

Long-term evolution of discs around magnetic stars

Caroline R. D’Angelo and Hendrik C. Spruit

Max Planck Institute for Astrophysics, Karl-Schwarzschildstr.1 D-85741 Garching, Germany

27 July 2011

ABSTRACT

We investigate the evolution of a thin viscous disc surrounding magnetic star, including the spindown of the star by the magnetic torques it exerts on the disc. The transition from an accreting to a non-accreting state, and the change of the magnetic torque across the corotation radius r_c are included in a generic way, the widths of the transition taken in the range suggested by numerical simulations. In addition to the standard accreting state, two more are found. An accreting state can develop into a ‘dead’ disc state, with inner edge r_{in} well outside corotation. More often, a ‘trapped’ state develops, in which r_{in} stays close to corotation even at very low accretion rates. The long-term evolution of these two states is different. In the dead state the star spins down incompletely, retaining much of its initial spin. In the trapped state the star asymptotically can spin down to arbitrarily low rates, its angular momentum transferred to the disc. We identify these outcomes with respectively the rapidly rotating and the very slowly rotating classes of Ap stars and magnetic white dwarfs.

Key words: accretion, accretion discs – instabilities – MHD – stars: oscillations – stars: magnetic fields – stars:formation – stars:rotation

1 INTRODUCTION

Accreting stars with strong magnetic fields are generally observed to rotate more slowly than their less-magnetic or discless counterparts. In protostars, T Tauri systems (which often have strong surface magnetic fields of $\sim 10^2 - 10^3$ G) with discs rotate more rapidly than systems without discs (Getman et al. 2008). Most (but not all) mainsequence Ap stars (with surface fields of up to 10^4 G) are observed to rotate much slower than normal A stars (Stepień & Landstreet 2002), and recent work has suggests this relationship extends down to their pre-main-sequence progenitors, the Herbig Ae stars (Alecian et al. 2008). In high energy systems the result is similar: accreting neutron stars with weak ($\sim 10^8$ G) fields rotate up to 10^4 times faster than neutron stars with strong ($\sim 10^{12}$ G) fields. These observations suggest that the interaction between the accretion disc and stellar magnetic field plays a critical role in regulating the spin-rate of the star.

Early theoretical studies of accretion predicted that a strong stellar field would truncate the accretion disc some distance from the surface of the star, with the truncation radius located roughly where the magnetic pressure ($B^2/4\pi$) equals the ram pressure of the infalling gas ($\dot{m}v_r/2\pi r^2$), so that infalling matter is channelled onto the surface via magnetic field lines, causing the star to spin up. (Pringle & Rees 1972). This assumes that the disc is truncated inside the corotation radius ($r_c \equiv (GM_*/\Omega_*^2)^{1/3}$), where the star’s spin frequency is equal to the disc’s Keplerian frequency). If instead the magnetic field spins faster than the inner edge of the disc, a centrifugal barrier prevents accretion. Interaction between the magnetic field and the disc will then spin down the star

(Illarionov & Sunyaev 1975; Mineshige et al. 1991; Lovelace et al. 1999; Romanova et al. 2004; Ustyugova et al. 2006).

The presence of the centrifugal barrier is often equated in the literature with the idea that the accreting gas will be flung out, or ‘propellered’ out of the system so as to maintain a steady state. This assumption turns out to be both arbitrary and unnecessary. For example, in order for the accreting material to be flung out of the system, the disc must be truncated a sufficient distance away from r_c . Otherwise the rotational velocity difference between the disc and the magnetosphere is too small (Spruit & Taam 1993).

Steady disc solutions with a centrifugal barrier at the inner edge were first described by Sunyaev & Shakura (1977), who called them ‘dead discs’, because even though the disc is actively transporting angular momentum outwards, no accretion onto the star takes place and the disc itself is very dim.

As pointed out already in Sunyaev & Shakura (1977) and Spruit & Taam (1993), in a system with an externally imposed mass flux the likely effect of a centrifugal barrier is to cause the accretion onto the star to be *cyclic*. Accretion phases alternate with quiescent periods during which mass piles up outside the barrier, without mass having to leave the system. In the quiescent phase, the angular momentum extracted from the star by the disc-field interaction is carried outward through the disc by viscous stress. This alters the surface density profile of the disc from the usual accreting solution.

In our previous paper (D’Angelo & Spruit 2010; hereafter DS10) we studied this form of cyclic accretion with numerical solutions of the viscous diffusion equation for a thin disc subject to a magnetic torque. As in the (somewhat more ad hoc) model of Spruit & Taam (1993), limit cycles of the relaxation oscillator type were found. The cycle period of these oscillations depends on the

arXiv:1102.3697v2 [astro-ph.HE] 26 Jul 2011

accretion rate, from fast oscillations at higher mass flux to arbitrarily long periods at low accretion rates.

Instead of the two states: accreting and dead as suggested above, the results in DS10 are actually described better by including a third, intermediate state we call here the ‘trapped’ state:

- (i) $r_{\text{in}} < r_c$: accreting state, star spins up,
- (ii) $r_c - \Delta < r_{\text{in}} < r_c + \Delta$: trapped state, spinup or spindown,
- (iii) $r_{\text{in}} - r_c \gg \Delta$ star spins down, no accretion (dead disc),

where $\Delta \ll r_c$ is a narrow range around corotation, to be specified later. In state (ii), the inner edge of the disc remains close to corotation over a range of accretion rates onto the star, and the net torque on the star can be of either sign, depending on the precise location of the inner edge of the disc.

For a given accretion rate, a disc that starts in state (i) will gradually move into state (ii) or (iii) as the star spins up and r_c moves inward. In state (ii) accretion can proceed steadily or happen in bursts, depending on the disc-field interaction at r_{in} . For steady externally imposed accretion a disc in this state will eventually move into spin equilibrium with the star, so that the net torque on the star is zero. In the dead state (iii) a steady state can exist if the torque exerted by the star is taken up at the outer edge of the disc by a companion star. If we neglect the transition to the propeller regime, then in theory the dead disc solution can exist for a disc truncated at any distance outside r_c . Such a disc will remain static as the star spins down and r_c moves outward. Our model is thus qualitatively different from the conventional ‘propeller’ picture since at very low accretion rates a considerable amount of mass remains confined in the disc, and the star can be efficiently spun down.

In the following we study the long-term evolution of the star-disc system by using the description of magnetospheric accretion in DS10, allowing the star’s spin rate to evolve. Of special interest will be the trapped state (ii), since in many cases the evolution of the system ends in it. The accretion cycles found in DS10 also take place essentially within a trapped state. The inner edge of the disc is near corotation in the trapped state, as is the case also for a disc in spin equilibrium with the accreting star. Spin equilibrium is only a special case of a trapped state, however. In general a trapped state is not one of spin equilibrium, spinup is possible as well as spindown. We focus particular attention on cases in which the mean accretion rate through the disc is very low and the star cannot easily move into spin equilibrium with the disc. This is most applicable for isolated stars, or accreting binaries in quiescence.

This scenario poses a number of questions which we address in the course of this paper. These include: under what conditions does the disc get into a trapped state, and when does it instead evolve into a dead state? It will turn out that this is determined by the details of the disc-field interaction and the ratio of the spindown timescale of the star (T_{SD}) to the viscous timescale of the disc (T_{visc}). The initial conditions of the disc also significantly influence the outcome. In section 5.2 we ask how an initially trapped disc could become untrapped as a dead disc state. In particular, does this depend on the initial location of the inner edge of the disc, the initial accretion rate, the presence or absence of a companion or the size of the disc? Finally, in sec. 5 we discuss the physics that determines whether a disc will become trapped, and 5.3.1 we ask whether a trapped disc could plausibly regulate the slow spins observed in Ap stars, some of which have spin periods of up to decades.

In a companion paper we investigate the observable consequences of a trapped disc, focusing in particular on how the burst

instability studied in DS10 will change the spin evolution and observable properties of the star. In that paper we also discuss our model’s predictions in terms of observations of magnetospherically regulated accretion in both protostars and X-ray binaries.

We use the code developed in DS10, adding the star’s moment of inertia as a parameter of the problem in order to follow the spin evolution of the star in response to the disc interaction. We can then simultaneously follow the viscous evolution of the disc and spin evolution of the star as the star’s spin changes, and explore how these two interact with each other. We describe our model in more detail in the following section.

2 MAGNETOSPHERIC INTERACTIONS WITH A THIN DISC

2.1 Magnetic torque

The interaction between a strong stellar magnetic field and surrounding accretion disc truncates the disc close to the star, and forces incoming matter to accrete along closed field lines onto the surface of the star in a region called the magnetosphere. At the outer edge of the magnetosphere (termed here the magnetospheric radius), the field lines become strongly embedded in the disc over some small radial extent that we term the *interaction region*, Δr . The differential rotation between the star and the Keplerian disc will cause the field lines to be twisted, which will generate a toroidal component to an initially poloidal field (e.g. Ghosh et al. 1977). This will allow the transfer of angular momentum between the disc and star, with the torque per unit area exerted by the field on the disc given by $\tau = r S_{z\phi} \hat{\mathbf{z}}$, where:

$$S_{z\phi} \equiv \frac{B_\phi B_z}{4\pi} \quad (1)$$

is the magnetic stress generated by the twisted field lines. Both theoretical arguments (e.g. Aly 1985; Lovelace et al. 1995) and numerical simulations (such as Miller & Stone 1997; Goodson et al. 1997; Hayashi et al. 1996) suggest that the strong coupling between magnetic field lines and the disc will cause the field lines to inflate and open. The inflation and opening of field lines limits the growth of the B_ϕ component for the field to $B_\phi = \eta B_z$, with η of order unity, and reduces the radial extent of the interaction region, since beyond a given radius the field lines are always open and the disc-field connection will be severed. We take the interaction region to be narrow, $\Delta r/r < 1$ (as found in numerical simulations, see section 2 of DS10 for a more detailed discussion). Assuming the star’s dipole field strength $B_d(r)$ as an estimate of B_z , and taking into account that S acts on both sides of the disc, (1) yields the magnetic torque T_0 exerted on the disc:

$$T_0 = 4\pi r \Delta r r S_{z\phi} = \eta r^2 \Delta r B_d^2. \quad (2)$$

This torque exists only if the inner edge r_{in} of the disc is outside the corotation radius r_c . For $r_{\text{in}} < r_c$, we have instead a disc accreting on an object rotating slower than the Kepler rate at r_{in} . By the standard theory of thin viscous discs, the torque exerted on the disc by the accreting object then vanishes, independent of the nature of the object. The torque $T_B(r_i)$ thus changes over a narrow range around r_c . To model this transition we introduce a ‘connecting function’ y_Σ :

$$T_B(r_{\text{in}}) = y_\Sigma(r_{\text{in}}) T_0(r_{\text{in}}), \quad (3)$$

with the properties $y_\Sigma \rightarrow 0$ ($r_c - r_{\text{in}} \gg \Delta r$), $y_\Sigma \rightarrow 1$ ($r_{\text{in}} - r_c \gg$

Δr). As in DS10, we take for this function

$$y_\Sigma = \frac{1}{2} \left[1 + \tanh \left(\frac{r_{\text{in}} - r_c}{\Delta r} \right) \right]. \quad (4)$$

The width of the transition is thus described by Δr . We take the same value for it as used in eq. (2).

2.2 Model for disc-magnetosphere interaction

In DS10 we derived a description of the interaction between a disc and magnetic-field for a disc truncated either inside or outside r_c , and introduced two numerical parameters to connect the two regimes. To keep the problem axisymmetric, we assumed a dipolar magnetic field, with the dipole axis aligned with the stellar and disc rotation axis. Since the region of interaction between the disc and the field is small, we use our description of the interaction as a boundary condition for a standard thin accretion disc (Shakura & Sunyaev 1973).

To evolve a thin disc in time, we must choose a description for the effective viscosity (ν) that allows transport of angular momentum. We adopt an α prescription for the viscosity and assume a constant scale height (h) for the disc, so that:

$$\nu = \alpha (GM_*)^{1/2} (h/r)^2 r^{1/2}. \quad (5)$$

At the inner edge of the disc the behaviour is regulated by the disc-field interaction. However, since the interaction region is small, we incorporate the interaction as a boundary condition on the inner disc, and assume that the majority of the disc is shielded from the magnetic field and then evolves as a standard viscous disc, albeit with a very different inner boundary condition from the standard one. Below we summarize our analysis of the disc-field interaction and how these translate into boundary conditions on the disc. (For the detailed derivation of our boundary conditions, see sections 2.3, 2.4, and 3.2 of DS10).

2.2.1 Surface density at r_{in}

In a dead disc, the disc-field interaction prevents matter from accreting or being expelled from the system, instead retaining matter that interacts with the magnetic field. This implies that the angular momentum injected via magnetic torques in the interaction region Δr must be transported outwards by viscous torques in the disc. A dead disc will therefore have a maximum in surface density at r_{in} , and $\Sigma(r_{\text{in}})$ will depend on the amount of angular momentum being added by the disc-field interaction.

By equating the amount of angular momentum added by the field to the amount carried outwards by viscous processes, we can calculate the surface density at the inner boundary of the disc needed for the injected angular momentum to be carried away. This yields (see DS10) a value for the surface density Σ at the inner edge of a dead disc, proportional to the magnetic torque (3):

$$3\pi\nu\Sigma(r_{\text{in}}) = \left. \frac{T_B}{r^2\Omega_K} \right|_{r_{\text{in}}}, \quad (6)$$

where Ω_K is the Keplerian rotation frequency. If the stellar field is a dipole and we use (5) to describe the viscosity, then for $r_{\text{in}} > r_c$, $\Sigma(r_{\text{in}}) \propto r_{\text{in}}^{-4}$. $\Sigma(r_{\text{in}})$ thus decreases rapidly with increasing r_{in} .

2.2.2 Accretion rate across r_c

If the inner edge is well inside the corotation radius r_c , we use a standard result to estimate the location of r_{in} as a function of the

accretion rate \dot{m}_a . It is obtained from the azimuthal equation of motion for gas at the point at which it is forced to corotate with the star (c.f. Spruit & Taam 1993). This gives:

$$r_{\text{in}}\pi\langle S_{z\phi} \rangle / \Omega_* = \dot{m}. \quad (7)$$

[Note that we take the sign of \dot{m} positive for *inward* mass flow.] It is not necessary that stationarity holds: (7) can also be applied when the inner edge of the disc moves. However, since it describes the accretion through the magnetosphere-disc boundary r_{in} , it has to be applied in a frame comoving with r_{in} . If \dot{m} is the mass flow rate in a fixed frame, it is related to the accretion rate in this comoving frame (\dot{m}_{co}) by

$$\dot{m}_{\text{co}} = \dot{m} + 2\pi r_{\text{in}} \Sigma(r_{\text{in}}) \dot{r}_{\text{in}}, \quad (8)$$

where \dot{r}_{in} is the rate of change of the inner disc edge.

To connect the accreting case with the dead disc case, for which $\dot{m} = 0$, we need one more prescription, this time for the accretion rate as a function of the inner edge radius. We introduce a connecting function y_m for this (DS10):

$$\dot{m}_{\text{co}}(r_{\text{in}}) = y_m(r_{\text{in}}) \dot{m}_a(r_{\text{in}}), \quad (9)$$

with the properties $y_m \rightarrow 1$ ($r_c - r_{\text{in}} \gg \Delta r_2$), $y_m \rightarrow 0$ ($r_{\text{in}} - r_c \gg \Delta r_2$), with

$$y_m = \frac{1}{2} \left[1 - \tanh \left(\frac{r_{\text{in}} - r_c}{\Delta r_2} \right) \right], \quad (10)$$

where Δr_2 describes the width of the transition (different in general from Δr).

With the star's assumed field of dipole moment μ , $B_d = \mu/r^3$ and Keplerian orbits in the disc, (8) becomes, with the viscous thin-disc expression for \dot{m} :

$$6\pi r_{\text{in}}^{1/2} \frac{\partial}{\partial r} (\nu \Sigma r^{1/2}) \Big|_{r_{\text{in}}} = y_m \frac{\eta \mu^2}{4\Omega_* r_{\text{in}}^5} - 2\pi r_{\text{in}} \Sigma(r_{\text{in}}) \dot{r}_{\text{in}}, \quad (11)$$

where η is a numerical factor of order unity and μ the dipole moment of the star (see DS10 for details).

Along with our description for the viscosity, (6) and (11) define a boundary condition at r_{in} and an equation for $r_{\text{in}}(t)$, for a disc over a continuous range of accretion rates, from strongly accreting systems ($r_{\text{in}} \ll r_c$) to dead-disc systems ($\dot{m} \simeq 0$).

2.2.3 Evolution of corotation radius

In order to study the response of a disc to changes in spin of the star, we must incorporate the angular momentum exchange between the star and disc:

$$I_* \frac{d\Omega_*}{dt} = \frac{dJ}{dt}, \quad (12)$$

which introduces the moment of inertia of the star, $I_* = k^2 M_* R_*^2$ as an additional parameter of the problem.

The disc-star angular momentum exchange dJ/dt has two components: matter accreting onto the star adds angular momentum at a rate $\dot{m}_{\text{co}} r_{\text{in}}^2 \Omega_K(r_{\text{in}})$, while the disc-field coupling outside co-rotation extracts angular momentum spinning the star down. The rate of angular momentum exchange between the disc to the star will thus be (with 2, 3, 4):

$$\begin{aligned} \frac{dJ}{dt} &= \dot{m}_{\text{co}} r_{\text{in}}^2 \Omega_K(r_{\text{in}}) - T_B \\ &= \frac{\eta \mu^2}{r_{\text{in}}^3} \left[\frac{1}{4} \left(\frac{r_c}{r_{\text{in}}} \right)^{3/2} y_m - \frac{\Delta r}{r_{\text{in}}} y_\Sigma \right]. \end{aligned} \quad (13)$$

The corotation radius (a function of Ω_*) evolves as:

$$\frac{dr_c}{dt} = -\frac{2}{3} \frac{dJ}{dt} I_*^{-1} \left(\frac{GM_*}{r_c^5} \right)^{-1/2}. \quad (14)$$

Eq. (13) shows that there is a value of \dot{m} for which there is no net angular momentum exchange with the star. This is the ‘spin equilibrium’ state discussed in previous work. This zero-point will depend on our adopted connecting functions, as well as the size of the transition widths, Δr and Δr_2 . If $\dot{m} = 0$, there is no spin-equilibrium solution: the star will spin down by the magnetic torque.

2.2.4 Steady-state solutions

In the presence of a magnetic torque at the disc inner edge, the steady solutions ($\partial/\partial t = 0$) of the thin viscous disc diffusion equation with the above boundary conditions have the form (cf. DS10):

$$3\pi\nu\Sigma = \frac{T_B}{\Omega(r_{\text{in}})r_{\text{in}}^2} \left(\frac{r_{\text{in}}}{r} \right)^{1/2} + \dot{m} \left[1 - \left(\frac{r_{\text{in}}}{r} \right)^{1/2} \right], \quad (15)$$

where \dot{m} is the accretion rate onto the star, given by (9). If the inner edge is inside corotation ($T_B = 0$), Σ has the standard form for steady accretion on an object rotating below the Keplerian rate (second term on the RHS).

For r_{in} well outside corotation ($r_{\text{in}} - r_c \gg \Delta r$), $\dot{m} \downarrow 0$ and we have a dead disc. The surface density is then determined by the first term on the RHS. The steady outward flux of angular momentum in this case has to be taken up by a sink at some larger distance, otherwise the disc can not be stationary as assumed. This sink can be the orbital angular momentum of a companion star, or the disc can be approximated as infinite. The latter is a good approximation for changes in the inner regions of the disc, if time scales short compared with the viscous evolution of the outer disc are considered.

2.3 Numerical method

We use the one-dimensional numerical code described in DS10 to evolve the standard diffusive thin-disc equation with our viscosity prescription (5) and our description of the disc-field interaction (which gives the inner boundary conditions the boundary conditions 6 and 11). At the outer boundary a mass flux and a flux of angular momentum are specified in various combinations (described in sec. 2.3.1).

The calculations are done in dimensionless coordinates and variables. In DS10 we scaled all physical lengthscales to r_c , and physical time scales to $T_{\text{visc}}(r_c)$. Since in this paper we want to follow the evolution of r_c , we instead use the stellar radius r_* and $T_{\text{visc}}(r_*) \equiv T_*$ to scale our physical length and timescales. The grid is logarithmically spaced (to ensure sufficient resolution in the inner disc to capture the disc instability). It is an adaptive mesh, such that the inner boundary moves with r_{in} .

Since the grid used is time dependent, the outer boundary condition is also applied at a time-varying location. As discussed in DS10, the artefacts this causes are small, compared to specification at a fixed location (at least for the large discs studied in most cases).

The size of the discs studied range from 10 to 10^6 times the inner edge radius, and the number of grid points needed for sufficient resolution varies accordingly, from 90 for the smallest to 560 for the largest discs.

Time-stepping is done with an implicit method, so that the short timescales encountered during episodes of cyclic accretion

can be followed, as can the much slower viscous evolution of the disc as a whole and the spin down the star. It is adapted to the stiff nature of the equation to be solved (see DS10 for details).

2.3.1 Outer Boundary Condition

The lifetime and evolution of a star surrounded by a dead disc is an inherently time-dependent problem, so the initial conditions in the disc can be critical for its evolution. Since the spin-down timescale for the star can be much larger than viscous timescales throughout the disc, the conditions in the outer disc will also strongly influence the evolution of the system.

We thus consider the effect of varying the outer boundary conditions for the disc. The first condition we study is the simplest: a fixed mass flux $\dot{m} = \dot{m}_0 (> 0$, corresponding to accretion). As discussed in the introduction, a key aspect of disc-magnetosphere interaction is that accretion is possible even as the star is spun down. At fixed $\dot{m} > 0$, the angular momentum flux can be either inward or outward.

If the mass flux specified vanishes at r_{out} , the boundary condition is

$$\left. \frac{\partial}{\partial r} (r^{1/2} \nu \Sigma) \right|_{r_{\text{out}}} = 0. \quad (16)$$

On long evolution timescales, the finite extent of the disc itself could be relevant in a star without a companion where the disc can spread outwards. To model this, as our final boundary condition we take $\Sigma(r_{\text{out}}) = 0$, so that the angular momentum added at r_{in} is carried away by the outer parts of the disc, causing the disc to spread outwards. In section 5.2 we discuss the consequences of these assumptions in limiting the lifetime of a trapped disc.

2.3.2 The Evolution of r_c

The final modification to our code used in DS10 is to allow r_c to evolve as the spin rate of the star changes (14). The characteristic evolution timescale for \dot{r}_c , the spin-down timescale for the star, is much longer than the nominal viscous timescale in the disc (see the next section). The code updates r_c by an explicit time step, rather than implicitly, as we do the other variables. Rather than discretizing (14) and add it to our system of linearized equations that are solved numerically at each timestep, we instead approximate the evolution in r_c to first order in time, that is:

$$r_c(t_0 + \Delta t) = r_c(t_0) + \left. \frac{dr_c}{dt} \right|_{t_0} \Delta t \quad (17)$$

This scheme is simpler than adding additional equations to the code, and is sufficient to describe the co-evolution of the disc and stellar spin-rate. However, as we discuss in sec. 4.3, it is not accurate enough when r_{in} is close to r_c and the spindown timescale comparable to the viscous timescale in the inner regions of the disc.

3 CHARACTERISTIC TIMESCALES OF DISC-STAR EVOLUTION

Three kinds of timescale play a role in the evolution of a star coupled magnetically to an accretion disc. These are the spin period of the star, the timescale for changes in spin period of the star, and viscous evolution timescales of the disc. The viscous evolution does not have a single characteristic timescale; it can vary over many

orders of magnitude depending on which regions in the disc participate in the evolution.

The spin period of the star (P_*) determines the location of the corotation radius which in turn sets the accretion rate at which the magnetic torque changes sign. P_* also determines the timescale for magnetospheric variability (from processes like reconnection of field lines), which can lead to variability in \dot{m} , Δr , Δr_2 and the B_ϕ component of the magnetic field (which sets the magnitude of the torque). P_* is much shorter than the other characteristic timescales studied in this paper, and the complex variability processes are best studied with detailed MHD simulations, so in this work we assume time-averaged values for \dot{m} , Δr , Δr_2 and B_ϕ and neglect timescales much shorter than the characteristic viscous timescale at R_* .

A convenient unit of time for measuring changes in a viscous disc at a distance r from the center is $t_a = r^2/\nu(r)$, sometimes called the accretion- or viscous timescale at distance r . If α is the viscosity parameter and H the disc thickness, it is longer than the orbital timescale Ω_K^{-1} by a factor $\alpha^{-1}(r/H)^2$, a large number for most observed discs. Natural choices for r in this expression would be the disc's inner edge r_{in} (t_{in}) or the corotation radius $r_c(t_c)$. Both of these are functions of time. The actual timescales of variation in our discs can be much shorter than t_c and t_{in} , however, since the extent of the disc that participates in the variation can be much smaller than r_{in} . In the cyclic accretion mode described in DS10, for instance, cycle times as short as $0.01t_c$ are found. The timescale for viscous adjustment in the outer disc regions, on the other hand, can be very large compared to t_c .

The longest timescale is the rate at which the star's spin will change, which is determined both by the rate of angular momentum exchange with the disc and the star's moment of inertia. The spin-down torque of a dead disc (with $\dot{m} = 0$ and $r_{\text{in}} = r_c$) is, from eq. (13):

$$I_* \frac{d\Omega_*}{dt} = -\frac{\eta\mu^2\delta}{r_{\text{in}}^3}, \quad (18)$$

where $\delta = \Delta r/r_{\text{in}}$. The characteristic spindown time is:

$$T_{\text{SD}} \equiv P_*/\dot{P} \sim \frac{I_*\Omega_*r_{\text{in}}^3}{\eta\mu^2\delta}. \quad (19)$$

If the inner edge stays near corotation, $\Omega_K = \Omega_*$, replacing r_{in} by r_c , this yields:

$$T_{\text{SD}} = \frac{GM_*I_*}{2\pi\eta\mu^2\delta}P_*, \quad (\Omega_K(r_{\text{in}}) = \Omega_*). \quad (20)$$

The spindown timescale varies considerably between different sources. Adopting $\eta = 1$ and $\Delta r/r_{\text{in}} = 0.3$, this spindown timescale is short enough to account for spin-regulation in slowly rotating magnetic stars. In Table 3 we summarize the predicted spin-down timescales for a slowly rotating X-ray pulsar, a millisecond pulsar, a slowly rotating Ap star, and a typical T Tauri star. For all these examples but the millisecond pulsar the spin-down timescale is much shorter than the lifetime of the star. Provided the conditions are such that the inner edge of the disc can stay near corotation (i.e. what we have called the 'trapped disc' state), it will be able to spin down a star to very slow rotation periods. In the next sections we will explore how this could work in detail by evolving a viscous disc in time numerically.

The last column of Table 3 lists the ratio of the viscous timescale r_c^2/ν and (19). Note that for our description of viscosity, the viscous and spin-down timescales both scale as $r^{-3/2}$. The

quantity $T_{\text{visc}}/T_{\text{SD}}$ thus defines the ratio of the time that gas at that radius takes to travel inwards onto the star and the time it would take to spin-down so that $r_c = r_{\text{in}}$, independent of radius. In all cases, the spin-down timescale is much longer than the viscous timescales in the disc, so that at least part of the disc is able to adjust to the new spin rate of the star. However, the exact ratio between the two timescales varies over ten orders of magnitude, from 0.06 for a strongly magnetic Herbig Ae star to 10^{-17} for an accreting millisecond pulsar. This ratio implies that the extent of spin-down will be influenced by the viscous evolution of the disc itself in response to the disc-field interaction, and that this evolution will vary substantially between different systems, breaking the scale invariance usually assumed in disc-magnetospheric interactions. In section 4.3 we demonstrate how the ratio of these two timescales is critical in determining the ratio of r_{in} to r_c in a trapped disc.

3.1 Representative Model

In sections 4 and 5.2 we study how a trapped disc can form and evolve, as well as how it can become untrapped. In order to simplify comparison between different simulations, we adopt a set of parameters for a representative model, which we then vary between solutions as necessary.

For the dimensionless parameters we adopt $\Delta r/r_{\text{in}} = 0.1$, $\Delta r_2/r_{\text{in}} = 0.04$, and $B_\phi/B_z \equiv \eta = 0.1$. The values of $\Delta r/r_{\text{in}}$ and $\Delta r_2/r_{\text{in}}$ are small enough to provide an abrupt transition between an accreting and non-accreting disc, but do not show the cyclic instability discussed in DS10. Neglecting the star's spin change, the problem has a scale invariance (DS10), whereby the parameters μ , M_* , Ω_* and \dot{m} can be re-written as the ratio \dot{m}/\dot{m}_c , and \dot{m}_c is the accretion rate in (7) that puts the magnetospheric radius at r_c , a natural unit of \dot{m} for magnetospheric accretion. The results in DS10 were presented in this unit.

As discussed in the previous section, the variation of r_c with time during spindown of the star makes this unit impractical. Instead, we present the representative model in units suitable for a protostellar system with $T_{\text{visc}}/T_{\text{sd}} = 2.6 \times 10^{-3}$ (which is as large a ratio as the present version of the code allows), and explore the effect of varying this ratio. As unit of length we use r_* , the star's radius, and for timescale t_* , the nominal viscous timescale of the disc at the star's radius.

4 TRAPPED DISCS

4.1 Trapped disc evolving from an accreting disc

For our description of the disc-field interaction (which ignores outflows), once the accretion rate falls to zero, the inner edge of the disc could be located anywhere outside r_c , depending on the amount of mass in the disc. What then determines the location of the inner radius of a dead disc? To answer this question, we simulate an initially steadily accreting disc in which the accretion rate at r_{out} suddenly decreases to zero. As the reservoir of gas in the disc runs out, the accretion rate onto the star declines, and the inner radius of the disc moves outwards.

In the simulation we use our representative disc parameters described above, and set the initial inner radius of the disc to be just inside r_c , $r_{\text{in}}(t=0) = 0.88r_{c,0}$, and the outer radius $r_{\text{out}} = 100r_{\text{in}}$. We can calculate the corresponding accretion rate from (7), and use the static solution for Σ given by (15) as our initial surface density profile. At $t=0$, we set $\partial_r(\nu\Sigma r^{1/2})|_{r_{\text{out}}} = 0$, so that no

Table 1. Spindown and viscous timescales for different type of magnetic stars

Source	Mass (M_{\odot})	Radius (R_{\odot})	B_* (G)	P_*	I_* ($M_{\odot} R_{\odot}^2$)	T_{SD} (years)	$T_{\text{visc}}(r_c)/T_{\text{SD}}$
slow Pulsar	1.4	1.4×10^{-5}	10^{12}	5.0s	2.9×10^{-11}	4400	3×10^{-7}
ms Pulsar	1.4	1.4×10^{-5}	10^8	0.1s	2.9×10^{-11}	2.6×10^{11}	2×10^{-17}
Magnetic Ae star ^a	3.0	5.5	10^4	10 yrs	4.0	3×10^5	0.06
T Tauri Star ^b	0.6	3.0	1500	7 days	0.54	2.3×10^4	0.001

^a B_* and I_* from Stępień (2000)^b Sipos et al. (2009)

mass is added to the disc or allowed to escape. This sets a constant angular momentum flux at r_{out} .

The results are shown in Fig. 1. The bottom panel of Fig. 1 shows the change in accretion rate onto the star, scaled to \dot{m}_c . The top panel of Fig. 1 shows the evolution of the inner radius (solid black curve) and r_c (dashed red curve) in response to the changing accretion rate. After initial steady accretion over about $30 t_*$ (less than 1/10th the viscous timescale at r_{out}), \dot{m} through the inner edge of the disc begins to decrease as the reservoir of mass in the disc is accreted onto the star, and r_{in} begins to move outward. From $30 - 1500 T_*$, \dot{m} decreases exponentially with a decay timescale of about $240 t_*$ and the disc moves outwards. However, r_{in} increases by only about 20% as the accretion rate decreases by three orders of magnitude. (The structure of r_{in} around r_c is an artifact of the tanh connecting functions we adopted to describe r_{in} and \dot{m} across the transition region).

After $\sim 1500 t_*$ the star begins to spin down (so that r_c moves outwards), and the inner radius of the disc begins to track r_c , so that the ratio r_{in}/r_c remains nearly constant thereafter. The behaviour of the accretion rate at this point also changes. Although it continues to decrease exponentially, the decay timescale lengthens considerably and the accretion rate ($\sim 10^{-4}$ of the initial \dot{m}) is regulated by the spin-down rate of the star. Instead of continuing to move away from r_c into the ‘dead disc’ regime (in which $\dot{m} = 0$), the inner radius instead remains *trapped* at nearly a constant fraction of r_c while the star continues to spin down. We thus call this disc solution a ‘trapped disc’, since rather than continue to move outwards, r_{in} becomes trapped at a nearly constant fraction of r_c .

At the outer edge of the disc $\dot{m} = 0$ and there is an outward angular momentum flux. The accretion onto the star comes from the disc being slowly eroded (although at a very low rate) as r_{in} moves outward. The evolution of r_{in} and the inner parts of the disc is determined by the spin down rate of the star itself, which is itself influenced by how close r_{in} can stay near r_c . In Sec. 4.3 we demonstrate how r_{in}/r_c is mostly determined by the parameters Δr and Δr_2 , and the ratio $T_{\text{visc}}/T_{\text{SD}}$. However, the main conclusion of this section is clear: if a trapped disc forms and can efficiently carry away the angular momentum of the star, over spin-down timescales the disc will accrete at such a rate so as the inner edge of the disc can move outwards together with r_c and the star could in theory spin down completely.

4.2 Trapped disc evolving from a dead disc

Consider next a case where the initial condition is a dead disc, ($r_{\text{in}} > r_c$) given by the steady profile ((15) with $\dot{m} = 0$ and $y_{\Sigma} = 1$). As the star spins down, r_c moves out until it catches up

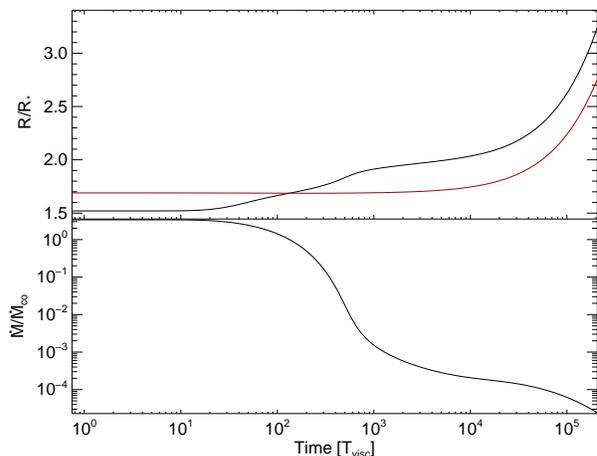


Figure 1. Transition from an accreting to a ‘trapped disc’ state. The initial Σ profile of the disc sets $r_{\text{in}} = 1.5r_*$, but the accretion rate through r_{out} is set to zero at $t = 0$. As a result of mass loss by accretion through r_c , the disc quickly evolves away from a steady accretion state and after about $1500 t_*$ settles into a slowly evolving state in which r_{in} tracks r_c . Top: The black solid curve shows the evolution of the inner radius in time, while the red dashed curve shows the evolution of r_c . Bottom: The accretion rate onto the star, which decreases sharply as r_{in} moves outwards across the corotation radius.

with the inner edge r_{in} . From then on, the same evolution is as in the previous case: r_{in} and r_c move outward together indefinitely. A small amount of mass accretes while the star’s angular momentum is transferred to the disc.

The asymptotic evolution of this dead disc can be compared with the case when a fixed mass flux is imposed at the outer edge. The asymptotic state is then a steady state with *spin equilibrium*: the spin-up torque due to the accreted mass is balanced by the magnetic torque at r_c transferring angular momentum outward.

We illustrate the distinction between these two cases in figure 2. This shows the evolution of an initially dead disc (black, thick curves) and accreting disc (red, thin curves). Both discs have the same representative parameters (Sec. 3.1) and $r_{\text{out}} = 100 r_{\text{in}}$, but with different initial inner radii (a few times the stellar radius), accretion rates and appropriate initial surface density profiles given by (15). For the dead disc, we take $r_{\text{in}} = 1.3r_{c,0}$ (where $r_{c,0}$ is the initial corotation radius) which corresponds to $\dot{m} \simeq 0$ for our chosen value of $\Delta r_2/r_{\text{in}}$. For the accreting disc, we choose $r_{\text{in}} = 1.1r_{c,0}$, which corresponds to a low but non-zero accretion rate ($\dot{m} = 8 \times 10^{-3} \dot{m}_c$).

Since \dot{m} for the accreting disc is initially low compared to

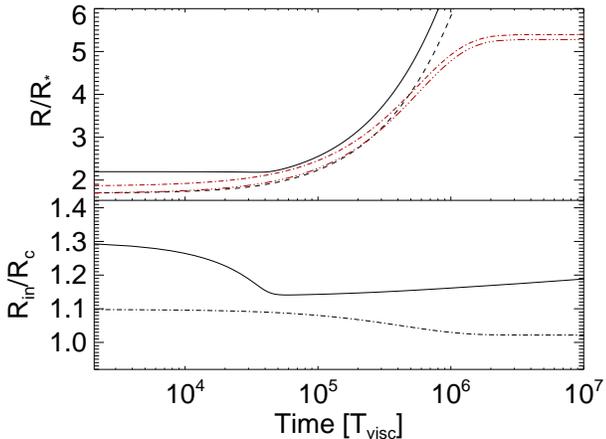


Figure 2. Comparison of the evolution of r_{in} and r_c between a dead disc (black solid and dashed curves) and accreting disc (red dot-dashed curves). Top: The evolution of r_c and r_{in} in units of the stellar radius. Black curves: r_{in} (solid) and r_c (dashed) in the dead disc; red curves: the accreting disc (r_{in} , dot-dashed curve; r_c , triple-dot dashed curve). Bottom: The ratio r_{in}/r_c for the dead disc (solid) and the accreting disc (dot-dashed). The dead disc keeps evolving indefinitely, the accreting case reaches a steady state in spin equilibrium with the star around $t \sim 10^6$.

\dot{m}_c , at early times r_c evolves at the same rate in both simulations (accreting: red, triple-dashed curve; non-accreting: black, dashed curve), and the star spins down. Eventually, however, the amount of angular momentum added by the accreted gas becomes comparable to the amount removed, and spin equilibrium is reached at $\sim 10^6 t_*$. In contrast, for the $\dot{m} = 0$ case, the disc at first remains unaffected, while r_c moves outwards. (This is because the magnetic torque depends only on distance, not on the rotation rate of the star). After r_c moves close enough to r_{in} that accretion can begin (around $4 \times 10^4 t_*$), the two start to move outwards at approximately the same rate. The (low) accretion rate onto the star is determined by the (slow) rate at which r_c moves outwards, and the star continues to spin down indefinitely. The bottom panel of fig. 2 shows the ratio of r_{in}/r_c for the accreting (dashed) and non-accreting disc (solid). After the non-accreting disc passes out of the dead disc phase (at $\sim 4 \times 10^4$), in both systems the ratio changes by less than 10%, and r_{in} always remains close to r_c . The non-accreting disc, however, differs in that it never reaches spin equilibrium.

The main difference between the evolution of a dead disc and an accreting disc is the behaviour of the inner edge radius. As seen in sec. 4.2, in the initially dead disc the accretion rate onto the star is determined by the disc's behaviour when it reaches $r_{\text{in}} \simeq r_c$, while in an accreting disc the accretion rate is governed by the value set at the outer boundary.

Both the accretion rate on the star and the outward angular momentum flux in our trapped discs depend sensitively on the distance between r_{in} and r_c compared with the transition widths Δr and Δr_2 . In the results of figure 2, $r_{\text{in}} - r_c$ is of the order $0.5 - 2 \Delta r$. In the next section we develop an analytic estimate of this number and compare it with the numerical results.

4.3 Analytic estimates for a trapped disc

As we showed above, an initially dead disc will eventually start accreting at a low rate, in such a way that r_{in} moves outwards together with r_c at a nearly constant ratio. The accretion rate onto the star is

determined by how close r_c can move to r_{in} before the disc moves outwards in response. The actual distance on which r_{in} settles in cases like those show in the previous section depends on the details of the disc-field interaction (namely the parameters Δr and Δr_2) and the ratio of timescales, $T_{\text{visc}}/T_{\text{SD}}$.

The spin-down timescale derived above assumes that r_{in} moves steadily outwards at the same rate as r_c . This timescale is an upper limit, since as r_c approaches r_{in} there is reduced transport of angular momentum through r_{in} and accretion onto the star begins. In addition, in order for spin-down to remain efficient, the angular momentum added by the disc-field interaction can be transported through the disc and carried away at r_{out} , otherwise r_{in} will move quickly away from r_c and spin-down will effectively cease.

When r_{in} is far enough from r_c that $\dot{m} \simeq 0$, r_{in} stays fixed as the star is spun down and r_c moves outwards (15). However, once r_{in} moves closer to r_{in} (within Δr or Δr_2), this static state is no longer possible: either matter at r_{in} starts accreting onto the star ($y_m \neq 0$), or the surface density at r_{in} declines ($y_\Sigma \neq 1$), which causes r_{in} to move closer to r_c until accretion through r_{in} can begin. Since the viscous timescale in the inner part of the disc is much shorter than the spindown timescale, after accretion through r_{in} begins, a pseudo-steady-state develops, and r_{in} moves slowly outwards with r_c . If the disc can maintain a steady-state for the given \dot{m} , then r_{in} will track r_c , and the disc will remain a nearly dead disc as the star spins down to a small fraction of its initial spin period.

We can study this quantitatively by considering the equations for \dot{r}_{in} and \dot{r}_c . The evolution of the inner edge of the disc (11 from Sec. 2.2.2), defining $u \equiv \Sigma r$ for convenience is:

$$2\pi u(r_{\text{in}})\dot{r}_{\text{in}} = y_m \frac{\eta \mu^2}{4\Omega_* r_{\text{in}}^5} - 6\nu_0 \pi r_{\text{in}}^{1/2} \frac{\partial u}{\partial r} \Big|_{r_{\text{in}}}. \quad (21)$$

As long as the two terms on the right of (21) balance, $\dot{r}_{\text{in}} = 0$ even after accretion through r_{in} begins. This will continue until the surface density profile near r_{in} no longer satisfies (15). Since the change in r_c is the only source of variability in the problem, r_{in} will approximately track r_c .

Eq. (21) cannot be solved as is, since $\partial u/\partial r$ depends on solving the full time dependent diffusion problem. As an estimate we assume that u changes as a result of the changing boundary condition (which will increase the surface density gradient), divided by the rate at which the rest of the disc can respond to that change (which will smooth it out). This can be approximated by:

$$\frac{\partial u}{\partial r} \Big|_{r_{\text{in}}} \sim - \frac{\partial u(r_{\text{in}})}{\partial t} v_{\text{visc}}^{-1}, \quad (22)$$

where v_{visc} is the viscous speed at r_{in} , of order: $v_{\text{visc}} \sim \nu/r$.

The time derivative for $u(r_{\text{in}})$ follows from the boundary condition for u :

$$\frac{\partial u}{\partial t} \Big|_{r_{\text{in}}} = \frac{\partial u}{\partial r_{\text{in}}} \dot{r}_{\text{in}} + \frac{\partial u}{\partial r_c} \dot{r}_c. \quad (23)$$

The equation for \dot{r}_{in} then becomes:

$$\dot{r}_{\text{in}} = \left(\frac{\nu_0 y_m f^{3/2}}{8 r_{\text{in}} \partial_{r_{\text{in}}} y_\Sigma} \left(\frac{\Delta r}{r} \right)^{-1} r_{\text{in}}^{-1/2} - \dot{r}_c \right) \left(\frac{10}{3} \frac{y_\Sigma}{\partial_{r_{\text{in}}} y_\Sigma r_{\text{in}}} - 1 \right)^{-1}, \quad (24)$$

where $f \equiv (r_c/r_{\text{in}})$, and we have used the definition of y_Σ from Sec. 2.2.1 so that $\partial_{r_{\text{in}}} y_\Sigma = -\partial_{r_c} y_\Sigma$.

The evolution of \dot{r}_c depends on the rate of angular momentum exchange with the star. Matter falling onto the star spins it up, while the interaction with the disc outside r_c transfers angular momentum

outward and spins the disc down. The equation for this evolution is given by (14), which can be re-written:

$$\dot{r}_c = \frac{2}{3} \frac{\eta\mu^2}{(GM_*)^{1/2} I_*} f^{7/2} \left[\frac{\Delta r}{r_{\text{in}}} y_\Sigma - \frac{y_m}{4} f^{1/2} \right] r_{\text{in}}^{-1/2}. \quad (25)$$

We can study the evolution of \dot{r}_c and \dot{r}_{in} in two limiting cases. In the limit where $r_{\text{in}} - r_c \gg \Delta r, \Delta r_2$:

$$\dot{r}_c \rightarrow \frac{2}{3} \frac{\eta\mu^2 r_{\text{in}}^{-4}}{(GM_*)^{1/2} I_*} \frac{\Delta r}{r_{\text{in}}} r_c^{7/2} \quad (26)$$

$$\dot{r}_{\text{in}} \rightarrow 0,$$

so that:

$$r_c = \left(-\frac{5}{3} \frac{\eta\mu^2 r_{\text{in}}^{-4}}{(GM_*)^{1/2} I_*} \frac{\Delta r}{r_{\text{in}}} t + r_{c,0}^{-5/2} \right)^{-2/5} \quad (27)$$

$$r_{\text{in}} = r_{\text{in},0}.$$

This is the limiting ‘dead disc’ case, where the amount of angular momentum being injected at r_{in} can be extracted at r_{out} and the disc remains steady while the star is spun down. It predicts a slightly smaller spin-down torque than was estimated in Sec. 3 because the torque scales with r_{in} .

The inner radius of the disc will remain approximately constant until either $\frac{y_m}{y_\Sigma}$ or $\frac{\partial_r y_\Sigma}{y_\Sigma} \left(\frac{\Delta r}{r_{\text{in}}} \right)^{-1}$ become non-negligible (24). Based on our assumption that the disc will remain in a quasi-steady-state while r_{in} moves, r_{in} will evolve only in response to changes in r_c , which means that f is a constant. This simplifies (24) and (25) considerably:

$$\dot{r}_c = (A_0 f^{7/2} - A_1 f^4) r_{\text{in}}^{-1/2} \quad (28)$$

$$\dot{r}_{\text{in}} = \frac{B_0 f^{3/2} r_{\text{in}}^{-1/2} - \dot{r}_c}{B_1 - 1},$$

where,

$$A_0 = \frac{2}{3} \frac{\eta\mu^2}{(GM_*)^{1/2} I_*} \frac{\Delta r}{r_{\text{in}}} y_\Sigma \quad (29)$$

$$A_1 = \frac{1}{6} \frac{\eta\mu^2}{(GM_*)^{1/2} I_*} y_m$$

$$B_0 = \frac{\nu_0 y_m}{8 r_{\text{in}} \partial_{r_{\text{in}}} (y_\Sigma)} \left(\frac{\Delta r}{r_{\text{in}}} \right)^{-1}$$

$$B_1 = \frac{10 y_\Sigma}{3 r_{\text{in}} \partial_{r_{\text{in}}} (y_\Sigma)}.$$

The solution is then:

$$r_c = \left(\frac{3}{2} \left(A_0 f^3 - A_1 f^{9/2} \right) t + r_{c,0}^{3/2} \right)^{2/3} \quad (30)$$

$$r_{\text{in}} = \left(\frac{3}{2} \left(\frac{B_0 f^{3/2} - A_0 f^{7/2} + A_1 f^4}{B_1 - 1} \right) t + r_{\text{in},0}^{3/2} \right)^{2/3}.$$

We can use (28) to calculate f , that is, how close r_c can move towards r_{in} before the disc will start moving outwards in response. Setting $\dot{r}_c = f \dot{r}_{\text{in}}$, we can re-express the constants in (30) as:

$$\left(2 \frac{\Delta r}{r_{\text{in}}} f y_\Sigma - \frac{y_m f^{3/2}}{2} \right) \left(\frac{10 y_\Sigma}{3 r_{\text{in}} \partial_{r_{\text{in}}} y_\Sigma} + f - 1 \right) \left(\frac{r_{\text{in}} \partial_{r_{\text{in}}} y_\Sigma}{y_m} \right) = \frac{3}{8} \frac{T_{\text{SD}}}{T_{\text{visc}}}, \quad (31)$$

and solve f numerically to give the approximate evolution for r_c and r_{in} in time.

In Fig. 3 we compare our estimates for the evolution of r_c

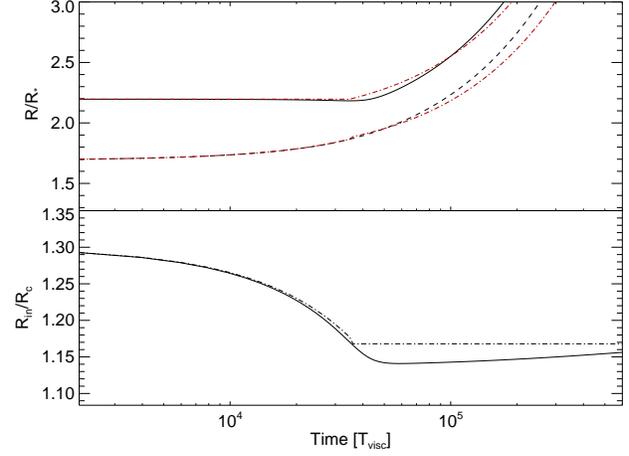


Figure 3. Comparison between numerical solution and analytic estimate for an idealized dead disc around a star spinning close to break-up $r_c = 1.7$, with $\Delta r/r_{\text{in}} = 0.1$ and $\Delta r_2/r_{\text{in}} = 0.05$. Top: Numerical solution of the evolution of r_c (black, dashed curve) and r_{in} (black, solid curve) in time. Overplotted is our analytic estimate for r_{in} (red dash-dotted curve) and r_c (red dash-triple dotted curve) for the same physical conditions. Bottom: The ratio r_c/r_{in} for our numerical solution (solid curve) and analytic estimate (dash-dotted curve). The disagreement in the two solution arises from our simplified treatment of the viscous disc.

and r_{in} to the solution from our numerical simulation. We consider a rapidly spinning star ($r_c = 1.7$), with $\Delta r/r_{\text{in}} = 0.1$ and $\Delta r_2/r_{\text{in}} = 0.05$. In the top panel we compare the two solutions for r_{in} (estimate: dot-dashed red curve vs. numerical solution: black solid curve) and r_c (estimate: triple-dotted-dashed red curve vs. numerical solution: black dashed curve). In the bottom panel we plot the ratio r_{in}/r_c for the numerical (solid curve) and analytic (dashed curve) result. At early times, $r_c < r_{\text{in}}$ (27), and the solutions match exactly. However, at late times the solutions disagree somewhat. Most obviously, the value for f calculated by (30) is smaller than the numerical solution, that is that $\dot{r}_{\text{in}} = 0$ for longer than we predict, and there is some evolution in f over long timescales. This mismatch comes from our simplified treatment for $\partial_r u|_{r_{\text{in}}}$, which over long timescales will depend on the surface density gradients in the entire disc.

By comparing the sizes of each term on the left-hand side of (31), some insightful approximations can be made. Since the solution is nearly a dead disc, the disc’s accretion rate will be very low, so that $y_m \ll 1$, $y_\Sigma \sim O(1)$, and $\partial_{r_{\text{in}}} y_\Sigma \ll 1$. As well, r_{in} is close to r_c so that $f \sim O(1)$. Since in general $T_{\text{visc}}/T_{\text{SD}} \ll 1$, (31) can be approximated:

$$\frac{160}{9} \frac{\Delta r}{r_{\text{in}}} f \frac{y_\Sigma^2}{y_m} \simeq \frac{T_{\text{SD}}}{T_{\text{visc}}}, \quad (32)$$

and used to estimate f . The left-hand side of (32) is dominated by y_m^{-1} , which quickly grows as r_{in} moves away from r_c , and must balance the right-hand side. The ability to sustain a dead disc will thus depend on *both* the interaction between the disc and the field (through the parameter Δr_2), *and* the ratio between the spin-down timescale and the accretion timescale.

For a given $T_{\text{SD}}/T_{\text{visc}}$, if $\Delta r_2/r_{\text{in}} \ll 1$, then y_m is nearly a step function, and f will stay close to 1 even if $T_{\text{SD}}/T_{\text{visc}}$ is very large. On the other hand, if $\Delta r_2/r_{\text{in}} \sim 1$, and accretion continues even when r_{in} moves a fair distance from r_c , then it is possible that the solution to (31) will predict a larger value for r_{in} than can

support a dead disc. In this case no dead disc solution exists: even if the disc initially begins as a dead disc, once r_c moves close enough to r_{in} that accretion begins, the disc will begin to move outwards until matter at r_{in} can be expelled from the system. The outflow of material will then proceed at a moderate rate as the surface density profile of the disc evolves away from the dead disc solution [(15) with $\dot{m} = 0$] to the standard disc solution, with outflow rather than accretion onto the star.

5 TRAPPED AND UNTRAPPED

5.1 Accreting discs evolving to trapped or dead disc states

The results found so far and in DS10 indicate the strong tendency for the inner edge of the disc to track the corotation radius, what we call here a ‘trapped disc’. This does not happen in all cases, however. We would like to find out under what conditions a disc gets stuck in this way, and when instead the inner edge proceeds to move well outside corotation into the ‘dead disc’ state.

Armed with qualitative understanding from the previous section, we can address this question with a few numerical experiments. We take the case of a neutron star with field strength $B_S = 10^{12}$, initial spin period $P_* = 5\text{s}$, and initial inner edge radius $r_{\text{in}} = 0.95r_{c,0}$. The disc is thus initially in an accreting state. The (initial) outer boundary is located at $r_{\text{out}} = 100r_{\text{in}}$, and we set $\dot{m} = 0$ there so that no matter can escape the system. The other parameters are the same as for our representative model.

We first investigate the effect of varying the viscosity on the way the transition from an accreting to a dead disc takes place. This is shown in fig. 4. The spindown timescale (20) for the initial r_c is $T_{\text{SD}} = 10^5$ years. From top to bottom, we plot this transition for decreasing values of viscosity (and hence increasing viscous timescales). To compare these two quantities we define T_{visc} as the viscous timescale at the initial r_c . In the different plots, the ratio $T_{\text{visc}}/T_{\text{SD}}$ increases from 2.5×10^{-9} to 10^{-4} .

The most striking change in behaviour occurs between the top panel and the second panel. At the shortest viscous timescale, the disc does not get into a trapped state, but evolves directly through corotation, while at lower viscosity the it always settles into the trapped state, with the inner edge moving in step with with the corotation radius.

In all the discs, the initial evolution is the same: as \dot{m} decreases, r_{in} moves outwards, crossing r_c over about 10^3 years. Once that happens, however, the subsequent evolution of the disc and star differs substantially between simulations. In the most viscous discs (top), r_{in} continues moving steadily outwards over 10^{10} years – roughly 10^5 times longer than the nominal spin-down timescale of the disc. Since r_{in} keeps moving outwards, the torque on the star decreases too, so that the disc moves far away from r_c before it is able to spin down the star.

As the viscosity is reduced, the ratio between the two timescales becomes smaller, and r_{in} does not move so far away from r_c before $\dot{r}_{\text{in}} \sim \dot{r}_c$. Thus for the disc with the lowest viscosity (bottom), r_{in} and r_c begin to move outwards after about 10^5 years, and the trapped disc (where \dot{m} is regulated by \dot{r}_c) has a much larger accretion rate onto the star (seven orders of magnitude larger after 10^4 years) than for higher viscosities.

By the initial condition chosen, the magnetic torque pushes the inner edge out across corotation. This causes mass to pile up outside r_{in} . The higher the viscosity, the faster this pile is reduced again by outward spreading, and the faster the inner edge can continue to move outward in response. The experimental result is then

that trapping behavior is avoided when the transition takes place fast enough. We return to this in the discussion.

The pile up is also influenced by the way in which accretion on the star changes as r_{in} crosses r_c , hence we expect that the parameter controlling this, Δr_2 , will have a strong effect as well.

In fig. 5 we show three discs in which the initial ratio $T_{\text{SD}}/T_{\text{visc}}$ is kept fixed, but the value of Δr_2 changes, from top to bottom, $\Delta r_2/r_{\text{in}} = [0.4, 0.04, 0.004]$. The larger Δr_2 , the further away r_{in} must move from r_c in order for the accretion rate to decrease sufficiently to form the trapped disc. In the top panel (when Δr_2 is largest), the disc must move out a considerable distance before becoming trapped, sufficiently far to significantly decrease the efficiency of the spin-down torque (and hence increase the spin-down timescale of the star). As well as decreasing the spin-down efficiency, if Δr_2 is large enough, the inner radius could move far enough away from r_c that material could begin to be launched from the disc in an outflow, and the disc would become untrapped.

As was shown in Section 3, the ratio $T_{\text{visc}}/T_{\text{SD}}$ itself can vary over many orders of magnitude in different systems, from 10^{-17} in neutron stars with weak magnetic fields to 10^{-2} in discs around massive young stars. The size of this ratio will also determine whether a trapped disc can form.

This analysis would suggest that, assuming $\Delta r_2/r_{\text{in}}$ does not vary much from system to system, trapped discs are much more likely to form in protostellar discs than in strongly ionized discs around neutron stars. Furthermore, the closer r_{in} is to r_c , the higher the accretion rate in the trapped disc. Conceivably, especially if the viscosity in the disc were very low, this accretion rate could be larger than the average accretion rate in the disc itself, so that the disc could spin down the star for a long time without ever reaching spin equilibrium, even with a finite accretion rate onto the star.

As the results reported above show, the evolution can end either in a dead disc state in which the star has lost only a fraction of its angular momentum, or a trapped state in which corotation is maintained and the star can spin down much further. Which outcome results depends details of the interaction between the star and the disc, parametrized in our model by the transition widths Δr and Δr_2 . It also depends on the rate at which the disc can respond viscously compared to the spin change rate of the star. A fast response of the disc makes the transition through corotation faster than r_c changes, and the disc is more likely to enter the dead state. This makes it far more likely to occur in young stellar systems than in neutron star binaries. In the results presented above, the initial conditions were taken from steady solutions of the viscous thin disc equation. These included dead discs in which a steady state was made possible by a sink of angular momentum at the outer boundary of the numerical grid, which takes up the angular momentum added by the magnetic torques at the inner edge.

5.2 Dead discs evolving into trapped discs

In this example we investigate the opposite case of section 5.1: discs with r_{in} initially outside corotation, and conditions chosen such that the disc begins by spreading inward. The mass flux at the outer boundary is set to zero. Varying the initial outer radius, $r_{\text{out},0}$ varies the amount of mass in it, and the timescale of its long-term evolution can change.

We adopt the representative model parameters used before, with an initial inner radius set to $1.3r_c$ (corresponding to a negligible accretion rate for our chosen $\Delta r_2/r_{\text{in}}$), and set $r_{\text{out},0} = [10, 100, 10^3, 10^4] r_{\text{in},0}$. The evolution of r_{in} and r_c is plotted in Figs. 6 and 7. Fig. 6 compares the evolution of r_{in}/r_c for different

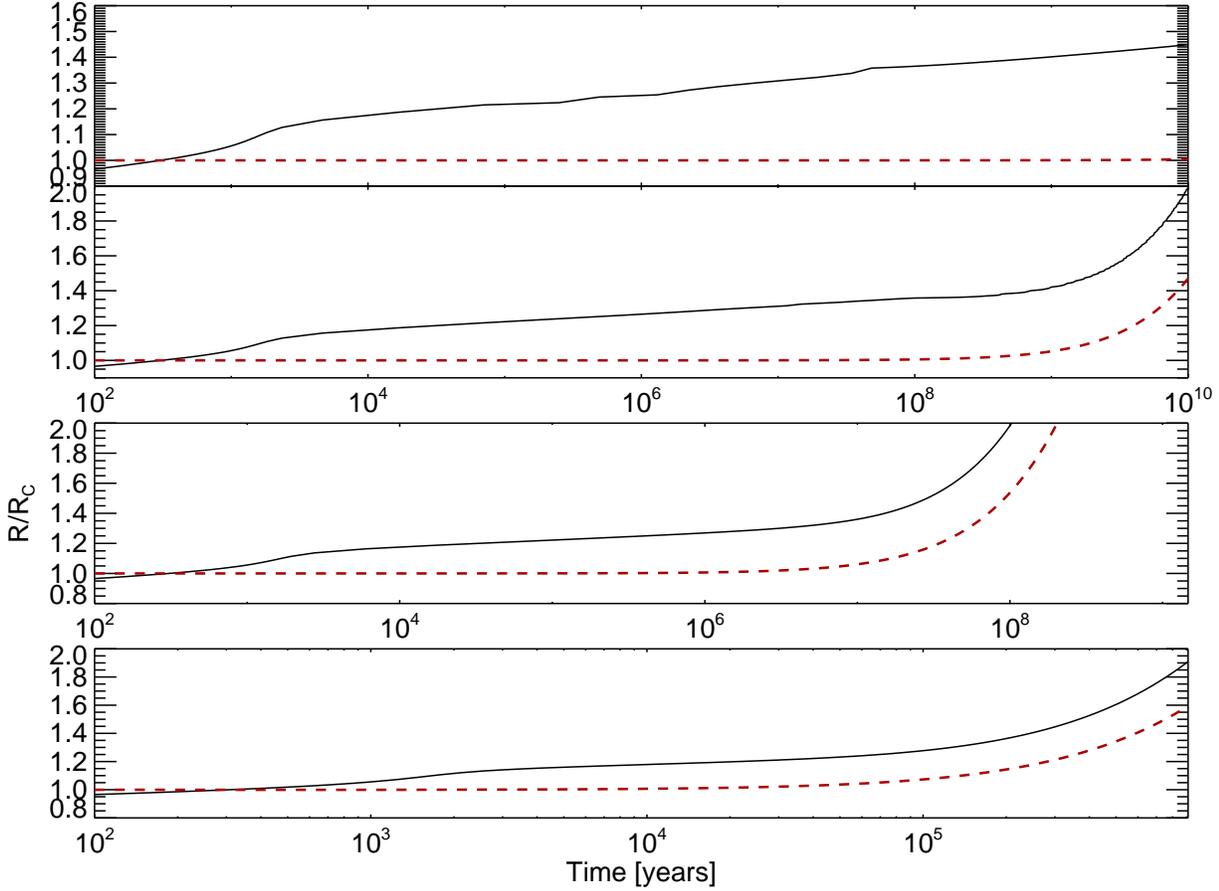


Figure 4. The evolution from an accreting to a non-accreting disc, for increasing (top to bottom) ratios of $T_{\text{visc}}/T_{\text{SD}}$ (with $\Delta r/r_{\text{in}} = 0.1$, $\Delta r_2/r_{\text{in}} = 0.04$). From top to bottom, the ratio $T_{\text{visc}}/T_{\text{SD}}$ is $2.5 \times [10^{-9}, 10^{-7}, 10^{-5}, 10^{-4}]$. As the ratio between the two timescales decreases, the disc is not able to move outwards as quickly before the star begins to spin down, so that r_{in} will always remain close to r_c . Black solid curve: evolution of r_{in} . Red dashed curve: evolution of r_c .

sizes of disc, while fig. 7 compares the evolution of r_{in} and r_c for different disc sizes to the simulation where $r_{\text{out},0} = 10r_{\text{in},0}$. In both simulations, the different curves correspond to different initial r_{out} : $10 r_{\text{in}}$ (dotted curve), $100 r_{\text{in}}$ (dashed curve), $1000 r_{\text{in}}$ (dash-dotted curve), $10^4 r_{\text{in},0}$ (dash-triple-dotted curve).

Fig. 6 shows the evolution of r_{in}/r_c in time for different initial r_{out} . For the simulations with $r_{\text{out},0}/r_{\text{in},0} = [10, 100, 1000]$, the ratio r_{in}/r_c declines to a minimum value that decreases as r_{out} is taken larger, before again increasing approximately logarithmically. In the largest disc, $r_{\text{out},0}/r_{\text{in},0} = 10^4$, the evolution is the same as in the smaller discs at early times, but r_{in}/r_c continues to decline for much longer until it reaches a minimum at around $10^7 t_*$ when it finally turns over.

The minimum value of r_{in}/r_c is thus determined by the amount of mass in the disc available for accretion. The larger discs have more mass, which sustains the accretion rate onto the star for a longer time before the drop in surface density causes r_{in} to move outward again.

The bottom panel of Fig. 7 shows the evolution of r_{in} and r_c for $r_{\text{out},0} = 10 r_{\text{in},0}$. The evolution is qualitatively the same as we derived in the analytic approximation in Sec. 4.3. Initially r_{in} remains fixed as r_c starts to evolve outwards, until r_c moves close enough to r_{in} that accretion can begin. The inner radius then

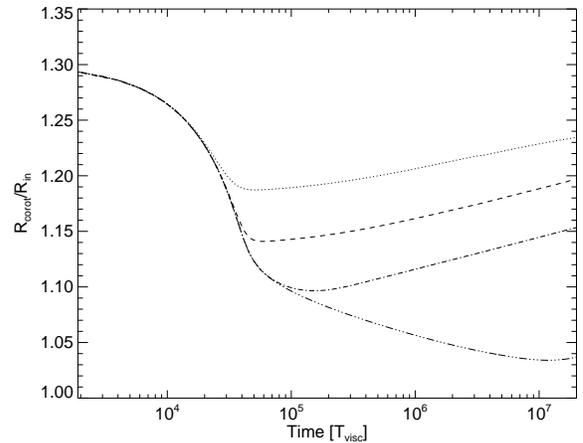


Figure 6. Discs starting outside corotation and spreading inward. Evolution of r_{in}/r_c for different initial r_{out} . From top to bottom: $r_{\text{out},0} = 10r_{\text{in},0}$ (dotted curve), 10^2 (dashed curve), 10^3 (dash-dotted curve), and 10^4 (triple dash-dotted curve) r_{in} at $t = 0$. Larger discs have a larger reservoir of matter, so that they can sustain a larger \dot{m} (so smaller r_{in}) as r_c increases due to spindown of the star.

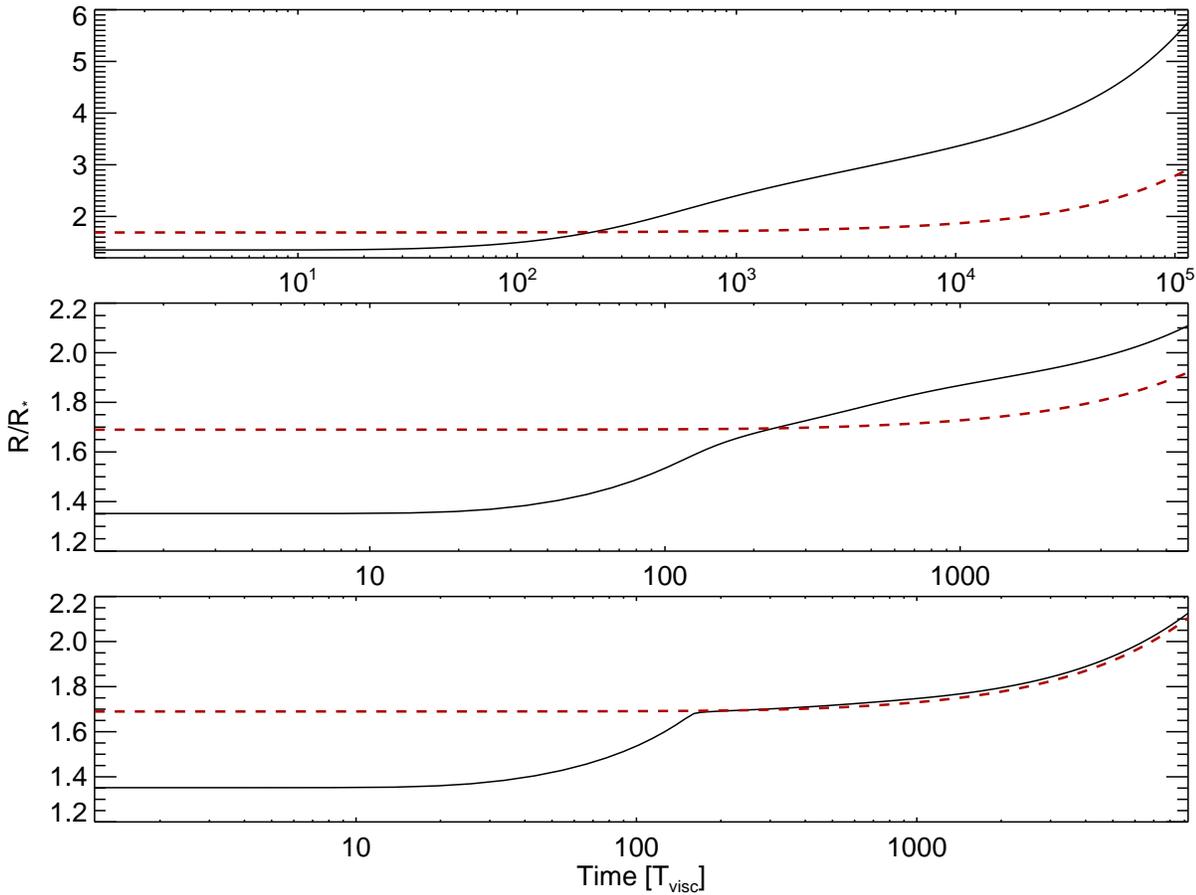


Figure 5. The evolution from an accreting to a non-accreting disc, for stable discs ($\Delta r/r_{\text{in}} = 1$) with different Δr_2 . From top to bottom, $\Delta r_2/r_{\text{in}} = [0.4, 0.04, 0.004]$. Curves show r_{in} and r_c as in fig. 4.

evolves outward at approximately the same rate as r_c as the star spins down. The variation between different simulations is emphasized in the top panel of Fig. 7. Here we plot the evolution of r_c (thick curves, red) and r_{in} (thin curves, black) for the discs with $r_{\text{out},0}/r_{\text{in},0} = [10^2, 10^3, 10^4]$, divided by the solution for $r_{\text{out},0}/r_{\text{in},0} = 10$. For larger discs the accretion rate is higher, so that r_{in} can move closer to r_c . In the three smaller discs, the accreted mass adds a negligible amount of angular momentum to the star, so that as r_{in} moves closer to r_c . As a result, the spin-down torque simply becomes more efficient, and r_c spins down faster. After $10^7 t_*$ r_c for the disc with $r_{\text{out},0}/r_{\text{in},0} = 10^3$ is more than 10% larger than in the smallest disc. However, for the largest disc, the accretion rate puts r_{in} close enough to r_c that the spin-down torque starts to drop in efficiency and the spin-up from accretion becomes non-negligible. Although r_c still increases, after $10^7 t_*$ r_c is 30% smaller than for a small disc.

These results show that size of the disc can considerably influence the efficiency of spin-down, emphasizing the fact that the spin-down of a star is an initial value problem. The initial size of the disc can be as important as the ratio $T_{\text{SD}}/T_{\text{visc}}$ and the parameter Δr_2 in determining whether the disc can become trapped, and the efficiency of the spin-down torque. The results of this section would suggest that larger discs (with their larger reservoirs of mass) are more likely to become trapped than smaller discs.

5.3 Long-term behaviour of discs of finite size

As we found in section 5, the long-term evolution of the disc+star system tends to ‘bifurcate’. The end state is either a star that has lost little angular momentum, surrounded by a dead disc, or a star continuously spun down by a disc trapped at corotation with the star.

We investigate this further by a set of simulations in which the initial state is a disc of finite size r_0 , evolving in a grid that is 10 times larger, $r_{\text{out}} = 10 r_0$. Apart from this change we use the standard parameters (section 3.1), setting the initial value of r_{in}/r_c at 1.3 (for comparison with the results of the previous section). Whether the disc will be able to substantially spin down the star depends on the ratio the timescales T_{visc} at r_0 to T_{SD} . If the disc is very small ($T_{\text{visc}}(r_0) \ll T_{\text{SD}}$), r_{in} will move outward too quickly to spin down the star, while if $T_{\text{visc}}(r_0) \gg T_{\text{SD}}$, the moment of inertia in the disc is sufficiently high to be able to absorb much more angular momentum from the star, so that the disc can operate as an efficient sink for the star’s angular momentum.

Fig. 8 shows results for three different sizes of disc. In each curve, the dashed red line shows the evolution of r_c , while the solid black curve shows the evolution of r_{in} . The top panel of fig. 8 shows the disc evolution when the viscosity is chosen such that $T_{\text{visc}}(r_0) \ll T_{\text{SD}}$, with $r_0 = 100 r_{\text{in}}$. Since the angular momentum injected at r_{in} is carried away by viscous spreading of the disc,

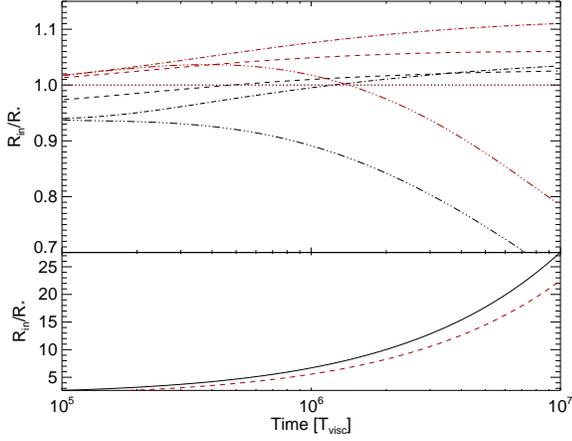


Figure 7. Evolution of r_{in} and r_c in a dead disc with different r_{out} . Bottom: The evolution of r_{in} (solid black curve) and r_c (dashed red curve) for the smallest disc, $r_{\text{out},0} = 10 r_{\text{in},0}$. Top: The evolution of r_{in} (thin black curves) and r_c (thick red curves) for different sizes of disc, divided by the $r_{\text{out},0} = 10 r_{\text{in},0}$ solution. The individual curves are the same as in Fig. 6.

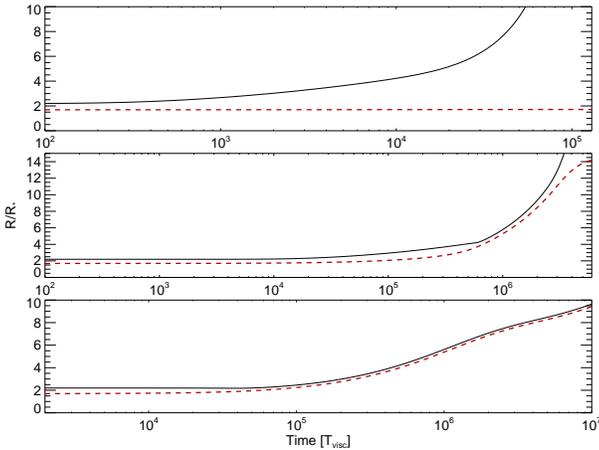


Figure 8. Top: Evolution of r_{in} and r_c in a dead disc with mass transport through r_{out} , where $T_{\text{visc}}(r_{\text{out},0}) \gg T_{\text{SD}}$. The moment of inertia in the disc is sufficiently large to prevent r_{in} from moving out as r_c evolves. Middle: Evolution of r_{in} and r_c in a dead disc with mass transport through r_{out} , where $T_{\text{visc}}(r_{\text{out},0}) \sim T_{\text{SD}}$. The disc is initially massive enough to spin down the star, but after some time evolves away from the equilibrium solution and r_{in} moves rapidly out. Bottom: Evolution of r_{in} and r_c in a dead disc with mass transport through r_{out} , where $T_{\text{visc}}(r_{\text{out},0}) \gg T_{\text{SD}}$. The moment of inertia in the disc is sufficiently large to prevent r_{in} from moving out as r_c evolves. Curves show r_{in} and r_c as in fig. 4.

the disc quickly evolves away from its initial configuration. Since the moment of inertia in the disc is much smaller than in the star, the disc becomes too spread out to absorb the angular momentum injected at r_{in} , and r_{in} moves outward before the star is able to slow down substantially.

In the middle panel of 8 we show the evolution of r_{in} and r_c when the spin-down timescale is comparable to the viscous timescale at r_0 . The disc also initially diffuses outwards (as seen by the increase in r_{in} around $t = 5 \times 10^5$), so that the rate of angular momentum exchange from the star to r_{in} decreases. This allows r_c to catch up, and the two radii start to evolve together. Eventu-

ally, however, viscous spreading of the disc wins, and r_{in} begins to move outwards.

The bottom panel of 8 shows the evolution of r_{in} and r_c when $T_{\text{visc}}(r_0) \gg T_{\text{SD}}$. The result is essentially the same as in Sec. 5.2, since the disc is now so large that additional angular momentum from the star does not reach r_{out} on the spindown timescale. In other words, the moment of inertia of the disc itself is large enough that the star is able to spin down without causing r_{in} to move rapidly outward.

5.3.1 The rotation of Ap stars and magnetic white dwarfs

An intriguing clue to the spindown of magnetic stars comes from slowly rotating Ap stars. As a class these stars are observed to have very strong dipolar magnetic fields (up to 10 kG). A few of them have extremely long rotation periods (up to 10-100 years), while others have rotation periods as short as 0.5 d. A similar phenomenon is observed in the magnetic white dwarfs. Most of these have periods of a few days or weeks, but some rotate as slowly as the slowest Ap stars. We suggest that the bifurcation of outcomes we found in the above is the underlying reason for the remarkable range of spin periods of magnetic stars.

6 CONCLUSIONS

As found before in Sunyaev & Shakura (1977) and DS10, a disc in contact with the magnetosphere of a star can be in a ‘dead’ state, with its inner edge well outside the corotation radius so the accretion rate onto the star vanishes, and the torque exerted by the magnetic field transmitted outward by viscous stress. In the calculations reported in DS10, an additional state was found, intermediate between the accreting and dead state. In this state, the inner edge of the disc stayed close to the corotation radius r_c , even as the accretion rate onto the star varied by large factors. We call this phenomenon *trapping* of the disc. Accretion can be stationary or in the form of a limit cycle in this state.

One of the goals of this investigation was to find out under what conditions this trapping takes place, using a series of numerical experiments with varying initial and boundary conditions. If initial conditions are such that the inner edge starts inside corotation and slowly moves outward, we find that the disc gets stuck in a trapped state for a long time if the disc viscosity ν is low (up to about 10^3 times shorter than the spindown timescale of the star). The accretion rate then slowly vanishes but the inner edge always stays close to corotation. At higher viscosity ($> 10^5 \times$ the spindown timescale) on the other hand, the disc evolves through corotation into a ‘dead’ disc state, with inner edge well outside the corotation radius. In terms of the standard viscosity parametrization $\nu = \alpha(H/r)^2$, compared with the spin-down timescale of the disc, a trapped state is more likely to happen in strongly magnetic ($B_S \sim 10^{12} G$) X-ray pulsars and protostellar discs than the weaker magnetic fields ($B_S \sim 10^8 G$) of millisecond X-ray pulsars.

Our second goal was to find out how a star spins down in the long term, under the influence of the angular momentum it loses by the magnetic torque exerted on the disc. The results show an interesting ‘bifurcation’ of long-term outcomes: if the disc evolves into a dead state, the star loses only a fraction of its initial angular momentum, and can remain spinning rapidly throughout its life. If on the other hand it enters a trapped state at some point, it remains in this state. The star can then slow down to very low rotation rates, the inner edge of the disc tracking the corotation radius outward.

We suggest that these two outcomes can be identified respectively with the rapidly rotating and slowly rotating classes of magnetic Ap stars and magnetic white dwarfs. The evolution of the trapped state could also be reproduced with a simplified model that does not require solving the full viscous diffusion equation.

This picture of magnetosphere-disc interaction differs from the standard view that mass will be ‘propellered’ out of the system instead of accreting, once the star rotates more rapidly than the inner edge of the disc. As shown already by Sunyaev & Shakura (1977) and again argued in Spruit & Taam (1993) and DS10 this assumption is not necessary, in many cases unlikely, and ignores some of the theoretically and observationally most interesting aspects of the disc-magnetosphere interaction.

Though the dead state is a regime where a significant fraction of the disc mass could in principle be expelled ($r_{\text{in}} \gg r_c$), the results presented here show that magnetospherically accreting systems often avoid this regime. Instead, they end up in the trapped state, in which the disc-field interaction keeps the inner radius truncated very close to the co-rotation radius, even at very low accretion rates. Both the trapped and ($r_{\text{in}} \approx r_c$) and the dead state ($r_{\text{in}} \gg r_c$) allow the disc to efficiently spin down the star. The disc retains a large amount of mass, but in the absence of accretion onto the central star appears quiescent.

7 DISCUSSION

By assuming a given dependence of viscosity on distance, we have bypassed the physics that determines it. In terms of the standard α -parametrization of viscosity, we have left out the physics that determines the disc temperature and hence its thickness H/r . Additional time dependence or instabilities may arise from feedback between accretion and disc temperature. The radiation produced by matter accreting on the star could be large enough to change temperature, the ionization state and hence the thickness at the inner edge of the disc. In a trapped disc, this is just the region that controls the accretion rate onto the star. The size of the transition region, which we have parametrized with the widths Δr , Δr_2 may well depend on disc thickness. Positive feedback may be thus possible. We leave this possibility for future work.

In systems such as the accreting X-ray pulsars the accretion is episodic on long timescales. This is attributed to the instability of viscous discs that is also responsible for the outbursts of cataclysmic variables. In these cases, in which there is a large change in \dot{m} and r_{in} is far from r_c in the quiescent phase, there is the possibility of hysteresis: the same accretion rate will lead to a different value of r_{in} (and therefore disc torque) depending on whether the source is moving into or out of outburst. As the source goes into outburst, the disc will not have as much mass in its inner regions, so that r_{in} will move inward gradually from large radii until it crosses r_c and the outburst begins. In the decline phase, however, the disc will become trapped around the inner radius of the disc when the accretion rate drops, allowing for a larger spin-down in the disc and accretion bursts via the instability of DS10. The net effect of such episodic accretion on the spin history of the star, as compared with the case of steady accretion, is not obvious. We discuss this in more depth in the companion paper.

Some work on disc-magnetosphere interaction assumes magnetic torques to act over a significant part of the disc (Königl 1991; Armitage & Clarke 1996). More recent work (and in particular numerical MHD simulations of the disc-field interaction) finds the interaction region to be much narrower (as we have also as-

sumed here), and spindown torques on the star are correspondingly smaller. The difference for the long-term spin evolution is not dramatic, however, as our trapped disc results demonstrate.

Interestingly, Armitage & Clarke (1996) also observed that their discs would become trapped around r_c as the accretion rate in the disc decreased by several orders of magnitude. In their model, the magnetic field-disc interaction also acts like a boundary condition at low accretion rates and the disc evolved viscously in response. This is presumably because although the disc in their model is threaded by a magnetic field everywhere, the disc-field interaction is by far the strongest in the inner regions, so that they see a similar behaviour to the one described in this paper.

MHD simulations of interaction between the disc and the magnetic field are becoming increasingly realistic. These simulations can only run for very short timescales (the longest of order T_{visc} at r_{in}), so they tend to emphasize initial transients. Still, they offer insight in how the disc and magnetic field will interact. To date, most simulations have concentrated on strongly accreting (Hayashi et al. 1996; Goodson et al. 1997; Miller & Stone 1997) or propeller (Romanova et al. 2004) cases. Simulations that come closest to the conditions of a trapped disc are the study of a so-called ‘weak propeller’ regime by Ustyugova et al. (2006). The authors found that discs in which r_{in} was initially truncated close to r_c launch much weaker outflows than discs truncated further away. They also found some evidence of the field changing the disc structure (as shown from mass piling up in the inner regions), although in their simulation the majority of the angular momentum in the star was carried away via a wind, rather than through the disc. These simulations also did not run for very long, however, so it is hard to separate transient behaviour due to the initial conditions from the longer-term systematic effects we are interested in.

Perna et al. (2006) did calculations were done for a case where the star’s field is inclined with respect to the disc axis. The authors’ results suggest that the transition width which we have parametrized by Δr_2 would increase with the inclination of the magnetic field, since there will be some values for r_{in} at which which both accretion and disc mass trapping can occur. This then would suggest that dead discs are more likely to form in systems with small inclinations between the spin axis and magnetic axis, and also that the disc instability studied in DS10 (which also tends to occur for smaller values of Δr_2). This prediction is supported by the observation that Ap stars with the longest periods tend to have the lowest inclination angles for the magnetic field (Landstreet & Mathys 2000).

The transition widths of disc-magnetosphere interaction that can be inferred from these simulations are significant, and are in the range we have assumed here. They are much larger than the very narrow interaction regions assumed by Matt et al. (Matt & Pudritz 2004, 2005; Matt et al. 2010).

We have found that the distance from r_c at which the inner edge of the disc gets trapped is determined by the ratio of two timescales for the disc’s evolution: the viscous evolution timescale in the disc (which determines the rate at which disc density profile can change) and the star’s spin-down timescale (which sets the rate at which r_c moves outwards). The viscous timescale is in general much shorter than the spin-down timescale, and the ratio of the two timescales varies from $\sim 10^{-3}$ (in protostellar discs) to $\sim 10^{-17}$ (in millisecond X-ray pulsars). The larger the ratio, the longer the disc will take to respond to changes in r_c . As a result r_{in} remains closer to r_c than it would if the ratio were smaller, and the trapped disc has a higher accretion rate onto the star. If the ratio is too low,

then r_{in} moves much further away from r_c , and the system likely enters the dead disc regime.

For the parameters characterizing the interaction region between disc and magnetosphere in our model, we have used here values such that the cyclic accretion behavior found in DS10 does not develop. This was done for convenience, since the short time steps needed to follow these cycles makes it harder to calculate the long-term evolution. The time-averaged effect of these cycles is not expected to make a big difference for the long-term evolution.

These cyclic accretion bursts can persist in the trapped state, when the star is spinning down efficiently. They are observed to occur both over several orders of magnitude of accretion rates and transiently (over a small range of accretion rate), and depend on the ratio of timescales discussed above. Whether or not the instability occurs is determined by the detailed disc-field interaction (the parameters Δr and Δr_2 in our model). The peak of the accretion bursts is typically much larger ($> 10\times$) than the mean accretion rate for the system, and the period is typically between $0.01 - 10^2 T_{\text{visc}}(r_{\text{in}})$. The properties and conditions for occurrence of these cycles are studied further in a companion paper.

7.1 ‘Propellering’

In our calculations we have left out the possibility that interaction of the magnetosphere with the disc will cause of mass ejection from the system. The point being that, contrary to common belief, such interaction can function without mass ejection by ‘propellering’, as pointed out already by Sunyaev & Shakura (1977). Understanding of this restricted case, as we have developed here, is prerequisite for understanding the much less well defined case of mass losing discs.

On energetic grounds, mass loss from the system is necessarily limited, unless the inner edge is well outside corotation (Spruit & Taam 1993). This point has also been made by Perna et al. (2006), who propose that mass lifted at r_{in} may fall back on the disc at some finite distance. This would create a feedback loop in the mass flux through the disc, opening the possibility of additional forms of time-dependent behaviour that do not exist in accretion onto non-magnetic stars. In the trapped disc state we have studied here the difference in rotation between disc and star is small, so any significant amount of mass kicked up from the interaction region cannot move very far before returning to the disc. Its effects are then secondary, at least for the long-term evolution of the disc.

The possibility of significant effects of mass loss is more realistic for the dead disc states, where the distance of the inner edge from corotation can become much larger.

Real propellering is expected to happen when mass transfer from a companion star sets in for the first time onto a rapidly spinning magnetic star. The cataclysmic binary AE Aqr is evidently such a case (Pearson et al. 2003). A disc is absent in this CV, and all mass transferred appears to be ejected in a complex outflow associated with strong radio emission.

8 ACKNOWLEDGEMENTS

CD’A acknowledges support from the National Science and Engineering Research Council of Canada.

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