

The statistics of the subhalo abundance of dark matter haloes

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23 June 2010

ABSTRACT

We study the population statistics of the surviving subhaloes of Λ CDM dark matter haloes using a set of very high resolution N -body simulations. These include both simulations of representative regions of the Universe and ultra-high resolution resimulations of individual dark matter haloes. We find that more massive haloes tend to have a larger mass fraction in subhaloes. For example, cluster size haloes typically have 7.5 percent of the mass within R_{200} in substructures of fractional mass larger than 10^{-5} , which is 25 percent higher than galactic haloes. There is, however, a large variance in the subhalo mass fraction from halo to halo, whereas the subhalo abundance shows much higher regularity. For dark matter haloes of fixed mass, the subhalo abundance decreases by 30 percent between redshift 2 and 0. The subhalo abundance function correlates with the host halo concentration parameter and formation redshift. However, the intrinsic scatter is not significantly reduced for narrow ranges of concentration parameter or formation redshift, showing that they are not the dominant parameters that determine the subhalo abundance in a halo.

Key words: methods: N-body simulations – dark matter – galaxies: haloes

1 INTRODUCTION

In the standard Λ CDM model of cosmogony, cold dark matter haloes form from primordial Gaussian fluctuations by accretion of diffuse dark matter and mergers with other haloes. During this hierarchical process, accreted haloes often survive as self-bound substructures or subhaloes orbiting around the main system (e.g., Tormen et al. 1998; Ghigna et al. 2000; Springel et al. 2001; De Lucia et al. 2004; Gao et al. 2004a; Diemand et al. 2004; Reed et al. 2005; Diemand et al. 2007; Springel et al. 2008a). The most massive subhaloes are likely hosts of luminous satellite galaxies.

The evolution of subhaloes is determined by the interplay of phenomena such as accretion, mergers, dynamical friction and tidal stripping. Thus, subhalo properties are most directly and accurately investigated using simulations. So long as one is exclusively interested in evolution of the dark matter, this is a purely gravitational problem that is

ideally suited to N-body techniques. Indeed, most work on subhaloes has exploited this approach. However, since subhaloes represent a small fraction of the total mass of the halo, most studies have been based, for computational resource reasons, on high resolution simulations of a handful of individual haloes (e.g., Ghigna et al. 2000; Springel et al. 2001; Stoehr et al. 2003; Gao et al. 2004a; Diemand et al. 2004, 2007; Springel et al. 2008a).

Statistical studies are much more demanding because of the need to simulate large volumes with a large dynamic range and, to date, have focused on subhaloes in relative massive haloes (e.g., De Lucia et al. 2004; Gao et al. 2004a; Kravtsov et al. 2004; Shaw et al. 2006; Elahi et al. 2009; Angulo et al. 2009). Inspired by, and calibrated from, the results of simulations, semi-analytic models have been developed to calculate certain properties of subhalo populations (e.g., Benson 2005; Taylor & Babul 2005; van den Bosch et al. 2005; Zentner et al. 2005; Giocoli et al. 2008a,b; Li & Mo 2009).

In this short paper, we exploit the unprecedented statistical power and high resolution of the *Millennium-II Simula-*

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tion (Boylan-Kolchin et al. 2009), together with other recent cosmological simulations and a suite of very high resolution resimulations of individual galaxy haloes (Springel et al. 2008a) to study the abundance of subhaloes with much better statistics and over a significantly larger dynamic range than has been possible in the past. We address three specific questions concerning the subhalo mass function: (i) how does it depend on the mass of the parent halo? (ii) how does it depend on redshift, for a fixed halo mass? (iii) how does it depend, at fixed mass and redshift, on the properties of the halo, specifically the concentration and formation redshift? Some of these questions have been addressed in some of the earlier studies mentioned above. Our results extend these studies into a regime of subhalo mass not previously probed and with much greater statistical power.

2 NUMERICAL SIMULATIONS

The main advance in this paper comes from analysing two sets of very recent, high resolution simulations. One is the 10-billion particle Millennium-II Simulation (MS-II, Boylan-Kolchin et al. 2009) of a $100h^{-1}\text{Mpc}$ cubic volume. The other one is the set of 6 resimulations in the Aquarius Project (Springel et al. 2008a,b), each of which follow the evolution of a galaxy size halo with more than 10^8 particles inside the virial radius. All these simulations represent a significant improvement over the previous generation of N-body simulations. We supplement these data with older simulations, the original 10-billion particle Millennium Simulation (MS), Springel et al. 2005) of a $500h^{-1}\text{Mpc}$ cubic volume and the hMS simulation (Gao et al. 2008), which has matching phases in the initial conditions to those of the MS-II, but with about 14 times lower mass resolution and 2.4 times lower spatial resolution.

All the simulations assume the same values of the cosmological parameters: mean matter density, $\Omega_m = 0.25$; cosmological constant term, $\Omega_\Lambda = 0.75$; Hubble parameter (in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$), $h = 0.73$; fluctuation amplitude, $\sigma_8 = 0.9$; and primordial spectrum power-law index, $n = 1$; These values are consistent with the first year WMAP data (Spergel et al. 2003) but differ at about the $2 - \sigma$ level from more recent determinations. This small off-set is of no consequence for the topics addressed in this paper. All the simulations were run with either the Gadget-2 (Springel 2005) or the more recent, Gadget-3, code used for the Aquarius Project.

The parameters of the simulations are summarized in Table 1. Note that the MS and the MS-II differ in mass resolution by a factor of 125, while the MS and the hMS differ by a factor of 9, allowing numerical convergence tests to be carried out.

Dark matter haloes were found using the friends-of-friends algorithm (Davis et al. 1985), and their self-bound subhaloes were identified using the SUBFIND algorithm (Springel et al. 2001). Haloes and their subhaloes were tracked across the simulation outputs and linked together in merger trees. In what follows, we restrict our attention to subhaloes contained within the virial radius, R_{200} , of their parent host halo, defined as the radius within which the enclosed mass density is 200 times the critical value. We note that Boylan-Kolchin et al. (2010) used a different def-

Table 1. Simulation parameters: (1) name of the simulation; (2) the side of the simulated cubic volume; (3) the number of particles in the simulation; (4) the mass per particle; (5) the gravitational softening length. The initial conditions for hMS and MS-II simulations have matching phases.

Name	Box size [$h^{-1}\text{Mpc}$]	N_p	m_p [$h^{-1}M_\odot$]	ϵ [$h^{-1}\text{kpc}$]
MS	500	2160^3	8.6×10^8	5.0
hMS	100	900^3	9.5×10^7	2.4
MS-II	100	2160^3	6.9×10^6	1.0
Aquarius (level 2)	–	–	$\sim 1 \times 10^4$	0.048

inition of the virial radius, based on the spherical collapse model (Eke, Cole & Frenk 1996).

3 RESULTS

We start by considering the cumulative subhalo mass functions for samples of haloes with mass in the range $[1, 3] \times 10^{14} h^{-1} M_\odot$ in the MS-II, hMS and MS simulations at $z = 0$. The first two samples comprise the same 12 haloes at different resolutions, while the third contains 1676 haloes.

The mean cumulative mass functions of subhaloes for all three samples are plotted in Fig. 1. Here, and below, we show cumulative mass functions multiplied by M_{sub}/M_{200} in order to take out the dominant mass dependence and make the differences between curves more apparent. Following previous work (e.g., Stoehr et al. 2003; Gao et al. 2004a; Diemand et al. 2007; Springel et al. 2008a), we restrict the samples to include only subhaloes with 100 particles or more in order to avoid numerical effects. There is good convergence between the results from the MS-II and the hMS and, within the errors, there is good agreement with the cumulative mass function of the MS. The error bars show the rms scatter about the mean for the MS and MS-II haloes. (They are plotted at a selection of well-separated points to reduce the effect of correlations between adjacent bins.) This scatter, which reflects the varied formation histories of haloes, is substantial, particularly for large values of M_{sub}/M_{200} .

In a recent study, Boylan-Kolchin et al. (2010) demonstrated that the subhalo occupation distribution is well described by a negative binomial, independently of host halo mass. In this case, the scatter at the high mass end of the subhalo mass function is well described by a Poisson distribution, but at the low mass end, $m_{\text{sub}}/M_{200} < 10^{-4}$, intrinsic scatter of fractional amplitude $\sigma \sim 0.15$ dominates. The authors further found that the cumulative subhalo mass function of galactic size haloes can be well fitted with the following form:

$$N(> \mu \equiv m_{\text{sub}}/M_{200}) = \left(\frac{\mu}{\mu_1}\right)^a \exp\left[-\left(\frac{\mu}{\mu_{\text{cut}}}\right)^b\right]. \quad (1)$$

The dashed line in Fig. 1 is a fit to the mean subhalo cumulative mass function for the cluster size haloes from the MS using eqn. (1) with parameters $\tilde{\mu}_1 = 0.01$, $\mu_{\text{cut}} = 0.1$,

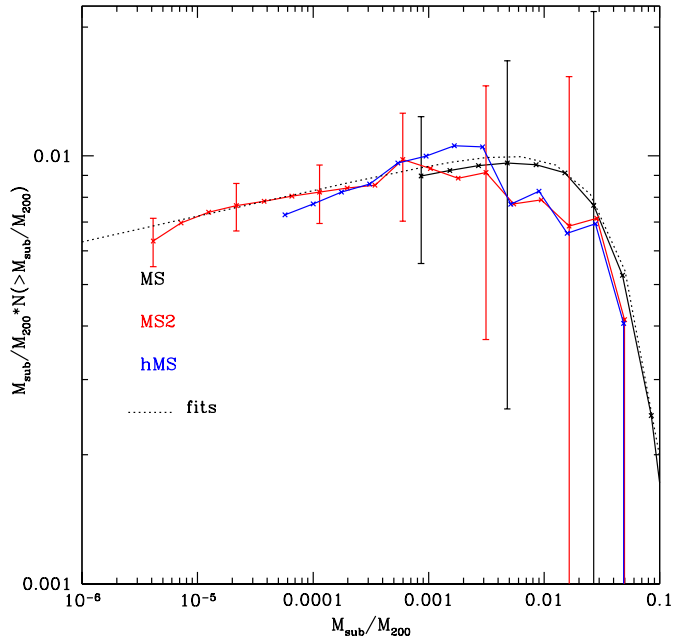


Figure 1. The mean cumulative mass function of subhaloes in hosts with masses in the range $[1, 3] \times 10^{14} h^{-1} M_{\odot}$, as a function of subhalo mass fraction, M_{sub}/M_{200} . The y -axis has been multiplied by the mass fraction in order to expand the dynamic range. The 3 solid lines show the averaged subhalo mass function for samples of 12 haloes in the MS-II (red) and the hMS (blue) and of 1676 haloes in the MS (black), as indicated in the legend. An analytic fit to the MS data using eqn. (1) is shown as a dotted line. The error bars on selected points show the *rms* scatter about the mean for the MS and MS-II haloes.

$b = 1.2$ and power-law index $a = -0.94$. Clearly, the fit is very good. The best-fit parameters differ slightly from the values given by Boylan-Kolchin et al. (2010) because a different definition of virial radius has been used here and the range of halo masses is different.

3.1 Dependence on host halo mass

Gao et al. (2004a) noted that the abundance of relatively massive subhaloes ($M_{\text{sub}}/M_{200} > 0.001$) increases systematically with host halo mass. This trend reflects the fact that in the CDM cosmology more massive haloes are, in the mean, both less centrally concentrated and younger than less massive haloes. Thus, they exert weaker tidal forces and have had less time to disrupt their substructure. This result has been confirmed by subsequent numerical and semi-analytical studies (Zentner et al. 2005; van den Bosch et al. 2005; Shaw et al. 2006; Giocoli et al. 2008b).

The much larger samples of well resolved haloes in our set of simulations allows us to explore the trend with host halo mass to much smaller mass ratios than have been resolved in earlier simulations which, for the most part, have focused on cluster size haloes (e.g. Gao et al. 2004a; Zentner et al. 2005). Combining data from the Aquarius and MS-II simulations, we have a sample of haloes spanning the

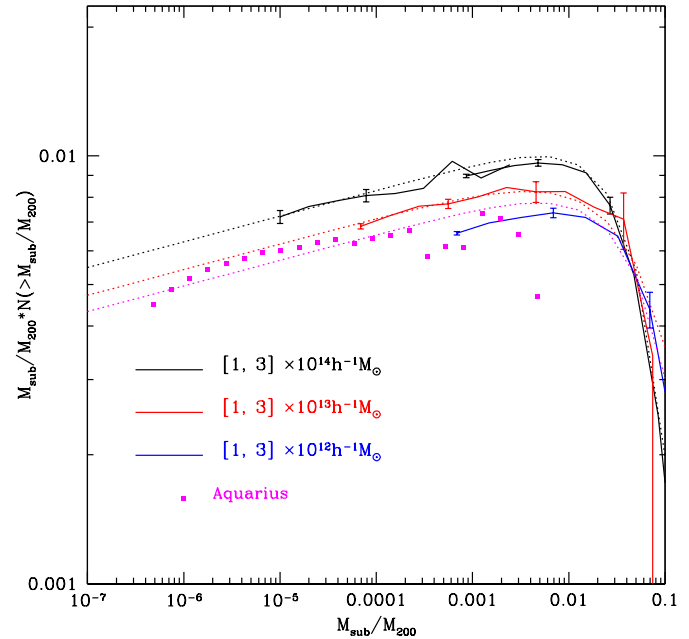


Figure 2. The dependence of the subhalo mass function on the host halo mass. The solid lines show the averaged cumulative subhalo mass functions for three intervals of host halo mass: $[1, 3] \times 10^{14} h^{-1} M_{\odot}$ (black), $[1, 3] \times 10^{13} h^{-1} M_{\odot}$ (red) and $[1, 3] \times 10^{12} h^{-1} M_{\odot}$ (blue). Haloes in the first mass range come from the MS and MS-II, while those in the latter two ranges come from the MS-II. The error bars on selected points show the error on the mean for the three mass ranges indicated. The filled squares show the mean of the cumulative subhalo mass functions of the 6 Aquarius haloes. The dotted lines show fits to eqn. (1); the fit parameters are listed in eqn. (2).

range $\sim 10^{12} - 3 \times 10^{14} h^{-1} M_{\odot}$, each resolved with at least 10^5 particles within the virial radius, and often with 10 to 1000 times more. This represents an improvement by a factor of 100-1000 over the simulations analyzed by Gao et al. (2004a).

Fig. 2 shows cumulative subhalo mass functions for host haloes of different mass. The magenta squares and the solid lines respectively represent galactic size haloes ($[1, 3] \times 10^{12} h^{-1} M_{\odot}$) from Aquarius and MS-II, the blue lines represent group size haloes ($[1, 3] \times 10^{13} h^{-1} M_{\odot}$) from the MS-II and the black lines represent cluster size haloes ($[1, 3] \times 10^{14} h^{-1} M_{\odot}$) from the MS-II and the MS. The corresponding best fits of the form of eqn. (1) are overplotted as dotted lines. Clearly, eqn. (1) provides an excellent fit to the cumulative subhalo mass function for all three ranges of host halo mass, albeit with varying best-fit parameters. We fixed the subhalo function slope index to $a = -0.94$ when fitting all three curves. Of the remaining three free parameters in eqn. (1), $\tilde{\mu}_1$ determines the overall amplitude of the subhalo abundance function and is largely independent of the parameters μ_{cut} and b which combine to determine the shape of the cut-off at the high mass end. We find that neither μ_{cut} nor b can be well constrained on their own, but are largely degenerate. Choosing μ_{cut} in the range $[0.04, 0.1]$

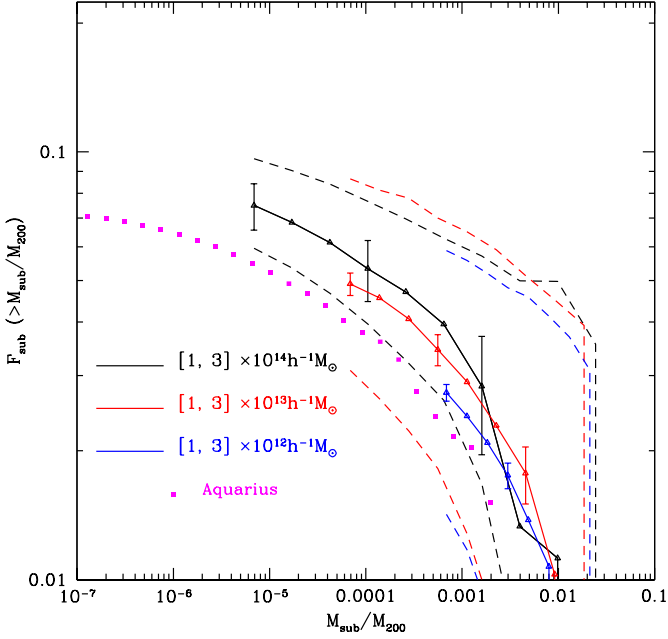


Figure 3. The mass fraction in substructures in haloes of different mass as a function of the normalised subhalo mass. The lines show the **medians** of the cumulative subhalo mass fractions for three ranges of host halo mass in the MS-II simulation: $[1, 3] \times 10^{14} h^{-1} M_{\odot}$ (black solid lines), $[1, 3] \times 10^{13} h^{-1} M_{\odot}$ (red solid lines) and $[1, 3] \times 10^{12} h^{-1} M_{\odot}$ (blue solid lines). The magenta squares show the averaged subhalo mass fraction in the 6 Aquarius haloes. The dashed lines show the 20 and 80% of the corresponding distribution. The error bars on selected points show the error on the median.

or b in the range $[0.8, 1.5]$ gives fits which match the overall subhalo halo mass function reasonably well. To break the degeneracy between μ_{cut} and b in our fits we set $b = 1.2$ and fitted eqn. (1) by varying just $\widetilde{\mu}_1$ and μ_{cut} . The best-fit values for $\widetilde{\mu}_1$ and μ_{cut} are:

$$\begin{aligned} M_{200} \in [1, 3] \times 10^{14} h^{-1} M_{\odot} : \quad & \widetilde{\mu}_1 = 0.0110, \quad \mu_{\text{cut}} = 0.10. \\ M_{200} \in [1, 3] \times 10^{13} h^{-1} M_{\odot} : \quad & \widetilde{\mu}_1 = 0.0092, \quad \mu_{\text{cut}} = 0.07. \\ M_{200} \in [1, 3] \times 10^{12} h^{-1} M_{\odot} : \quad & \widetilde{\mu}_1 = 0.0085, \quad \mu_{\text{cut}} = 0.08. \end{aligned} \quad (2)$$

At a given M_{sub}/M_{200} , there is a weak trend in the abundance of subhaloes with host halo mass. In the region of overlap in M_{sub}/M_{200} , the cluster haloes have a mass fraction of substructures that is about 25% higher than in the galaxy haloes. The mean of the group sample is intermediate between these two. The 15% difference between group and cluster halo identified by Gao et al. (2004a) is visible at the smaller mass ratios plotted, $M_{\text{sub}}/M_{200} \sim 10^{-4}$. The mass dependence of the subhalo abundance with host halo mass in our simulations is much weaker than a theoretical expectation based on semianalytical modelling of the subhalo population (e.g., van den Bosch et al. 2005).

The mass fractions in subhaloes as a function of relative mass for host haloes of different size are shown in Fig. 3. The curves are very steep at high values reflecting the fact that

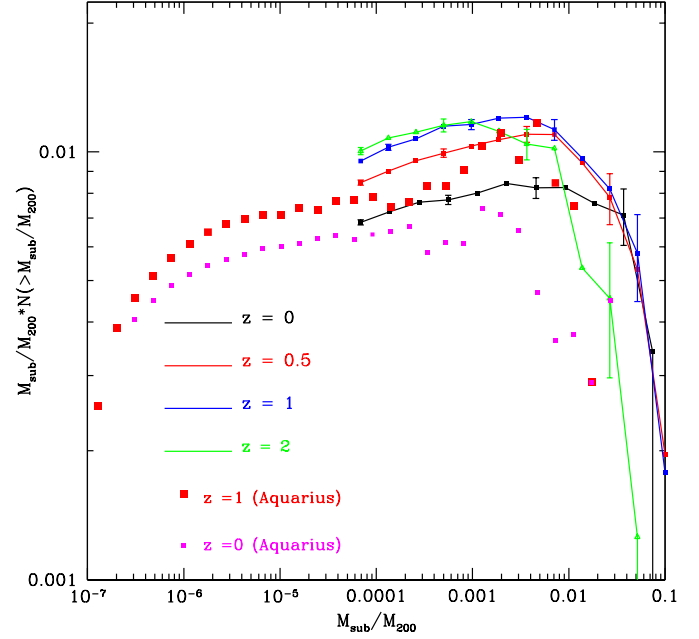


Figure 4. The dependence of the subhalo mass function on redshift. The lines show the averaged cumulative subhalo mass functions for MS-II haloes in the mass range $[1 - 3] \times 10^{13} h^{-1} M_{\odot}$ at redshift $z = 0$ (black), $z = 0.5$ (red), $z = 1$ (blue) and $z = 2$ (green). The filled squares show results for the Aquarius haloes at $z = 1$ (large red) and $z = 0$ (magenta). The error bars on selected points show the error on the mean.

most of a typical halo’s mass in subhaloes is contributed by a relatively small number of very massive subhaloes. As shown by Springel et al. (2008a) for the Aquarius haloes, these massive subhaloes tend to be in the outer parts of their parent halo. The subhalo mass fraction above a given value of M_{sub}/M_{200} depends on the mass of the parent halo. Typically, cluster haloes contain about 25% more subhalo mass than galaxy haloes. The Aquarius galactic haloes resolve substructure with relative mass as small as 10^{-6} . Subhaloes above this mass contain about 7% of the total halo mass within R_{200} . For clusters, the MS-II resolves subhaloes with relative mass just below 10^{-5} . Subhaloes more massive than this already amount to about 7% of the mass within R_{200} . Note, however, that there is a rather large variance from halo to halo in the subhalo mass fraction above a given value of M_{sub}/M_{200} , particularly at the high mass end, where much of the subhalo mass is typically contributed by one or two objects.

3.2 Redshift dependence

We expect haloes of a *given mass* to contain more subhaloes at earlier times because the earlier counterparts are both less concentrated and dynamically younger. This trend was seen in the simulations of Gao et al. (2004a). To investigate the redshift dependence of the subhalo mass function in our simulations, we use a sample of group haloes of mass $[1, 3] \times$

$10^{13}h^{-1}M_{\odot}$ in the MS-II. We also use the Aquarius galaxy haloes at $z = 0$ and their main progenitors at $z = 1$, the mean mass of these halo is $M_{200} = 1.19 \times 10^{12}h^{-1}M_{\odot}$ at $z = 0$ and $M_{200} = 6.73 \times 10^{11}h^{-1}M_{\odot}$ at $z = 1$.

The results are displayed in Fig 4. In the MS-II sample, there are a total of 219, 204, 169 and 41 haloes at the four redshifts shown, $z = 0, 0.5, 1, 2$. Clearly, the subhalo mass function evolves with redshift weakly, but systematically, in the expected sense: at high redshift haloes of a given mass contain more subhaloes. For example, the subhalo abundance in groups at $z = 0$ is typically 18% lower than at $z = 0.5$, 25% lower than at $z = 1$, and 30% lower than at $z = 2$. The subhalo mass function of galaxy haloes evolves slightly more slowly than this, reflecting the earlier formation epoch of galaxy haloes. In this case the abundance is only 15% higher at $z = 1$ than at $z = 0$. We note that using eqn. (1) with a and b fixed to -0.94 and 1.2 , respectively, fits the data shown at each redshift very well. The corresponding parameters are:

$$\begin{aligned} z = 0.0: \widetilde{\mu}_1 &= 0.0092, \mu_{\text{cut}} = 0.07 \\ z = 0.5: \widetilde{\mu}_1 &= 0.0118, \mu_{\text{cut}} = 0.06 \\ z = 1.0: \widetilde{\mu}_1 &= 0.0130, \mu_{\text{cut}} = 0.05 \\ z = 2.0: \widetilde{\mu}_1 &= 0.0140, \mu_{\text{cut}} = 0.02 \end{aligned} \quad (3)$$

For the galaxy haloes the trend persists down to the smallest subhalo masses resolved in the set of six Aquarius simulations, $M_{\text{sub}}/M_{200} = 10^{-6}$.

3.3 Dependence on host halo properties

We now examine how the subhalo mass function depends on two basic properties of the parent halo: the concentration parameter and the formation redshift. We also consider whether any such dependence contributes to the scatter in the subhalo mass function seen in previous figures, as suggested by Zentner et al. (2005). Earlier studies have shown that, at least for relatively massive haloes, the subhalo abundance does depend on the concentration and formation redshift of the parent halo (Gao et al. 2004a; Zentner et al. 2005; Shaw et al. 2006). With the MS-II, we can re-examine the dependence of subhalo abundance on the properties of the parent halo with much better statistics than was possible before and also explore the low mass end of the subhalo mass distribution.

We select 219 haloes in the MS-II in the mass range $[1, 3] \times 10^{13}h^{-1}M_{\odot}$, and subdivide this sample into three equal size subsamples ranked according to concentration parameter or formation redshift. We evaluate the concentration parameter of a halo as V_{max}/V_{200} (as was done in Gao et al. 2004a) and we define its formation redshift as the time when half the mass was assembled. The results shown in Fig. 5 confirm the conclusion of Gao et al. (2004a) that the subhalo abundance decreases with increasing parent halo concentration and formation redshift. At the low subhalo mass end, the third least concentrated and most recently formed haloes have about 25% more substructures than the most concentrated and earliest forming third. At the high subhalo mass end, $M_{\text{sub}}/M_{200} > 0.01$, the difference between the two extreme thirds is about a factor of 2, significantly larger than at the lower subhalo mass end.

The variance around the mean of the subhalo mass func-

tion for each subsample and for the sample as a whole are listed in Table 2, in units of the mean value. The size of the trends seen in Fig. 5 is comparable to the scatter in the relations. Nevertheless, the dependence of subhalo abundance on the concentration and formation time of the parent halo, at a fixed host halo mass, is much stronger than the dependence on host halo mass seen in Fig. 2.

The data presented in Table 2 show that the scatter in subhalo abundance among haloes chosen to have a narrow range of concentration parameters or formation times is not significantly smaller than the scatter for the halo population as a whole. Thus, neither concentration nor formation time appear to be the primary halo property responsible for the observed scatter in the subhalo mass function.

4 CONCLUSION

We have used a new set of very large cosmological N -body simulations to investigate the statistics of subhalo abundance in Λ CDM dark matter haloes. Our results may be summarized as follows:

- (i) The subhalo abundance function of dark matter haloes can be well fitted with the functional form proposed by Boylan-Kolchin et al. (2010), which is a power-law at the low subhalo mass end and has an exponential cut-off at the high mass end, independently of host halo mass and redshift.
- (ii) The subhalo abundance function depends weakly on host halo mass. The difference between typical cluster and galaxy size haloes is about 25 percent at the lower subhalo mass end, substantially weaker than expected from previous studies. The scatter in the subhalo abundance function of different haloes is larger than the systematic trend.
- (iii) For a given mass halo, the subhalo abundance evolves systematically with redshift. The evolution is largest at lower redshift (~ 18 percent between $z = 0$ and $z = 0.5$) and becomes very small at high redshift (a few percent between $z = 1$ and $z = 2$). Over the range $z = 0 - 2$, the substructure abundance increases by 30% for haloes corresponding to bright galaxies and poor galaxy groups.
- (iv) At fixed mass, dark matter haloes with higher concentration parameters and earlier formation redshifts contain fewer subhaloes. The difference is a factor of 25 percent between the top and bottom thirds of the population ranked by concentration parameter or formation redshift. However, this dependence on concentration and formation redshift is not enough to explain the scatter in the subhalo mass function, suggesting that these are not the dominant parameters determining the subhalo mass fraction.

ACKNOWLEDGEMENTS

The Millennium Simulation and the Millennium-II Simulation were carried out as part of the programme of the Virgo Consortium on the Regatta and VIP supercomputers at the Computing Centre of the Max-Planck Society in Garching. The hMS simulation was carried out using the Cosmology Machine at Durham. LG acknowledges support from the one-hundred-talents program of the Chinese academy of science (CAS), the National basic research program of China (program 973 under grant No. 2009CB24901), NSFC grants

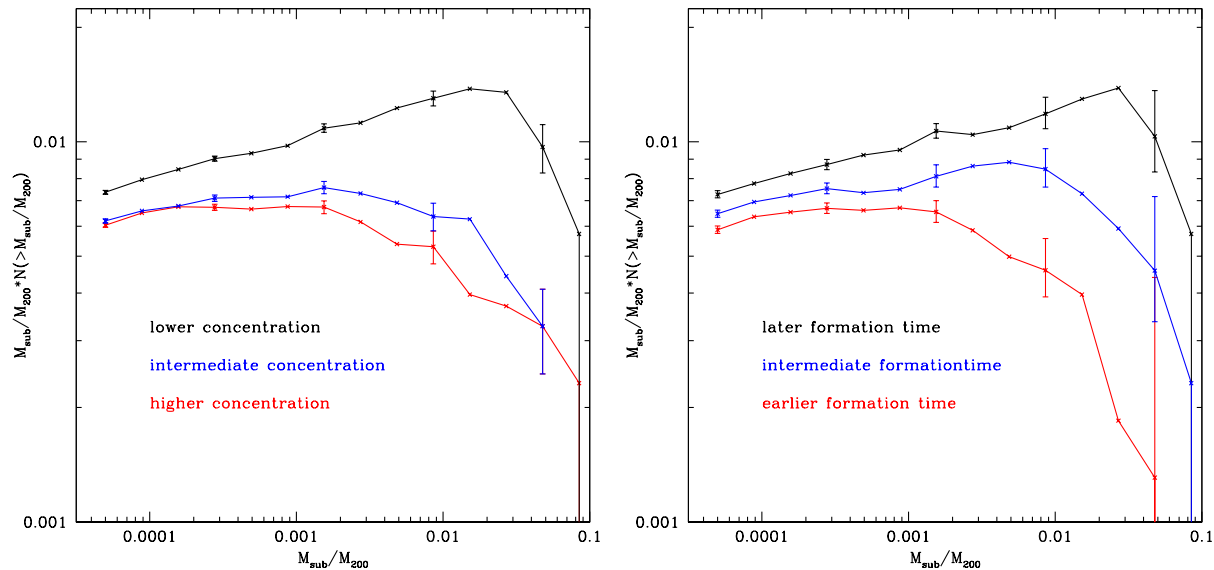


Figure 5. The dependence of the subhalo mass function on the properties of the parent halo. The left panel shows the dependence on the concentration parameter of the halo and the right panel on the formation redshift. The lines show the mean of the cumulative subhalo mass functions for 219 MS-II haloes in the mass range $[1 - 3] \times 10^{13} h^{-1} M_{\odot}$. In the two panels, the red solid lines show the averaged subhalo abundance function of the top third of the sample, that with the highest concentration parameter (left), and earliest formation redshift (right). The blue lines correspond to the intermediate third of the sample and the black lines to the bottom third, with the lowest concentration parameter and the latest formation redshift. The error bars on the selected points show the error on the mean.

Table 2. The standard deviation from the mean of the subhalo mass function at a given fractional mass, $m_{\text{sub}}/M_{200} = 0.01, 0.001, 0.0001$, in units of the mean.

μ	high c	intermediate c	low c	high z	intermediate z	low z	whole sample
0.01	1.52	1.23	0.67	1.5	1.01	0.81	1.08
0.001	0.47	0.44	0.32	0.48	0.43	0.33	0.48
0.0001	0.21	0.21	0.18	0.20	0.20	0.20	0.22

(Nos. 10973018) and an STFC Advanced Fellowship, as well as the hospitality of the Institute for Computational Cosmology in Durham, UK. CSF acknowledges a Royal Society Wolfson Research Merit Award. This work was supported in part by an STFC rolling grant to the ICC.

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