

# Constraints on star formation driven galaxy winds from the mass-metallicity relation at $z = 0$

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## ABSTRACT

We extend a chemical evolution model relating galaxy stellar mass and gas-phase oxygen abundance (the mass-metallicity relation) to explicitly consider the mass-dependence of galaxy gas fractions and outflows. Using empirically derived scalings of galaxy mass with halo virial velocity in conjunction with the most recent observations of  $z \sim 0$  total galaxy cold gas fractions and the mass-metallicity relation, we place stringent global constraints on the magnitude and scaling of the efficiency with which star forming galaxies expel metals. We demonstrate that under the assumptions that metal accretion is negligible and the stellar initial mass function does not vary, efficient outflows are required to reproduce the mass-metallicity relation; without winds, gas-to-stellar mass ratios  $\gtrsim 0.3$  dex higher than observed are needed. Moreover,  $z = 0$  gas fractions are low enough that while they have some effect on the magnitude of outflows required, the slope of the gas fraction–stellar mass relation does not strongly affect our conclusions on how the wind efficiencies must scale with galaxy mass. Because theoretical descriptions of the mass loading factor  $\eta_w \equiv \dot{M}_w / \dot{M}_{\text{SFR}}$ , where  $\dot{M}_w$  is the mass outflow rate and  $\dot{M}_{\text{SFR}}$  is the star formation rate, are often cast in terms of the depth of the galaxy potential well, which is in turn linked to the host halo virial velocity  $v_{\text{vir}}$ , we use one of the latest abundance matching analyses to describe outflow efficiencies in terms of  $v_{\text{vir}}$  rather than stellar mass. Despite systematic uncertainties in the normalization and slope of the mass-metallicity relation, we show that the metal expulsion efficiency  $\zeta_w \equiv (Z_w/Z_g)\eta_w$  (where  $Z_w$  is the wind metallicity and  $Z_g$  is the interstellar medium metallicity) must be both high and scale steeply with mass. Specifically, we show that  $\zeta_w \gg 1$  and  $\zeta_w \propto v_{\text{vir}}^{-3}$  or steeper. In contrast, momentum- or energy-driven outflow models suggest that  $\eta_w$  should scale as  $v_{\text{vir}}^{-1}$  or  $v_{\text{vir}}^{-2}$ , respectively, implying that the  $Z_w$ - $M_*$  relation should be shallower than the  $Z_g$ - $M_*$  relation.

**Key words:** ISM: abundances — ISM: jets and outflows — galaxies: abundances — galaxies: evolution — galaxies: fundamental parameters — galaxies: ISM

## 1 INTRODUCTION

Star-forming galaxies follow a tight ( $\sim 0.1$  dex scatter) correlation between their gas phase oxygen abundance (hereafter referred to as “metallicity”) and stellar mass (Tremonti et al. 2004). This mass-metallicity relation is primarily understood to be a sequence of oxygen *suppression*, rather than enrichment (Tremonti et al. 2004; Dalcanton 2007; Erb 2008; Finlator & Davé 2008). The production of oxygen traces the production of stars, implying that the observed trend in the oxygen-to-gas ratio reflects either a trend in the galaxy gas-to-stellar mass ratio or in processes that affect gas-phase metals but not stars. A consensus is emerging that although galaxy

gas fractions can and do affect the mass-metallicity relation, if the stellar initial mass function (IMF) is the same in all galaxies, then outflows that are more efficient at removing metals from low-mass galaxies are required in order to reproduce the observations (e.g., Dalcanton 2007; Finlator & Davé 2008; Spitoni et al. 2010). However, the global properties of these outflows and the physics underlying how star formation drives them are not well understood—and winds are expensive and difficult to observe directly. In this paper, we incorporate the most recent observations of galaxy gas fractions and the mass-metallicity relation at  $z \sim 0$  into a simple chemical evolution model to explore what constraints can be placed on how the efficiencies and composition of star formation driven galactic winds scale with galaxy stellar mass and halo virial velocity.

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Several analytic studies have concluded that star formation

driven outflows are crucial to reproducing the observed mass-metallicity relation. Erb (2008) used a simple analytic chemical evolution model to argue that the star formation rate,  $\dot{M}_{\text{SFR}}$ , and the outflow rate,  $\dot{M}_{\text{w}}$ , should be roughly equal. While  $\dot{M}_{\text{w}}$  and the gas accretion rate  $\dot{M}_{\text{acc}}$  vary with the star formation rate (and thus gas fraction),  $\eta_{\text{w}} \equiv \dot{M}_{\text{w}}/\dot{M}_{\text{SFR}}$  and  $\eta_{\text{a}} \equiv \dot{M}_{\text{acc}}/\dot{M}_{\text{SFR}}$  are constant universal parameters, a common practice in analytic models of galaxy chemical evolution (see also Samui et al. 2008, and references therein). Though models specifically aimed at duplicating observations of the mass-metallicity relation commonly assume  $Z_{\text{w}} = Z_{\text{g}}$ , Dalcanton (2007) argues that metal-enriched outflows (those comprised predominantly of Type II supernova ejecta, and thus with  $Z_{\text{w}} > Z_{\text{g}}$ ) are required if the rate of gas accretion is to be reasonable. More recently, Spitoni et al. (2010) have argued that the  $z = 0$  mass-metallicity relation together with gas fractions derived by inverting the Kennicutt-Schmidt (K-S, Kennicutt 1998; Schmidt 1959) law imply that not only are outflows required, but that they must be more efficient at removing metals from low-mass galaxies than from more massive ones. Finlator & Davé (2008) drew a similar conclusion by analyzing a suite of cosmological smoothed particle hydrodynamics (SPH) simulations evolved with GADGET-2 (Springel 2005) in conjunction with detailed analytic models. They showed that, in general,  $Z_{\text{g}} \propto \eta_{\text{w}}^{-1}$  for  $\eta_{\text{w}} \gg 1$ . Their favored model that reproduces the Erb et al.  $z \sim 2.2$  mass-metallicity relation is one in which  $\eta_{\text{w}} \propto \sigma^{-1}$ , where  $\sigma$  is the galaxy velocity dispersion.<sup>1</sup> In this simulation,  $\sigma^{-1} \propto M_{\text{halo}}^{-1/3} \propto M_{\star}^{-1/3}$ , which naturally explains why this  $\eta_{\text{w}}$  scaling is able to reproduce a mass-metallicity relation with  $Z_{\text{g}} \propto M_{\star}^{0.3}$ . These simple scaling relations highlight a link between a galaxy’s stellar mass, its halo mass, and its potential well: wind models aimed at successfully reproducing the mass-metallicity relation also need to correctly reproduce (or incorporate) the  $M_{\star}$ - $M_{\text{halo}}$  relation. Moreover, this analysis shows that the mass-metallicity relation is a potentially powerful tool for constraining how star formation driven outflows scale with galaxy and halo properties; this is particularly interesting as such scalings are currently not well constrained through either direct observations or theoretical considerations.

On the other hand, several models focus instead on the efficiency of star formation as a function of stellar mass. In such models, an increase in the star formation efficiency with galaxy mass—without the need for outflows—is sufficient to reproduce the observed mass-metallicity relation (Calura et al. 2009). Brooks et al. (2007) used a set of SPH simulations evolved with Gasoline (Wadsley et al. 2004)—and therefore a different recipe for star-formation feedback<sup>2</sup> than Finlator & Davé (2008)—to argue that preferentially expelling gas from the low-mass galaxies is insufficient for reproducing the observed mass-metallicity relation. These authors claim that it is instead the reduced star-formation efficiency (and thus differences in galaxy gas fractions) induced by such feedback that is primarily responsible for driving the relation’s mor-

phology. In the context of the mass-metallicity relation, variations in the star formation efficiency affect galaxy gas fractions (as well as the  $M_{\star}$ - $M_{\text{halo}}$  relation). We do not directly address star formation efficiency here because we take both galaxy gas fractions and the  $M_{\star}$ - $M_{\text{halo}}$  relation as givens rather than something to be constrained by the model; we discuss in Appendix B the implications our choices for these relations have on how star formation efficiency varies with galaxy mass.

Finally, letting the IMF (and thus the amount of oxygen produced per unit stellar mass) vary with galaxy mass provides a straightforward way to reproduce the mass-metallicity relation (Tinsley 1974; Köppen et al. 2007; Recchi et al. 2009; Calura & Menci 2009; Spitoni et al. 2010). We primarily assume here that the IMF is the same in all galaxies, with a brief discussion in §4.2 of how uncertainties in yields in the presence of a variable IMF affect our results. In general, if the IMF is top-light in low-mass galaxies then this will imply that outflow efficiencies do not need to scale as steeply with mass as suggested by the non-varying IMF case.

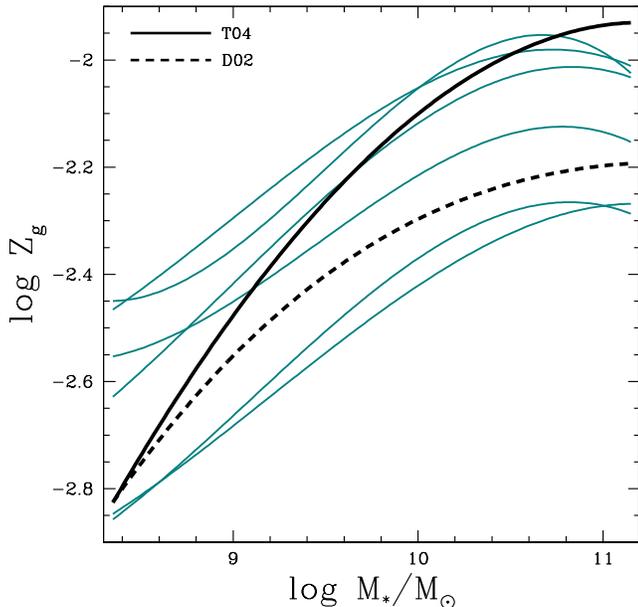
We apply here a simple model with which to understand the mass-metallicity relation to the mass-metallicity relation at  $z \sim 0$ , where external constraints such as gas fractions and the stellar mass function are best measured. We do not assume a particular form for the mass-metallicity relation; we instead base our conclusions on the range of parameter space allowed by the range of systematic uncertainties in interpreting strong nebular emission lines as oxygen abundances (§2.1). Our main simplifying assumptions are that the metallicity of gas accreted from the intergalactic medium is negligible and that the nucleosynthetic yield is constant with galaxy mass. With these constraints and assumptions, the only free parameters are those describing outflows, which we are able to describe as a function of halo virial velocity.

This paper is organized as follows. In §2, we discuss the relevant observations. The slope and normalization of the mass-metallicity relation strongly affect the interpreted properties of galaxy winds. Unfortunately, while there exist exquisite data on emission line ratios of star forming galaxies at  $z = 0$ , the correct way to interpret these line ratios in terms of oxygen abundances is not agreed upon; we therefore consider several measurements of the  $z = 0$  mass-metallicity relation, as outlined in §2.1. It is commonly assumed in chemical evolution models, and we assume here, that the gas is well-mixed; we address the differences between galaxies’ cold gas reservoirs and the gas traced by star formation in §2.2 (see also Appendix A). As the purpose of this paper is to place constraints on how galaxy outflows scale with galaxy mass, we briefly outline observed properties of galaxy outflows (and theoretical models thereof) in §2.3. We lay out the formalism in §3.1 and its derivation in Appendix C, along with how we connect galaxy stellar masses to host halo properties (§3.2). In §4, we show how gas dilution and outflows must combine in order to yield the observed mass-metallicity relation, and what this implies about galaxy outflows in order for predicted gas fractions to be consistent with the data; further details are presented in Appendix D. How these conclusions are affected by uncertainties in the yield is addressed in §4.2. We then present in §5 what constraints wind metallicity and entrainment fraction considerations place on viable outflow models, with a summary and further discussion in §6. Appendix B describes the connection between gas masses, accretion, and star formation rates in our approach, with implications for star formation efficiency.

Throughout we adopt a cosmology of  $(\Omega_m, \Omega_b, \sigma_8, h) = (0.26, 0.047, 0.77, 0.72)$  and a Chabrier (2003a) initial mass func-

<sup>1</sup> This parameterization is motivated by the observations of Martin (2005) and the theory of momentum-driven winds (Murray et al. 2005); see §2.3 for more details.

<sup>2</sup> Because of the resolution of cosmological SPH simulations, star-formation feedback must be included using “recipes” instead of directly modelling the underlying physics. The winds in Finlator & Davé’s simulations are implemented by physically moving gas particles away from star-forming regions. In Brooks et al.’s simulations, star formation thermally heats neighboring particles. In both prescriptions, the relevant particles are not allowed to interact hydrodynamically (Finlator & Davé) or radiatively cool (Brooks et al.) for some physically-motivated amount of time.



**Figure 1.** Mass-metallicity relations listed in Table 1 (Kewley & Ellison 2008, and equation 1). The scatter about any given one of these curves is 0.1–0.15 dex, which is much less than the differences in normalization; that is, the normalization differences are systematic. The mass-metallicity relations in black (T04, solid; D02, dashed) are modeled in detail §§ 4, 5, and Appendix D.

tion (IMF), unless otherwise noted. Varying the cosmological parameters within the ranges allowed from observations (e.g., Hinshaw et al. 2009) does not alter our conclusions. The impact of varying  $\Omega_m$  or  $\Omega_b$ , has, for example, little effect on the shape of the  $M_* - M_{\text{halo}}$  relation or on the determination of the stellar masses in SDSS. Though varying  $\sigma_8$  does change the number density of massive halos, it has little impact on the range of halo masses of interest here. Finally, we note that the virial relations only have a mild change in normalization when varying cosmological parameters, without having much impact on our overall results.

## 2 RELEVANT OBSERVATIONS

### 2.1 The observed $z \sim 0$ mass-metallicity relation

Since oxygen is effectively produced only in Type II SNe—the deaths of massive, short-lived stars—and H II regions are associated with ongoing star formation, the gas-phase “mass-metallicity relation” typically refers to only the galaxy’s oxygen abundance in gas that is currently forming stars; we therefore will use “metals” and “oxygen” interchangeably unless otherwise noted. However, though  $12 + \log(\text{O}/\text{H})$  is measured at the sites of star formation, the measured abundances are the *birth* abundances of the H II regions; supernovae (the sites of oxygen production) destroy their nascent clouds, rendering so-called “self enrichment” of H II regions extremely rare. We therefore assume that the galaxy gas is well-mixed, i.e., that the mixing time is short relative to the timescale for star formation.

Observationally, oxygen abundance increases with galaxy stellar mass. This relation has very little scatter ( $\sim 0.1$  dex in  $12 + \log[\text{O}/\text{H}]$  at fixed stellar mass), though severe outliers do exist (Peeples et al. 2008, 2009). The amplitude and slope of the

ID	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
<b>T04</b>	<b>-0.759210</b>	<b>1.30177</b>	<b>0.003261</b>	<b>-0.00364112</b>
Z94	73.0539	-20.9053	2.23299	-0.0783089
KK04	28.1404	-7.02595	0.812620	-0.0301508
KD02	28.4613	-7.32158	0.855119	-0.0318315
M91	46.1480	-12.3801	1.33589	-0.0471074
<b>D02</b>	<b>-8.91951</b>	<b>4.18231</b>	<b>-0.323383</b>	<b>0.00818179</b>
PP04O3N2	32.5769	-8.61049	0.981780	-0.0359763
PP04N2	24.1879	-5.69253	0.648668	-0.0235065

**Table 1.** Kewley & Ellison (2008) fits to the mass-metallicity relation, where  $\log Z_g = a + b \log M_* + c(\log M_*)^2 + d(\log M_*)^3$ , sorted by decreasing  $\max(Z_g)$ . The two fits we consider in the main text (T04 and D02) are in bold. See text for abbreviations.

mass-metallicity relation, however, are not well constrained, despite exquisite and extensive data from the Sloan Digital Sky Survey (SDSS; Adelman-McCarthy et al. 2006). This ambiguity is due to the theoretical uncertainties in how to convert emission-line fluxes to  $12 + \log(\text{O}/\text{H})$ , as assumptions must be made about both the gas temperature and ionization structure. While the electron temperature can be estimated directly using the [O III]  $\lambda 4363$  auroral line, this line is extremely weak and usually only detectable in very metal-poor environments. Thus, it is common to calibrate measurement methods using much stronger forbidden emission lines such as [O II]  $\lambda\lambda 3726, 3729$ , H $\beta$ , [O III]  $\lambda\lambda 4959, 5007$ , H $\alpha$ , and [N II]  $\lambda 6584$  based on the so-called direct [O III]  $\lambda 4363$   $T_e$  method. However, since [O III]  $\lambda 4363$  preferentially emits in high-temperature regions, this calibration can lead to an over-estimate of the electron temperature based on this line and thus an under-estimate of the oxygen abundance (Kewley & Ellison 2008). It is therefore common to instead calibrate strong-line measurement methods based on theoretical photoionization models. On the other hand, there are arguments that such strong-line methods *over-estimate* the true abundance (Kennicutt et al. 2003). Moreover, most indicators are either double-valued at low metallicities (such as the popular  $R_{23}$  indicator) or saturated at high metallicities as emission-line cooling shifts to the near-infrared (Bresolin 2006).

Kewley & Ellison (2008) highlight many of these issues, and derive  $12 + \log(\text{O}/\text{H})$  for a large set of galaxies from SDSS using ten indicators (eight of which we consider here: T04, Tremonti et al. 2004; D02, Denicoló et al. 2002; KK04, Kobulnicky & Kewley 2004; Z94, Zaritsky et al. 1994; KD02, Kewley & Dopita 2002; M91, McGaugh 1991; PP04O3N2 and PP04N2, using the Pettini & Pagel 2004 ( $[\text{O III}]/\text{H}\beta)/([\text{N II}]/\text{H}\alpha)$  and  $[\text{N II}]/\text{H}\alpha$  flux ratios, respectively). The Kewley & Ellison fits to the mass-metallicity relation are given in Table 1, where we have converted from a Kroupa (2001) to a Chabrier (2003a) IMF and from  $12 + \log(\text{O}/\text{H})$  to  $\log Z_g$ , where

$$\begin{aligned} \log Z_g &= [12 + \log(\text{O}/\text{H})] - 12 - \log \left[ \frac{M_{\text{O}}/M_{\text{H}}}{X M_{\text{H}} + Y M_{\text{He}}} \right] \\ &= \log(\text{O}/\text{H}) - \log \left[ \frac{15.999/1.0079}{0.75 \times 1.0079 + 0.25 \times 4.0026} \right]. \end{aligned} \quad (1)$$

These mass-metallicity relations are plotted in Figure 1; the scatter in  $Z_g$  at fixed  $M_*$  for each mass-metallicity relation is smaller by a factor of 2–3 than the spread in normalizations, implying that the differences are caused by the systematics discussed above.

We consider in detail the two relations in black in Figure 1 and in bold in Table 1 (T04, Tremonti et al. 2004 and D02, Denicoló et al. 2002). The D02 indicator is a linear relation between the [N II]  $\lambda 6584/\text{H}\alpha$  ratio and  $12 + \log(\text{O}/\text{H})$  calibrated against  $T_e$  metallicities. The relatively low normalization of this

$\langle \log M_\star \rangle$	$\langle \log F_g \rangle$	$\sigma_{\log F_g}$
8.3298	0.5153	0.07867
8.7265	0.3084	0.06500
9.0892	0.2062	0.06359
9.5141	-0.07142	0.06220
9.8941	-0.3230	0.04817
10.298	-0.5548	0.06666
10.664	-0.8389	0.06212
11.053	-0.8303	0.06566

**Table 2.** Cold gas fractions  $\log F_g = \log(M_g/M_\star)$  in bins of  $\Delta \log M_\star = 0.4$  dex and the uncertainty in the mean  $\sigma_{\log F_g}$  for the McGaugh (2005), Leroy et al. (2008), and Garcia-Appadoo et al. (2009) data sets.

method is common for  $T_e$ -calibrated indicators. The T04 method is based on theoretical stellar population synthesis and photoionization models combined with a Bayesian analysis of many more strong emission lines than used in most methods. While we do not favor any one  $12 + \log(\text{O}/\text{H})$  indicator, we take these two mass-metallicity relations as representative of the normalizations and slopes observations as a whole.

## 2.2 Observed gas fractions of $z \sim 0$ galaxies

Figure 2 shows how the gas-to-stellar mass ratio  $F_g$  (left panel) and gas mass  $M_g$  (right panel) vary with galaxy stellar mass. The open diamonds are total H I gas masses measured from 21 cm line fluxes (McGaugh 2005). The crosses are also H I gas masses, with stellar masses measured from SDSS (Garcia-Appadoo et al. 2009; West et al. 2009, 2010). The filled circles represent the total H I + H<sub>2</sub> gas masses (including a correction for helium) from The H I Nearby Galaxy Survey (THINGS), with the H<sub>2</sub> masses derived from HERA CO-Line Extragalactic Survey (HERACLES) and the Berkeley-Illinois-Maryland Association Survey of Nearby Galaxies (BIMA SONG) CO measurements (Leroy et al. 2008). Though there is large scatter in the gas fraction at a fixed stellar mass, gas fractions clearly decrease as  $M_\star$  increases; this behavior is found in cosmological hydrodynamic simulations (e.g., Hopkins et al. 2008). The mean  $\log F_g$  in bins of  $\Delta \log M_\star = 0.4$  dex for  $8.1 \leq \log M_\star \leq 11.3$  is overplotted with the large solid orange squares; we list these means and uncertainties in Table 2. Each of these data sets focus on star-forming galaxies similar to those in which  $12 + \log(\text{O}/\text{H})$  is measurable; surveys not restricted to actively star-forming galaxies lead to much lower average gas fractions (Catinella et al. 2010).

We parameterize  $F_g$  as power-law of the form

$$F_g \equiv \frac{M_g}{M_\star} = \left( \frac{M_\star}{M_{\star,0}} \right)^{-\gamma} = K_f M_\star^{-\gamma}, \quad (2)$$

with  $\gamma > 0$ . Table 3 lists  $\log M_{\star,0}$ ,  $K_f$  and  $\gamma$  for our adopted gas relations. As we show in §3.1,  $F_g$  is a more convenient parameterization than the commonly used and more arguably intuitive  $\mu_g$ , the gas mass as a fraction of the total baryonic galaxy mass  $M_\star + M_g$ ,

$$\mu_g \equiv \frac{M_g}{M_g + M_\star} = \frac{F_g}{1 + F_g}. \quad (3)$$

The “total” gas fraction relation is a power-law fit to the combined McGaugh, Leroy et al., and Garcia-Appadoo et al. data sets, offset by +0.2 dex so that the total gas fractions are greater than those implied by the K-S law (see below). In order to understand the con-

Name	$\log M_{\star,0}$	$K_f$	$\gamma$
Total	9.6	316228	0.57
SDSS	6.0	15.85	0.20
Fiber	2.7	2.24	0.13
Flat	—	0.50	0.00

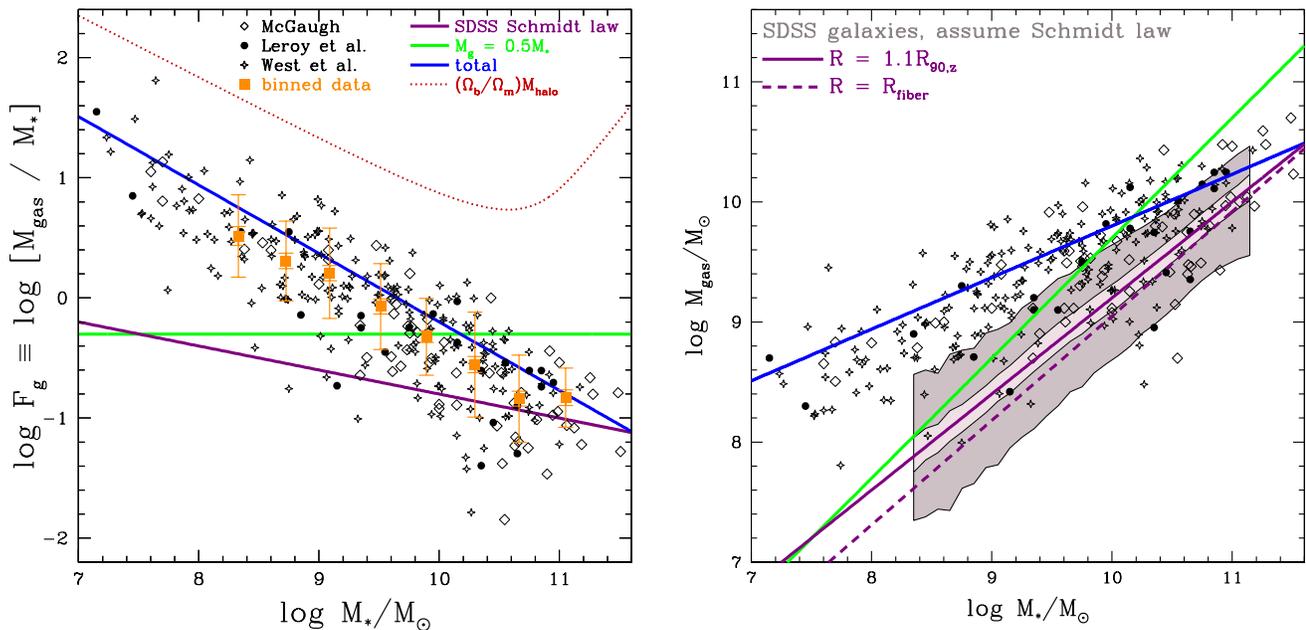
**Table 3.** Gas fraction relation parameters,  $F_g = M_g/M_\star = K_f M_\star^{-\gamma}$ .

tribution of a sloped gas fraction relation to the mass-metallicity relation, we also consider a flat gas relation of  $M_g = 0.5M_\star$ , shown in green in Figure 2.

For reference, Figure 2 shows how the total baryonic halo mass,  $(\Omega_b/\Omega_m)M_h$ , varies with stellar mass (halo mass as a function of  $M_\star$  is calculated as discussed in §3.2). The offset between the baryonic halo mass and  $M_\star + M_g$  is evidence of the so-called “missing baryon” problem; the missing baryons are either hot or have been expelled from the halos by  $z = 0$  (Crain et al. 2007). Figure 2 further highlights the fact that for  $M_\star \lesssim 10^{10} M_\odot$ , the fraction of baryons in the form of cold gas is roughly constant (i.e., the blue and red lines are roughly parallel). Moreover, while massive galaxies are gas poor, galaxies with stellar masses below  $\sim 10^{9.5} M_\odot$  have most of their mass in the form of gas: the processes responsible for the missing baryons in  $z = 0$  halos must also account for this inefficiency of star formation in low mass halos. We discuss this issue further in Appendix B.

The solid line in the right panel of Figure 2 shows the median gas fractions obtained by inverting the Kennicutt-Schmidt (K-S, Kennicutt 1998; Schmidt 1959) relation, as explained in detail in Appendix A. The shaded contours denote the 1- $\sigma$  and 2- $\sigma$  gas masses derived for the entire galaxy ( $R_g = 1.1R_{90,z}$ ) in running bins of  $\log M_\star$  from  $\log M_\star = 8.3$  to 11.1; for clarity, galaxies falling outside this region are not shown. The solid line is an eyeball power-law fit to the median  $R_g = 1.1R_{90,z}$  gas masses while the dashed line is the same for the gas (and stellar) masses within the SDSS fiber. The fact that these relations are quite similar to one another indicates that aperture corrections are relatively small and/or that gas fractions are relatively scale-invariant within  $1.1R_{90,z}$ .

The gas masses estimated from the K-S law and the measurements of total cold gas masses roughly agree with one another on the low gas fraction of  $F_g \sim 0.1$  at  $\log M_\star \sim 11$ , and that  $F_g$  increases with decreasing stellar mass. The amount of this increase in gas fraction, however, is in stark disagreement, with a range of over an order of magnitude in  $F_g$ . The K-S law only traces star-forming gas and therefore traces molecular gas more closely than atomic, and dwarf galaxies are deficient in molecular gas (Leroy et al. 2008). At large radii in more massive galaxies, the gas is predominantly atomic, i.e., the H I radii of galaxies is often much larger than the optical (star-forming) radii (Boomsma et al. 2008; Walter et al. 2008). For the purposes of the mass-metallicity relation, what matters is the total amount of gas that is able to effectively mix and dilute metals. A lower limit to this gas mass is the gas that is able to collapse and form stars—the gas traced by the K-S law. If on the other hand the atomic and molecular gas are well mixed (as opposed to, e.g., molecular gas only populating the galaxy center and atomic gas being at large radii), then the total gas fractions are more applicable. Finally, neither of these gas fraction estimates include ionized gas; if such gas is not only prevalent in typical galaxies but also has efficient mass transfer with both supernova ejecta and gas that will cool to form molecular clouds (and subsequently H II regions), then even the “total” gas fraction



**Figure 2.** Left: Gas fractions  $F_g$  as a function of  $M_*$ . Right: Gas masses  $M_g$  as a function of  $M_*$ . The open black diamonds are H I gas fractions and masses from McGaugh (2005); the crosses are the same from West et al. (2009, 2010). The filled circles are H I + H<sub>2</sub> gas fractions and masses (Leroy et al. 2008), who find that there is very little H<sub>2</sub> below  $\log M_* \sim 9.5$ , which is consistent with the comparison to the H I samples. The red dotted line shows the maximum baryonic mass  $(\Omega_b/\Omega_m)M_{\text{halo}}$ , while the green “flat” line shows  $M_g = 0.5M_*$ . The blue “total” line is a fit to these data with the normalization increased by 0.2 dex; the orange squares are the mean  $\log F_g$  of these same data in bins of  $\Delta \log M_* = 0.4$  dex with the inner and outer errorbars denoting the uncertainty in and dispersion about the mean, respectively. Gas fractions and masses derived from SDSS data and inverting the K-S law, assuming a radius of  $1.1R_{90,z}$  (solid line) and the fiber radius (dashed line); in the right panel, the shaded region corresponds to the 1- and 2- $\sigma$  dispersions in moving bins of  $\log M_*$ .

relation will be an underestimate of the gas diluting the galaxies’ metals.

### 2.3 Galaxy outflows

Though observations of galaxy-scale outflows are notoriously difficult, galaxy winds observed in a range of star-forming galaxies display a complex, multiphase structure. Since detectability increases with the star formation rate density (Veilleux et al. 2005), however, the most detailed studies of galaxy winds have been of the outflows associated with extreme starbursts, namely, (ultra)luminous infrared galaxies ([U]LIRGs). Studies of blue-shifted absorption-lines reveal both neutral (Heckman et al. 2000; Rupke et al. 2002; Martin 2005) and photoionized gas (Grimes et al. 2009), often with several kinematically distinct components. In contrast, X-ray emission around local starbursts such as M82 indicates a hot ( $T \sim 10^{6.5} - 10^8$  K), tenuous ( $n \sim 10^{-4} - 10^{-3}$  cm<sup>-3</sup>) wind fluid (Strickland & Stevens 2000; Strickland & Heckman 2007, 2009). Wind velocities derived from both emission and absorption line studies are typically hundreds of km s<sup>-1</sup> (Martin 2005; Grimes et al. 2009). The outflow velocity  $v_w$  of the colder neutral gas is typically comparable to one to a few times the galaxy’s circular velocity  $v_{\text{circ}}$  (Martin 2005), which is comparable to the galaxy’s virial velocity  $v_{\text{vir}}$  (e.g., Diemand et al. 2007).

The scaling  $v_w \sim v_{\text{vir}}$  follows naturally if momentum transfer from radiation pressure is driving the wind (Murray et al. 2005). For radiation pressure to be effective, the starburst must be Eddington limited and the outflowing gas has an asymptotic velocity of

$$v_w(\infty) = 2v_{\text{esc}} \left( \frac{L}{L_{\text{edd}}} - 1 \right)^{1/2}, \quad (4)$$

where the escape velocity  $v_{\text{esc}}$  is comparable to the virial velocity. The wind velocity is therefore typically taken to be  $v_w = 3v_{\text{vir}}$ . In the single-scattering limit (Murray et al. 2005),

$$\dot{M}_w v_w = \frac{L_{\text{starburst}}}{c} = \frac{\epsilon_{\text{nuc}} \dot{M}_{\text{SFR}} c^2}{c}, \quad (5)$$

where  $L_{\text{starburst}}$  is the starburst luminosity and  $\epsilon_{\text{nuc}} = 8 \times 10^{-4}$  is the nuclear burning efficiency. Thus the mass-loading factor<sup>3</sup>  $\eta_w$  is proportional to the inverse of the virial velocity such that

$$\eta_w \Big|_{\text{momentum}} \equiv \frac{\dot{M}_w}{\dot{M}_{\text{SFR}}} = \frac{\epsilon_{\text{nuc}} c}{v_w} \sim \frac{80 \text{ km s}^{-1}}{v_{\text{vir}}}. \quad (6)$$

This same scaling is achieved if the wind is driven by cosmic rays (Socrates et al. 2008).

On the other hand, the outflow may be driven by energy transfer, perhaps from supernovae thermally heating the ISM (Chevalier & Clegg 1985; Dekel & Silk 1986; Silk & Rees 1998; Murray et al. 2005). In this popular scenario,

$$\frac{1}{2} \dot{M}_w v_w^2 \approx \xi E_{\text{SN}} \times [\text{\# of SNe per solar mass of stars formed}] \dot{M}_{\text{SFR}}, \quad (7)$$

<sup>3</sup> Definitions in the literature of the “mass-loading factor” vary; we take it to mean the *total* outflow mass rate divided by the *total* star formation rate (including short-lived stars).

where  $E_{\text{SN}} \sim 10^{51}$  erg is the typical energy per supernova and  $\xi$  is the efficiency with which supernovae transfer energy to the ISM. Letting  $\xi = 0.1$ , i.e., a 10% efficiency, and taking the number of supernovae per unit mass to be  $10^{-2}$ , this yields a mass-loading factor of

$$\eta_w \Big|_{\text{energy}} \equiv \frac{\dot{M}_w}{\dot{M}_{\text{SFR}}} \sim \left( \frac{73 \text{ km s}^{-1}}{v_{\text{vir}}} \right)^2, \quad (8)$$

where we have implicitly assumed  $v_w \approx 3v_{\text{vir}}$ . While we in general consider models in which  $\eta_w \propto v_{\text{vir}}^{-\beta}$  for  $\beta > 0$  (or, equivalently,  $\eta_w \propto M_{\text{halo}}^{-\beta/3}$ , see §3.2); it is helpful to keep the normalizations suggested by equations (6) and (8) in mind.

Except via the impact of outflows on galaxy gas fractions (see Appendix B), the mass-metallicity relation is insensitive to the *total* mass outflow rate  $\dot{M}_w$ . Instead, as we show in §3.1, oxygen depletion due to winds is governed by the rate of metal loss,  $Z_w \dot{M}_w$ , where  $Z_w$  is the metallicity of the outflow; in our case (see §2.1), the mass ratio of oxygen in the outflowing material. While many metals (oxygen, as well as, e.g., iron, sodium, carbon, magnesium, and neon; Heckman et al. 2000; Martin 2005; Strickland & Heckman 2007; Martin & Bouché 2009; Grimes et al. 2009; Spoon & Holt 2009) are observed in galaxy outflows, there are relatively few observations of outflowing oxygen, and elemental abundances in the wind fluid are rarely reported. Strickland & Heckman (2009), however, find that the X-ray emitting outflow from M82 has a high enough metal content that it is consistent with containing nearly all of the freshly produced metals in the starburst with an inferred velocity of  $\sim 1000\text{--}2000 \text{ km s}^{-1}$ . Combined with their interpretation that the outflow has very little entrained gas (i.e., that it is essentially comprised solely of supernova ejecta), this implies that the metallicity of the outflow is quite high. (We note that in this interpretation of the data, supernova explosions surprisingly have no radiative energy losses when interacting with the ambient ISM [ $\xi = 1$  in equation 7]; see also Heckman 2003.) This picture is further complicated by the fact that outflows are likely multi-phase, and the metallicities and escape fractions in, e.g., the cold and ionized phases may be different. From the perspective of the mass-metallicity relation, however, what matters is the total amount of expelled oxygen relative to the total amount of expelled gas, where “expelled” oxygen or gas is just the oxygen or gas that has either been physically ejected from the galaxy or simply heated up such that it cannot efficiently transfer mass to the gas that is able to cool and form stars and thus be observed contributing to the mass-metallicity relation.

### 3 THE FORMALISM

#### 3.1 The mass-metallicity relation

The three galaxy masses relevant to the mass-metallicity relation are the total galaxy mass in stars,  $M_*$ , the galaxy gas mass,  $M_g$ , and the mass of gas-phase metals,  $M_Z$ . The model is based on relating the instantaneous change in these masses via their sources and sinks to the instantaneous galaxy star formation rate,  $\dot{M}_{\text{SFR}}$ , ignoring environmental effects such as mergers and tidal stripping (see also, e.g., Tinsley 1974, 1980; Matteucci 2002; Recchi et al. 2009; Finlator & Davé 2008; Spitoni et al. 2010). Observationally, Mannucci et al. (2010) and Lara-López et al. (2010) have recently shown that  $Z_g$  has less scatter at fixed  $M_*$  and  $\dot{M}_{\text{SFR}}$  than at just fixed  $M_*$  (i.e., the mass-metallicity relation); there is no evidence for evolution of this surface up to  $z \sim 2.5$ . This finding implies that

the  $M_*\text{-}\dot{M}_{\text{SFR}}\text{-}Z_g$  hypersurface provides a more physical description of the underlying physics than just the  $M_*\text{-}Z_g$  plane. In the formalism, the star formation rate is closely linked with outflow efficiencies, and observationally, gas fractions and star formation rates are tightly correlated. We review the relevant equations here and their derivation in Appendix C.

The mass-metallicity relation is described as

$$Z_g = y \left[ \zeta_w - \zeta_a + \right. \quad (9)$$

$$\left. F_g(1 - f_{\text{recy}}) \left( \frac{d \log M_g}{d \log M_*} + \frac{d \log Z_g}{d \log M_*} \right) + 1 \right]^{-1}$$

$$= y [\zeta_w - \zeta_a + \alpha F_g + 1]^{-1}, \quad (10)$$

where

$$\alpha \equiv (1 - f_{\text{recy}}) \left( \frac{d \log M_g}{d \log M_*} + \frac{d \log Z_g}{d \log M_*} \right) \quad (11)$$

is a factor of order unity. Equation (10) shows that the metallicity  $Z_g$  is proportional to the nucleosynthetic yield  $y$ . Because the IMF and Type II supernova yields are highly uncertain,  $y$  is only constrained to be  $0.08 \lesssim y \lesssim 0.023$  (e.g., Finlator & Davé 2008); we adopt a mid-range value of  $y = 0.015$ . We further justify this value and discuss models with a varying yield in §4.2.

The denominator of equation (10) includes terms for metal accretion ( $\zeta_a$ ), metal expulsion ( $\zeta_w$ ), and dilution from gas ( $\alpha F_g$ ). The metallicity-weighted mass-loading factors  $\zeta_a$  and  $\zeta_w$  in equation (C6) describe the relative rates at which metals are being accreted and expelled from the system, and are defined as

$$\zeta_a \equiv \frac{Z_{\text{IGM}}}{Z_g} \times \frac{\dot{M}_{\text{acc}}}{\dot{M}_{\text{SFR}}} = \left( \frac{Z_{\text{IGM}}}{Z_g} \right) \eta_a, \text{ and} \quad (12)$$

$$\zeta_w \equiv \frac{Z_w}{Z_g} \times \frac{\dot{M}_w}{\dot{M}_{\text{SFR}}} = \left( \frac{Z_w}{Z_g} \right) \eta_w. \quad (13)$$

The metallicity of accreting gas,  $Z_{\text{IGM}}$ , is typically taken to be zero, though SPH simulations indicate that due to previous episodes of enrichment of the intergalactic medium (IGM) from metal-containing galaxy outflows, the effective  $Z_{\text{IGM}}$  may be non-negligible (Finlator & Davé 2008; Oppenheimer et al. 2009). Because a self-consistent model of an enriched IGM will be based on the evolution of the mass-metallicity relation, we will for now take  $Z_{\text{IGM}}$  and thus  $\zeta_a = 0$ , though we will return to the ramifications of this assumption in §6.3. The wind metallicity,  $Z_w$ , is often assumed to be the ISM metallicity (Finlator & Davé 2008; Erb 2008), giving  $\zeta_w = \eta_w$ . However,  $Z_g$  is simply a lower-limit to the possible outflow metallicity (if the wind is driven by supernovae, then it can be metal-enriched relative to the ambient ISM, but not metal-depleted). The actual wind metallicity will depend on the fraction  $f_e$  of the outflow that is entrained interstellar gas, which has a generic metallicity  $Z_g$ , and the fraction  $1 - f_e$  that is from newly exploded supernovae and therefore has a metallicity  $Z_{\text{ej,max}} \sim 0.1$  (Woosley & Weaver 1995). The wind metallicity is thus

$$Z_w = (1 - f_e) Z_{\text{ej,max}} + f_e Z_g, \quad (14)$$

where we note that  $f_e$  may vary with galaxy mass and must satisfy  $0 \leq f_e < 1$ .

In this model, as long as galaxies have a given  $F_g\text{-}M_*$  relation (§2.2), how they got that gas (i.e.,  $\eta_a$  and  $\eta_w$ ) is irrelevant. However, for any given wind model  $\eta_w$ , the accretion rate as a function of the

star formation rate  $\eta_a$  is uniquely determined. We explore this point and its implications in Appendix B.

By finding combinations of the yield, outflow strength, and gas fractions that combine as stated on the right-hand side of Equation (10) to give  $Z_g(M_*)$  on the left-hand side, we can explicitly reproduce the observed mass-metallicity relation. This is the tack we take in § 4.

### 3.2 Connecting galaxies and halos

A number of methods have been developed to empirically connect galaxies to halos. One straightforward approach is the cumulative matching of galaxy ( $n_{\text{gal}}$ ) and halo ( $n_{\text{halo}}$ ) number counts (Vale & Ostriker 2004; Shankar et al. 2006; Conroy & Wechsler 2009), i.e.,

$$n_{\text{gal}}(> M_*) = n_{\text{halo}}(> M_{\text{halo}}). \quad (15)$$

Assuming that each halo (and subhalo) contains a galaxy, equation (15) determines the average mapping between halo mass and galaxy mass.

We adopt one of the latest determinations of the  $M_*$ - $M_{\text{halo}}$  relation by Moster et al. (2009, top panel of Figure 3),

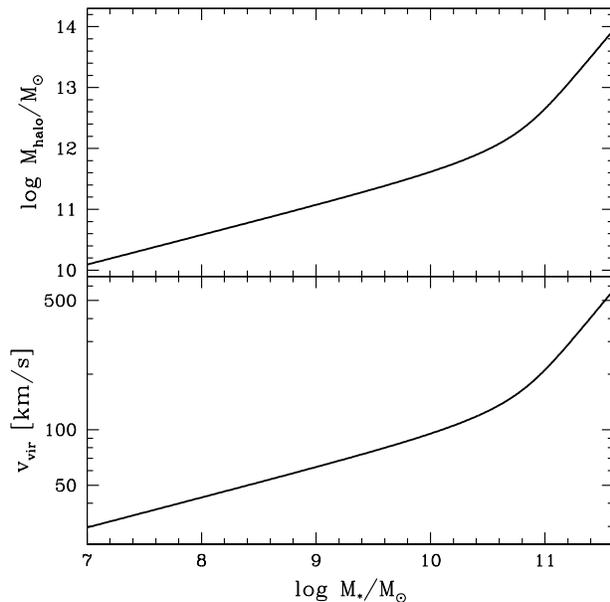
$$\frac{M_*}{M_{\text{halo}}} = 0.0633(1+z)^{-0.72} \times \left[ \left( \frac{M_{\text{halo}}}{M_{h,0}} \right)^{-1.06-0.17z} + \left( \frac{M_{\text{halo}}}{M_{h,0}} \right)^{0.556(1+z)^{-0.26}} \right]^{-1} \quad (16)$$

with the zero point increased by 0.05 dex to correct from a Kroupa (2001) to a Chabrier (2003b) IMF (Bernardi et al. 2010), and where

$$\log M_{h,0}/M_\odot = [\log 11.88] (1+z)^{0.019}. \quad (17)$$

The Moster et al. (2009)  $M_*$ - $M_{\text{halo}}$  mapping is in good agreement with constraints from galaxy-galaxy lensing, galaxy clustering, and predictions of semi-analytic models. Following the scaling relations in Tonini et al. (2006, and references therein), we have verified that equation (17) yields a Tully-Fisher relation (Tully & Fisher 1977) consistent with the more recent calibrations by Pizagno et al. (2007), as long as the dynamical contribution of the dark matter within a few optical radii is less than the one predicted by a pure Navarro, Frenk, & White (1996) mass profile, in line with many other studies (e.g., Salucci et al. 2007). Also note that the subhalo masses quoted by Moster et al. refer to *unstripped* quantities, which represent more reliable indicators of the intrinsic potential well in which satellites formed.

The Moster et al. relation is in broad agreement with other studies, such as the ones by Guo et al. (2010) and Shankar et al. (2006), although the latter relied on a stellar mass function based on dynamical mass-to-light ratios that cannot be directly used in the present study based on SDSS stellar masses. Despite the different techniques adopted, most of the studies find consistent results on the  $M_*$ - $M_{\text{halo}}$  relation, especially in the mass range of interest here, i.e., star-forming galaxies with stellar mass  $\lesssim 2 \times 10^{11} M_\odot$  and hence halos with mass  $\lesssim 5 \times 10^{12} h^{-1} M_\odot$  (Firmani et al. 2009; More et al. 2010). We caution that Neistein et al. (2011) have recently described how the  $M_*$ - $M_{\text{halo}}$  relation could be quite different from expected by the basic abundance matching technique. Allowing satellite galaxy masses to depend both on host subhalo mass at infall *and* on the friends-of-friends (FOF) group mass, many distinct galaxy-halo correlations are found to satisfy all basic statistical and clustering constraints. In particular, successful mod-



**Figure 3.** Halo mass,  $M_{\text{halo}}$ , and virial velocity,  $v_{\text{vir}}$ , as a function of stellar mass,  $M_*$ , at  $z = 0$  (Moster et al. 2009). See § 3.2 for more details.

els are found where satellite galaxies are hosted by much lower mass halos than we assume here.

If winds depend on the potential well depth of the halo rather than the mass itself, then the halo virial velocity  $v_{\text{vir}}$  is more relevant than  $M_{\text{halo}}$ . Roughly speaking,

$$v_{\text{vir}}^2 \sim \Phi \sim \frac{GM_{\text{halo}}}{R_{\text{halo}}}, \quad (18)$$

where the dependence of the halo radius  $R_{\text{halo}}$  on the halo mass is a function of both cosmology and the structure of the halo (Łokas & Mamon 2001; Ferrarese 2002; Loeb & Peebles 2003; Baes et al. 2003). We connect  $v_{\text{vir}}$  to  $M_{\text{halo}}$  via

$$\begin{aligned} v_{\text{vir}} &= \left( \frac{GM_{\text{halo}}}{R_{\text{vir}}} \right)^{1/2} \\ &= 112.6 \left( \frac{M_{\text{halo}}}{10^{12} M_\odot} \right)^{1/3} \\ &\times \left[ \frac{\Omega_m}{0.25} \frac{1}{\Omega_m^z} \frac{\Delta}{18\pi^2} \right]^{1/6} (1+z)^{1/2} \text{ km s}^{-1}, \end{aligned} \quad (19)$$

where the mean density contrast (relative to the critical density) within the virial radius  $R_{\text{vir}}$  is  $\Delta = 18\pi^2 + 82d - 39d^2$ , with  $d \equiv \Omega_m^z - 1$ , and  $\Omega_m^z = \Omega_m(1+z)^3 / [\Omega_m(1+z)^3 + \Omega_\Lambda]$  (Bryan & Norman 1998; Barkana & Loeb 2001). The bottom panel of Figure 3 shows how  $v_{\text{vir}}$  varies with stellar mass in this model. We have verified that our  $M_*$ - $v_{\text{vir}}$  relation is in good agreement with the  $M_*$ - $v_{200}$  relation recently derived by Dutton et al. (2010a).

## 4 MODELS OF THE MASS-METALLICITY RELATION

We now turn to what is required to reproduce the observed mass-metallicity relation. Rearranging equation (10), we find

$$\frac{y}{Z_g} - 1 = \zeta_w - \zeta_a + \alpha F_g, \quad (20)$$

where we hereafter take  $\zeta_a = 0$  (i.e., metal accretion is negligible; see § 6.3 for a discussion of this choice). Expressed this way, the metallicity  $Z_g$  is related explicitly to a sum of  $\zeta_w$  (a term describing outflows) and  $F_g = M_g/M_*$  (a term describing the galaxy gas content). Equation (20) is the principal theoretical result of this paper, connecting gas-phase metallicities to gas fractions, outflows, and accretion. Functionally, one can use equation (20) to find working models for a given  $Z_g(M_*)$  in several ways:

(i) Assume  $y$  and  $Z_g(M_*)$  are known; use trial and error to find combinations of  $\zeta_w(v_{\text{vir}})$  and  $[\alpha F_g](M_*)$  that satisfy equation (20).

(ii) Assume  $y$ ,  $Z_g(M_*)$ , and  $\zeta_w(v_{\text{vir}})$  are known; solve for  $d \log M_g / d \log M_*$  in equation (9) and integrate to find  $M_g(M_*)$ .

(iii) Assume  $y$ ,  $Z_g(M_*)$ , and  $M_g(M_*)$  are known; equation (20) then says  $\zeta_w = y/Z_g - 1 - \alpha F_g$ .

Method (i) works well for developing intuition regarding tensions in the data and theoretical wind models, while methods (ii) and (iii) yield models that exactly produce the observed mass-metallicity relation. In § 4.1, we explore models with a constant yield  $y = 0.015$ , focusing in § 4.1.2 on what constraining the model to match observed gas fractions implies about the efficiency of metal expulsion. In Appendix D, we go into some of the more subtle details of how different scalings of  $\zeta_w$  with  $v_{\text{vir}}$  are and are not consistent with observed gas fractions. In particular, we show that the no-winds model ( $\zeta_w = 0$ ) requires gas fractions that are  $\sim 0.3$  dex higher than observed for all galaxy masses, implying that if the yield is constant, then the mass-metallicity relation is direct evidence of outflows. Finally, in § 4.2, we consider how variable yields affect our conclusions.

## 4.1 Models with constant yield

### 4.1.1 Choice of $F_g$ - $M_*$ relation

Figure 4 shows outflow models  $\zeta_w(v_{\text{vir}})$  for different given  $F_g$ - $M_*$  relations [method (iii)]. As discussed in § 2.2, we consider the total gas fractions (blue, solid lines),  $M_g = 0.05(\Omega_b/\Omega_m)M_{\text{halo}}$  (red, dotted), gas fractions as inferred by inverting the Schmidt law for SDSS galaxies (purple), and  $M_g = 0.5M_*$  (green). The  $M_g \propto M_{\text{halo}}$  model is included because it might provide a natural explanation for the observed turnover in the mass-metallicity relation near  $M^*$ . We find that for the observed normalization of  $F_g(M_*)$ , the *slope* of the gas fraction relation is largely irrelevant in setting the mass-metallicity relation morphology. That is,  $z = 0$  galaxies have little enough gas that the mass-metallicity relation is shaped by how  $\zeta_w$  rather than  $F_g$  scales with galaxy mass. This can be seen visually in the right-hand panel of Figure 4: at low masses, even the flat gas fraction relation has approximately the same  $\zeta_w$  slope as those models with steep  $F_g$ - $M_*$  relations.

### 4.1.2 Best-fit models

We quantify what  $\zeta_w(v_{\text{vir}})$  scalings are required in order to reproduce the mass-metallicity relation while remaining consistent with the observed gas fractions by using method (ii): by taking a given  $\zeta_w$  we can compare the corresponding  $F_g$  to binned gas fractions (§ 2.2, Table 2) to calculate a  $\chi^2$ . Parameterizing  $\zeta_w$  as  $(v_0/v_{\text{vir}})^{-b} + \zeta_{w,0}$ , the best-fit model for the T04 mass-metallicity relation is  $\zeta_w = (78 \text{ km s}^{-1}/v_{\text{vir}})^{-3.81} + 0.19$ , as shown in Figure 5. We show the  $\Delta\chi^2$  contours for 1-, 2-, and 3- $\sigma$  using the  $\Delta\chi^2$ -to- $\sigma$  conversion from Press et al. (1992) for 5 degrees-of-freedom

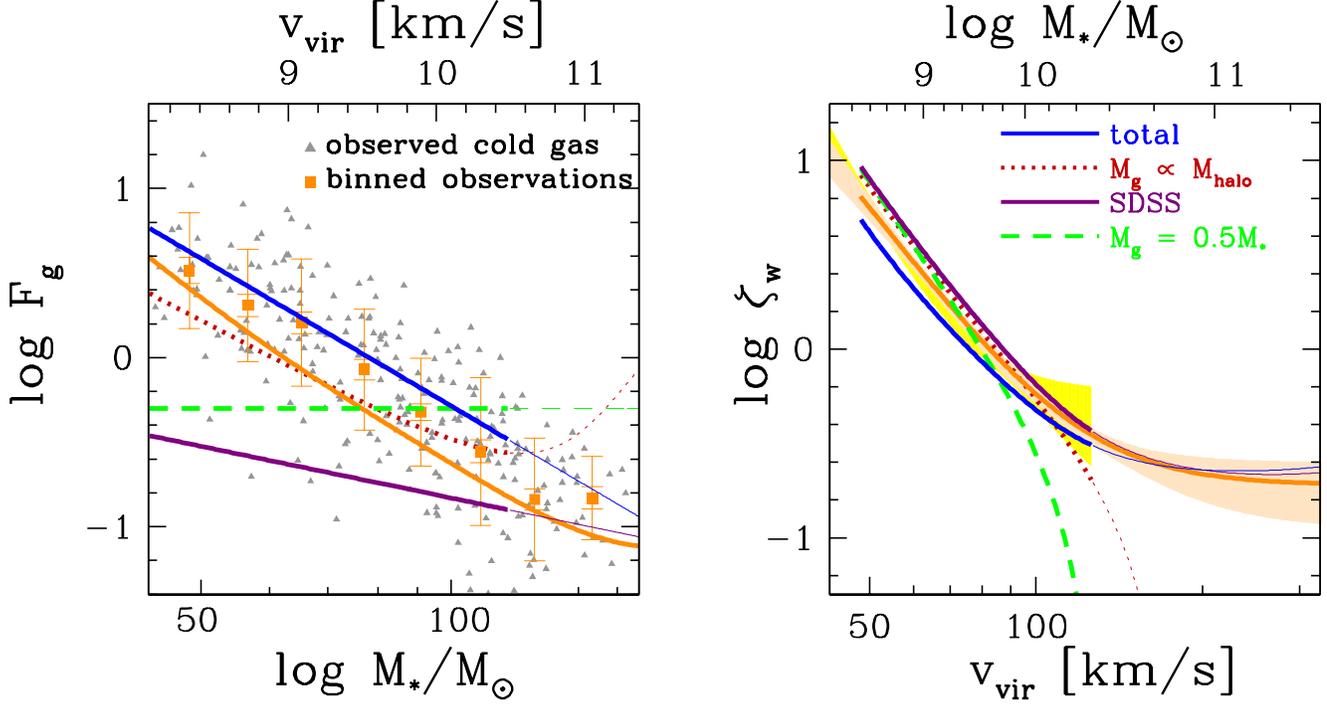
ID	$v_0$	$b$	$\zeta_{w,0}$
T04	78.0	3.81	0.19
Z94	63.5	3.20	0.23
KK04	55.5	3.04	0.32
KD02	71.0	3.18	0.39
M91	73.0	2.47	0.77
D02	79.0	3.42	1.25
PP04O3N2	90.0	3.15	1.50
PP04N2	111.8	2.31	1.35

**Table 4.** Best-fit parameters for  $\zeta_w = (v_0/v_{\text{vir}})^b + \zeta_{w,0}$  the fits to the mass-metallicity relation calculated by Kewley & Ellison (2008) and listed in Table 1 and the binned gas fractions plotted in Figure 2. These  $\zeta_w$  are plotted next to the corresponding  $Z_g(M_*)$  in Figure 6.

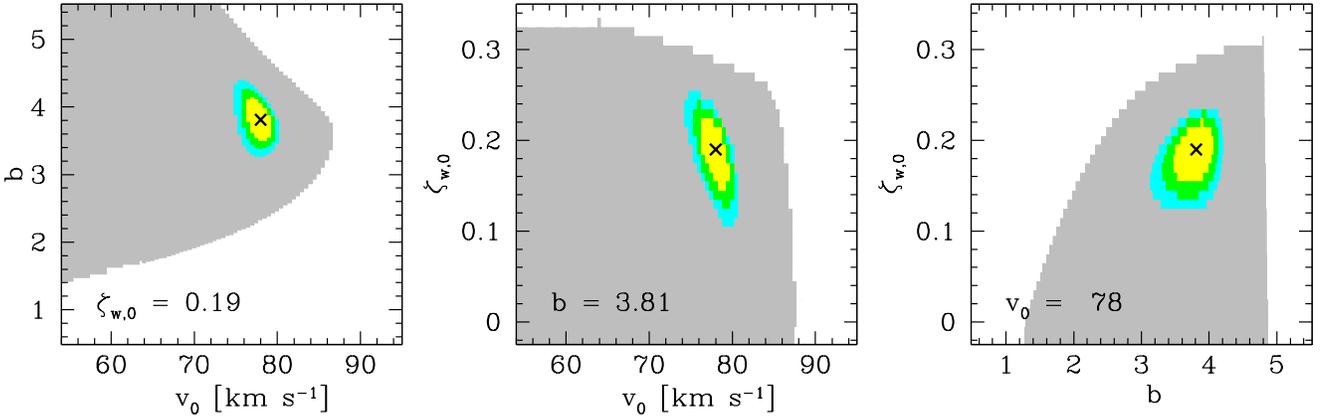
(8 data points and 3 parameters). The best-fit values do not change significantly if the dispersion about the mean is used instead of the uncertainties when calculating  $\chi^2$ , and we safely consider the points and errors for the binned data to be uncorrelated because the measurements for individual galaxies do not depend on one another. We bin  $F_g$  instead of  $Z_g$  because the mass-metallicity relation has been more rigorously measured than the  $F_g$ - $M_*$  relation. The white regions in Figure 5 correspond to models that are unphysical because they require negative gas fractions. The best-fit models are always close to the border between physical and unphysical regions in parameter space, reflecting the fact that gas fractions at  $z = 0$  are relatively small; it takes only a small change in  $\zeta_w$  to go from a small  $F_g$  to a negative one.

The best-fit  $\zeta_w$  can be strongly driven by the turnover of the mass-metallicity relation and change in slope of the  $M_*$ - $v_{\text{vir}}$  relation above  $\log M_* = 10.5$ . For example, for the T04 mass-metallicity relation, if we instead only consider the data at  $\log M_* < 10.5$ , the best-fit  $\zeta_w$  is instead  $(72 \text{ km s}^{-1}/v_{\text{vir}})^{-4.69} + 0.41$ ; that is, the velocity normalization  $v_0$  does not change much, but the slope steepens and the constant offset  $\zeta_{w,0}$  increases. Whether the best-fit  $\zeta_w$  shifts to higher  $b$  and  $\zeta_{w,0}$  (T04 and D02), lower  $b$  and  $\zeta_{w,0}$  (M91, Z94, PP04O3N2, and PP04N2), or doesn't change (KD02 and KK04) when only modeling  $\log M_* < 10.5$  depends on the subtle details of the particular fit to the mass-metallicity relation under consideration. In all cases, however,  $\Delta\chi^2$  for the parameters for the best fitting  $\zeta_w$  for a given mass-metallicity relation when the entire mass range is modeled fall within 1- $\sigma$  of the  $\log M_* < 10.5$  best fitting model for that indicator (but not necessarily vice-versa, since the best fitting low-mass model often requires negative gas fractions if extrapolated above  $10^{10.5} M_\odot$ ). The 1- $\sigma$  range of  $\zeta_w$  for the T04 mass-metallicity relation is shown by the shaded yellow and beige regions in the right-hand panel of Figure 4 for the  $\log M_* < 10.5$  and entire mass range, respectively.

Other metallicity indicators lead to mass-metallicity relations that are generally shallower and have a lower normalization than the Tremonti et al. mass-metallicity relation. This translates into  $\zeta_w + \alpha F_g$  needing to be larger and to scale slightly less steeply with mass than seen in Figure 4; the best-fit  $\zeta_w$  for all of the mass-metallicity relations shown in Figure 1 are plotted in Figure 6. (Detailed example models for the shallow, low-normalization Denicoló et al. (2002) mass-metallicity relation are shown in Figure D2.) Numerically, observed gas fractions require  $2.3 \lesssim b \lesssim 4$ ; this scaling with  $v_{\text{vir}}$  is much steeper than the canonical models for the unweighted mass-loading parameter discussed in § 2.3. Furthermore,  $\zeta_w$  must be large ( $\gtrsim 1$ ) at all relevant masses. The only way



**Figure 4.** Required  $\zeta_w$  to reproduce the T04 mass-metallicity relation with varying gas fraction relations: total (blue, solid),  $M_g = 0.5M_*$  (green, dashed), inverting the K-S law from SDSS data (purple, solid), and  $M_g = 0.05(\Omega_b/\Omega_m)M_{\text{halo}}$  (red, dotted); see § 2.2 for the motivations behind these relations. *Left:* Gas fractions as a function of stellar mass. The grey triangles in are the gas fractions plotted in Figure 2 (McGaugh 2005; Leroy et al. 2008; West et al. 2009, 2010) and the orange squares are the binned data (§ 2.2). *Right:*  $\log \zeta_w$  as a function of virial velocity corresponding to the gas fractions in the left panel. The orange lines are the best-fitting models based on the binned data (see Figure 5); the beige and yellow shaded regions in the right-hand panel show the 1- $\sigma$  range in  $\zeta_w$  for the entire mass range and  $\log M_* \leq 10.5$ , respectively.

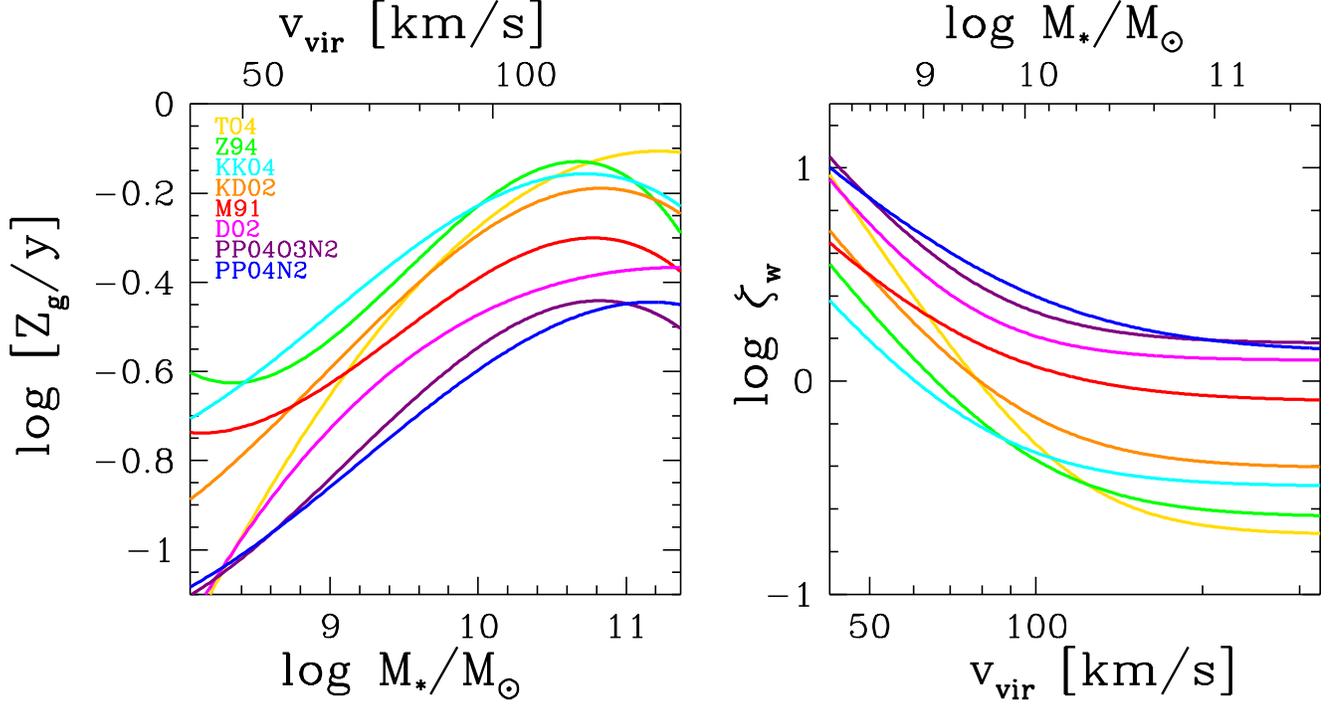


**Figure 5.**  $\Delta\chi^2$  contours for the T04 mass-metallicity relation with  $\zeta_w = (v_0/v_{\text{vir}})^{-b} + \zeta_{w,0}$ . The black “X” marks the parameters with the lowest  $\chi^2$ ; the yellow, green, cyan, and grey regions denote solutions with  $\Delta\chi^2 \leq 1\text{-}\sigma$ ,  $1\text{-}\sigma < \Delta\chi^2 \leq 2\text{-}\sigma$ ,  $2\text{-}\sigma < \Delta\chi^2 \leq 3$ , and  $\Delta\chi^2 > 3\text{-}\sigma$ , respectively, using the  $\Delta\chi^2$ -to- $\sigma$  conversion from Press et al. (1992). The white regions correspond to unphysical ( $M_g \leq 0$ ) models.

around a large  $\zeta_w$  is if a significant fraction of the gas that is diluting the metals is ionized (and thus not included in the observations of cold gas our  $F_g$ - $M_*$  relations).

In the limit of small  $F_g$  and large  $\zeta_w$ , one can see from equation (10) that  $Z_g \propto \zeta_w^{-1}$  (Finlator & Davé 2008). We are using cubic fits to the mass-metallicity relation (Table 1, Kewley & Ellison 2008), but for the relevant mass range, the mass-metallicity relation has  $0.2 \lesssim \text{slope} \lesssim 0.45$  for most of the relations plotted in

Figure 6. Our  $M_*$ - $M_{\text{halo}}$ - $v_{\text{vir}}$  relation (Figure 3) has  $M_* \propto v_{\text{vir}}^6$  for  $\log M_* \lesssim 10$  (and  $M_* \propto v_{\text{vir}}^{1.5}$  for  $\log M_* \gtrsim 10.6$ ). Thus, the metallicity  $Z_g$  is roughly proportional to  $v_{\text{vir}}^{1.2}$  to  $v_{\text{vir}}^{2.7}$ , implying that for  $\zeta_w \propto v_{\text{vir}}^{-b}$ ,  $b$  should be in the range 1.2 to 2.7. The large constant offset  $\zeta_{w,0}$ , however, means that the parameterization presented here (see, e.g., Figure 5) cannot be directly interpreted in terms of the simple power-law scalings presented in § 2.3.



**Figure 6.** *Left:* The mass-metallicity relation as derived from different metallicity indicators (§ 2.1, Kewley & Ellison 2008), relative to the nucleosynthetic yield  $y = 0.015$ . *Right:* The corresponding best fitting  $\zeta_w = (v_0/v_{\text{vir}})^b + \zeta_{w,0}$  under the requirement that the models’ gas fractions are consistent with observations. The  $\zeta_w$  parameters are listed in Table 4.

We also caution that  $\zeta_w \neq \eta_w$ , and we explore the consequences of metallicity-weighting the mass-loading parameter below (§ 5).

A crucial step in this analysis is the assignment of virial velocities to stellar masses. For example, Finlator & Davé (2008) found that  $\zeta_w \propto v_{\text{vir}}^{-1}$  was sufficient to reproduce the  $z \sim 2.2$  mass-metallicity relation (which does not differ significantly in slope from the shallow relations at  $z = 0$ ). In their simulations, however,  $M_* \propto M_{\text{halo}}$ , which is a shallower relation than our  $M_* \propto M_{\text{halo}}^2$ , a slope which Moster et al. (2009) finds to approximately hold to  $z \sim 2$  (see their Figure 14). Because  $M_{\text{halo}} \propto v_{\text{vir}}^3$ , these differences have extreme consequences for the interpretation of how  $\zeta_w$  scales with  $v_{\text{vir}}$ .

## 4.2 Implications of uncertain or varying yields

There is increasing evidence that the IMF may vary with star formation rate, and thus with galaxy mass (Meurer et al. 2009; Lee et al. 2009). Kroupa & Weidner (2003) suggest that if all stars form in clusters and if more massive stars are more likely to form in more massive star clusters, the integrated galactic initial mass function (IGIMF) depends on the embedded cluster mass function (the ECMF,  $\xi_{\text{ecl}}(M_{\text{ecl}}) \propto M_{\text{ecl}}^{-\beta}$ , where  $M_{\text{ecl}}$  is the mass of a cluster). Köppen et al. (2007) showed that for certain choices of  $\xi_{\text{ecl}}(M_{\text{ecl}})$ , the  $m_{\text{max}}-M_{\text{ecl}}$  relation (where  $m_{\text{max}}$  is the most massive star a cluster of mass  $M_{\text{ecl}}$  can produce), and SN II yields, the mass-metallicity relation could be explained without the need to invoke outflows. Like many previous models, however, Köppen et al. derive stellar and gas masses from their star formation rates under various assumptions of closed box with inflows. We re-examine here the effects of a varying IGIMF on the mass-metallicity relation using observed gas fractions. We connect star formation rates and

stellar masses from observations; the median SSFR of SDSS DR4 star-forming galaxies can be fit with a power law

$$\log[\dot{M}_{\text{SFR}}/M_*] = -9.83 - 0.12(\log[M_*/M_\odot] - 10), \quad (21)$$

as shown as a histogram in Figure 10 of Peeples et al. (2009).

Like Köppen et al. (2007), we follow Weidner et al. (2004) and take the minimum mass a star cluster can have to be  $5M_\odot$  and the maximum mass to be governed by the galaxy-wide star formation rate such that

$$\log(M_{\text{ecl,max}}/M_\odot) = 4.93 + 0.75 \log(\dot{M}_{\text{SFR}}/[M_\odot \text{ yr}^{-1}]). \quad (22)$$

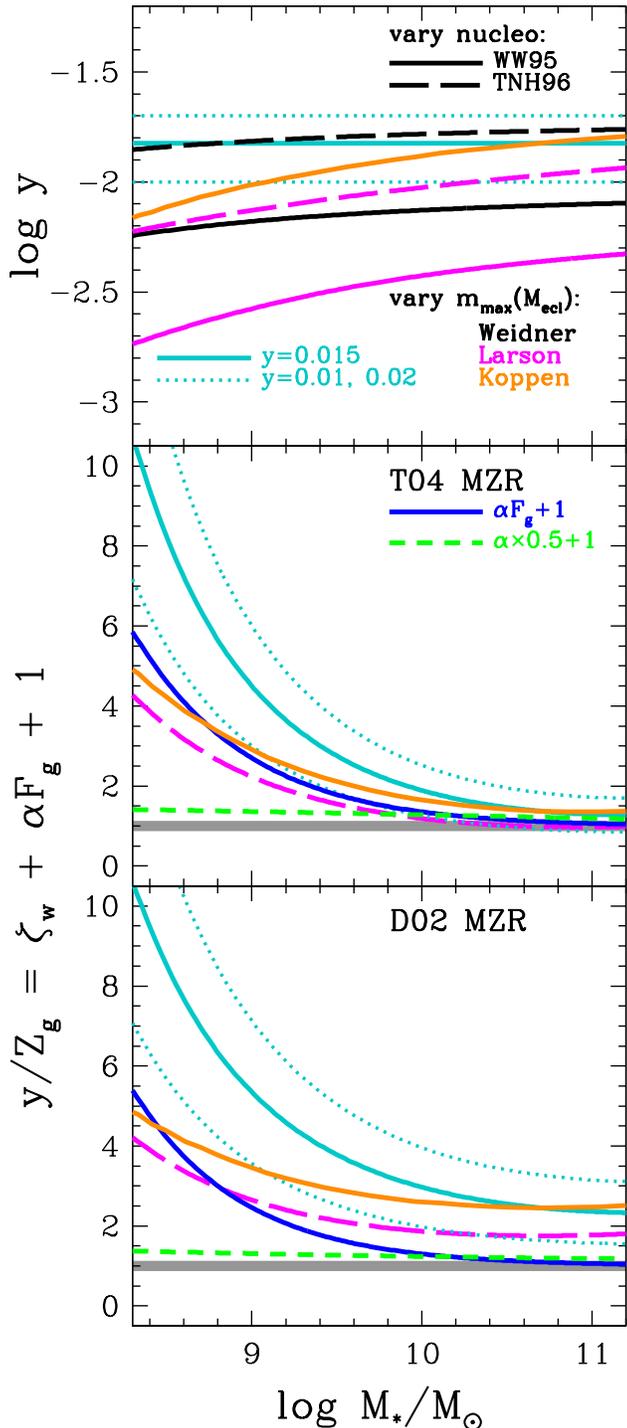
We adopt a power-law slope of the ECMF to be  $\beta = 2$  (Lada & Lada 2003; Köppen et al. 2007). The IGIMF is thus

$$\xi_{\text{IGIMF}}(m) = \int_{5.0M_\odot}^{M_{\text{ecl,max}}} \xi(m \leq m_{\text{max}}(M_{\text{ecl}})) \xi_{\text{ecl}}(M_{\text{ecl}}) dM_{\text{ecl}}, \quad (23)$$

where  $\xi(m)$  is the IMF. The oxygen yield<sup>4</sup> as a function of stellar mass,  $y(M_*)$ , will therefore be determined by  $\xi(m)$ , the  $m_{\text{max}}(M_{\text{ecl}})$  relation, and the Type II supernova yields, as shown in the top panel of Figure 7 (where we have adopted the Kroupa 2001 IMF). The Köppen et al. (2007) yields are shown as the solid orange line. The purple lines show models with the Larson (2003)  $m_{\text{max}}(M_{\text{ecl}})$ ,

$$m_{\text{max}} = 1.2M_{\text{ecl}}^{0.45}, \quad (24)$$

<sup>4</sup> The definition of yield used in this section,  $y \equiv M_{\text{oxy}}/M_*$ , is slightly different from the one given in Equation C7, but that for most purposes these are interchangeable.



**Figure 7.** *Top:* oxygen yields as a function of  $\log M_*$  for different choices of  $m_{\max}(M_{\text{ecl}})$  (Weidner et al. 2004, black; Larson 2003, magenta) and nucleosynthetic yields (Woosley & Weaver 1995, solid; Thielemann et al. 1996, long-dashed). Effects of different yields on models of the mass-metallicity relation as  $y/Z_g = \zeta_w + \alpha F_g + 1$  for the T04 and D02 mass-metallicity relations are shown in the middle and bottom panels, respectively. Constant yields are shown in cyan ( $y = 0.015$ , solid;  $\pm 0.005$ , dotted); gas fraction line types and colors are the same as in previous figures, with the total gas fractions as solid blue lines and  $M_g = 0.5M_*$  as short-dashed green lines. Details are given in § 4.2.

while the black lines show models with the same for models with  $m_{\max}(M_{\text{ecl}})$  derived from the semi-analytic model of Weidner et al. (2004, c.f., the thick solid line in Figure 1 of Weidner & Kroupa 2006); the Weidner et al. relation gives a shallower dependence of the yield on  $M_*$ . The thin solid lines show models derived using Woosley & Weaver (1995) nucleosynthetic models ( $Z = Z_\odot$ ), while the dotted lines show the same using the Thielemann et al. (1996) models; as discussed in detail by Thomas et al. (1998), Thielemann et al. gives oxygen abundances that are higher than Woosley & Weaver’s. Our fiducial value of  $y = 0.015$  is shown in cyan; this is very similar to the yields from an IGMF with the Thielemann et al. models with the Weidner et al.  $m_{\max}-M_{\text{ecl}}$  relation.

The bottom two panels of Figure 7 show how these uncertainties in the yield translate to uncertainties in outflows when modeling the mass-metallicity relation, where we have plotted  $y/Z_g$  for the various yields and for the Tremonti et al. (2004, middle panel) and Denicoló et al. (2002, bottom) mass-metallicity relations. The thick grey lines at  $y/Z_g = 1$  denote the boundary below which the yields are not high enough to produce the observed metallicities. As shown in § 3, the gas fractions and outflows must balance to give  $y/Z_g$ , we also show  $\alpha F_g + 1$  for two gas fraction models: our fiducial “total” gas fractions in blue and a toy  $M_g = 0.5M_*$  model in green. The difference between the  $y/Z_g$  curves and the gas fraction curves shows how much outflows are needed. For example, our fiducial yield of 0.015 gives  $y/Z_g$  that is greater than the  $\alpha/F_g + 1$  curves at all galaxy masses, with an decreasing difference at high  $M_*$ ; these differences are what’s explicitly plotted in Figures 4 and D2. The dotted  $y = 0.015 \pm 0.05$  lines in Figure 7 show how these results qualitatively do not change for a large range of constant yields; this range roughly shows the uncertainty in the yield from uncertainties in the IMF. Note that by exploring a range of normalizations of the mass-metallicity relation in § 4.1, we are exploring a range of  $y/Z_g$ —the parameter to which our results are sensitive.

The closeness of the orange line (the Köppen et al. yields) and the blue line (total gas fractions) in the middle panel shows that, within reasonable uncertainties, outflows are not strictly needed in that model. If the normalization of the mass-metallicity relation is lower, however, then even with the Köppen et al. yields, outflows are required. This can be explicitly seen by comparing the blue and orange lines in the bottom panel for the D02 mass-metallicity relation; moreover, because the difference between these two curves is greater at larger galaxy mass, it implies that in this case outflows would have to be *more* efficient at removing metals from massive galaxies. Unfortunately, however, the uncertainties in the oxygen production of core-collapse supernovae,  $m_{\max}(M_{\text{ecl}})$ , the IMF, and the normalization of the mass-metallicity relation all conspire to make drawing strong conclusions over the nature of outflows in the case of a varying yield extremely difficult.

## 5 OUTFLOW METALLICITY AND ENTRAINMENT

Supernova-driven galaxy outflows are comprised of some combination of supernova ejecta and ambient interstellar medium entrained in the outflow. The fraction  $f_e$  of entrained gas determines wind metallicity  $Z_w$ . As mentioned in § 3.1, the wind metallicity  $Z_w$  is usually assumed to be equal to the ISM metallicity  $Z_g$  when modeling the mass-metallicity relation, but if the outflowing supernova ejecta entrains very little gas (which would dilute the wind metallicity) then  $Z_w$  could be much higher than  $Z_g$ .

We showed in §4 that models of the observed  $z = 0$  mass-metallicity relation are more consistent with observations of  $z = 0$  galaxy gas fractions when the metallicity-weighted mass-loading factor  $\zeta_w \equiv (Z_w/Z_g)\eta_w$  scales steeply with the halo virial velocity, i.e.,  $\zeta_w = (v_0/v_{\text{vir}})^b + \zeta_{w,0}$  with  $b \gtrsim 3$ . Theoretical models for how supernovae drive galaxy-scale outflows, however, generally predict that the *unweighted* mass-loading factor  $\eta_w \equiv \dot{M}_w/\dot{M}_{\text{SFR}} = (\sigma_0/v_{\text{vir}})^\beta$  will scale much more shallowly, with  $\beta = 1$  or 2 (§2.3). Reconciling these disparate scalings therefore requires that  $Z_w/Z_g$  and hence the wind fluid composition varies with galaxy mass.

For any given  $\zeta_w$  that reproduces the mass-metallicity relation, additionally assuming the form of  $\eta_w(v_{\text{vir}})$  uniquely constrains the wind metallicity  $Z_w(M_*)$ . If  $\eta_w$  is constant with galaxy mass, then  $Z_w/Z_g$  must increase sharply in lower mass galaxies (Spitoni et al. 2010). Figure 8 shows  $Z_w$  for the best-fit  $\zeta_w = (78 \text{ km s}^{-1}/v_{\text{vir}})^{3.81} + 0.19$  for the T04 mass-metallicity relation (left) and  $\zeta_w = (79 \text{ km s}^{-1}/v_{\text{vir}})^{3.42} + 1.25$  for the D02 mass-metallicity relation (right). The dotted, short-dashed, and long-dashed lines are for  $\eta_w \propto v_{\text{vir}}^{-1}$ ,  $v_{\text{vir}}^{-2}$ , and  $v_{\text{vir}}^{-3}$  models, respectively. If  $\eta_w$  has a similar scaling with mass as  $\zeta_w$ , then  $Z_w \sim Z_g$  for all masses. However, a less steep dependence of  $\eta_w$  on  $v_{\text{vir}}$  implies that outflow metallicities should depend less on galaxy mass than  $Z_g$ . Moreover, determining  $Z_w$  from galaxy wind observations has different systematics than determining  $\eta_w$ , and  $Z_w$  clearly depends sensitively on the scaling of  $\eta_w$ . Figure 8 shows how measurements of  $Z_w(M_*)$  can therefore be used to place unique constraints on  $\eta_w$ .

Physically, different scalings of  $\eta_w$  and  $\zeta_w$  (and thus  $Z_g$  and  $Z_w$ ) indicate that the entrainment fraction  $f_e$  (equation 14) varies with galaxy mass, offering a clue to the physics of galaxy outflows. If, for example,  $f_e$  increases with increasing gas mass (and thus galaxy mass), it would indicate that the wind fluid does not “punch” through a blanketing column density of gas but instead sweeps up this material and expels it from the galaxy. On the other hand,  $f_e$  decreasing with increasing galaxy mass, would indicate that the ability of supernova ejecta to collect the surrounding ISM into the wind fluid depends on the depth of the galaxy potential well. We find the former to be the case: to reconcile a steep  $\zeta_w$  scaling with a shallower  $\eta_w$  scaling, then winds driven from deeper potential wells must be *more* efficient at entraining the ambient ISM than those driven from shallow potential wells. We also find that in order to have the normalization of  $\eta_w$  be consistent with the normalizations suggested in §2.3 (i.e.,  $v_0 \sim 70 \text{ km s}^{-1}$ ) then the entrainment fraction must be  $\sim 1$ , though the exact value is dependent on the value of  $Z_{\text{ej,max}}$ . This is particularly interesting in light of interpretations of X-ray emitting outflows in which the wind fluid is almost entirely comprised of supernova ejecta, i.e.,  $f_e \sim 0$  (Strickland & Heckman 2009). Because iron is primarily not made in Type II supernovae and thus likely affected differently by star formation driven outflows than  $\alpha$ -elements, stellar  $[\alpha/\text{Fe}]$  variations could also be used to shed light on the oxygen expulsion efficiency of galaxy winds (Recchi et al. 2009).

## 6 SUMMARY AND DISCUSSION

### 6.1 The approach: modeling a system of galaxies

We use a simple model of the  $z = 0$  mass-metallicity relation to place constraints on star formation driven galaxy outflows. In this formalism (§3.1 and equation 9), the gas phase (oxygen) metallicity  $Z_g$  of star forming galaxies is

$$Z_g = y[\zeta_w - \zeta_a + \alpha F_g + 1]^{-1}, \quad (25)$$

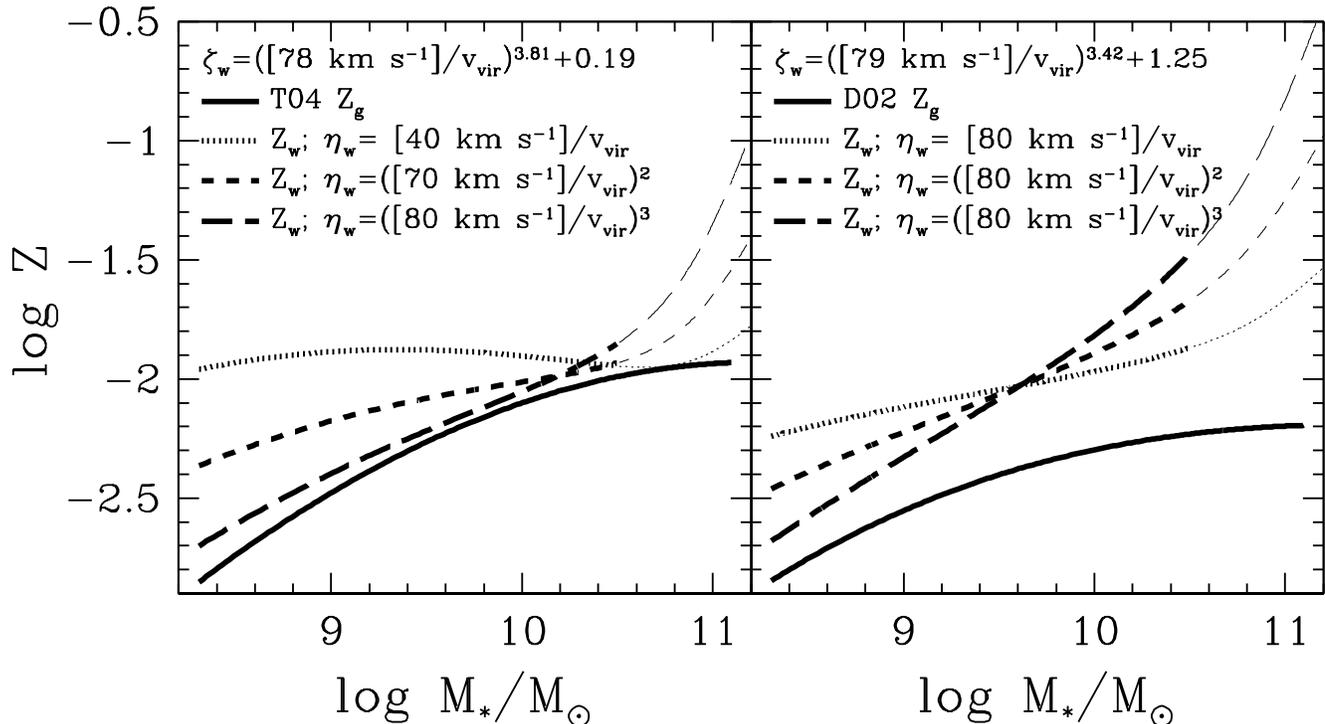
where  $y$  is the nucleosynthetic yield,  $\zeta_a$  describes accreting metals,  $\zeta_w$  describes the efficiency of metal expulsion,  $F_g$  describes dilution by gas, and  $\alpha$  is a factor of order unity (see equation 11). In the absence of metal accretion ( $\zeta_a = 0$ ), equation (25) shows that the metallicity  $Z_g$  is set by a balance of outflows ( $\zeta_w$ ) and gas dilution ( $\alpha F_g$ ), with the normalization set by the nucleosynthetic yield  $y$ . This equation represents a general result: each piece can vary with galaxy mass, halo mass, and redshift. To the extent that the star formation history is not bursty, i.e.,  $\dot{M}_{\text{SFR}}$  varies slowly on timescales of 10 Myr then the yield  $y$  can be taken as constant with time, letting equation (25) describe the instantaneous state of a sequence of galaxies. Galaxies at  $z = 0$  are assumed to live on a hypersurface described by their stellar masses, gas fractions, metallicities, outflow and host halo properties. By taking gas fractions and metallicities from observations, we are therefore able to uniquely solve for outflow properties in terms of galaxy masses or metallicities (that are therefore easily comparable to observations) or in terms of the galaxy potential (and therefore easily comparable to models of the underlying wind physics). The only fitting of models to data in this approach is that of functional forms to observations of the mass-metallicity relation (e.g., Kewley & Ellison 2008) and either models or parameterizations to gas fractions as a function of stellar mass (§2.2). Because there is theoretical uncertainty in which metallicity indicator(s) to use when calculating the mass-metallicity relation from data, we do not favor a particular indicator when drawing our conclusions, and specifically state which constraints come from which pieces of the mass-metallicity relation.

### 6.2 Resulting constraints

We consider implications for both the efficiency of star-formation driven galaxy outflows and for the content of the outflowing material. The two relevant outflow efficiencies are the efficiency with which a galaxy expels its metals,  $\zeta_w \equiv (Z_w/Z_g)(\dot{M}_w/\dot{M}_{\text{SFR}})$ , which we parameterize as  $\zeta_w = (v_0/v_{\text{vir}})^b + \zeta_{w,0}$ . The second relevant efficiency is that with which a galaxy expels its gas, the unweighted mass-loading parameter  $\eta_w \equiv \dot{M}_w/\dot{M}_{\text{SFR}}$ , which we similarly parameterize as  $\eta_w = (\sigma_0/v_{\text{vir}})^\beta$ , where  $\beta$  is predicted to be  $\sim 1$  or  $\sim 2$  with  $\sigma_0 = 70\text{--}80 \text{ km s}^{-1}$  (§2.3). The content of the wind is observed by its metallicity  $Z_w$ , which can be expressed in terms of the fraction of entrained ISM in the outflow,  $f_e$ , where  $Z_w = (1 - f_e)Z_{\text{ej,max}} + f_e Z_g$  (equation 14 in §3.1). Under the assumption that  $Z_{\text{IGM}} = 0$ , we draw the following conclusions by requiring that viable models reproduce both the  $z = 0$  mass-metallicity relation and are consistent with observed cold gas fractions.

#### 6.2.1 The necessity of outflows

Models with no outflows ( $\dot{M}_w = 0 \Rightarrow \zeta_w = 0$ ) are inconsistent with observed galaxy gas fractions, if the yield  $y$  is constant. Specifically, in the absence of winds, the gas masses needed to dilute the produced metals are higher at all galaxy masses than the total observed cold gas masses; the magnitude of this offset is as great as  $\sim 0.3$  dex in  $F_g \equiv M_g/M_*$ , depending on the particular mass-metallicity relation being modeled.



**Figure 8.** Wind metallicities  $Z_w$  for the best-fit  $\zeta_w$  T04 (left) and D02 (right) mass-metallicity relations (see Figure 6). The solid line corresponds to  $\zeta_w = \eta_w$  and therefore  $Z_w = Z_g$  and  $f_e = 1$ ; different scalings for  $\eta_w = (\sigma_0/v_{\text{vir}})^\beta$  are shown as the dotted ( $\beta = 1$ ), short-dashed ( $\beta = 2$ ), and long-dashed ( $\beta = 3$ ) lines.

### 6.2.2 Constraints from the normalization of the mass-metallicity relation

Equation (25) makes it clear that the nucleosynthetic yield sets the normalization of the mass-metallicity relation. From a modeling perspective, it is useful to consider the mass-metallicity relation normalization relative to the yield (rather than their absolute values) because the true nucleosynthetic yield is unknown to a factor of two due to uncertainties in both the IMF and in Type II supernova physics (e.g., Thomas et al. 1998). Likewise, the overall normalization of the mass-metallicity relation (§ 2.1) is unknown at the  $\sim 0.3$  dex level.<sup>5</sup> In the constant  $y$  framework, the normalization of  $y/Z_g$  sets the value of the constant offset  $\zeta_{w,0} > 0$  (which is set by the turnover of the mass-metallicity relation, see below). The typical required velocity normalization  $v_0 \sim 70\text{--}80 \text{ km s}^{-1}$  is consistent with expectations.

Low normalization mass-metallicity relations require  $\zeta_w > 1$  for all relevant masses; if the true nucleosynthetic yield is larger than our fiducial value ( $y > 0.015$ ), then the efficiency with which galaxies expel metals will have to be even stronger. Thus if normal quiescently star forming galaxies are not expelling winds with  $\zeta_w \gg 1$ , then the data prefer a low nucleosynthetic yield and a high normalization of the mass-metallicity relation. Furthermore, because the mass-metallicity relation shifts to lower normalizations at

higher redshifts, galaxies at these epochs must have either stronger winds or higher gas fractions than their  $z = 0$  counterparts.

In § 4.2, we explored the possibility that the nucleosynthetic yield  $y$  could vary with star formation rate and thus  $M_*$ . While the possible  $y(M_*)$  relations are still highly uncertain and the models are much more susceptible to the  $F_g\text{--}M_*$  relation, we find that a wide range of such models have outflows that are more efficient in high mass galaxies than in lower mass galaxies. Observations of such a trend would be compelling evidence that  $y$  varies strongly with galaxy mass.

### 6.2.3 Constraints from the morphology of the mass-metallicity relation

The morphology of the mass-metallicity relation has two main features: the slope below  $\sim M^*$  and the turnover at higher masses. In the constant  $y$  case, the slope of the mass-metallicity relation largely determines how  $\zeta_w$  scales with galaxy mass, though with some degeneracies with the normalization and constant offset. For small  $F_g$ , as is the case at  $z = 0$ ,  $\zeta_w$  should scale roughly as  $Z_g^{-1} \sim M_*^{-0.3}$  to  $M_*^{-0.4}$ . The power-law scaling of  $\zeta_w$  with respect to  $v_{\text{vir}}$  is typically  $b \sim 3$ . The required scalings with respect to  $M_*$  are fairly robust, while the scaling with respect to  $v_{\text{vir}}$  depends on our assumed  $M_*\text{--}M_{\text{halo}}$  relation; if this relation is significantly different from that derived from the abundance matching technique, then the  $\zeta_w \propto v_{\text{vir}}^{-3}$  scaling might be able to be relaxed. Regardless, this need for a high and mass-dependent wind efficiency is in broad agreement with previous studies (Dekel & Woo 2003; Dutton et al. 2010b; Sawala et al. 2010; Spitoni et al. 2010).

<sup>5</sup> Though neither the nucleosynthetic yield nor the normalization of the mass-metallicity relation are well determined, the scatter in  $\log Z_g$  at fixed  $M_*$  is known to be  $\pm 0.1$  dex (Kewley & Ellison 2008). In light of the formalism presented here, this small scatter implies that either the scatter in both  $\alpha F_g$  and  $\zeta_w$  are small, or they are highly correlated.

The turnover<sup>6</sup> in the mass-metallicity relation at  $\log M_* \sim 10.5$  may be an observational artifact of the metallicity indicators saturating at high  $Z_g$  (§ 2.1); however, if oxygen abundances do asymptote to a particular value at high masses, then this behavior can be used to place strong constraints on galaxy outflow properties. Specifically, both the normalization of the mass-metallicity relation relative to the yield ( $\max[Z_g/y]$ ) and the effects of  $v_{\text{vir}}$  increasing sharply above  $M^* \sim 10^{11} M_\odot$  (Figure 3) must be then taken into consideration; moreover, the interplay between these effects can place stronger constraints on viable models than just considerations of the mass-metallicity relation below  $10^{10.5} M_\odot$ . Morphologically, a turnover in the mass-metallicity relation means that either  $\alpha F_g$  or  $\zeta_w$  cannot be approximated as a power-law. Because cold gas fractions are observed to roughly follow a power-law with respect to  $M_*$ , then  $\zeta_w$  needs a constant offset  $\zeta_{w,0} \sim 0.2\text{--}1.5$ , depending on which indicator is used to calculate the mass-metallicity relation and/or the yield. In several cases, if  $M_g \propto (\Omega_b/\Omega_m) M_{\text{halo}}$ , then  $\zeta_w$  can be described as a power-law; physically, this would imply that galaxies above  $M^*$  have large reservoirs of ionized gas that are able to efficiently transfer mass with colder, star-forming gas.

If  $\zeta_w$  and  $\eta_w$  scale differently, then the fraction of entrained ISM in the wind fluid will vary with galaxy mass. Observationally this will be seen as  $Z_w/Z_g$  varying with mass. As the morphology of the mass-metallicity relation constrains the scaling of  $\zeta_w$  with mass, the scaling of  $Z_w$  and thus  $\eta_w$  with mass therefore depends on the slope of the mass-metallicity relation. For example (see Figure 8), for a fixed  $\eta_w$ , a steep mass-metallicity relation will lead to a shallower  $Z_w$ - $M_*$  relation than a shallower mass-metallicity relation will. However, since current uncertainties in the slope of the mass-metallicity relation are smaller than uncertainties in how (or if)  $\eta_w$  scales with mass, measurements of  $Z_w$  across a large range in galaxy mass, especially above  $M^*$ , will be particularly useful for constraining how  $\eta_w$  (and  $\zeta_w$ ) scale.

### 6.3 The role of metal-(re)accretion

At  $z = 0$ , the assumption that accreting material has a negligible metal content (i.e., that  $Z_{\text{IGM}} = 0$  and therefore  $\zeta_a = 0$ ) may not be entirely safe. The IGM is enriched as early as  $z > 3$  (Songaila & Cowie 1996; Ellison et al. 2000; Schaye et al. 2003), and if this material is re-accreted onto galaxies at later epochs it could have a significant effect on the shape and normalization of the  $z = 0$  mass-metallicity relation. The re-accretion of winds (i.e., gas with  $Z_{\text{IGM}} > 0$ ) is a significant component of accreted gas in cosmological SPH simulations (Oppenheimer et al. 2009). Though the total accretion rate scales with halo mass ( $\dot{M}_{\text{acc}} \propto M_{\text{halo}} \propto v_{\text{vir}}^3$ , see Appendix B), the contribution of accreted metals to the mass-metallicity relation may not scale so steeply (Finlator & Davé 2008). Moreover, an extra source of metals  $\zeta_a$  will imply that the overall efficiency of outflows (i.e., the amplitude of  $\zeta_w$ ) will need to be even higher than the ones presented here. However, the re-accretion of wind material seen in SPH simulations may be sensitive to numerical issues in the wind implementation; more detailed investigations are needed to verify the importance of wind-recycling. The metal budget available for re-accretion depends on both the amount of metals expelled at higher redshifts and the recycling timescale. We will address the metal content of winds at  $z > 0$  as

implied by the evolution of the mass-metallicity relation in a later paper.

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### REFERENCES

- Adelman-McCarthy, J. K., et al. 2006, *ApJS*, 162, 38  
 Baes, M., Buyle, P., Hau, G. K. T., & Dejonghe, H. 2003, *MNRAS*, 341, L44  
 Barkana, R. & Loeb, A. 2001, *PhysRep*, 349, 125  
 Bernardi, M., Shankar, F., Hyde, J. B., Mei, S., Marulli, F., & Sheth, R. K. 2010, *MNRAS*, 436  
 Boomsma, R., Oosterloo, T. A., Fraternali, F., van der Hulst, J. M., & Sancisi, R. 2008, *A&A*, 490, 555  
 Bresolin, F. 2006, *ArXiv Astrophysics e-prints*  
 Brinchmann, J., Charlot, S., White, S. D. M., Tremonti, C., Kauffmann, G., Heckman, T., & Brinkmann, J. 2004, *MNRAS*, 351, 1151  
 Brooks, A. M., Governato, F., Booth, C. M., Willman, B., Gardner, J. P., Wadsley, J., Stinson, G., & Quinn, T. 2007, *ApJL*, 655, L17  
 Bryan, G. L. & Norman, M. L. 1998, *ApJ*, 495, 80  
 Calura, F. & Menci, N. 2009, *ArXiv e-prints*  
 Calura, F., Pipino, A., Chiappini, C., Matteucci, F., & Maiolino, R. 2009, *A&A*, 504, 373  
 Catinella, B., Schiminovich, D., Kauffmann, G., Fabello, S., Wang, J., Hummels, C., Lemonias, J., Moran, S. M., Wu, R., Giovanelli, R., Haynes, M. P., Heckman, T. M., Basu-Zych, A. R., Blanton, M. R., Brinchmann, J., Budavári, T., Gonçalves, T., Johnson, B. D., Kennicutt, R. C., Madore, B. F., Martin, C. D., Rich, M. R., Tacconi, L. J., Thilker, D. A., Wild, V., & Wyder, T. K. 2010, *MNRAS*, 403, 683  
 Chabrier, G. 2003a, *PASP*, 115, 763  
 —. 2003b, *ApJL*, 586, L133  
 Chevalier, R. A. & Clegg, A. W. 1985, *Nature*, 317, 44  
 Conroy, C. & Wechsler, R. H. 2009, *ApJ*, 696, 620  
 Crain, R. A., Eke, V. R., Frenk, C. S., Jenkins, A., McCarthy, I. G., Navarro, J. F., & Pearce, F. R. 2007, *MNRAS*, 377, 41  
 Dalcanton, J. J. 2007, *ApJ*, 658, 941  
 Dekel, A. & Birnboim, Y. 2006, *MNRAS*, 368, 2  
 Dekel, A. & Silk, J. 1986, *ApJ*, 303, 39  
 Dekel, A. & Woo, J. 2003, *MNRAS*, 344, 1131

<sup>6</sup> The turn-“up” at low masses for the Z94 mass-metallicity relation is unphysical and due to the cubic fit to the data.

- Denicoló, G., Terlevich, R., & Terlevich, E. 2002, *MNRAS*, 330, 69
- Diemand, J., Kuhlen, M., & Madau, P. 2007, *ApJ*, 667, 859
- Dutton, A. A., Conroy, C., van den Bosch, F. C., Prada, F., & More, S. 2010a, *ArXiv e-prints*
- Dutton, A. A., van den Bosch, F. C., & Dekel, A. 2010b, *MNRAS*, 608
- Ellison, S. L., Songaila, A., Schaye, J., & Pettini, M. 2000, *AJ*, 120, 1175
- Erb, D. K. 2008, *ApJ*, 674, 151
- Erb, D. K., Shapley, A. E., Pettini, M., Steidel, C. C., Reddy, N. A., & Adelberger, K. L. 2006a, *ApJ*, 644, 813
- Erb, D. K., Steidel, C. C., Shapley, A. E., Pettini, M., Reddy, N. A., & Adelberger, K. L. 2006b, *ApJ*, 646, 107
- Ferrarese, L. 2002, *ApJ*, 578, 90
- Finlator, K. & Davé, R. 2008, *MNRAS*, 385, 2181
- Firmani, C., Avila-Reese, V., & Rodriguez-Puebla, A. 2009, *ArXiv e-prints*
- Garcia-Appadoo, D. A., West, A. A., Dalcanton, J. J., Cortese, L., & Disney, M. J. 2009, *MNRAS*, 394, 340
- Genel, S., Genzel, R., Bouché, N., Sternberg, A., Naab, T., Schreiber, N. M. F., Shapiro, K. L., Tacconi, L. J., Lutz, D., Cresci, G., Buschkamp, P., Davies, R. I., & Hicks, E. K. S. 2008, *ApJ*, 688, 789
- Grimes, J. P., Heckman, T., Aloisi, A., Calzetti, D., Leitherer, C., Martin, C. L., Meurer, G., Sembach, K., & Strickland, D. 2009, *ApJS*, 181, 272
- Guo, Q., White, S., Li, C., & Boylan-Kolchin, M. 2010, *MNRAS*, 404, 1111
- Heckman, T. M. 2003, in *Revista Mexicana de Astronomia y Astrofisica*, vol. 27, Vol. 17, *Revista Mexicana de Astronomia y Astrofisica Conference Series*, ed. V. Avila-Reese, C. Firmani, C. S. Frenk, & C. Allen, 47–55
- Heckman, T. M., Lehnert, M. D., Strickland, D. K., & Armus, L. 2000, *ApJS*, 129, 493
- Hinshaw, G., Weiland, J. L., Hill, R. S., Odegard, N., Larson, D., Bennett, C. L., Dunkley, J., Gold, B., Greason, M. R., Jarosik, N., Komatsu, E., Nolte, M. R., Page, L., Spergel, D. N., Wollack, E., Halpern, M., Kogut, A., Limon, M., Meyer, S. S., Tucker, G. S., & Wright, E. L. 2009, *ApJS*, 180, 225
- Hopkins, P. F., Hernquist, L., Cox, T. J., Dutta, S. N., & Rothberg, B. 2008, *ApJ*, 679, 156
- Kennicutt, Jr., R. C. 1998, *ApJ*, 498, 541
- Kennicutt, Jr., R. C., Bresolin, F., & Garnett, D. R. 2003, *ApJ*, 591, 801
- Kereš, D., Katz, N., Fardal, M., Davé, R., & Weinberg, D. H. 2009, *MNRAS*, 395, 160
- Kereš, D., Katz, N., Weinberg, D. H., & Davé, R. 2005, *MNRAS*, 363, 2
- Kewley, L. J. & Dopita, M. A. 2002, *ApJS*, 142, 35
- Kewley, L. J. & Ellison, S. L. 2008, *ApJ*, 681, 1183
- Kobulnicky, H. A. & Kewley, L. J. 2004, *ApJ*, 617, 240
- Köppen, J., Weidner, C., & Kroupa, P. 2007, *MNRAS*, 375, 673
- Kroupa, P. 2001, *MNRAS*, 322, 231
- Kroupa, P. & Weidner, C. 2003, *ApJ*, 598, 1076
- Lada, C. J. & Lada, E. A. 2003, *ARA&A*, 41, 57
- Lara-López, M. A., Cepa, J., Bongiovanni, A., Pérez García, A. M., Ederoclite, A., Castañeda, H., Fernández Lorenzo, M., Póvic, M., & Sánchez-Portal, M. 2010, *ArXiv e-prints*
- Larson, R. B. 2003, in *Astronomical Society of the Pacific Conference Series*, Vol. 287, *Galactic Star Formation Across the Stellar Mass Spectrum*, ed. J. M. De Buizer & N. S. van der Blied, 65–80
- Lee, J. C., Gil de Paz, A., Tremonti, C., Kennicutt, R. C., Salim, S., Bothwell, M., Calzetti, D., Dalcanton, J., Dale, D., Engelbracht, C., Funes, S. J. J. G., Johnson, B., Sakai, S., Skillman, E., van Zee, L., Walter, F., & Weisz, D. 2009, *ApJ*, 706, 599
- Leroy, A. K., Walter, F., Brinks, E., Bigiel, F., de Blok, W. J. G., Madore, B., & Thornley, M. D. 2008, *AJ*, 136, 2782
- Loeb, A. & Peebles, P. J. E. 2003, *ApJ*, 589, 29
- Lokas, E. L. & Mamon, G. A. 2001, *MNRAS*, 321, 155
- Mannucci, F., Cresci, G., Maiolino, R., Marconi, A., & Gnerucci, A. 2010, *ArXiv e-prints*
- Martin, C. L. 2005, *ApJ*, 621, 227
- Martin, C. L. & Bouché, N. 2009, *ApJ*, 703, 1394
- Matteucci, F. 2002, *ArXiv Astrophysics e-prints*
- McGaugh, S. S. 1991, *ApJ*, 380, 140
- . 2005, *ApJ*, 632, 859
- Meurer, G. R., Wong, O. I., Kim, J. H., Hanish, D. J., Heckman, T. M., Werk, J., Bland-Hawthorn, J., Dopita, M. A., Zwaan, M. A., Koribalski, B., Seibert, M., Thilker, D. A., Ferguson, H. C., Webster, R. L., Putman, M. E., Knezek, P. M., Doyle, M. T., Drinkwater, M. J., Hoopes, C. G., Kilborn, V. A., Meyer, M., Ryan-Weber, E. V., Smith, R. C., & Staveley-Smith, L. 2009, *ApJ*, 695, 765
- More, S., van den Bosch, F. C., Cacciato, M., Skibba, R., Mo, H. J., & Yang, X. 2010, *ArXiv e-prints*
- Moster, B. P., Somerville, R. S., Maulbetsch, C., van den Bosch, F. C., Maccio, A. V., Naab, T., & Oser, L. 2009, *ArXiv e-prints*
- Murray, N., Quataert, E., & Thompson, T. A. 2005, *ApJ*, 618, 569
- Navarro, J. F., Frenk, C. S., & White, S. D. M. 1996, *ApJ*, 462, 563
- Neistein, E., Li, C., Khochfar, S., Weinmann, S. M., Shankar, F., & Boylan-Kolchin, M. 2011, *ArXiv e-prints*
- Neistein, E., van den Bosch, F. C., & Dekel, A. 2006, *MNRAS*, 372, 933
- Oppenheimer, B. D., Davé, R., Kereš, D., Fardal, M., Katz, N., Kollmeier, J. A., & Weinberg, D. H. 2009, *ArXiv e-prints*
- Peebles, M. S., Pogge, R. W., & Stanek, K. Z. 2008, *ApJ*, 685, 904
- . 2009, *ApJ*, 695, 259
- Pettini, M. & Pagel, B. E. J. 2004, *MNRAS*, 348, L59
- Pizagno, J., Prada, F., Weinberg, D. H., Rix, H., Pogge, R. W., Grebel, E. K., Harbeck, D., Blanton, M., Brinkmann, J., & Gunn, J. E. 2007, *AJ*, 134, 945
- Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, *Numerical Recipes in C* (Cambridge University Press: Cambridge)
- Recchi, S., Calura, F., & Kroupa, P. 2009, *A&A*, 499, 711
- Rupke, D. S., Veilleux, S., & Sanders, D. B. 2002, *ApJ*, 570, 588
- Salpeter, E. E. 1955, *ApJ*, 121, 161
- Salucci, P., Lapi, A., Tonini, C., Gentile, G., Yegorova, I., & Klein, U. 2007, *MNRAS*, 378, 41
- Samui, S., Subramanian, K., & Srianand, R. 2008, *MNRAS*, 385, 783
- Sawala, T., Scannapieco, C., Maio, U., & White, S. 2010, *MNRAS*, 402, 1599
- Schaye, J., Aguirre, A., Kim, T., Theuns, T., Rauch, M., & Sargent, W. L. W. 2003, *ApJ*, 596, 768
- Schiminovich, D., Catinella, B., Kauffmann, G., Fabello, S., Wang, J., Hummels, C., Lemonias, J., Moran, S. M., Wu, R., Giovanelli, R., Haynes, M. P., Heckman, T. M., Basu-Zych, A. R., Blanton, M. R., Brinchmann, J., Budavari, T., Goncalves, T., Johnson, B. D., Kennicutt, R. C., Madore, B. F., Martin, C. D.,

Rich, M. R., Tacconi, L. J., Thilker, D. A., Wild, V., & Wyder, T. K. 2010, ArXiv e-prints

Schmidt, M. 1959, ApJ, 129, 243

Shankar, F., Lapi, A., Salucci, P., De Zotti, G., & Danese, L. 2006, ApJ, 643, 14

Silk, J. & Rees, M. J. 1998, A&A, 331, L1

Socrates, A., Davis, S. W., & Ramirez-Ruiz, E. 2008, ApJ, 687, 202

Songaila, A. & Cowie, L. L. 1996, AJ, 112, 335

Spitoni, E., Calura, F., Matteucci, F., & Recchi, S. 2010, ArXiv e-prints

Spoon, H. W. W. & Holt, J. 2009, ApJL, 702, L42

Springel, V. 2005, MNRAS, 364, 1105

Strickland, D. K. & Heckman, T. M. 2007, ApJ, 658, 258

—. 2009, ApJ, 697, 2030

Strickland, D. K. & Stevens, I. R. 2000, MNRAS, 314, 511

Thielemann, F., Nomoto, K., & Hashimoto, M. 1996, ApJ, 460, 408

Thomas, D., Greggio, L., & Bender, R. 1998, MNRAS, 296, 119

Tinsley, B. M. 1974, ApJ, 192, 629

—. 1980, FCPH, 5, 287

Tonini, C., Lapi, A., Shankar, F., & Salucci, P. 2006, ApJL, 638, L13

Tremonti, C. A., Heckman, T. M., Kauffmann, G., Brinchmann, J., Charlot, S., White, S. D. M., Seibert, M., Peng, E. W., Schlegel, D. J., Uomoto, A., Fukugita, M., & Brinkmann, J. 2004, ApJ, 613, 898

Tully, R. B. & Fisher, J. R. 1977, A&A, 54, 661

Vale, A. & Ostriker, J. P. 2004, MNRAS, 353, 189

Veilleux, S., Cecil, G., & Bland-Hawthorn, J. 2005, ARA&A, 43, 769

Wadsley, J. W., Stadel, J., & Quinn, T. 2004, New Astronomy, 9, 137

Walter, F., Brinks, E., de Blok, W. J. G., Bigiel, F., Kennicutt, R. C., Thornley, M. D., & Leroy, A. 2008, AJ, 136, 2563

Weidner, C. & Kroupa, P. 2006, MNRAS, 365, 1333

Weidner, C., Kroupa, P., & Larsen, S. S. 2004, MNRAS, 350, 1503

West, A. A., Garcia-Appadoo, D. A., Dalcanton, J. J., Disney, M. J., Rockosi, C. M., & Ivezić, Ž. 2009, AJ, 138, 796

West, A. A., Garcia-Appadoo, D. A., Dalcanton, J. J., Disney, M. J., Rockosi, C. M., Ivezić, Ž., Bentz, M. C., & Brinkmann, J. 2010, AJ, 139, 315

Woosley, S. E. & Weaver, T. A. 1995, ApJS, 101, 181

Zaritsky, D., Kennicutt, Jr., R. C., & Huchra, J. P. 1994, ApJ, 420, 87

## APPENDIX A: INVERTING THE K-S RELATION

The observed Kennicutt-Schmidt (K-S, Kennicutt 1998; Schmidt 1959) relation is commonly used to indirectly estimate gas masses in star-forming galaxies in chemical evolution models (e.g., Spitoni et al. 2010), when direct gas masses are expensive (or currently impossible) to achieve (such as at high redshifts, Erb et al. 2006b), or for large samples of galaxies (e.g., Tremonti et al. 2004). Furthermore, since  $12+\log(\text{O}/\text{H})$  is measured only in star-forming gas, it is reasonable to consider gas fractions that trace this same gas. The purple lines in Figure 2 (see also Figures 4, and D2) are the gas masses we derive from applying the K-S law to star-forming Data Release 4 SDSS galaxies with  $z$ -band magnitude errors of  $< 0.01$  mag (Brinchmann et al. 2004; Adelman-McCarthy et al.

2006). Specifically, we relate the star formation rate surface density  $\Sigma_{\text{SFR}}$  to the gas surface density  $\Sigma_g$  by

$$\begin{aligned} \Sigma_{\text{SFR}} &\equiv \frac{\dot{M}_{\text{SFR}}}{A_g} = K_g \Sigma_g^\alpha & (\text{A1}) \\ &= 1.67 \times 10^{-4} \left( \frac{\Sigma_g}{1 \text{M}_\odot \text{pc}^{-2}} \right)^{1.4} \text{M}_\odot \text{yr}^{-1} \text{kpc}^{-2} \end{aligned}$$

from Kennicutt (1998), where we have corrected for the fact that the Brinchmann et al. (2004) star formation rates are based on a Kroupa (2001) IMF while the Kennicutt relation is based on a Salpeter (1955) IMF. SDSS spectra are taken within a  $3''$  aperture; therefore, to measure *total* galaxy properties (e.g., star formation rates and stellar masses), the fact that the aperture does not subtend the entire galaxy must be corrected for. We therefore consider  $\Sigma_{\text{SFR}}$  and  $M_*$  both for the full galaxy-light radius (which we take to be 1.1 times the 90th percentile  $z$ -band isophotal radius  $R_{90,z}$ ) and only within the fiber, i.e., we take

$$A_g = \pi R_g^2 = \pi R_{\text{light}}^2 = \pi \times \begin{cases} 1.1^2 \times R_{90,z}^2; & \text{solid lines.} \\ R_{\text{fiber}}^2; & \text{dashed line.} \end{cases} \quad (\text{A2})$$

The galaxy gas mass is then simply

$$M_g = \left( \dot{M}_{\text{SFR}} \times \frac{A_g^{\alpha-1}}{K_g} \right)^{1/\alpha}. \quad (\text{A3})$$

## APPENDIX B: OUTFLOWS, INFLOWS, AND STAR FORMATION: GETTING THE GAS MASSES

As shown in § 3.1,

$$\dot{M}_g = \dot{M}_{\text{acc}} - \dot{M}_{\text{SFR}} + \dot{M}_{\text{recy}} - \dot{M}_w \quad (\text{B1})$$

$$= \dot{M}_{\text{SFR}}(\eta_a - 1 + f_{\text{recy}} - \eta_w), \quad (\text{B2})$$

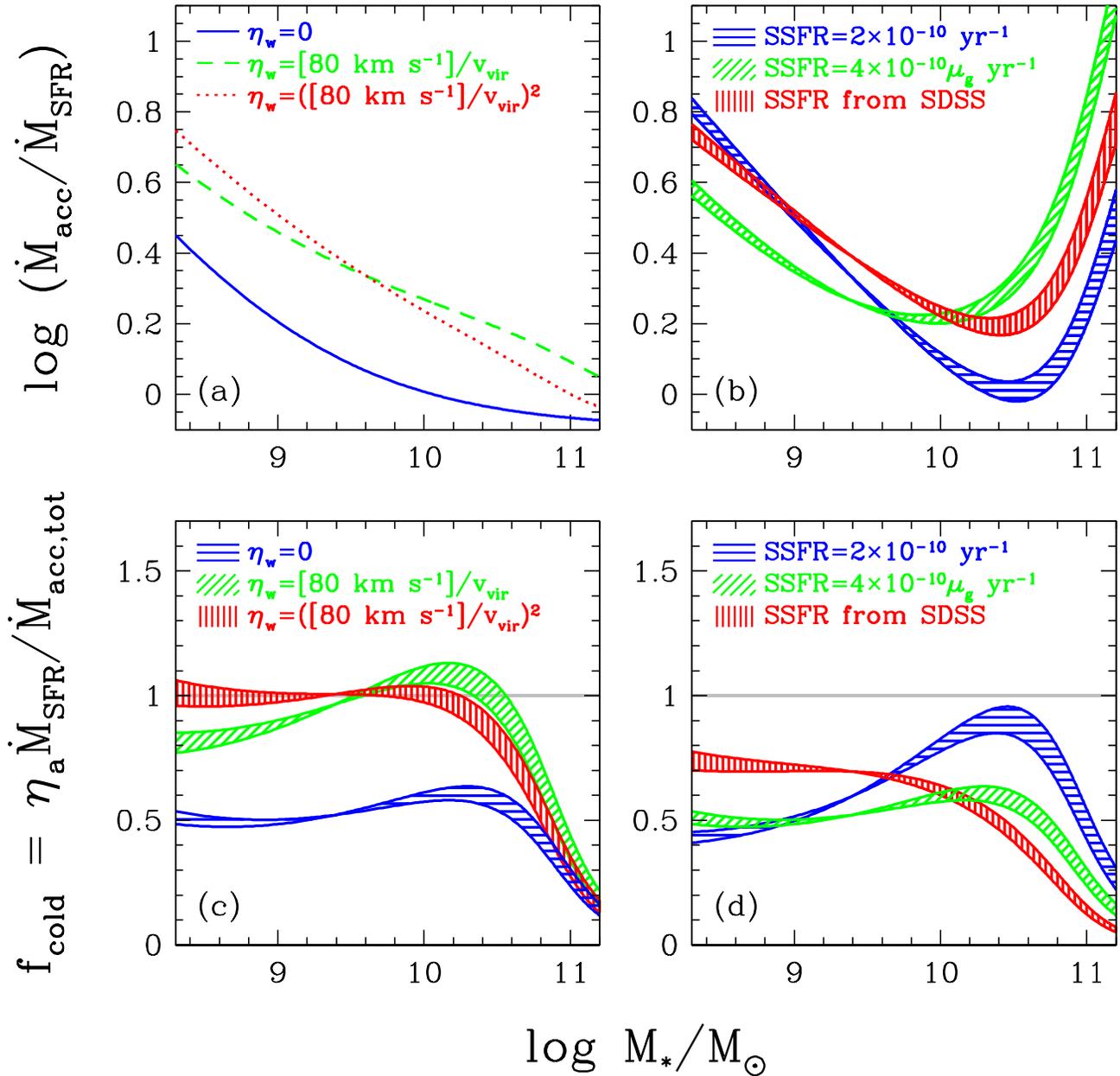
and

$$\frac{dM_g}{dM_*} = \frac{\eta_a - \eta_w - 1 + f_{\text{recy}}}{1 - f_{\text{recy}}} = F_g(1 - \gamma). \quad (\text{B3})$$

In § 4 we assumed an  $F_g$ - $M_*$  relation existed and that as galaxies evolve they remain on such a relation. Here we consider, for a given  $\eta_w$ , what implications such a relation has on the gas accretion rate and how efficiently galaxies are able to turn this accreted gas into stars. The above equations imply that the gas inflow and outflow rates must be balanced by

$$\eta_a - \eta_w = (1 - f_{\text{recy}})F_g(1 - \gamma) - f_{\text{recy}} + 1. \quad (\text{B4})$$

Thus, for a given combination of  $\eta_w$  and  $F_g$ , we can uniquely determine  $\eta_a \equiv \dot{M}_{\text{acc}}/\dot{M}_{\text{SFR}}$ , i.e., the efficiency with which a galaxy turns its accreted gas into stars. For example, if the star formation rate is higher than the accretion rate ( $\log \eta_a < 0$ ), then the galaxy is forming stars more quickly than it is accreting gas, i.e. it is very efficient at forming stars. We plot  $\log \eta_a$  for the no wind,  $\eta_w = [70 \text{ km s}^{-1}]/v_{\text{vir}}$ , and  $\eta_w = ([70 \text{ km s}^{-1}]/v_{\text{vir}})^2$  cases as a function of stellar mass in the upper-left panel of Figure B1 for the total gas fraction relation (see Table 3 and Figure 2). Strikingly,  $\eta_a$  always decreases significantly with increasing mass—even in the absence of winds (solid blue line). This behavior follows directly from the steepness of the gas fraction relation (equation B4). When outflows that preferentially remove gas from low-mass galaxies ( $\eta_w \propto v_{\text{vir}}^{-1}$ , green dashed line;  $v_{\text{vir}}^{-2}$ , red dotted line) are taken into account,  $\eta_a$  likewise increases and steepens to compensate. Therefore, while winds may affect how star-formation efficiency varies

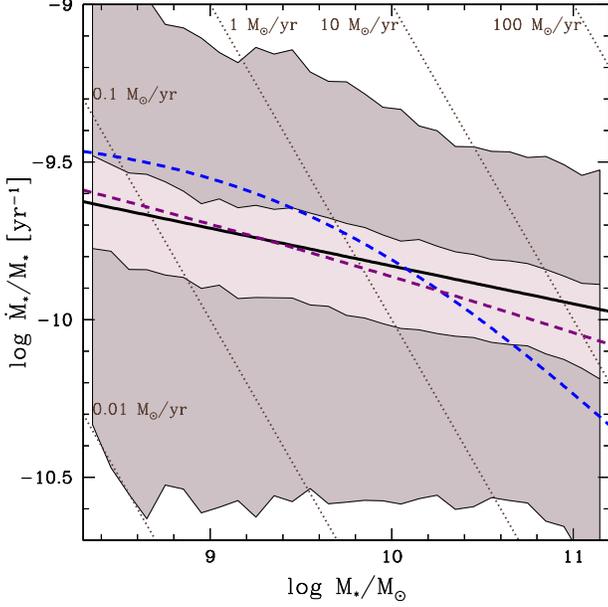


**Figure B1.** Gas accretion rates, star formation rates, and cold gas accretion fractions as a function of stellar mass with varying outflows and specific star formation rates. All panels assume the total cold gas fractions described in § 2.2. Panel (a) shows how  $\eta_a$  varies according to equation (B4) for different  $\eta_w$  models: no wind (solid blue), a momentum-driven scaling (green dashed), and an energy-driven scaling (red dotted). In all cases, high mass galaxies accrete less gas per unit star formation than less massive galaxies. Panel (b) shows the expected range in  $\dot{M}_{\text{acc,tot}}/\dot{M}_{\text{SFR}}$  between the Neustein et al. (2006) and Genel et al. (2008)  $\dot{M}_{\text{acc}}$  models (shaded regions) and with three scalings of  $\dot{M}_{\text{SFR}}/M_*$  with stellar mass: constant (blue),  $\propto \mu_g$  (green), and the median values from SDSS (red). These  $\dot{M}_{\text{acc,tot}}/\dot{M}_{\text{SFR}}$  are qualitatively similar at low masses to the  $\eta_a$  shown in panel (a), but increase rapidly at high masses. Panels (c) and (d) show the ratio  $f_{\text{cold}}$  of these two estimates, with varying  $\eta_w$  and the SDSS SSFRs and with varying the SSFR and no winds, respectively.

as a function of galaxy mass, they are not necessary to explain the trend, implying that additional physics is at play.

This analysis does not entirely reveal what drives the  $\eta_a$ - $M_*$  relation. However, the nature of  $\eta_a$  can be unraveled by appealing to  $\dot{M}_{\text{SFR}}$  and  $\dot{M}_{\text{acc}}$  from independent sources. For example, as shown in Figure B2, the median specific star formation rate (SSFR,  $\dot{M}_{\text{SFR}}/M_*$ ) in SDSS DR4 star-forming galaxies decreases with increasing  $M_*$ , though there is large scatter in the SSFR at fixed  $M_*$ .

We consider here three scalings for how the SSFR may vary with  $M_*$ . The median SSFRs from SDSS are shown as the solid line in Figure B2. A physically-motivated way to have the SSFR to decrease with mass is to postulate that it is proportional to the total gas fraction,  $\mu_g$ . The blue dashed line shows  $\mu_g \times 4 \times 10^{-10} \text{ yr}$  for the total gas fractions, while the purple dashed line shows  $\mu_g \times 1 \times 10^{-9} \text{ yr}$  for the SDSS gas fractions (note that the SDSS gas fractions were derived largely from these same  $\dot{M}_{\text{SFR}}$  and thus this



**Figure B2.** Specific star formation rates. The shaded regions 1- and 2- $\sigma$  dispersions in running bins of  $\log M_*$  of the aperture-corrected specific star formation rates from SDSS (Brinchmann et al. 2004); the black solid line is a power-law fit to median (equation 21). The purple dashed is the SDSS  $\mu_g \times 1 \times 10^{-9}$  yr and blue dashed line is the total  $\mu_g \times 4 \times 10^{-10}$  yr; these offsets imply a star formation timescale of 1–2.5 Gyr. The shaded regions are dotted lines are constant  $\dot{M}_{\text{SFR}}$ .

is a somewhat degenerate comparison). Finally, we consider a constant SSFR,  $\dot{M}_{\text{SFR}}/M_* = 2 \times 10^{-10}$  yr ( $\log[\dot{M}_{\text{SFR}}/M_*] = -9.7$  in Figure B2).

Using extended Press-Schechter theory, Neistein et al. (2006) parameterize the baryonic accretion rate onto halos by

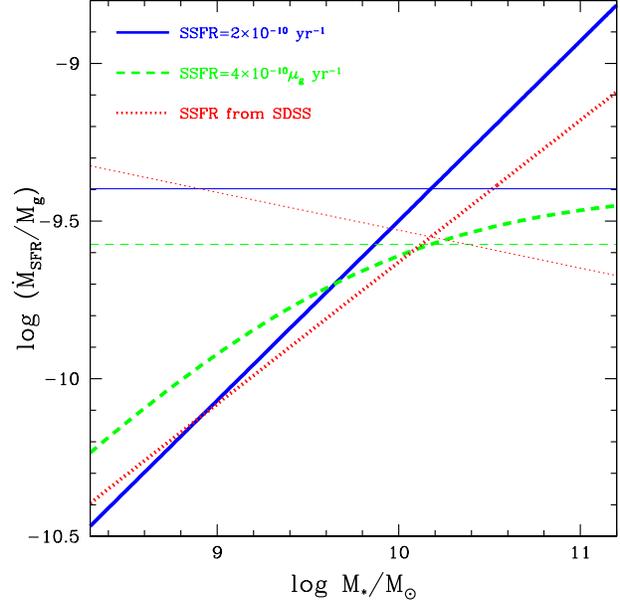
$$\dot{M}_{\text{acc, tot}} = 7.23 \left( \frac{M_{\text{halo}}}{10^{12} M_{\odot}} \right)^{1.15} \left( \frac{f_b}{0.181} \right) (1+z)^{2.25} M_{\odot} \text{ yr}^{-1}, \quad (\text{B5})$$

where  $f_b \equiv \Omega_b/\Omega_m$ . Genel et al. (2008) find a similar accretion rate of dark matter onto halos in the Millineum Simulation, which implies a baryonic accretion rate of

$$\dot{M}_{\text{acc, tot}} = 6.34 \left( \frac{M_{\text{halo}}}{10^{12} M_{\odot}} \right)^{1.07} \left( \frac{f_b}{0.181} \right) (1+z)^{2.2} M_{\odot} \text{ yr}^{-1}. \quad (\text{B6})$$

These accretion rates are for matter being accreted into the *halo*, not the galaxy, and can be safely considered as upper limits to  $\dot{M}_{\text{acc}}$ .

The range of  $\dot{M}_{\text{acc, tot}}/\dot{M}_{\text{SFR}}$  allowed between these two  $\dot{M}_{\text{acc}}$  models and three SSFRs (constant, solid;  $\propto \mu_g$ , dashed; SDSS median, dotted) are plotted in the top-right panel of Figure B1. At low stellar masses,  $M_* \propto M_{\text{halo}}^{0.5}$  (equation 17 and Figure 3), which when combined with the nearly linear mass-dependence of the accretion rate with halo mass, provides  $\dot{M}_{\text{acc}}/\dot{M}_{\text{SFR}} \sim M_{\text{halo}}/M_* \sim M_*^{-0.5}$ , which is the approximate trend found at  $M_* \lesssim 10^{10} M_{\odot}$ . Equations (B5) and (B6) state that the overall “efficiency” of mass accretion  $\dot{M}_{\text{acc}}/M_{\text{halo}}$  is roughly constant with halo mass. Therefore, although the host halos of lower mass galaxies accrete a proportionally equal baryon mass, they are less capable at converting this gas into stars. At high masses, however, the opposite is true: galaxies become more efficient at converting accreted gas into stars. For  $M_* \gtrsim 10^{10} M_{\odot}$ ,  $M_* \propto M_{\text{halo}}^{0.5}$  (equation 17), implying  $\dot{M}_{\text{acc}}/\dot{M}_{\text{SFR}} \sim$



**Figure B3.** Star formation efficiency  $\dot{M}_{\text{SFR}}/M_g$  as a function of  $M_*$ , taking  $M_g$  to be the total cold gas masses (thick lines, § 2.2) and  $M_g = 0.5 M_*$  (thin lines) and three choices of the specific star formation rate: constant (solid blue lines),  $\propto \mu_g$  (dashed green lines), and the median values from SDSS (dotted red lines). In all cases, a steeply decreasing  $F_g$ - $M_*$  relation is required for the star formation efficiency to increase with stellar mass.

$M_{\text{halo}}/M_* \sim M_*$ , which is close to the observed  $\eta_a$ - $M_*$  slope at high masses. This combined double mass-dependent behaviour of  $\eta_a$  with stellar mass produces the characteristic “U” shape observed in Figure B1.

The Neistein et al. and Genel et al. estimates of  $\dot{M}_{\text{acc, tot}}$  are for baryonic accretion into the halo. However, only a fraction of this infalling gas may be usable for star formation; for example, if this infalling gas is shock-heated as it is accreted, then it will neither be detected in H I+H<sub>2</sub> observations nor contribute towards star formation (since we are sensitive to  $\eta_a$  rather than  $\dot{M}_{\text{acc, tot}}$  proper, the gas participating in star formation is relevant). Therefore, to better characterize the fraction of gas that is accreted “cold”—and therefore able to further cool and form stars—we combine the estimates of  $\dot{M}_{\text{acc}}$ ,  $\dot{M}_{\text{SFR}}$ , and  $\eta_a$ , defining this cold fraction as

$$f_{\text{cold}} \equiv \eta_a \frac{\dot{M}_{\text{SFR}}}{\dot{M}_{\text{acc, tot}}}, \quad (\text{B7})$$

where  $\dot{M}_{\text{acc, tot}}$  and  $\dot{M}_{\text{SFR}}$  are generally defined. For illustrative purposes, we let  $\dot{M}_{\text{acc, tot}}$  be defined as in equations (B5) and (B6). Note that to be physical,  $0 \leq f_{\text{cold}} \leq 1$ . The lower-left panel of Figure B1 shows how  $f_{\text{cold}}$  varies with different  $\eta_w$  scalings, assuming the median SDSS SSFRs, while the lower-right panel shows how  $f_{\text{cold}}$  depends on the SSFR in the absence of winds.

There are several interesting behaviors in the lower panels of Figure B1 worth noting. First, the morphology of  $f_{\text{cold}}(M_*)$  is fairly robust against variations in the SSFR and  $\eta_w$ : it is roughly constant, perhaps with a slight rise, for  $\log M_* \lesssim 10.5$ , i.e., below about  $M^*$ , and then drops precipitously at higher masses. Physically, this is a restatement of galaxies with masses near  $M^*$  being more efficient at turning gas accreted by their halos into stars, relative to either more or less massive galaxies (Shankar et al. 2006; Guo et al. 2010).

Second,  $f_{\text{cold}}(M_\star) \sim 1$  for low-mass galaxies. At face value, this would imply that all the accreting gas is available for star formation. This closely resembles so-called ‘‘cold-mode’’ accretion scenario in which gas falling into lower mass halos along filaments do not experience significant shock-heating, thereby easily accreting onto the central galaxy (Dekel & Birnboim 2006; Kereš et al. 2005, 2009; Dutton et al. 2010b). At higher masses, on the other hand, accreting gas may be shock-heated and subsequently unable to cool and contribute to star formation. Despite this neat picture, however, we find it intriguing that  $f_{\text{cold}}(M_\star)$  is so close to unity at low masses. Figure 2 clearly shows that  $M_\star + M_g$  in these same galaxies falls short of accounting for all of the baryons in the halo by at least a factor of two. Thus, a large part of the accreted baryons must be removed from the halo via strong winds, even if the star formation is reasonably inefficient in these galaxies, possibly induced by a particularly strong supernova feedback efficiency.

Finally, Figure B3 builds on this analysis to show the star formation efficiency, traditionally-defined as  $\dot{M}_{\text{SFR}}/M_g$ , as a function of stellar mass for the total cold gas fractions and the three choices of SSFR. In all cases, star formation is more efficient in more massive galaxies: they are forming more stars per unit gas (though see Schiminovich et al. 2010). Several previous analyses of the mass-metallicity relation have suggested that a varying star formation efficiency with galaxy mass is required in order to reproduce the mass-metallicity relation (e.g., Brooks et al. 2007; Calura et al. 2009). Figure B3 shows that this condition is implicitly passed as long as gas fractions are decreasing with galaxy mass and star formation rates vary reasonably with stellar mass, as is observed for  $z = 0$  galaxies. We note, however, that with proper choices of  $\zeta_w$ , the mass-metallicity relation is *theoretically* able to be reproduced with a constant  $F_g$  and therefore constant star formation efficiency.

### APPENDIX C: DERIVING THE MASS-METALLICITY RELATION

The instantaneous change in the stellar mass,

$$\dot{M}_\star = \dot{M}_{\text{SFR}} - \dot{M}_{\text{recy}} \quad (\text{C1})$$

$$= \dot{M}_{\text{SFR}}(1 - f_{\text{recy}}), \quad (\text{C2})$$

is given by the creation of stars ( $\dot{M}_{\text{SFR}}$ ) and the rate at which stars return mass to the ISM when they die,  $\dot{M}_{\text{recy}}$ . (We include the mass of stellar remnants in  $M_\star$ .) The relative rate of these two effects,  $f_{\text{recy}} \equiv \dot{M}_{\text{recy}}/\dot{M}_{\text{SFR}}$ , depends on the star formation history and therefore varies somewhat with time; its effect on our results, however, is small, and we are safe to adopt  $f_{\text{recy}} = 0.2$ .

The gas mass is also regulated by the star formation rate and  $f_{\text{recy}}$ , with gas accretion adding gas and outflows removing gas from the system. The instantaneous change in  $M_g$  is therefore

$$\dot{M}_g = \dot{M}_{\text{acc}} - \dot{M}_{\text{SFR}} + \dot{M}_{\text{recy}} - \dot{M}_w \quad (\text{C3})$$

$$= \dot{M}_{\text{SFR}}(\eta_a - 1 + f_{\text{recy}} - \eta_w), \quad (\text{C4})$$

where  $\dot{M}_{\text{acc}}$  is the gas accretion rate and  $\dot{M}_w$  is the mass rate of outflowing gas. As introduced in §2.3, we define the mass-loading factor  $\eta_w$  as  $\dot{M}_w/\dot{M}_{\text{SFR}}$ ; analogously,  $\eta_a \equiv \dot{M}_{\text{acc}}/\dot{M}_{\text{SFR}}$ . The sources and sinks of metals are essentially the same as for gas, except that each component can have a different metallicity. Hence,

$$\dot{M}_Z = Z_{\text{IGM}}\dot{M}_{\text{acc}} - Z_g\dot{M}_{\text{SFR}} + Z_{\text{ej}}\dot{M}_{\text{recy}} - Z_w\dot{M}_w \quad (\text{C5})$$

$$= \dot{M}_{\text{SFR}}(y + Z_g(\zeta_a - \zeta_w - 1)), \quad (\text{C6})$$

where  $Z_{\text{IGM}}$  is the metallicity of accreting gas,  $Z_g$  is the ISM

metallicity,  $Z_{\text{ej}}$  is the metallicity of gas being returned to the ISM by dying stars, and  $Z_w$  is the metallicity of outflowing gas. The yield  $y$  is the nucleosynthetic yield, which is defined as the rate at which metals are being returned to the ISM relative to the current star formation rate, i.e.,

$$y = \frac{\dot{M}_{\text{new metals}}}{\dot{M}_{\text{recy}}} \times \frac{\dot{M}_{\text{recy}}}{\dot{M}_{\text{SFR}}} = Z_{\text{ej}}f_{\text{recy}}. \quad (\text{C7})$$

After the first generation of Type II supernovae explode ( $\sim 10^7$  yr),  $y$  is constant for continuous star formation; we adopt a mid-range value of  $y = 0.015$ .

Since we are interested in the mass-metallicity relation at  $z = 0$ , and not its rate of change, it is useful to eliminate the time-dependence in equations (C1–C6). We assume galaxies live on a hypersurface of  $M_g$ ,  $M_\star$ ,  $Z_g$ , halo, accretion and wind properties. Dividing out the time-dependence in these equations allows us to solve for the shape of this surface, with observations setting the amplitude. Combining equations (C2) and (C4),

$$\frac{dM_g}{dM_\star} = \frac{\eta_a - \eta_w - 1 + f_{\text{recy}}}{1 - f_{\text{recy}}} = F_g(1 - \gamma), \quad (\text{C8})$$

where we include  $dM_g/dM_\star = F_g(1 - \gamma)$  based on our parameterization of  $F_g = M_g/M_\star$  (equation 2) introduced in §2.2.

The rate of change of the gas phase metallicity  $Z_g$  is

$$\dot{Z}_g = \frac{d}{dt} \frac{M_Z}{M_g} = \frac{\dot{M}_Z}{M_g} - \frac{Z_g}{M_g} \dot{M}_g = \frac{1}{M_g} \left[ \dot{M}_Z - Z_g \dot{M}_g \right]. \quad (\text{C9})$$

We can now combine equations (C2), (C4), (C6), and (C9) to find

$$\frac{dZ_g}{dM_\star} = \frac{y + Z_g \left( \zeta_a - \zeta_w - 1 - \frac{\dot{M}_g}{\dot{M}_{\text{SFR}}} \right)}{M_g(1 - f_{\text{recy}})} \quad (\text{C10})$$

$$= \frac{y + Z_g[\zeta_a - \zeta_w - 1 - F_g(1 - \gamma)]}{M_g(1 - f_{\text{recy}})}. \quad (\text{C11})$$

Equation (C10) can be integrated with respect to  $M_\star$  to find  $Z_g(M_\star)$ . Furthermore, using the Kewley & Ellison (2008) fits (§2.1, Table 1), we can turn the problem around: by assuming we know the mass-metallicity relation (and  $dZ_g/dM_\star$ ), we can infer the required relation between, e.g.,  $F_g$  and  $\zeta_w$ . Specifically, by rearranging equation (C10), we find

$$Z_g = y \left[ \zeta_w - \zeta_a + \right. \quad (\text{C12})$$

$$\left. F_g(1 - f_{\text{recy}}) \left( \frac{d \log M_g}{d \log M_\star} + \frac{d \log Z_g}{d \log M_\star} \right) + 1 \right]^{-1}$$

$$= y [\zeta_w - \zeta_a + \alpha F_g + 1]^{-1}, \quad (\text{C13})$$

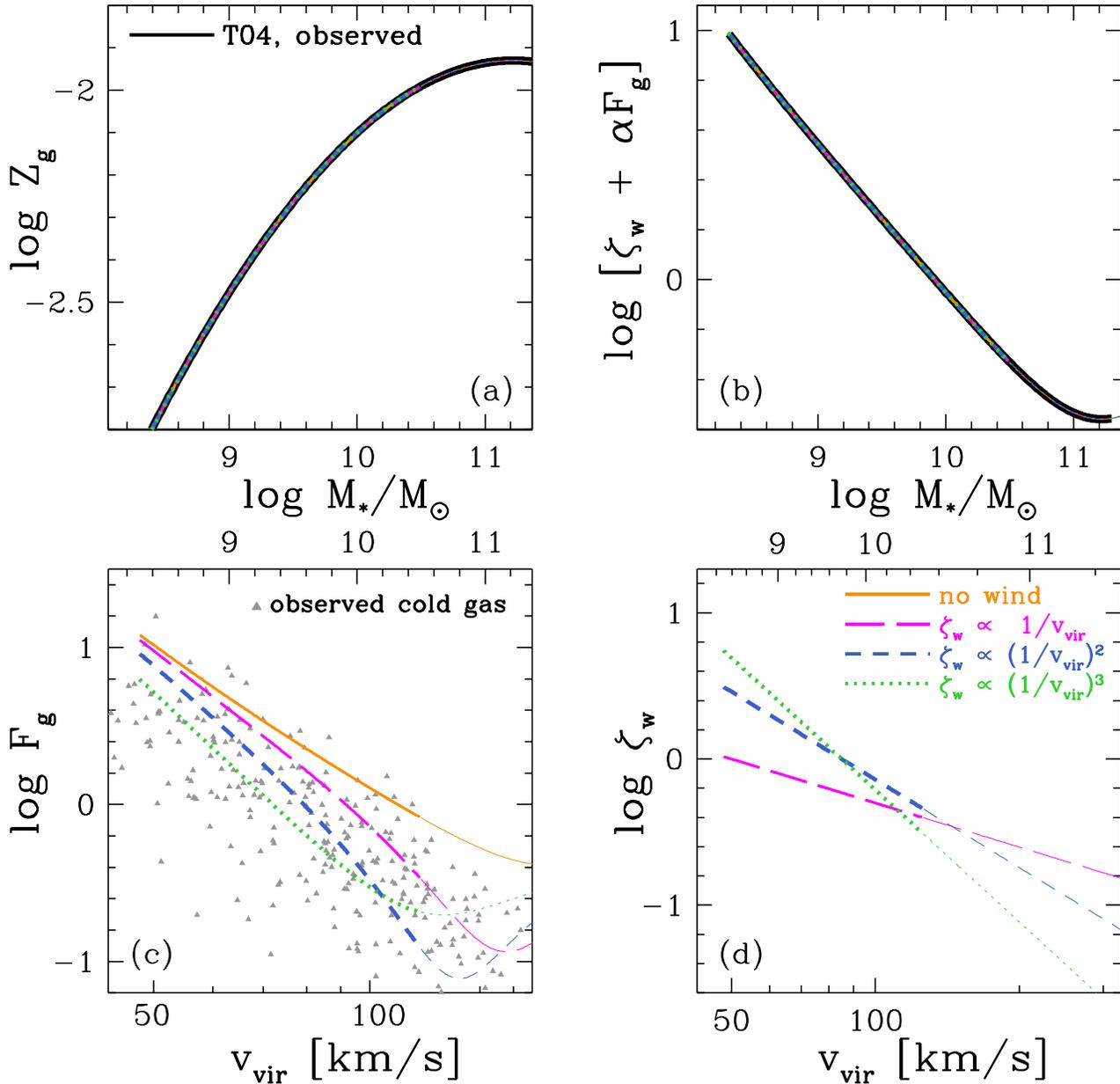
where

$$\alpha \equiv (1 - f_{\text{recy}}) \left( \frac{d \log M_g}{d \log M_\star} + \frac{d \log Z_g}{d \log M_\star} \right) \quad (\text{C14})$$

is a factor of order unity, as given in §3.1.

### APPENDIX D: GENERAL MODELS OF THE MASS-METALLICITY RELATION

Following methods (ii) and (iii) in §4, Figures D1 and 4 show different combinations of gas fractions and outflow efficiencies that explicitly reproduce the Tremonti et al. (2004) mass-metallicity relation; Figure D2 shows the same for the Denicoló et al. (2002)

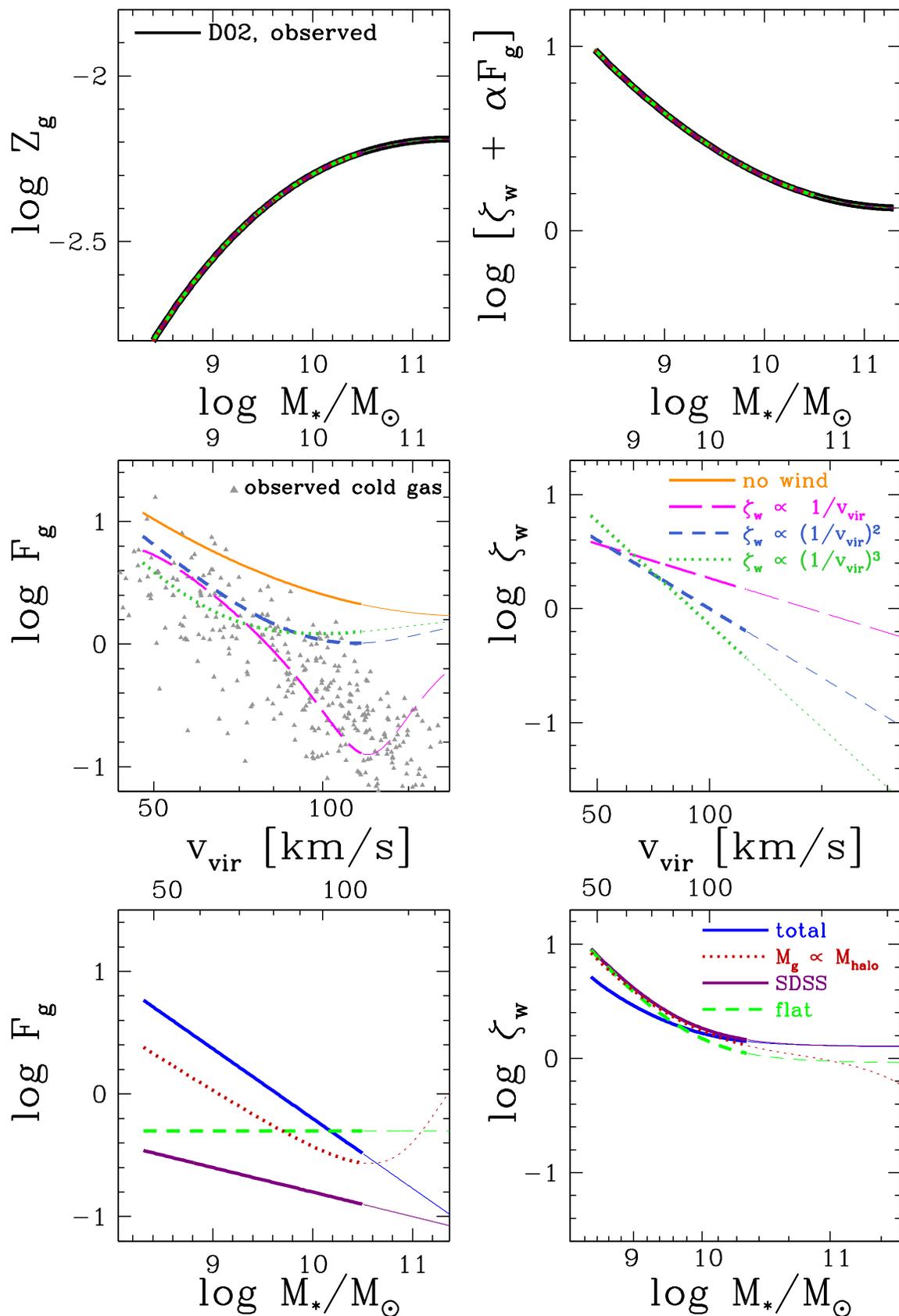


**Figure D1.** Required gas fractions to reproduce the T04 mass-metallicity relation with varying power-law slopes of  $\zeta_w(v_{\text{vir}})$ :  $\zeta_w = 0$  (orange, solid),  $[50 \text{ km s}^{-1}]/v_{\text{vir}}$  (pink, long dashed),  $([85 \text{ km s}^{-1}]/v_{\text{vir}})^2$  (blue, short dashed),  $([85 \text{ km s}^{-1}]/v_{\text{vir}})^3$  (green, dotted); these normalizations are chosen to give gas fractions that are as compatible with the observations as possible. Note that all models fit data in (a) and (b) by construction: panel (a) shows the T04 mass-metallicity relation (black, solid) and models (colored lines) while panel (b) shows  $\log[\zeta_w + \alpha F_g]$  as a function of stellar mass for the four models (colored lines) and  $\log[y/Z_g - 1]$  for the T04 mass-metallicity relation in black. Panel (c) shows the model  $\log F_g$  as a function of stellar mass (colored lines) and the observed gas fractions as grey triangles; these are the same observations plotted in Figure 2 (McGaugh 2005; Leroy et al. 2008; West et al. 2009, 2010). The model  $\log \zeta_w$  as a function of virial velocity are plotted in panel (d) (the  $\zeta_w = 0$  case is unplotted because of the logarithmic  $\zeta_w$  axis).

relation. In the top two panels of Figures D1 and D2, the observations are shown as the solid black curves; the colored lines denote models with different scalings of  $\zeta_w$  with  $v_{\text{vir}}$ . Panel (a) shows the mass-metallicity relation ( $\log Z_g$  as a function of  $\log M_*$ ). The models are chosen so that they give  $\zeta_w + \alpha F_g$  to equal the observed  $y/Z_g - 1$  (panel b). Gas fractions and  $\zeta_w(v_{\text{vir}})$  are plotted in panels (c) and (d), respectively. Because of uncertainties in the nucleosynthetic yield, the normalization of the mass-metallicity

relation, and possible saturation of metallicity indicators at high  $12 + \log(\text{O}/\text{H})$  (§ 2.1), we will consider both the mass-metallicity relation across the mass range  $8.1 \leq \log M_* \leq 11.3$  and restricted to below  $M_* \sim 10.5 M_\odot$ .

The gas fractions needed to dilute the metals in the absence of winds ( $\zeta_w = 0$ ) are shown as the solid orange line; these gas fractions are higher by a factor of  $\gtrsim 3$  than observed cold gas fractions in typical  $z \sim 0$  galaxies. For a non-varying yield, outflows are



**Figure D2.** Same as Figures D1 and 4, but for the Denicoló et al. (2002) mass-metallicity relation. The normalizations for  $\zeta_w \propto v_{\text{vir}}^{-b}$  in the middle two panels are chosen to give gas fractions that are as compatible with the observations as possible and are:  $[185 \text{ km s}^{-1}/v_{\text{vir}}]$  (pink, long dashed),  $[(100 \text{ km s}^{-1})/v_{\text{vir}}]^2$  (blue, short dashed),  $[(90 \text{ km s}^{-1})/v_{\text{vir}}]^3$  (green, dotted).

therefore required in order to keep the observed mass-metallicity relation consistent with galaxy gas fraction observations. This conclusion holds even more strongly for the other mass-metallicity relations plotted in Figure 1: in the absence of winds, lower metallicities imply higher gas fractions.

The other colored lines show the required gas fractions if we assume  $\zeta_w \propto v_{\text{vir}}^{-1}$  (pink, long-dashed),  $\propto v_{\text{vir}}^{-2}$  (blue, short-dashed), or  $\propto v_{\text{vir}}^{-3}$  (green, dotted). For the T04 mass-metallicity relation, both the momentum-driven and energy-driven  $\zeta_w$  scalings require  $F_g$  to scale more steeply with mass than is observed; lower normalizations of  $\zeta_w$  force  $F_g$  to asymptote to the no winds case. A steeper scaling of  $\zeta_w$  with  $v_{\text{vir}}$ , however, leads to more reasonable gas fractions.