

Are the superstructures in the two-degree field galaxy redshift survey a problem for hierarchical models?

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ABSTRACT

We introduce an objective method to assess the probability of finding extreme events in the cold dark matter such as voids, overdensities or very high mass haloes. Our approach uses an ensemble of N-body simulations of the hierarchical clustering of dark matter to find extreme structures. The frequency of extreme events, in our case the cell or smoothing volume with the highest count of cluster-mass dark matter haloes, is well described by a Gumbel distribution. This distribution can then be used to forecast the probability of finding even more extreme events, which would otherwise require a much larger ensemble of simulations to quantify. We use our technique to assess the chance of finding concentrations of massive clusters or superclusters, like the two found in the two-degree field galaxy redshift survey (2dFGRS), using a counts-in-cells analysis. The Gumbel distribution gives an excellent description of the distribution of extreme cell counts across two large ensembles of simulations covering different cosmologies, and measuring the clustering in both real and redshift space. We find examples of structures like those found in the 2dFGRS in the simulations. However, the chance of finding such structures in a volume equal to that of the 2dFGRS is less than 1%.

Key words: galaxies:high-redshift – galaxies:evolution – cosmology:large scale structure – methods:numerical – methods:N-body simulations

1 INTRODUCTION

The discovery of extreme objects, such as large voids or highly overdense regions in which many galaxy clusters are found close together, called superclusters, has often been presented as a challenge to the hierarchical structure formation paradigm. However, the main drawback of using the presence of unusual structures to rule out a particular model is that it is not always clear how to assess the probability of finding such objects. In this paper we introduce a new methodology to address this problem in which we use N-body simulations and extreme value theory to provide a quantitative assessment of the probability of finding rare structures in a given cosmology.

Claims of rare structures are common in the literature. Frith et al. (2003) argued that the shape of the local galaxy counts implies a underdense volume in the southern sky $\sim 300h^{-1}\text{Mpc}$ across (see also Busswell et al. 2004; Frith et al. 2005, 2006). Cruz et al. (2005) found a cold spot in the

cosmic microwave background radiation that is much bigger than expected in a Gaussian distribution. Rudnick, Brown & Williams (2007) suggested that this cold spot is a secondary anisotropy, coinciding with the angular position of a void in a survey of radio galaxies. Swinbank et al. (2007) found a large association of galaxy clusters, a supercluster of galaxies, at $z \sim 0.9$ in the UK Infrared Deep Sky Survey. There are two common problems with the interpretation of such results. Firstly, what is the selection function, which would allow us to fix the frequency of finding such structures? And, secondly, what exactly are we looking at? For example, have we seen one massive cluster or are we looking at a projection of smaller structures along the line of sight? How should we compare the observations to theoretical predictions?

In this paper we assess how common the superclusters found in the two-degree field galaxy redshift survey (2dFGRS; Colless et al. 2001, 2003) are in the CDM cosmology. These structures were identified as “hotspots” in the distribution of galaxy counts-in-cells (Baugh et al. 2004; Croton et al. 2004). One structure is in the NGP part of the 2dFGRS at a redshift of $z = 0.08$ and a right ascension of 3.4

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hours, and the other is in the SGP region at $z = 0.11$ at a right ascension of 0.2 hours. The higher order moments of the counts are strongly influenced by the presence of these structures (Croton et al. 2004; Nichol et al. 2006). A subsequent analysis of galaxy groups in the 2dFGRS revealed that these regions contain a surprisingly large fraction of all the massive clusters in the survey (Eke et al. 2004a). Of the 94 groups in the full flux limited 2dFGRS out to $z \sim 0.15$ with 9 or more members and estimated masses above $5 \times 10^{14} h^{-1} M_{\odot}$, 20 percent reside in these superclusters (Padilla et al. 2004). One of the superclusters is part of the ‘‘Sloan Great Wall’’ (Gott et al. 2005).

The 2dFGRS superclusters are by no means the largest superclusters in the local universe (for a list of superclusters, see Einasto et al. 2001). The Shapley cluster, for example, contains more clusters than either of the 2dFGRS structures (Raychaudhury et al. 1991; Proust et al. 2006; Munoz & Loeb 2008). However, not all of the clusters contained within Shapley and similar mass concentrations have measured redshifts. Many of the member clusters have been identified in projection, and so their actual size is open to debate (Sutherland & Efstathiou 1991). The advantage of focusing on the 2dFGRS structures is that they have been identified from an unbiased redshift survey which was designed to map a particular volume of the Universe, rather than by targeting known structures. The volume of space in which the superclusters are found is therefore well defined. Furthermore, through the construction of the 2dFGRS Percolation Inferred Galaxy Group (2PIGG) catalogue (Eke et al. 2004a), there is a clear, objective way to connect the observed properties of the galaxy groups which make up the superstructures to dark matter haloes in N-body simulations.

In this paper we use extreme value theory to assess the probability of finding structures like the 2dFGRS superclusters in the CDM cosmology. Previous attempts to address the probability of finding such structures have used small numbers of simulations, and so have not been able to make definitive statements. For example, Croton et al. (2004) measured the moments of the galaxy cell count distribution in the 22 mock catalogues whose construction was described by Norberg et al. (2002). None of these mocks displayed higher order moments that looked like those measured in the 2dFGRS, giving a probability of less than 5% that such a structure could arise in a CDM model. One possible way around this problem is to generate estimates of the error on a measurement from the data itself (see Norberg et al. 2009). Around 50 such estimates are required to get an accurate estimate of the variance on a measurement in the case of Gaussian statistics, and this method is clearly not applicable to a structure which appears once or twice in the dataset. The method we describe in this paper is calibrated against N-body simulations and can be extrapolated to very low probabilities, without requiring any assumption about the detailed form of the underlying distribution, just its asymptotic behaviour.

The layout of this paper is as follows. In Section 2 we first give a very brief overview of extreme value theory, before discussing how we connect dark matter haloes from an N-body simulation to galaxy groups in the 2PIGG catalogue. Finally in Section 2 we describe the ensembles of N-body simulations that we use to measure the counts-in-cells dis-

tribution of massive haloes. The results of the paper are presented in Section 3, in which we show the impact of mass errors on the distribution of halo counts-in-cells and demonstrate how well extreme value theory describes the probability of finding a ‘‘hot’’ cell. Our conclusions are given in Section 4.

2 METHOD

In this section we first give a brief introduction to extreme value theory (§ 2.1), before describing how we will look for superclusters like those in the 2dFGRS in N-body simulations (§ 2.2). The simulations we use are outlined in § 2.3.

2.1 Extreme value theory

Extreme value theory is used extensively by weather forecasters, seismologists and actuaries, but less so by cosmologists. However, there are some recent examples of applications to features in the cosmic microwave background (Mikelsons, Silk & Zuntz 2009) and the fluctuations in the density around galaxies (Antal et al. 2009).

The main result of interest for this paper is the probability distribution of extreme events. In our application the extreme event is a high count of galaxy clusters within a volume of space covered by a cell. Imagine a population of galaxy redshift surveys. For each survey, the distribution of the counts in cells of galaxy clusters is measured for a particular cell size. The maximum count obtained within a cell is recorded for each survey. The cumulative distribution of the maximum count across the population of surveys is given by the Gumbel distribution:

$$F(x; \mu, \beta) = \exp^{-\exp^{(\mu-x)/\beta}}, \quad (1)$$

where the mean of the distribution is $\mu + \gamma\beta$, where $\gamma = 0.577216$ is the Euler-Mascheroni constant. The standard deviation is given by $\beta\pi/\sqrt{6}$.

There is no need to specify the form of the underlying probability distribution of cluster counts. The only requirement for the extreme value distribution to be applicable to the extrema of this process is that the underlying distribution is well behaved, which means that it is continuous and the cumulative distribution has an inverse. There are three types of extreme value distribution, which are distinguished by the shape of the tail of the underlying distribution. The Gumbel distribution is a type one extreme value distribution, in which case the ‘‘shape parameter’’ of the distribution tends to zero (e.g. as is the case for a Gaussian or exponential distribution).

2.2 The extreme value data: 2dFGRS superclusters

As mentioned in the previous section, the extreme value data we will assess are the two ‘‘hot’’ cells in the 2dFGRS identified by Baugh et al. (2004) and Croton et al. (2004). These hot cells were initially identified in a volume limited sample of L_* galaxies. On leaving out the galaxies within the two hot cells, the higher order moments of the count distribution had the expected form on large scales. The cell radius used by Croton et al. was equivalent to a cube of side $40.6 h^{-1}$ Mpc.

On cross-matching the hot cells in the galaxy distribution with the 2PIGG catalogue, there are 10 groups in each cell with an estimated mass in excess of $5 \times 10^{14} h^{-1} M_{\odot}$ (Padilla et al. 2004).

Rather than construct mock galaxy catalogues to compare with the 2dFGRS, we shall consider instead the counts of dark matter haloes. This removes a layer of theoretical uncertainty, as there is no longer a need to include a model of galaxy formation, which may in any case have produced a group catalogue with different properties from those of the 2PIGG sample. Furthermore, there is an objective prescription which relates the mass of a galaxy group in the 2PIGG catalogue to the mass of a dark matter halo in an N-body simulation (Eke et al. 2004b). Careful tests using simulations show that there is a scatter and a small systematic bias between the true mass of the halo in the N-body simulation m_{true} and the estimated mass inferred from the galaxy groups found using a percolation algorithm (Eke et al. 2004b):

$$m_{\text{estimated}} = m_{\text{true}} \times 10^{0.1+0.3\sigma}, \quad (2)$$

where the systematic bias is 0.1 dex and σ is a Gaussian deviate with zero mean and unit variance. Hence, given the true halo mass obtained from the simulation, we can generate an estimated mass using Eq. 2, to mimic the mass that would have been recovered for this halo by the 2PIGG algorithm.

2.3 The N-body simulations

We use two ensembles of 50 large volume N-body simulations called L-BASICC (Angulo et al. 2008). The ensembles correspond to different choices of the values of the cosmological parameters within a flat Λ CDM cosmology (Sanchez et al. 2006, 2009). The L-BASICC ensemble uses the same cosmological parameters as the Millennium Simulation of Springel et al. (2005): a matter density parameter, $\Omega_m = 0.25$, an energy density parameter for the cosmological constant, $\Omega_{\Lambda} = 0.75$, a normalization of density fluctuations, $\sigma_8 = 0.9$, a Hubble constant of $h = 0.73$ and a baryon density parameter of $\Omega_b = 0.045$. The L-BASICC II ensemble uses a parameter set which is in somewhat better agreement with the latest observations of the cosmic microwave background and the local large scale structure of the galaxy distribution (Sanchez et al. 2006): $\Omega_m = 0.237$, $\Omega_b = 0.041$, scalar spectral index, $n_s = 0.954$, $\sigma_8 = 0.77$, and $h = 0.735$. Each of the L-BASICC and L-BASICC II simulations covers a comoving cubical region of side $1340 h^{-1}$ Mpc using 448³ particles. This gives a particle mass comparable to that employed in the Hubble Volume simulation (Evrard et al. 2002). The equivalent Plummer softening length in the gravitational force is $\epsilon = 200 h^{-1}$ kpc. The volume of each computational box, $2.41 h^{-3}$ Gpc³, is almost twenty times that of the Millennium Simulation, and more than three times the volume of the luminous red galaxy sample from the SDSS used to make the first detection of the acoustic peak by Eisenstein et al. (2005). The computational box is 300 times the volume of the region covered by the volume limited 2dFGRS sample of L_* galaxies. The total volume of the ensemble is $120 h^{-3}$ Gpc³, more than four times that of the Hubble Volume simulation. We use the friends-of-friends halo catalogue extracted from the $z = 0$ simulation output in our analysis, retaining objects with ten or more particles (corresponding

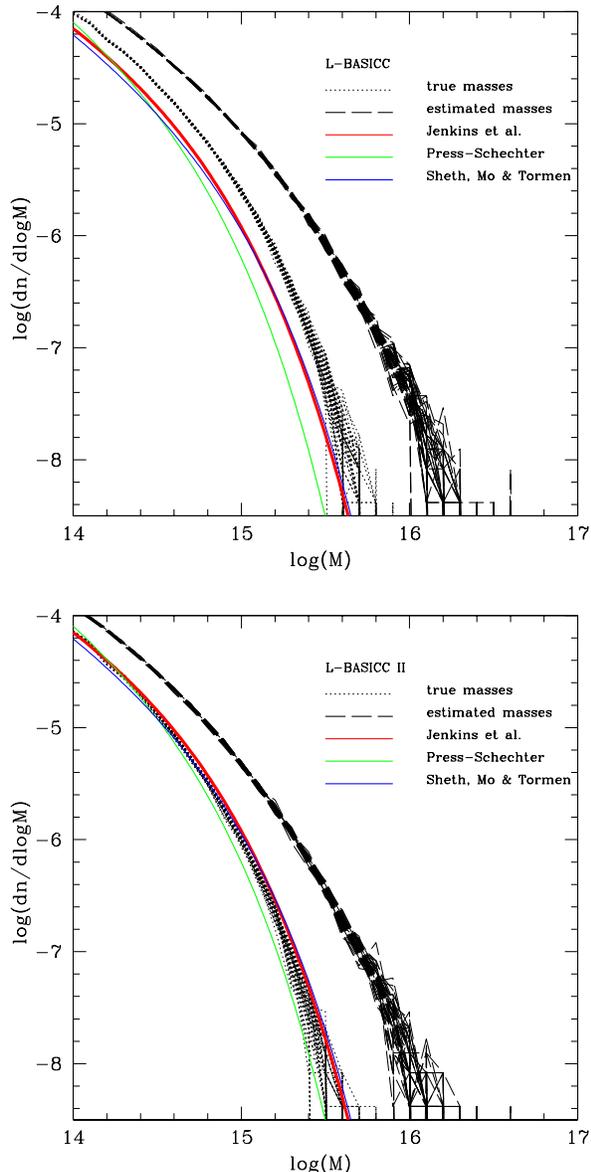


Figure 1. The abundance of dark matter haloes as a function of mass in the L-BASICC (top) and L-BASICC II (bottom) simulation ensembles. In both cases, the dotted black lines show the mass function using the true halo mass, as determined by the friends-of-friends group finder (Eq. 2). The black dashed lines show the mass function after applying the prescription to generate the “estimated” mass, as would be returned by applying the 2PIGG algorithm. Each line corresponds to a simulation from the ensemble. The coloured lines are the *same* in both panels and correspond to theoretical predictions for the true mass function in the L-BASICC II cosmology, with the red curve showing the Jenkins et al. (2001) empirical fit, the green curve the Press & Schechter (1974) mass function and the blue curve the prediction of Sheth, Mo & Tormen (2001). In the lower panel, these curves show how well the theoretical predictions agree with the simulation results; in the upper panel the same theoretical curves are plotted to show how much the simulation results have shifted following the change in cosmological parameters.

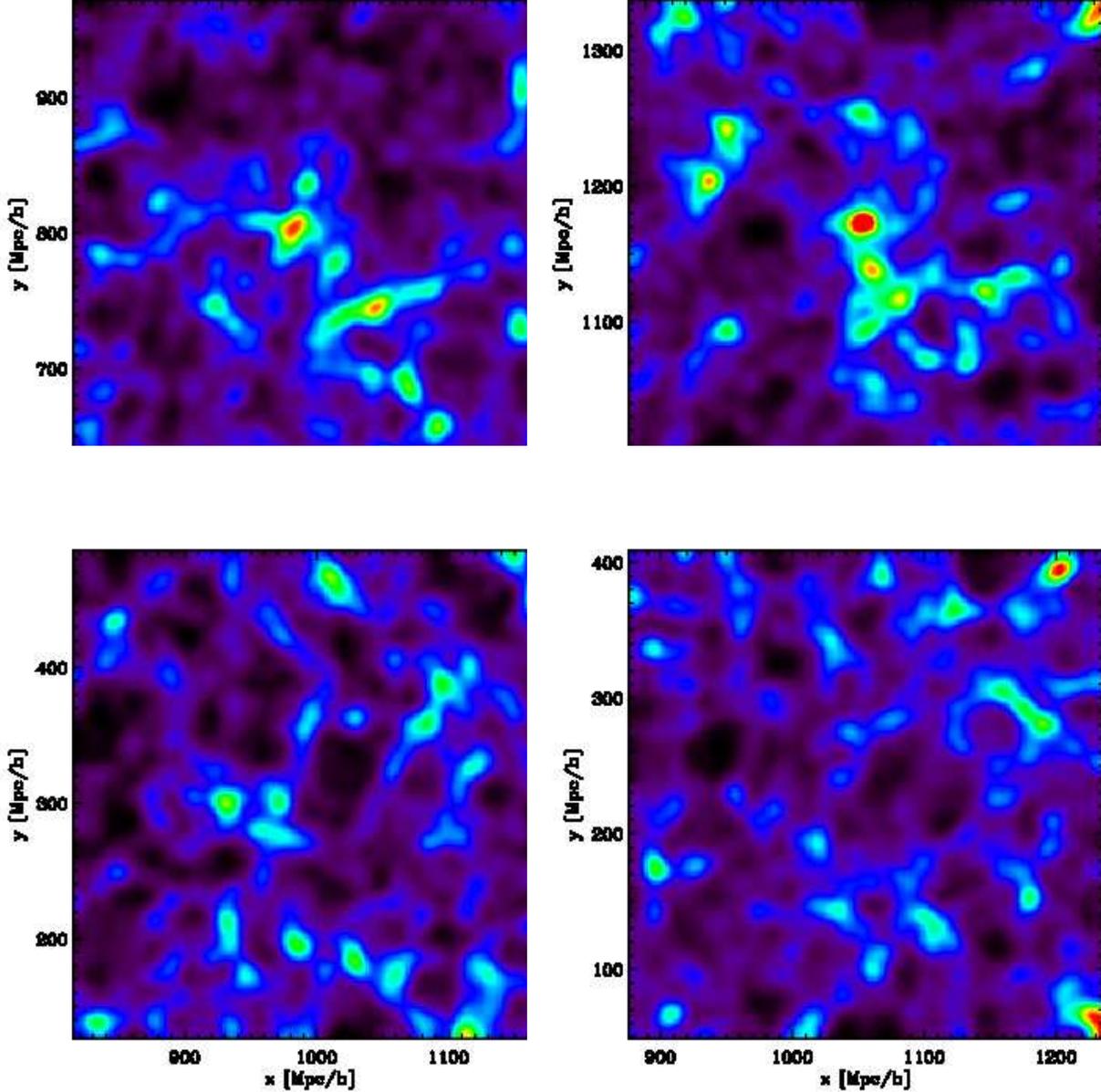


Figure 2. The projected density of dark matter in selected simulations. The color scale is the same in each panel and covers the range from 1 (black) to 16 (red) dark matter particles per pixel, with 256×256 pixels per image. Red corresponds to a projected dark matter density of $1.5 \times 10^{13} h^{-1} M_{\odot} / (h^{-1} \text{Mpc})^2$. The thickness of the slices is $40 h^{-1} \text{Mpc}$. The top two panels are centred on the location of “hot” cells in the distribution of massive haloes, and the lower panels show regions chosen at random.

to a mass limit of $1.75 \times 10^{13} h^{-1} M_{\odot}$) to analyze the likelihood of finding rare massive structures.

3 RESULTS

In this section we present the results for the distribution of the counts in cells of dark matter haloes in the L-BASICC and L-BASICC II simulation ensembles.

We first examine the impact on the halo mass function of including the bias and error expected if the 2PIGG group finding algorithm was to be applied to a mock galaxy catalogue made from the N-body simulations. Fig. 1 shows the abundance of haloes at $z = 0$, both without and with in-

corporating the error implied by the distribution given by Eq. 2. There is a considerable spread in the halo mass function between the individual realisations of the ensembles for the most massive haloes. There is a significant change in the abundance of haloes at the high mass end of the mass function upon including the mass error. The abundance of haloes of mass $\log_{10}(M_{\text{halo}}/h^{-1} M_{\odot}) \sim 15.5$ increases by an order of magnitude when the mass errors are included. This is readily understood in terms of the exponential shape of the mass function for haloes of these masses. There are many more haloes of low mass than high mass. Hence, on applying a perturbation in halo mass corresponding to a symmetrical error distribution, there is a net transfer of haloes from lower mass bins to higher mass bins. This effect is compounded

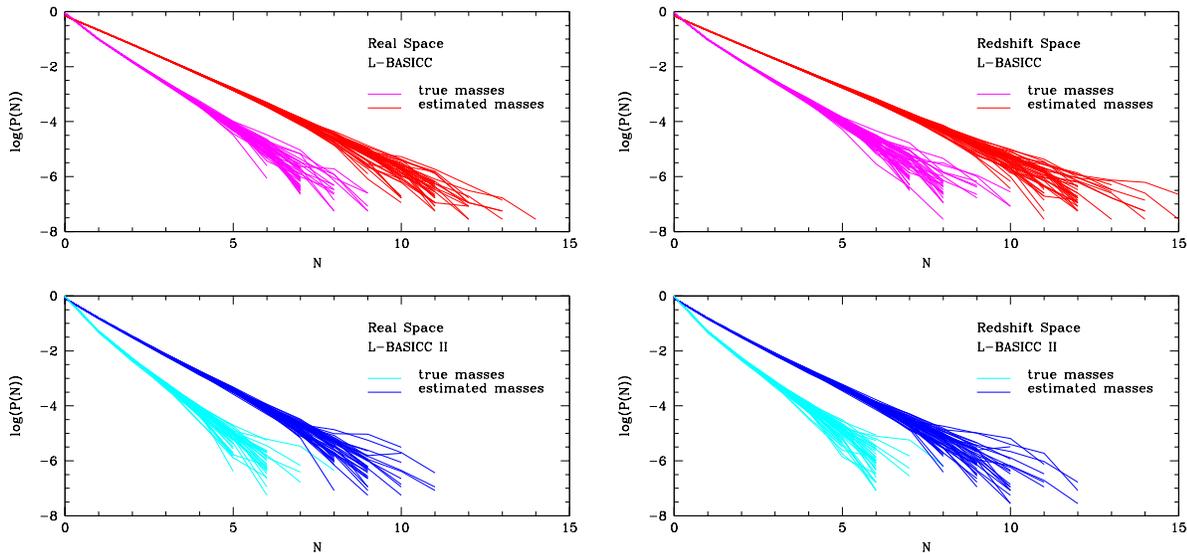


Figure 3. The count probability distribution for haloes with mass in excess of $5 \times 10^{14} h^{-1} M_{\odot}$ in cubical cells of side $40.6 h^{-1}$ Mpc. In the left hand panels, the cell counts are measured using the real-space positions of the haloes and in the right-hand panels using the redshift-space positions. The upper row shows the measurement in the L-BASICC ensemble and the lower row in the L-BASICC-II simulations. Each curve shows the count distribution in one of the realisations. The different coloured lines in each panel shows the measurements without and with mass errors as indicated by the key.

by the small systematic overestimate of halo mass resulting from the 2PIGG algorithm. The analytic mass functions give a reasonable match to the unperturbed mass function, except in the case of the Press & Schechter theory, which predicts too few massive haloes. This discrepancy has been noted before (see e.g. Efstathiou et al. 1988). Finally, we note that there is an appreciable reduction in the abundance of haloes of a given mass in the L-BASICC II cosmology compared with the L-BASICC cosmology, as is clear by comparing the simulation results with the analytical models in Fig. 1, which are the same in each panel.

We now measure the distribution of the counts in cells of dark matter haloes in the simulations. We consider haloes with mass in excess of $5 \times 10^{14} h^{-1} M_{\odot}$, using both the true mass estimated directly from the simulation by a friends-of-friends halo finder using the dark matter particles and the perturbed mass, which attempts to mimic the mass the halo would have been assigned by the 2PIGG algorithm. We use cubical cells of side $40.6 h^{-1}$ Mpc. To find cells of a similar halo overdensity to those corresponding to the 2dFGRS superclusters, we need to find cells which contain 10 or more haloes of the above mass. The halo distribution is oversampled by throwing down many more cells than would fit independently into the simulation volume. This is important because the halo count within a cell could change significantly with a small change in the location of the cell. This oversampling is taken into account when plotting the counts-in-cells probability distribution. We oversample the density field 1000 times; in practice the count probability distribution converges after a few tens of regriddings.

We show some images of selected regions in the L-BASICC simulations in Fig. 2. These plots show the projected density of dark matter in slices of $40 h^{-1}$ Mpc. The volume in these slices is 50% of the volume of the 2dFGRS

L_* sample. The top panels are centred on “hot” cells which contain 10 or more haloes more massive than $5 \times 10^{14} h^{-1} M_{\odot}$ and the lower panels show randomly selected regions.

We first measure the cell counts using the true positions of the haloes, without taking into account the impact of peculiar velocities. The cell counts for the two ensembles of simulations are plotted in Fig. 3. In all 100 realisations covering both cosmologies, we did not find any cells with the required occupancy of massive haloes to match the 2dFGRS superclusters, when using the true halo masses. The conclusion is very different if we consider the estimated halo masses rather than the true halo masses. Now there are many cells with 10 or more haloes of the required mass. In the L-BASICC cosmology, in the majority of realisations the hottest cells contains in excess of 10 massive haloes. Around a quarter of the realisations in the L-BASICC-II ensemble contain cells with counts above this threshold. This difference in the tail of the count distribution is driven primarily by the difference in the value of σ_8 between L-BASICC and L-BASICC II.

The impact of peculiar velocities on the appearance of large scale structure in the halo distribution is taken into account using the distant observer approximation. One of the cartesian axes of the simulation cube is taken to be the line of sight. The peculiar motion of the halo along this axis is added to its position, after applying a suitable scaling to convert the velocity into an equivalent displacement in Mpc. The change in the cell count probability is quite dramatic, as can be seen from Fig. 3. There is a shift in the maximum cell count and an increase in the spread of the maximum cell count across the realisations of the simulation ensembles. The impact of redshift space distortions on the cell counts is akin to changing the value of σ_8 .

In order to quantify these changes more clearly, we plot in Fig. 4 the distribution of the maximum cell count over

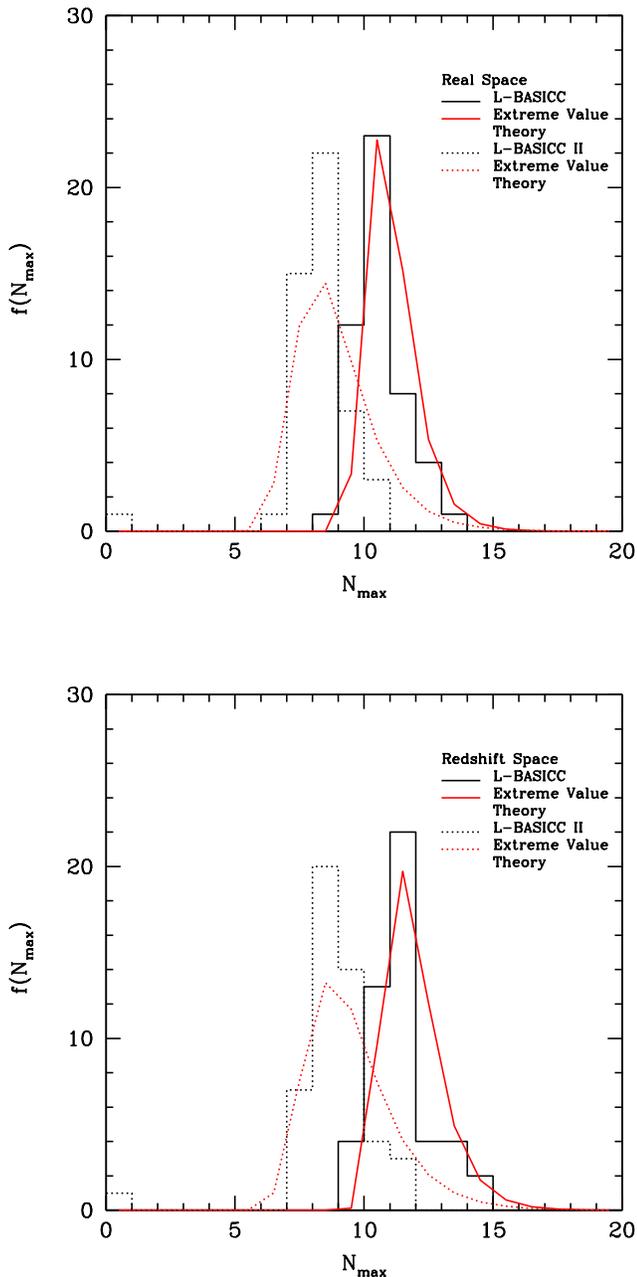


Figure 4. The distribution of the cell count in the hottest cells from each realisation in the simulation ensembles. The number plotted on the x-axis is the number of haloes with mass $> 5.e \times 10^{14} h^{-1} M_{\odot}$ in the cell. The upper panels show the cell counts measured in real-space and the lower panel shows the measurement in redshift-space. The results for the L-BASICC ensemble are shown using solid histograms and for the L-BASICC-II ensemble using dotted histograms. The corresponding Gumbel distributions are plotted in red.

the different realisations. The histograms show the distribution of hot cell counts extracted from the simulations. The curves show the Gumbel distribution of Eq. 1, plotted using the corresponding mean and variance of the hot cell distributions as measured from the simulations. In real-space, the mean and variance of the hot cells for the L-BASICC ensemble is 11.1 and 0.995 respectively, which correspond to $\mu = 10.65$ and $\beta = 0.78$; in the case of L-BASICC-II these values become a mean of 8.92 and variance of 1.562, which become $\mu = 8.21$ and $\beta = 1.22$. The agreement between the Gumbel distributions and the simulation results is impressive, being equally good in real and redshift space and with and without mass errors.

So far we have assessed the chance of finding a hot cell within the full simulation volume which is many times larger than the volume of the 2dFGRS L_* volume limited sample. We have found that cells with as many massive haloes as found in the 2dFGRS are not uncommon in the current best fitting CDM models, particularly when the clustering is measured in redshift space and the effects of mass errors introduced by the group finding procedure are included. The volume of the L_* sample is equivalent to around 120 of the cubical cells of side $40.6 h^{-1}$ Mpc. Fig. 4 shows that in the L-BASICC-II ensemble, 21 out of 50 simulations contain hot cells with 10 or more clusters. Hence within one box, the probability of finding a hot cell is 42%. The chance of then finding a hot cell within a volume corresponding to that of the 2dFGRS L_* sample is 300 times smaller, 0.14%. In the case of the L-BASICC ensemble, all of the realisations contain at least one hot cell. The chance of finding such a cell within the 2dFGRS L_* sample volume is 0.3%. Hence, although we can find examples of superclusters similar to those seen in the 2dFGRS in our simulations, the chance of coming across such a structure in a volume equal to that covered by the real survey is less than 1%.

4 CONCLUSIONS

We have introduced a new objective methodology for assessing the likelihood of finding extreme structures in hierarchical structure formation models. Quite often the probability of finding an unusual structure such as a large void or an overdensity is estimated using a Gaussian distribution, because the smoothing scale in question is large. This is a good approximation for events which represent small departures from the mean density. However, for extreme events this is a poor assumption. The probability distribution of the density contrast on a particular smoothing scale, although assumed to be initially Gaussian in most models, rapidly evolves away from this form due to gravitational instability (e.g. Gaztañaga, Fosalba & Elizade 2000). Assuming a Gaussian probability distribution rather than the true distribution would lead to a misestimation of the probability of finding a cell with an extreme density of many orders of magnitude.

The advantage of our approach is that we do not need to specify the actual form of the probability distribution of cell counts. We have shown that the distribution of extreme cell counts is well described by a Gumbel distribution in a range of different situations: real-space, redshift-space and both with and without errors in halo mass. The simulations

give the mean and variance of the Gumbel distribution. The analytic form can then be extrapolated into the tails to give the probability of events which would require hundreds or thousands of realisations of N-body simulations to find. By using N-body simulations, we can assess the probability of events which cannot be calculated analytically, such as the largest Einstein ring expected in a CDM model (e.g. Oguri & Blandford 2009).

Cells with the number of massive haloes seen in the 2dFGRS can be found within our simulations, provided that the clustering of these haloes is measured in redshift-space and the mass errors introduced by the group finding algorithm are taken into account. However, if we consider a volume of the size of the 2dFGRS L_* sample, which is 300 times smaller than our simulation volume, then we expect to find such an overdensity of cluster mass haloes in fewer than one in a hundred cases.

Norberg et al. (2010) have carried out a related analysis using the final release of the SDSS. These authors applied a different technique to identify overdense regions. They split the galaxy distribution into zones, as would be done to carry out a jackknife estimation of the error on clustering statistics (Norberg et al. 2009). By comparing the distribution of two and three point correlation functions measured from the jackknife resamplings, a zone whose omission produced an outlying estimate of the clustering was found. However, when applying the same analysis to the same ensemble of N-body simulations used in this paper, Norberg et al. (2010) found that such outliers were quite common. There are some key differences between our analyses. The SDSS volume limited sample is an order of magnitude larger than the 2dFGRS sample considered in this paper. Norberg et al. found one “unusual” structure in the SDSS volume limited sample for L_* galaxies. Also, the method for quantifying unusual structures is different from the one we use, and will pick up a very different type of structure. The zones used by Norberg et al. sample conical volumes of space, covering a large baseline in radial distance. The superstructure in their case could be a projection of independent structures along the line of sight. In our case, we use compact cells. Norberg et al. (2010) conclude that in the larger volume of the SDSS L_* sample, the structures found by their correlation function study are consistent with those found in CDM. Our results do not contradict this; we have applied a different test to search for overdense regions with a different structure.

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