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# General Relativistic Explosion Models of Core-Collapse Supernovae

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**Abstract.** We present results from the first generation of multi-dimensional general relativistic neutrino hydrodynamics simulations of core-collapse supernovae. A comparison with models computed using either the purely Newtonian approximation or the "effective gravitational potential" approach reveals appreciable quantitative differences in the heating conditions and the gravitational wave spectra. Our results underscore the important role of general relativity in the supernova problem (which appears to be on par with other important factors such as the dimensionality and the equation of state) both for our understanding of the explosion dynamics as well as for predictions of observable signatures.

## 1. Introduction

A core-collapse supernova occurs at the end of the life of a massive star when it has exhausted its nuclear fuel at its centre. The nuclear ashes form an iron core which undergoes a gravitational collapse once it reaches a mass of roughly $1.4 M_\odot$. When the core reaches supranuclear densities, the collapse is halted due the stiffening of the equation of state, and a shock wave is launched as the core rebounds. The initial kinetic energy of the shock is quickly spent for disintegrating the nuclei in the infalling material material into free nucleons, and the shock is further weakened by the rapid emission of several $10^{51}$ erg in neutrinos from the post-shock region during the neutronization burst. The stalled shock typically hovers at a radius of $\sim 100 \ldots 200$ km – possibly for hundreds of milliseconds – until it is revived, and expels the envelope of the progenitor.

The nature of the explosion mechanism has been a subject of intense research in computational astrophysics for several decades. Several mechanisms have been proposed to explain the revival of the shock, the most prominent one being the "delayed neutrino-driven mechanism" (Bethe & Wilson 1985; Wilson 1985), which relies on the deposition of energy by neutrinos in the "gain layer" behind the shock to re-energize the shock. Except for a special class of progenitors with an O-Ne-Mg core (Kitaura et al. 2006), this mechanism only works in concert with multi-dimensional hydrodynamic effects such as convection and the so-called "standing accretion shock instability" (SASI, Blondin et al. 2003; Foglizzo et al. 2006; Ohnishi et al. 2006), which both increase the efficiency of neutrino heating in the gain region. While some of the most ambitious supernova simulations in axisymmetry (Buras et al. 2006a; Marek & Janka 2009; Bruenn et al. 2010; Suwa et al. 2010) indicate that the neutrino-driven mechanism may indeed work, there are also alternative scenarios such a magnetohydrodynamicallydriven explosions (Bisnovatyi-Kogan 1970; Akiyama et al. 2003; Burrows et al. 2007), acoustically-powered supernovae (Burrows et al. 2006; see however Weinberg & Quataert 2008 for criticism), and explosions triggered by a QCD phase transition (Sagert et al. 2009).

From a computational point of view, core-collapse supernovae present a number of challenges: Not only are they inherently multi-dimensional phenomena due to the operation of hydrodynamical instabilities, but in order to accurately capture the crucial effects of neutrino cooling and heating in the optically thick and thin regimes, the problem needs to be treated within the framework of kinetic theory. As the direct solution of the full Boltzmann equation for neutrinos is currently only feasible under the assumption of spherical symmetry (Yamada et al. 1999; Liebendörfer et al. 2004), a variety of different approximation strategies for the neutrino transport are used in the most sophisticated multi-dimensional supernova simulations, including "ray-by-ray" transport combined with variable Eddington factor techniques (Buras et al. 2006b) or flux-limited multi-group diffusion schemes (Bruenn et al. 2010), multi-angle Boltzmann transport without energy-bin coupling (Ott et al. 2008), two-moment schemes with an analytic closure (Obergaulinger & Janka 2011), and the isotropic diffusion source approximation (Liebendörfer et al. 2009).

Moreover, *general relativity* (GR) plays a major role in the supernova problem due to the compactness of the proto-neutron star ($GM/Rc^2 \sim 0.1 \ldots 0.2$) and the occurrence of high velocities ($\sim 0.3c$). A general relativistic treatment is also required for precise predictions of the gravitational wave signal from core-collapse supernovae. However, relativistic supernova simulations with up-to-date neutrino transport have long been limited to the case of spherical symmetry (Bruenn et al. 2001; Yamada et al. 1999; Liebendörfer et al. 2004). In order to study the effects of GR in the context of multi-dimensional models, and in particular its impact on the gravitational wave emission from the supernovae core, we have recently (Müller et al. 2010) introduced a generalization of the ray-by-ray-plus variable Eddington factor method (Rampp & Janka 2002; Buras et al. 2006b). In this paper, we present results from the first generation of axisymmetric relativistic supernova simulations using our Vertex-CoCoNuT code and summarize the results of a comparison with the Newtonian case as well as with the pseudo-Newtonian "effective potential" approximation.

## 2. Relativistic Variable Eddington Factor Method

In our approach to neutrino transport in core-collapse supernovae, we solve the equations of GR hydrodynamics in the formulation of Banyuls et al. (1997) and use the xCFC approximation of Cordero-Carrión et al. (2009) for the space-time metric, which is particularly suitable for our purpose because of its excellent stability properties and high accuracy in the core-collapse case (Cordero-Carrión et al. 2009). The hydrodynamics solver is based on the CoCoNuT code (Dimmelmeier et al. 2002, 2005), an implementation of a HRSC scheme with PPM reconstruction (Colella & Woodward 1984) and second-order Runge-Kutta time-stepping. Different from the original code (Müller et al. 2010), we rely on an improved scheme for maintaining total energy conservation and employ the relativistic HLLC Riemann solver of Mignone & Bodo (2005) to resolve contact discontinuities in the convective post-shock region more accurately.

In order to capture the effects of neutrino heating and cooling the equations of hydrodynamics need to be coupled with a kinetic equation for the neutrino distribution

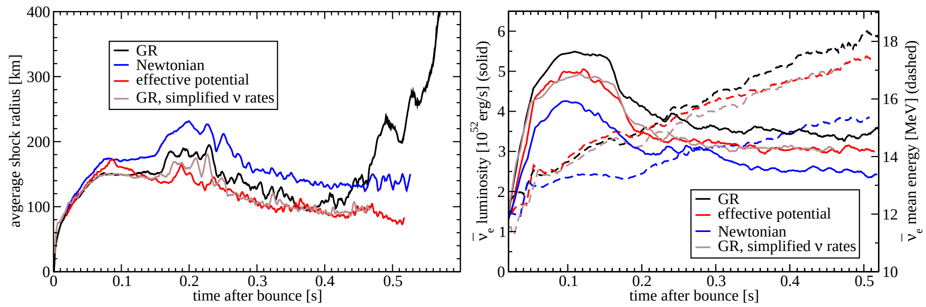


Figure 1. Left: Evolution of the average shock radius for simulations of a $15M_\odot$ progenitor with general relativistic hydrodynamics and neutrino transport (black), with an effective potential (red), in the purely Newtonian approximation (blue), and with a simplified set of neutrino interaction rates in the GR case (light brown). Right: Electron antineutrino luminosities (solid) and mean energies (dashed) for the four different cases.

function $f$. Following the approach already used for the Newtonian version the Vertex code (Rampp & Janka 2002; Buras et al. 2006b), we simplify the full six-dimensional phase-space problem by considering the first two angular moments $J, H, K, L, \ldots$ of the energy-dependent radiation intensity and by requiring $f$ to be axially symmetric in momentum space (but not in real space) around the unit radius vector (the ray-by-ray approximation). The transport problem can thus be reduced to conservation equations with source terms for neutrino number, energy and momentum; e.g., the equation for the zeroth moment $J$ reads

$$
\begin{aligned}
&\frac{\partial \sqrt{\gamma} W (J + v_r H)}{\partial t} + \frac{\partial}{\partial r}\left[\left(W\frac{\alpha}{\phi^2} - \beta_r v_r\right)\sqrt{\gamma} H + \left(W v_r \frac{\alpha}{\phi^2} - \beta_r\right)\sqrt{\gamma} J\right] - \qquad (1)\\
&\varepsilon \frac{\partial}{\partial \varepsilon}\left\{W\sqrt{\gamma} J\left[\frac{1}{r}\left(\beta_r - \frac{\alpha v_r}{\phi^2}\right) + 2\left(\beta_r - \frac{\alpha v_r}{\phi^2}\right)\frac{\partial \ln\phi}{\partial r} - 2\frac{\partial \ln \phi}{\partial t}\right] + \right.\\
&W\sqrt{\gamma} H\left[v_r\left(\frac{\partial \beta_r \phi^2}{\partial r} - 2\frac{\partial \ln\phi}{\partial t}\right) - \frac{\alpha}{\phi^2}\frac{\partial \ln \alpha W}{\partial r} + \alpha W^2\left(\beta_r \frac{\partial v_r}{\partial r} - \frac{\partial v_r}{\partial t}\right)\right] - \\
&\left.\sqrt{\gamma} K\left[\frac{\beta_r W}{r} - \frac{\partial \beta_r W}{\partial r} + W v_r r \frac{\partial}{\partial r}\left(\frac{\alpha}{r\phi^2}\right) + W^3\left(\frac{\alpha}{\phi^2}\frac{\partial v_r}{\partial r} + v_r \frac{\partial v_r}{\partial t}\right)\right]\right\} = \alpha \sqrt{\gamma} C^{(0)}
\end{aligned}
$$

in the adopted gauge, with $\alpha$ denoting the lapse function, $\phi$ the conformal factor, $\beta_r$ the radial shift vector, $\gamma$ the determinant of the three-metric, $v_r$ the radial velocity, $W$ the Lorentz factor, and $C^{(0)}$ the zeroth moment of the collision integral. A similar equation is solved for $H$, and the higher moments $K$ and $L$ that are required to close the system are provided by means of a formal solution of a simplified "model" Boltzmann equation. It should be noted that we fully retain the energy-dependence of the moment equations, and fully include Doppler shift, gravitational redshift an energy redistribution by inelastic scattering of neutrinos off nucleons, nuclei, electrons, and other neutrinos.

## 3. Results

Relativistic supernova simulations have been conducted for two different non-rotating progenitors with $11.2M_\odot$ (Woosley et al. 2002) and $15M_\odot$ (Woosley & Weaver 1995).

For the $15M_\odot$ progenitor, additional models were computed to allow for a comparison with the purely Newtonian case and the "effective potential" approach (Marek et al. 2006) which mimics certain strong-field effects by modifying the Newtonian gravitational potential. Moreover, we also considered the case of slightly simplified neutrino interaction rates, neglecting the effect of recoil, high-density correlations and weak magnetism in neutrino-nucleon reactions and ignoring reactions between different neutrino flavours. As the $15M_\odot$ progenitor has already proved to be a marginal case in earlier studies relying on the effective potential approach (Marek & Janka 2009), it is ideally suited to illustrate the impact of slightly different heating conditions depending on the treatment of GR. However, the relativistic

Among the four $15M_\odot$ simulations, only the GR model with the best currently available set of neutrino opacities for our code develops an explosion around 400 ms after bounce (left panel of Fig. 1). The more optimistic evolution of the GR model compared to the Newtonian run has been traced to slightly higher surface temperatures of the more compact proto-neutron star, which result in higher neutrino luminosities and mean energies (right panel of Fig. 1) and hence allow for more effective heating. To a lesser extent, this effect is also present in the effective potential run, but here the faster advection (i.e. shorter exposure time) of the accreted material through the smaller gain region around the compact neutron star compensates for the increase in the local heating rate. On the other hand, the enhancement of the neutrino heating in the GR model is strong enough to overcome this competing effect and to shift the balance between neutrino heating and advection in the gain region far enough to achieve favourable conditions for the development of an explosion. We emphasize, however, that despite such differences, the effective potential approximation provides a remarkable improvement over the purely Newtonian treatment also in multi-dimensional supernova models.

Incidentally, we also find that the heating conditions depend quite sensitively on the neutrino microphysics (as already noted by Rampp et al. 2002 and Bruenn et al. 2010) as the GR run with simplified interactions rates fails to develop an explosion. The more optimistic evolution of the model with the improved rates stems primarily from the reduction of the $\bar{\nu}_e$ scattering cross-section on nucleons due to weak magnetism and nucleon correlations (Horowitz 2002), which helps to enhance $\bar{\nu}_e$ luminosities and mean energies (right panel of Fig. 1).

The treatment of GR also turns out to be a crucial factor for the prediction of the gravitational wave signal. While the wave signal for our relativistic explosion models qualitatively shows the typical features known from (pseudo-)Newtonian studies with sophisticated or simplified neutrino transport (Kotake et al. 2007; Marek et al. 2009; Murphy et al. 2009; Yakunin et al. 2010) with distinct phases of gravitational wave emission from prompt post-shock convection, hot-bubble convection, enhanced SASI sloshing motions, and asymmetric shock expansion (left panel of Fig. 2), the signal spectrum is rather sensitive to GR effects (right panel of Fig. 2). In the purely Newtonian case, the integrated signal (which is dominated by the contribution from hot-bubble convection) from the $15M_\odot$ progenitor peaks at distinctly lower frequencies around $\sim$ 500 Hz than in the GR case ($\sim$ 900 Hz), which is a consequence of a lower Brunt-Väisälä frequency and hence a less rapid deceleration of convective plumes in the stably stratified cooling region above the surface of a more extended proto-neutron star surface with lower surface gravity (cp. the analysis of Murphy et al. 2009). On the other hand, the median frequency is somewhat overestimated by the effective potential approach ($\sim$ 1100 Hz) because the lower neutrino luminosities and mean energies result

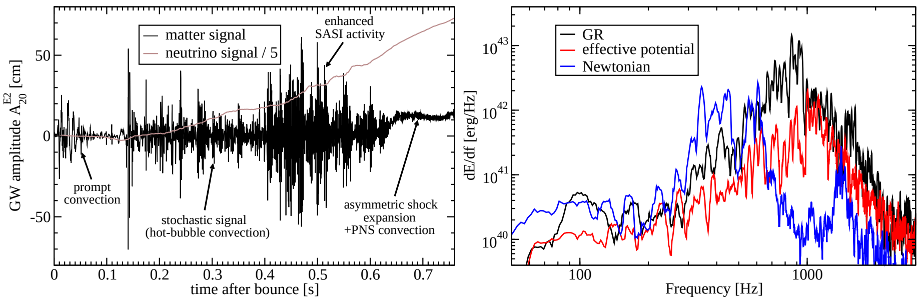


Figure 2. Left: Matter (black) and neutrino (light brown) gravitational wave signals for the general relativistic $15M_{\odot}$ explosion model. Right: Gravitational wave spectrum for the first 0.5 s of the post-bounce evolution of a $15M_{\odot}$ progenitor for simulations depending on the treatment of gravity (black: GR hydro, red: Newtonian hydro + effective potential, blue: purely Newtonian).

in a steeper density stratification in the cooling region, which in turn leads to a more abrupt braking of convective bubbles. The gravitational wave signal is thus at least as sensitive to GR effects as to other physical key parameters such as the equation of state (Marek et al. 2009).

## 4. Conclusions and Outlook

Multi-dimensional general relativistic simulations of core-collapse with a sophisticated treatment of the microphysics and the neutrino transport on par with the best currently available Newtonian models have only recently become possible, but the first available results presented here in this paper already serve to underscore the importance of general relativity in the supernova problem. For a $15M_{\odot}$ progenitor, we found that GR somewhat improves the heating conditions compared to models computed in the Newtonian and the effective potential approximations, which, unlike the GR model, fail to explode. The gravitational wave spectra are also considerably changed by GR effects, which shift the typical frequency of the time-integrated signal upward by $\sim 80\%$ compared to the purely Newtonian case. We therefore conclude that an accurate treatment of GR effects may be no less relevant for a better understanding of the neutrino-driven explosion mechanism and quantitative predictions of the signals from core-collapse supernovae than other (undoubtedly important) key factors that have recently been discussed such as dimensionality issues (Nordhaus et al. 2010; Hanke et al. 2011; Takiwaki et al. 2011) and the nuclear equation of state.

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