansion history for the sub-samples individually. The two main components of the Union2 data set are the ESSENCE and the SNLS samples, as they both cover a wide range in redshifts with a relatively large number of objects.

The expansion histories of both samples are compared with ΛCDM models based on the individual best-fit parameters in Fig. 8. Generally, the error bars for the ESSENCE data are wider because they concentrate on a redshift range of about $0.6 < a < 0.8$ ($0.25 < z < 0.66$), whereas the SNLS data extend up to $a \sim 0.5$ ($z \sim 1$). But, apart from that, both samples show consistency of the reconstructed expansion with a ΛCDM model. The tension at intermediate redshifts is present in both of them. Therefore, the deviation of the slope of the reconstructed $H(z)$ from the ΛCDM prediction seen in the Union2 data is not introduced by a mismatch of the main sub-samples or systematic errors in one of them.

8 CONCLUSIONS

In this work, we provided further constrains on the expansion history of the Universe in a model-independent way. The luminosity-distance measurements, obtained from SNe Ia, depend only on space-time geometry, and can be directly related to the Hubble function without need of assuming any dynamical model. To illustrate the power of this approach to investigate Dark Energy models, the method was used to reconstruct the expansion history from the Union2 sample (Amanullah et al. 2010) – the most complete compilation of SN Ia distances to date. Furthermore, a comparison with ΛCDM models with different values of the matter density, $\Omega_{\rm m}$, and the dark energy density, $\Omega_{\rm de}$, was presented. For the Union2 sample, we found good agreement between a ΛCDM model with the best fit parameters obtained specif-

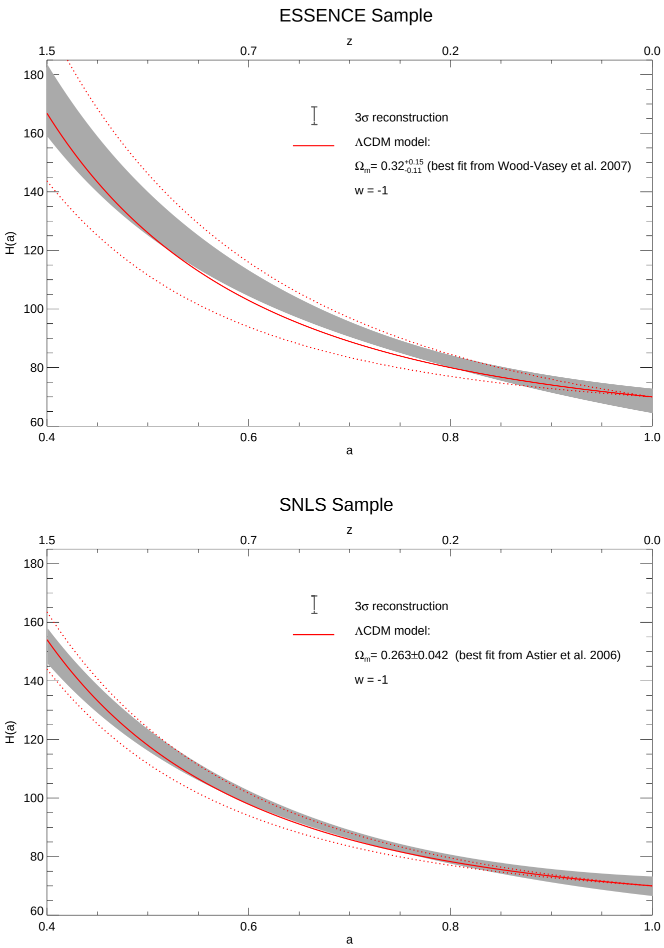

Figure 8. The reconstructed expansion history with 3σ errors for the ESSENCE (upper panel) and SNLS (lower panel) samples compared to ΛCDM models based on the individual best-fit parameters. The value of $\Omega_{\rm m} = 0.32^{+0.15}_{-0.11}$ was obtained by (Wood-Vasey et al. 2007) combining the ESSENCE, SNLS and the gold samples (Riess et al. 2004). In order to get a reliable reconstruction 57 nearby SN Ia from literature have been added to both samples.

ically from this sample and the data. However, the slope of the curve is not exactly the same and a systematic deviation seems to appear at intermediate values of the scale factor.

General relativity can accommodate the detected acceleration of the Universe but cannot give a deeper understanding about its cause. Non-standard cosmological models have been suggested as alternative explanations for acceleration without dark energy. We made use of the model-independent method to quantitatively compare the ability of some dark energy models and non-standard cosmologies to represent the data. Comparing the reconstruction recovered from the Union2 sample data with some braneworld, $f(R)$, and kinematic models we found that none of the considered cosmologies can be rejected rigorously on the basis of current SN Ia data. Particular braneworld models, such as the mDGP/DGP with some parameters, however, as well as some $f(R)$ and some kinematic models clearly disagree with our reconstruction and can be ruled out. Using an optimal set of functions based on principal component analysis instead of an arbitrary basis will strengthen this statement.

We argue that the model-independent method is also a promising tool in checking for biases and discrepancies in the SN Ia cosmology samples. This was tested by performing the reconstructions for sub-samples of the Union2 set – specifically the ESSENCE and the SNLS data – separately and comparing them with the corresponding best-fit ΛCDM models. We find a general agreement between the different data and the models, although some tension at intermediate redshifts seems to be present in both.

Although the method as employed here is indicative of viability of dark energy models, improvements are necessary to reach full decisive power. Apart from constructing an optimal basis for the series expansion of the luminosity distance, the model-dependence induced by the calibration of the SNe Ia as distance indicators has to be taken into account.

An increasing body of high quality data will help to reduce systematic uncertainties and will provide a better reconstruction of the expansion rate. To demonstrate this, we studied the effect that more nearby SNe Ia will have on the reconstruction of $H(a)$. We generated a sample of 200 SNe at very low redshifts, uniformly distributed in the range $0.86 < a < 1$ and with 30% smaller errors than those of Hicken et al. (2009). These mock data were added to the Union2 sample and we found a great decrease in the size of the error bars. This confirms the fact that not only an increased number of distant SNe, but also higher quality data and larger number of nearby SNe are indeed needed to reconstruct the expansion history more accurately.

The strength of the method employed here to derive the expansion history of the Universe is that it provides a purely geometrical test. Contrary to many other ways of analyzing cosmological data it does not revert to assumptions on the energy contents of the Universe nor its dynamics. It thus offers a complementary way of detecting possible systematic effects which could affect the data and be overlooked within a traditional analysis based on physically motivated parametrization. Moreover, it can be used to discriminate between different cosmological models and break the degeneracy in the cosmological parameters. In future work, we will consider not only SN Ia data but also other cosmological probes, such as the angular distances from BAO, in order to obtain tighter constrains on the expansion history of the Universe. Finally, is it worthwhile noting the potential of the method for the analysis of possible local inhomogeneities through comparison of the expansion history in different directions.

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