

# Local Kinematics and the Local Standard of Rest

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## ABSTRACT

We re-examine the stellar kinematics of the Solar neighbourhood in terms of the velocity  $\mathbf{v}_\odot$  of the Sun with respect to the local standard of rest. We show that the classical determination of its component  $V_\odot$  in the direction of Galactic rotation via Strömberg’s relation is undermined by the metallicity gradient in the disc, which introduces a correlation between the colour of a group of stars and the radial gradients of its properties. Comparing the local stellar kinematics to a chemodynamical model which accounts for these effects, we obtain  $(U, V, W)_\odot = (11.1^{+0.69}_{-0.75}, 12.24^{+0.47}_{-0.47}, 7.25^{+0.37}_{-0.36}) \text{ km s}^{-1}$ , with additional systematic uncertainties  $\sim (1, 2, 0.5) \text{ km s}^{-1}$ . In particular,  $V_\odot$  is  $7 \text{ km s}^{-1}$  larger than previously estimated. The new values of  $(U, V, W)_\odot$  are extremely insensitive to the metallicity gradient within the disc.

**Key words:** stars: kinematics – Solar neighbourhood – Galaxy: fundamental parameters – Galaxy: kinematics and dynamics – Galaxy: disc

## 1 INTRODUCTION

The Sun’s velocity  $\mathbf{v}_\odot$  with respect to the Local Standard of Rest (LSR)<sup>1</sup> is required to transform any observed heliocentric velocity to a local galactic frame. Since this transformation is often necessary for scientific interpretation of observed velocities in terms of Galactic structure, the determination of  $\mathbf{v}_\odot$  is a fundamental task of Galactic astronomy. The radial and vertical components  $U_\odot$  and  $W_\odot$  of  $\mathbf{v}_\odot$  are straightforwardly obtained from the mean heliocentric velocities of several different groups of Solar-neighbourhood stars:  $U_\odot$  and  $W_\odot$  are simply the negative radial and vertical components of these means<sup>2</sup>.

The component  $V_\odot$  of  $\mathbf{v}_\odot$  in the direction of Galactic rotation is much harder to determine, because the mean lag with respect to the LSR, the asymmetric drift  $v_a$ , depends on the velocity dispersion  $\sigma$  of the respective stellar population. The classical solution to this problem exploits the empirical linear relation between the

negative mean heliocentric azimuthal velocity of any stellar sample  $\bar{v}_s = v_a + V_\odot$  and its  $\sigma^2$  (Strömberg 1946). Hence, a straight-line fit yields  $V_\odot$  as the value of  $\bar{v}_s$  for a hypothetical population of stars on circular orbits, for which  $\sigma = 0$ .

The theoretical underpinning of this method is the asymmetric drift relation (see Binney & Tremaine 2008, eq. 4.228)

$$\bar{v}_s - V_\odot = v_a \approx \frac{\bar{v}_R^2}{2v_c} \left[ \frac{\sigma_\phi^2}{v_R^2} - 1 - \frac{\partial \ln(\nu \bar{v}_R^2)}{\partial \ln R} - \frac{R}{v_R^2} \frac{\partial(\bar{v}_R v_z)}{\partial z} \right], \quad (1)$$

where  $R$  is Galactocentric cylindrical radius,  $z$  the height above the plane,  $v_c$  the circular speed, and  $\nu$  the number density of stars, while a bar indicates a  $\nu$ -weighted local mean. The equation applies separately to each relaxed stellar population, for example to M stars or G stars. The idea behind the classical determination of  $V_\odot$  is that the square bracket in equation (1) takes essentially identical values for each stellar population, with the consequence that a plot of  $\bar{v}_s$  against  $\bar{v}_R^2$  should be linear.

Dehnen & Binney (1998, hereafter DB98) applied this method to a sample of  $\sim 15\,000$  main-sequence stars from the Hipparcos catalogue and their value of  $V_\odot = (5.25 \pm 0.62) \text{ km s}^{-1}$  has been widely used. Recent re-determinations using an improved reduction of the Hipparcos data (van Leeuwen 2007) confirm the DB98 value though with reduced error bars (van Leeuwen 2007; Aumer & Binney 2009).

However, two recent studies call the DB98 value for  $V_\odot$  into question. Binney (2009, hereafter B09) fitted distribution-function models (a) to velocity distributions inferred by Ivezić et al. (2008) from proper motions and photometric distances of stars in the Sloan Digital Sky Survey, and (b) to the space velocities of F and G in the Geneva-Copenhagen Survey (GCS, Nordström et al. 2004).

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<sup>1</sup> The LSR is the rest frame at the location of the Sun of a star that would be on a circular orbit in the gravitational potential one would obtain by azimuthally averaging away non-axisymmetric features in the actual Galactic potential.

<sup>2</sup> According to the above definition, the LSR’s radial and vertical motion w.r.t. the Galactic centre vanish. Therefore, the determination of  $U_\odot$  and  $W_\odot$  from such means implicitly assumes that the Solar neighbourhood as a whole does not move radially or vertically w.r.t. the Galaxy. That such motions are at most small is suggested by the proper motion of Sgr A\* (Reid & Brunthaler 2004) and the mean radial velocity of the stars orbiting it (e.g. Reid et al. 2007). Moreover, such motions should also obey an asymmetric-drift like relation (see below), i.e. the mean velocities depend systematically on velocity dispersion, which is not observed.

The GCS stars are a subset of the Hipparcos stars (analysed by DB98) for which radial velocities have been obtained. B09 was able to obtain satisfactory fits to these data only if  $V_{\odot}$  was larger than the DB98 value by  $\sim 6 \text{ km s}^{-1}$ , about ten times the formal error on  $V_{\odot}$ . Another body of evidence against the DB98 value for  $V_{\odot}$  originates from radio-frequency astrometry of masers in regions of massive-star formation (Rygl et al. 2009; Reid M. J. et al. 2009). If the DB98 value for  $V_{\odot}$  is correct, these sources systematically lag circular rotation by  $\sim 17 \text{ km s}^{-1}$  (Reid M. J. et al. 2009). Such a high systematic lag is unexpected for young stars and McMillan & Binney (2009) argued that a more plausible interpretation of the data is obtained if  $V_{\odot}$  exceeds the DB98 value by  $\sim 6 \text{ km s}^{-1}$ .

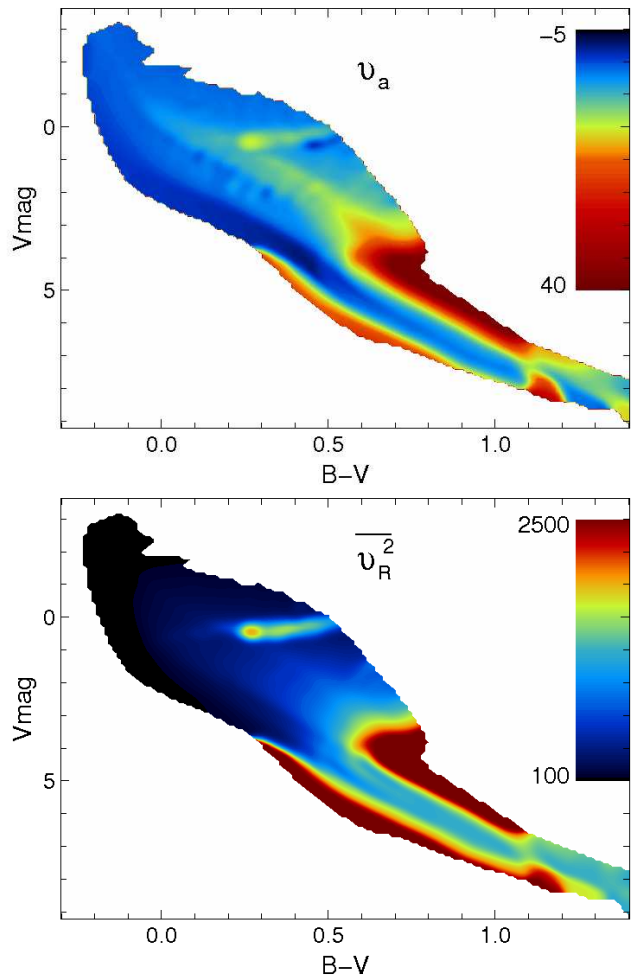
This paper does two things: (i) it explains why the approach to the determination of  $V_{\odot}$  by DB98 and subsequent studies is misleading, and (ii) it determines  $V_{\odot}$  from similar data but a different methodology. Both these tasks are accomplished with the help of a particular chemo-dynamical model of the Galaxy, that of Schönrich & Binney (2009a, hereafter SB09a), but the points that we make are general ones and the role played by the SB09 model is essentially illustrative. In Section 2 we show that a metallicity gradient in the disc gives rise to distributions of mean azimuthal velocity and velocity dispersion within the colour-magnitude plane that are much more complex than one naively expects, and we show that these distributions invalidate the methodology of DB98. In Section 3 we re-estimate  $V_{\odot}$  by fitting the entire velocity distribution of the GCS stars to the distribution predicted by the SB09 model without reference to the Strömberg relation.

## 2 KINEMATICS IN COLOUR AND MAGNITUDE

DB98 divided their sample of Hipparcos main-sequence stars into populations with different velocity dispersions by binning in  $B - V$  colour because colour is correlated with age and therefore with velocity dispersion. To examine the relation between colour, mean rotation velocity and velocity dispersion for stars near the Sun, we employ the SB09a model of the Galactic Disc. This model describes the chemodynamical evolution of the thin and thick Galactic discs and is a refinement of models pioneered by van den Bergh (1962) and Schmidt (1963). The disc is divided into 80 annuli, within each of which the chemical composition of the ISM evolves in response to the ejection of material by dying stars, while stars form continuously with the current composition of the ISM. The new features of the model are (a) stochastic stellar accelerations accounting for heating processes; (b) radial stellar migration accounting for both non-circular orbits and guiding-centre shifts caused by stochastic resonant scattering off spiral arms (Sellwood & Binney 2002); and (c) transfer of gas between annuli, both as result of resonant scattering by spiral arms and as a result of a secular tendency of gas to spiral inwards through the disc. Surprisingly, the model contains both thin and thick discs that are consistent with the available observational constraints (Schönrich & Binney 2009b).

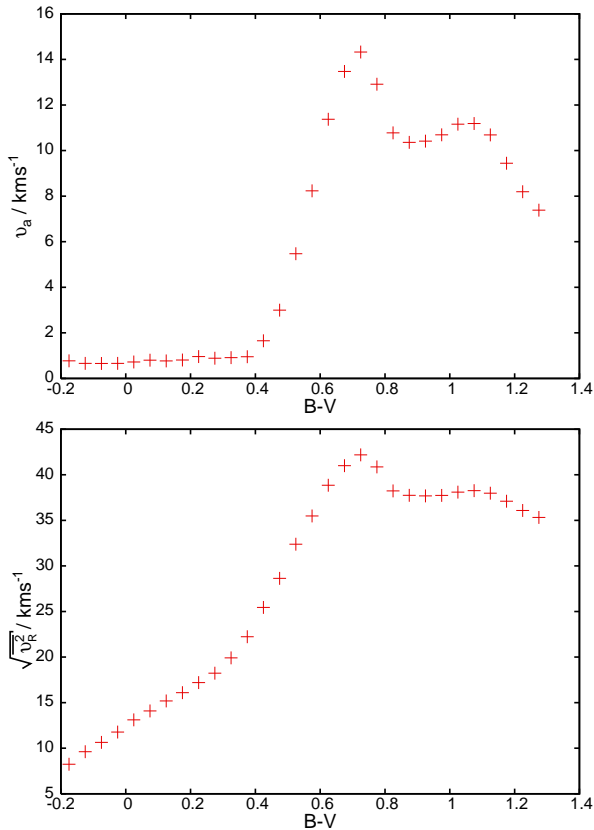
Fig. 1 shows the model kinematics in the colour-magnitude diagram. Each point in colour-magnitude space defines a separate sub-population whose asymmetric drift  $v_a \equiv v_c - \bar{v}_\phi$  and radial velocity dispersion are plotted via colour coding, such that dynamically cold and warm populations are shown with blue and red shades, respectively. The region in the colour-magnitude diagram shown in this figure corresponds to the cuts used by DB98 to define their sample.

Our naive expectation is that as we proceed down the main



**Figure 1.** The variation with colour and magnitude of the asymmetric drift  $v_a$  (top) and radial velocity dispersion (bottom) in the SB09a model of the Solar neighbourhood. Shown is the range of colours and magnitudes used by DB98 to generate their main-sequence sample. Note that the number density of stars is highly non-uniform across the region shown.

sequence from its blue end towards the main-sequence turnoff at  $B - V \sim 0.6$ , we encounter successively older stars with lower mean rotation velocities and higher velocity dispersions, so in both panels of Fig. 1 the shading should become redder as we move from left to right along the main sequence. The pattern actually found in Fig. 1 is more complex. Most notably, there is a pronounced velocity gradient *across* the lower main sequence. In the range  $0.6 < B - V < 1$  the lower edge of the main sequence is dynamically warm (orange in Fig. 1) on account of subdwarfs, which are metal-poor and therefore old with large velocity dispersions and low mean rotation rates. The number of these subdwarfs is small, however, so they will not have a significant impact on a sample binned by colour alone. More significant is the orange shading on the upper edge of the lower main sequence, which reflects the metallicity gradient within the disc: as metallicity increases, the main sequence shifts to the right, so in the upper panel the orange upper edge of the main sequence implies that the more metal-rich stars of the Solar neighbourhood are rotating more slowly because they formed at  $R < R_0$ . To the left of  $B - V \simeq 0.5$  this trend is weakened by contributions from old, sometimes metal-poor populations whose isochrones move up through this region. Still the more metal-rich main-sequence stars



**Figure 2.** The asymmetric drift (top) and radial velocity dispersion (bottom) for stellar samples of given  $B - V$  colour drawn from the model thus simulating the effects of the Hipparcos and DB98 selection criteria.

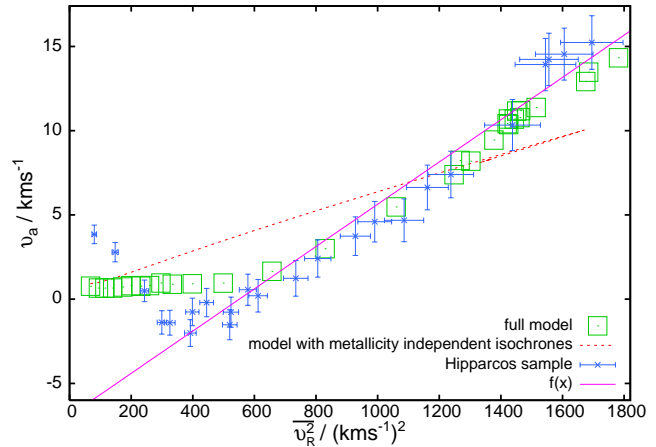
with smaller guiding centre radii give rise to slightly higher dispersions and asymmetric drifts to the red side of the main sequence.

The upper panel of Fig. 1 shows that in the crucial colour range  $0.4 < B - V < 0.6$ , the asymmetric drift is a complex function of colour and absolute magnitude because in this region stars of widely differing ages and metallicities are found as a result of old, metal-poor isochrones intersecting younger, metal-rich isochrones.

The horizontal branch is clearly visible on both panels of Fig. 1 as an almost horizontal feature just below  $M_V = 0$ . It is less pronounced in the upper panel because the blue end of the horizontal branch contains metal-poor stars, which tend to have large guiding-centre radii and therefore low  $v_a$  even at large  $\overline{v_R^2}$ .

Fig. 2 shows the asymmetric drift  $v_a$  and velocity dispersion  $\overline{v_R^2}$  obtained when stars are binned by colour alone. The lower panel can be compared with corresponding observational plots in DB98 and Aumer & Binney (2009). The model reproduces the structure of the data very well – in particular, the steepening in the slope around  $B - V = 0.4$  and the flatness redwards of  $B - V = 0.6$ . The peak in velocity dispersion seen in the lower panel of Fig. 2 is much less evident in Fig. 2 of Aumer & Binney (2009) but can be traced in their  $\sigma_R$  and  $\sigma_z$  data. Note that the rise with  $B - V$  in  $\overline{v_R^2}$  for  $B - V < 0.4$  is not accompanied by any change in  $v_a$ . This unexpected phenomenon arises because at these colours the contribution of old metal-poor stars is increasing with  $B - V$ , and because we see many of these stars near pericentre, they have small asymmetric drifts despite their large random velocities.

Since at its bright end the Hipparcos sample is close to being volume limited, the relative number of horizontal-branch stars is



**Figure 3.** Green squares: the asymmetric drift for synthetic stellar subsamples defined by  $B - V$  colour plotted against their radial velocity dispersion squared for the SB09a model. The red dots: the same relation obtained from a model with only one chemical composition (solar). Blue data points: the values of Aumer & Binney (2009) shifted by  $11 \text{ km s}^{-1}$  and with the radial velocity dispersion increased by 7%. Purple line: a linear fit to the Hipparcos points in the range of  $\overline{v_R^2}$  used by DB98.

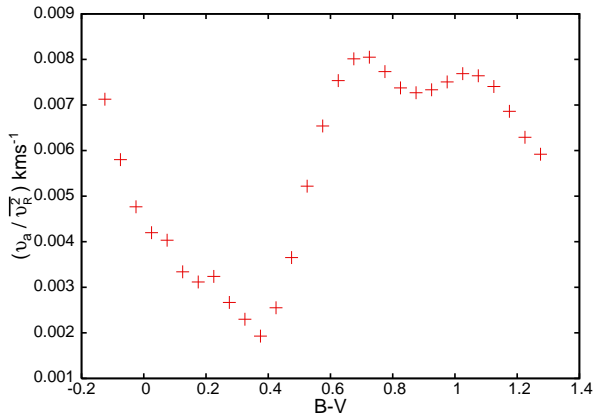
small and the complex structure above the main sequence in Fig. 1 has no significant effect in Fig. 2.

In Fig. 3, the squares show the resulting plot of the asymmetric drift  $v_a$  against velocity dispersion squared for the synthetic samples of Fig. 2; they do not lie on a straight line. The red dots show the plot one obtains if all stars are assigned solar metallicity. These dots *do* lie on a good approximation to a straight line.<sup>3</sup> The effect of re-assigning stars with large guiding-centre radii from solar metallicity to their true, low metallicities is to move them from redder to bluer bins. Since these stars have small or even negative values of  $v_a$  on account of the metallicity gradient in the disc, the transfer reduces  $v_a$  for the young, bluer bins and increases it in the old, redder bins. Consequently, the transfer morphs the near-straight line of the red points into the curve defined by the green squares.

DB98 estimated  $V_\odot$  by fitting a straight line to the observational analogue of Fig. 3, which is a plot of the solar velocity relative to a colour-selected group of stars versus the squared velocity dispersion of that group. The blue data points in Fig. 3 show such data for the Hipparcos sample in the re-analysis of Aumer & Binney (2009) after subtracting  $11 \text{ km s}^{-1}$  from each value of solar velocity. We see that for  $\overline{v_R^2} \gtrsim 600 [\text{km s}^{-1}]^2$  the Hipparcos data define the same straight line as the green crosses from the model. This straight line intercepts the  $v_a$  axis at  $\sim -7 \text{ km s}^{-1}$  rather than 0, causing  $V_\odot$  to be underestimated by this amount. For  $\overline{v_R^2} \lesssim 400 [\text{km s}^{-1}]^2$  the Hipparcos data points in Fig. 3 deviate from this straight line, but DB98 ignored samples with very low velocity dispersion on the grounds that such samples may not be in dynamical equilibrium. Indeed, both dissolving star clusters and the non-axisymmetric gravitational potentials of spiral arms are liable to distort the kinematics of stellar samples with low random velocities such that equation (1) does not hold.

The failure of the synthetic samples to follow a straight line in Fig. 3 implies that the square bracket in the asymmetric drift re-

<sup>3</sup> The slight deviation from a straight line of the model without metallicities is an effect of the approximations in SB09a and leads to a small underestimation of the real metallicity bias by the model.



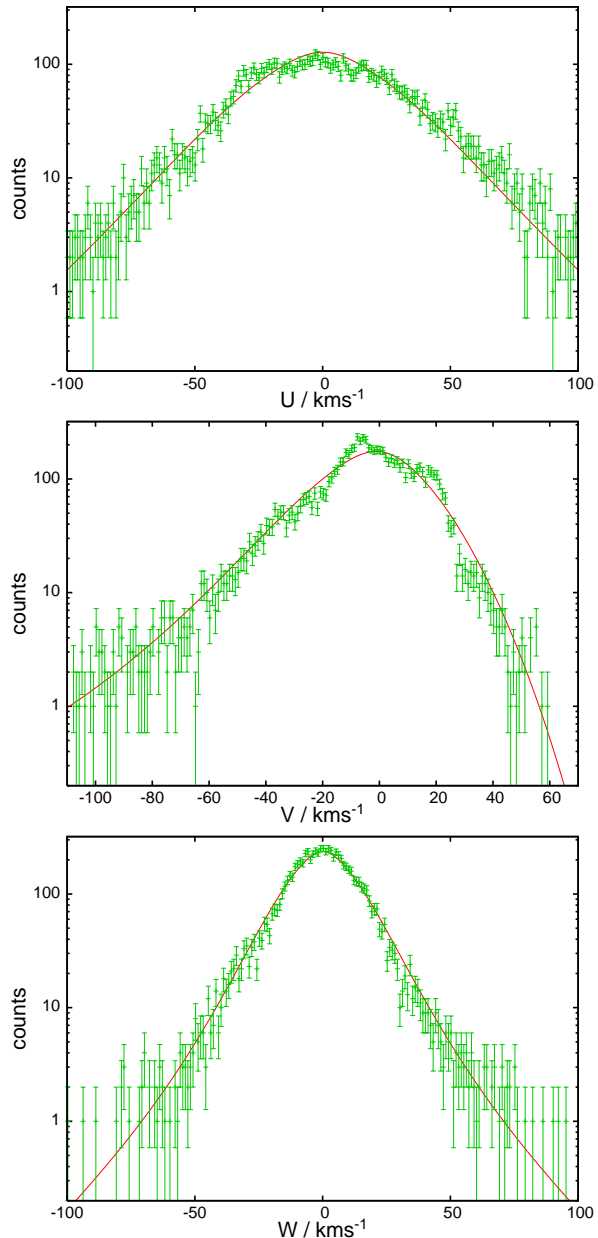
**Figure 4.** Asymmetric drift velocity divided by squared velocity dispersion for synthetic colour-selected samples using the SB09a model. This ratio should be proportional to the square bracket in equation (1).

lation (1) does depend on colour: it varies by a factor  $\sim 4$  in the colour range  $0.4 < B - V < 0.6$  as demonstrated in Fig. 4. A significant contribution to the value of the bracket comes from the first derivative term, which is smallest for metal-poor populations because their densities  $\nu$  decline more slowly outwards on account of the metallicity gradient in the disc. Physically, including metal-poor, thin-disc stars decreases  $v_a$  because such stars typically visit the Solar neighbourhood at pericentre, where they have  $u_\phi > v_c$ .

### 3 DETERMINING THE SOLAR MOTION FROM THE VELOCITY DISTRIBUTION

In view of the argument just presented, that the classical procedure cannot yield a reliable value of  $\nu_\odot$ , we now estimate  $\nu_\odot$  by fitting the observed distributions of heliocentric velocities to the velocity distribution of the SB09a model. That is, we seek the offset  $-\nu_\odot$  from the circular velocity at which the model velocity distributions provide the best match to the distribution of observed heliocentric velocities. B09 used an analogous procedure to argue that  $V_\odot \simeq 11 \text{ km s}^{-1}$ ; however, the model distributions he used were obtained from analytic distribution functions rather from a model of the Galaxy’s chemical evolution. By using velocity distributions that reflect much prior information about the chemodynamical history of the Galaxy in place of simple analytic functions, we hope to achieve a closer fit to the observed velocity distributions and therefore determine the requisite offset  $-\nu_\odot$  with greater precision.

In Fig. 5 the points with (Poisson) error bars are for a subsample of GCS stars for which Holmberg, Nordström & Andersen (2007, 2009) give reliable metallicities; thirty likely halo stars have been removed by requiring  $[\text{Fe}/\text{H}] > -1.2$ . This criterion is slightly stricter than that used in SB09b for the determination of the in-plane dispersion parameter ( $\sigma_R$  of a 10 Gyr old local population), such that we now use a marginally smaller value,  $43 \text{ km s}^{-1}$ , while lowering the vertical dispersion parameter to  $23 \text{ km s}^{-1}$ . The curves in Fig. 5 show the model distributions when offset by  $-\nu_\odot = (11.10^{+0.69}_{-0.75}, 12.24^{+0.47}_{-0.47}, 7.25^{+0.37}_{-0.36}) \text{ km s}^{-1}$  – we used cubic splines to interpolate between individual data points provided by the model in the  $V$  component and determined the offset by maximising the likelihood of the data given the model. We used only five parameters for all three distributions: the two dispersion parameters and the three components of Solar motion, yet the fit is of good quality. The small fluctuations of the data around the model  $V$  and to a lesser



**Figure 5.** Curves: The model distributions predicted by the SB09a model in the  $U$ ,  $V$  and  $W$  components of velocity (from top to bottom). Data points with Poisson error bars: the observed distributions of the GCS stars shifted by our estimate of  $\nu_\odot$  to optimise the fit of the data.

extent  $U$  distributions are readily accounted for by the well known stellar moving groups (Dehnen 1998), likely caused by the dynamical influence of the Galactic bar and spiral structure (Dehnen 1999; De Simone et al. 2004; Antoja et al. 2009; Minchev et al. 2009).

A significant advantage of determining  $\nu_\odot$  from the entire sample, as in this section, rather than from subsamples as done in the past, is the robustness of the result to changes in the modelled metallicity gradient. In fact, eliminating the model’s metallicity gradient changes  $V_\odot$  by less than  $0.1 \text{ km s}^{-1}$ .

One should note that the quoted errors on the components of  $\nu_\odot$  are purely formal. Sources of additional systematic error include the possible presence of halo stars in the sample, the dynamical approximations used in constructing the model, and the effects of stellar streams, which have a big impact on the observed distribu-

tion of stars near the circular velocity but are completely excluded from the model. Fortunately the likelihood of the data used here is not particularly sensitive to the fit of the data to the model around the peak density. In view of these uncertainties we roughly estimate systematic errors of  $\sim (1, 2, 0.5) \text{ km s}^{-1}$ , assuming that  $U$  and  $W$  are mostly affected by distortions by streams and  $V$  showing even more structure and having an additional uncertainty from the modelling. This is in perfect agreement with previous estimates as regards  $W_{\odot}$  and slightly higher in  $U_{\odot}$  compared to the DB98 value, which can be traced back of the larger influence of the Hercules stream at  $\sim -30 \text{ km s}^{-1}$  (lowering the estimate) on their statistics. However, it differs significantly from the value for  $V_{\odot} \approx 5.2 \pm 0.5 \text{ km s}^{-1}$  obtained by the classical technique (DB98, Aumer & Binney 2009). Our value for  $V_{\odot}$  is in good agreement with  $V_{\odot} \approx 11 \text{ km s}^{-1}$  proposed by B09. Given the residual uncertainties of  $v_{\odot}$ , it is questionable whether the standard practice of “correcting” observed (heliocentric) velocities for the Solar motion is useful, at least the adopted value should be explicitly provided.

#### 4 CONCLUSIONS

The metallicity gradient in the Galactic disc causes a systematic shift in the kinematics especially near the turnoff region. By the relationship between the colour and metallicity of a star, the more metal-rich populations, with on average lower angular momentum and thus higher asymmetric drifts, are displaced relative to their metal-poor counterparts, which have lower asymmetric drifts. When stars are binned by colour, the metallicity gradient in the Galactic disc prevents the relationship between mean rotation speed and squared velocity dispersion taking the linear form predicted by a naive application of the Strömberg relation. This breakdown in the conventionally assumed linearity invalidates the traditional technique for determining the Sun’s velocity with respect to the LSR, which involves a linear extrapolation to zero velocity dispersion of the empirical relation between the mean velocity and squared velocity dispersion. Moreover the SB09a model predicts that a treacherous linear relationship underestimating the solar azimuthal motion by  $\sim 7 \text{ km s}^{-1}$  will be mimicked redwards of the onset of the turnoff region, coinciding well with the behaviour observed in the Hipparcos data.

The Sun’s velocity with respect to the LSR may be alternatively determined from the velocity offset that optimises a model fit to the observed velocity distribution. Using the velocity distribution predicted by the SB09a model of the chemodynamical evolution of the Galaxy, we find  $v_{\odot} = (11.1_{-0.75}^{+0.69}, 12.24_{-0.47}^{+0.47}, 7.25_{-0.36}^{+0.37}) \text{ km s}^{-1}$  and roughly estimate the systematic uncertainties as  $\sim (1, 2, 0.5) \text{ km s}^{-1}$ . The radial and vertical components of this value of  $v_{\odot}$  agree with earlier estimates, but the  $V$  component is larger than the widely used value of DB98 by  $\sim 7 \text{ km s}^{-1}$ . This is in nice concordance with the model expectations for the systematic error arising from naively using the Strömberg relation and in good agreement with the value obtained by B09 using a similar method but with a less sophisticated distribution function. Curiously it agrees well with the result  $V_{\odot} \sim 10 - 13 \text{ km s}^{-1}$  obtained by Delhaye (1965) using the classical method with pre-Hipparcos data.

In this paper we have relied heavily on the SB09a model, so the question arises of how vulnerable our argument is to the model’s shortcomings. Our critique of the classical approach to the determination of  $v_{\odot}$  is secure so long as the disc has significant age and/or metallicity gradients. It is beyond question that such gradients ex-

ist, so the classical technique is certainly unreliable. Our proposed value of  $V_{\odot}$  is essentially independent of the assumed metallicity gradient, but does have some sensitivity to the dynamical approximations used in making the SB09a model – plausible variations of how one handles the secular acceleration of stars lead to changes in the estimated value of  $V_{\odot}$  by  $0.5$  to  $1 \text{ km s}^{-1}$ . Modest reassurance that the error of our value of  $V_{\odot}$  is less than  $2 \text{ km s}^{-1}$  is furnished by the fact that B09 favoured the same value using a distribution function that takes no account of the age and metallicity gradients in the disc. Consequently, our result is probably not sensitive to the assumptions about star formation and chemical evolution made by SB09a. However, as we develop more elaborate models of the Galaxy which fit a wider range of data, in particular more distant stars, we anticipate further small revisions in the value of  $V_{\odot}$ .

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