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Collisional interaction limits between dark matters and baryons in 'cooling flow' clusters

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ABSTRACT

Presuming weak collisional interactions to exchange the kinetic energy between dark matter and baryonic matter in a galaxy cluster, we re-examine the effectiveness of this process in several 'cooling flow' galaxy clusters using available X-ray observations and infer an upper limit on the heavy dark matter particle (DMP)−proton cross section $\sigma_{\rm xp}$. With a relative collisional velocity $V-$dependent power-law form of $\sigma_{\rm xp} = \sigma_0(V/10^3\ {\rm km\ s^{-1}})^a$ where $a \leq 0$, our inferred upper limit is $\sigma_0/m_{\rm x} \lesssim 2 \times 10^{-25}\ {\rm cm^2\ GeV^{-1}}$ with $m_{\rm x}$ being the DMP mass. Based on a simple stability analysis of the thermal energy balance equation, we argue that the mechanism of DMP−baryon collisional interactions is unlikely to be a stable nongravitational heating source of intracluster medium (ICM) in inner core regions of 'cooling flow' galaxy clusters.

Key words: cooling flows — cosmology: theory — dark matter — galaxy: clusters: general — radiation mechanisms: general — X−rays: galaxies: clusters

1 INTRODUCTION

Astrophysical and cosmological measurements together with numerical simulation experiments indicate that the cold dark matter (CDM) constitutes most of the matter in the Universe, even though the fundamental physical nature of such dark matter particles (DMPs) remains unknown. In the simplest scenario, these cold DMPs are presumed to be collisionless and they interact with each other or with other baryons only through the mutual gravity at the present epoch. While the collisionless CDM model is successful in explaining the formation of large-scale structures in the Universe, observational contradictions with numerical simulations appear inevitable regarding structures on sub-cluster scales, e.g., the prediction of a higher number of dwarf galaxies than that actually observed. Spergel & Steinhardt (2000) revitalized the concept of strongly interacting massive particles (SIMPs) to confront this issue and suggested a self-interacting cross section per unit DMP mass $\sigma_{\rm xx}/m_{\rm x} \sim 10^{-24}\ {\rm cm^2\ GeV^{-1}}$ where $\sigma_{\rm xx}$ is the collisional cross section among DMPs and $m_{\rm x}$ is the DMP mass. Along this line of reasoning, if DMPs are strongly self-interacting, then similar strong interactions would also be equally expected to exist between DMPs and baryons. Elastic scatterings between DMPs with nuclei could generate a recoil energy of the order of ~ 10 keV, which would then be detectable in underground and underwater particle experiments. Such 'direct detection' experiments currently limit the cross section to the order of $10^{-42} - 10^{-40}\ {\rm cm^2}$ for a cold DMP of mass in the range of $\sim 10-10^3$ GeV (e.g., Akerib et al. 2004). Limits on such collisional interactions in a wider mass range were investigated by various physical experiments and observations of several astrophysical processes, such as $\beta\beta$ decays, cosmic-ray detections, the galactic-halo stability, the cooling of molecular clouds, proton decay experiments, the existence of old neutron stars and the Earth (e.g., Starkman et al. 1990), satellite experiments in space (e.g., Wandelt et al. 2001), primordial nucleosynthesis and cosmic rays (e.g., Cyburt et al. 2002), and cosmic microwave background anisotropy and large-scale structure power spectrum (e.g., Chen et al. 2002). These experiments and observations have provided a limit on the mass-dependent cross section per unit DMP mass between DMPs and protons as $\sigma_{\rm xp}/m_{\rm x} \lesssim 10^{-26} - 10^{-24}\ {\rm cm^2\ GeV^{-1}}$ for a DMP mass $m_{\rm x} \gtrsim 1$ GeV.

Recent high-resolution observations of the hot X-ray emitting gaseous intracluster medium (ICM) by *Chandra*

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(e.g., Peterson et al. 2001) and *XMM-Newton* (e.g., Kaastra et al. 2001; Tamura et al. 2001) satellites have revealed deficits of cool gases (with gas temperatures much less than the virial temperature $T_{\rm vir}$) in the core of so-called 'cooling flow' galaxy clusters, inconsistent with predictions of the conventional radiative cooling models (e.g., Cowie & Binney 1977; Fabian & Nulsen 1977; Mathews & Bregman 1978; Stewart et al. 1984; Fabian 1994 and extensive references therein). Several heating mechanisms have been proposed to resolve this 'cooling flow dilemma', such as the inward thermal conduction from hot outer regions (e.g., Narayan & Medvedev 2001 and references therein), energy injections associated with central activities by an active galactic nucleus (e.g., Churazov et al. 2002), and outward or inward acoustic wave heating (e.g., Pringle 1989; Fabian et al. 2003a, b; Fujita et al. 2004; Feng, Zhang, Lou & Li 2004). The resonant excitation of internal gravity modes ($g-$modes) in the ICM by orbiting galaxies was explored by Balbus & Soker (1990); they examined the processes of excitation, propagation, amplification, damping of such galaxy cluster $g-$modes in the context of providing thermal energy in 'cooling flow' galaxy clusters.

Qin & Wu (2001) proposed collisional interactions between baryons and heavy DMPs ($m_{\rm x} \gg m_{\rm p}$ with $m_{\rm p}$ being the proton mass) as a major nongravitational heating mechanism for ICM in the core of a galaxy cluster. Assuming DMPs and baryons have comparable velocity dispersions, the kinetic energy of a single DMP would then be much larger than that of a baryon because of the presumed mass difference $m_{\rm x} \gg m_{\rm p}$. Therefore in numerous elastic collisions, kinetic energies of DMPs can be systematically transferred to the baryonic ICM to balance the radiative cooling in X-ray bands by hot electrons. By equating heating and cooling rates, they estimated a specific cross section of $\sigma_{\rm xp}/m_{\rm x} \sim 10^{-25}$ cm^2 GeV^{-1} for $m_{\rm x}/m_{\rm p} > 10^5$. This is actually a requirement of compensating the radiation cooling and should not be regarded as a limit of any sort. It should also be noted that they took strong lensing cluster CL0024+1654 to infer the DMP-baryon interaction cross section for balancing the cooling, which may not be proper for the following reason. The significant discrepancy by a factor of $3 \sim 4$ between the mass profiles derived from X-ray observations and gravitational lensing effects shows that the ICM may not be in a static equilibrium and might be still collapsing (e.g., Kneib et al. 2003; Zhang et al. 2005), and the bimodal velocity distribution of cluster galaxies indicates a merger of two systems with a mass ratio of 1 to 2 (e.g., Czoske et al. 2002). The main heating mechanism of CL0024+1654 could well be the gravitational collapse.

Chuzhoy & Nusser (2006) re-considered the ICM heating scenario of Qin & Wu (2001), corrected their calculations and derived a similar cross section for the heavy DMP-proton elastic collisional interaction. They found that, if $\sigma_{\rm xp}$ is independent of the relative velocity V of colliding particles, a thermal equilibrium state between heating by DMPs and radiative cooling by hot electrons of ICM would be always unstable. However, in galaxy clusters with $T > 2$ keV, a stable energetic balance may be achieved for a relative velocity $V-$dependent cross section $\sigma_{\rm xp} \propto V^a$ with $a \lesssim -3$.

There are two major simplifications in both analyses of Qin & Wu (2001) and Chuzhoy & Nusser (2006). First, they took the temperature and density of the ICM and the density of DMPs for typical values, rather than the relevant distributions determined by X-ray observations with high angular and spectral resolutions. Secondly, they estimated the velocity dispersion of DMPs either simply similar to that of baryons or by the results of numerical simulations, rather than a dynamically self-consistent 'true' value obtained by solving the Jeans equation (Binney & Tremaine 1987; Subramanian 2000; see also Ikebe et al. 2004 for the case of galaxy cluster A1795). Since the specific cross section limit they derived (at the centre or at the virial radius) is highly sensitive to the chosen parameters [see equation (8) of Qin & Wu (2001)], it is crucial to investigate this ICM heating mechanism more carefully using an actual sample of 'cooling flow' galaxy clusters, well observed in X-ray bands by the *XMM-Newton* and *Chandra* satellites in space.

This paper is structured as follows. In §2, we set the upper limits for DMP-proton elastic collisional cross section per unit DMP mass. In §3, we demonstrate that the DMP-proton collisional interaction alone is unlikely to be a stable ICM heating mechanism to compensate the radiative cooling of ICM. Discussion and conclusions are contained in §4. Details on the X-ray cooling function can be found in Appendix A.

In our theoretical model consideration, we have adopted the currently favoured standard ΛCDM cosmology with the cosmological parameters $h = 0.7$, $\Omega_{\rm m} = 0.3$ and $\Omega_\Lambda = 0.7$ in conventional notations.

2 DARK MATTER-BARYON COLLISIONAL INTERACTION CROSS SECTIONS

2.1 Model Description

The X-ray radiative cooling rate of the ICM can be inferred observationally and may be approximately represented by the cooling function Λ (i.e., radiative energy loss rate per unit volume) in terms of temperature T, baryon mass density $\rho_{\rm b}$ and abundances Z of the ICM as described in Appendix A. The radiative 'cooling time' $t_{\rm c}$ of a galaxy cluster is then defined by (e.g., Sarazin 1988)

$$t_{\rm c} \equiv \left| \left(\frac{{\rm d}\ln\varepsilon}{{\rm d}t} \right)^{-1} \right| = \frac{3\rho_{\rm b} k_{\rm B} T/(2\mu m_{\rm p})}{\Lambda} \ , \tag{1}$$

where ε is the ICM internal energy per unit volume, $k_{\rm B}$ is the Boltzmann constant, μ is the mean molecular weight of the ICM, and $m_{\rm p}$ is the proton mass. The cooling time t_c is a function of radius r in general. For 'cooling flow' clusters, the estimated central cooling time is shorter than their cosmic age, or the Hubble time $t_{\rm H} = H_0^{-1}$ with H_0 being the Hubble constant. The so-called 'cooling radius' $r_{\rm c}$ is defined as the radius such that $t_{\rm c}(r_{\rm c}) = t_{\rm H}$.

The thermal conduction in ICM across magnetic field may be negligible (e.g., Cowie & Binney 1977; Stewart et al. 1984; Sarazin 1988; Fabian 1994) due to small gyroradii given by $r_g = \gamma m v_\perp c/(Z_c e B)$ where $\gamma \equiv [1 - (v_\parallel^2 + v_\perp^2)/c^2]^{-1/2}$ is the relativistic Lorentz factor, m is the particle mass, $v_\parallel$ and $v_\perp$ are the particle velocity components parallel and perpendicular to the local magnetic field $\vec{B}$, $Z_c e$ is the particle electric charge, and c is the speed of light.[1]

[1] By numerical simulation analysis, Tao (1993) argued however

For thermal electrons and an average $|\vec{B}| \sim 1\mu G$ in a typical ICM, we estimate r_g to be a few thousand kilometers. The central regions of 'cooling flow' clusters will cool down substantially within a timescale of t_0 since their formation; in the absence of other effective heating mechanisms to compensate the radiative loss in X-ray bands, the hot ICM core would then collapse under self-gravity. Although the ICM behaviour in 'cooling flow' clusters is inhomogeneous on smaller scales due to thermal instabilities (e.g., Field 1965; Mathews & Bregman 1978; Malagoli et al. 1990), their main large-scale properties may be grossly modelled by smooth subsonic flows in a theoretical description (e.g., Fabian et al. 1984)[2]. Assuming a quasi-spherical symmetry and a quasi-steady state, the ICM evolution may be described by the following two equations, namely, the mass conservation of baryons

$$\dot{M} = 4\pi\rho_{\rm b} v r^2 \tag{2}$$

and the energy conservation of baryons in ICM (see Appendix B)

$$\rho_{\rm b} v \frac{\partial}{\partial r}\left(h_{\rm b} + \phi + \frac{v^2}{2}\right) = \Lambda - H\ , \tag{3}$$

where $\dot{M}$ is the baryon mass accretion rate, the ICM thermal pressure $p = \rho_{\rm b} k_{\rm B} T/(\mu m_{\rm p})$ follows the ideal gas law, ϕ is the total gravitational potential (including that of the dark matter halo), $v^2/2$ is the baryon kinetic energy per unit mass, and H represents the possible heating function (i.e., the heating rate per unit baryon volume). The specific baryon enthalpy $h_{\rm b}$ in energy conservation equation (3) is given by

$$h_{\rm b} \equiv \frac{\gamma p}{(\gamma - 1)\rho_{\rm b}} = \frac{5 k_{\rm B} T}{2\mu m_{\rm p}}\ , \tag{4}$$

where the ratio of specific heats γ is taken to be $\gamma = 5/3$ for the ICM. The estimated typical 'cooling flow' speed $v \sim 10$ km s^{-1} is much less than the ICM sound speed and thus $v^2/2$ is ignorable as compared to $h_{\rm b}$ in energy equation (3).

We could have added a magnetic term $\langle B_t^2\rangle/(4\pi\rho_{\rm b})$ within the parentheses on the left-hand side of energy equation (3) with $\langle B_t^2\rangle$ being the mean square of a completely random magnetic field $\vec{B}_t$ transverse to the radial direction (Yu & Lou 2005; Yu et al. 2006; Wang & Lou 2007; Lou & Wang 2007). For a diffuse ICM random magnetic field $\langle B_t^2\rangle^{1/2}$ of strength $\sim 10^{-6}$G away from the core region (e.g., Clarke et al. 2001; Carilli & Taylor 2002) and a typical proton number density of $n_p \sim 10^{-3}$ cm^{-3} (e.g., Sarazin 1988; Voit 2005), we estimate a typical Alfvén speed of ~ 100 km s^{-1} much less than the ICM sound speed. Such a magnetic field strength may give rise to an anisotropic distribution of electrons but may not be significant in the sense of bulk flow dynamics. By the equipartition argument, a random magnetic field $\langle B_t^2\rangle^{1/2}$ may reach strengths as strong as $\sim 10 - 30$ μG in the ICM core region of some galaxy clusters (e.g., Fabian 1994; Hu & Lou 2004). Depending on the actual proton number density in the range of $n_p \sim 10^{-2} - 10^{-3}$ cm^{-3}, magnetic pressure is less than (e.g., Dolag & Schindler 2000) or may be comparable to the thermal pressure in the core region for relaxed clusters of galaxies. For our main purpose of inferring the upper limit of DMP mass $m_{\rm x}$ based on data of galaxy clusters, it suffices to examine the purely hydrodynamic case.

Now we consider the possible ICM heating mechanism due to collisions between DMPs and baryons. As usual, we assume Maxwellian velocity distributions for both protons and DMPs with (one dimensional) velocity dispersions $v_{\rm p}$ and $v_{\rm x}$ respectively. For the case of V independent cross section, we have recalculated[3] the energy transfer rate per unit volume through elastic collisions as

$$H = 8\left[\frac{2}{\pi}\left(1 + \frac{v_{\rm p}^2}{v_{\rm x}^2}\right)\right]^{1/2} m_{\rm p} n_{\rm p} n_{\rm x} v_{\rm x}^3 \sigma_{\rm xp} \frac{1 - m_{\rm p} v_{\rm p}^2/(m_{\rm x} v_{\rm x}^2)}{(1 + m_{\rm p}/m_{\rm x})^2}\ , \tag{5}$$

where $n_{\rm p}$ and $n_{\rm x}$ are the number densities of protons and DMPs, respectively. For DMPs much heavier than protons (i.e., $m_{\rm x} \gg m_{\rm p}$) and as $v_{\rm x} \sim v_{\rm p}$ in a gravitationally virialized system, the last factor in the form of a division on the right-hand side of equation (5) approaches unity.

For the case of V dependent cross section $\sigma_{\rm xp}$, we take the case that the cross section has the form of a power law, namely $\sigma_{\rm xp} = \sigma_0 (V/V_0)^a$. To avoid complicated calculations and as an example of illustration, we only consider the case of heavy DMPs. Adopting expression (9) of Chuzhoy & Nusser (2006), we would then have

$$H \approx 6 \times 2^a m_{\rm p} n_{\rm p} n_{\rm x} v_{\rm x}^{3+a} V_0^{-a} \sigma_0 (1 + v_{\rm p}^2/v_{\rm x}^2)^{(1+a)/2} \tag{6}$$

as an approximate heating function.

In our model anaylsis, the scattering cross section $\sigma_{\rm xp}$ between a DMP and a proton is a free parameter to be constrained by several available X-ray observations of 'cooling flow' galaxy clusters. The scattering cross section between DMPs and helium nuclei $\sigma_{\rm xHe}$ is expected to be $4\sigma_{\rm xp}$ for incoherent scatterings, or $16\sigma_{\rm xp}$ for coherent scatterings, or 0 for the spin-dependent case (e.g., Chen et al. 2002). For simplicity, we shall take $\sigma_{\rm xHe} = 0$, noting that the other two alternatives would change the results only slightly.

The mass density and temperature distributions of the ICM can be inferred from X-ray observations independently. The one-dimensional velocity dispersion of ICM protons is $v_{\rm p} = (k_{\rm B} T/m_{\rm p})^{1/2}$. Under the approximations of quasi-spherical symmetry, quasi-hydrostatic equilibrium and ideal gas law, the total enclosed cluster mass distribution obeys the following condition

$$\frac{G M_r}{r^2} = -\frac{k_{\rm B} T}{\mu m_{\rm p}}\left(\frac{{\rm d}\ln T}{{\rm d}r} + \frac{{\rm d}\ln \rho_{\rm b}}{{\rm d}r}\right)\ , \tag{7}$$

where M_r is the total enclosed cluster mass (dark matter and baryonic matter together) inside radius r, $G = 6.67 \times 10^{-8}$ g^{-1} cm^3 s^{-2} is the gravitational constant. The mass density of DMPs can then be inferred from

$$\rho_{\rm x}(r) = \frac{1}{4\pi r^2}\frac{{\rm d}M_r}{{\rm d}r} - \rho_{\rm b}(r)\ . \tag{8}$$

that 'tangled' magnetic field may not be effective enough to suppress the thermal conduction in haloes of galaxy clusters.

[2] Chuzhoy & Nusser (2006) adopted different equations to describe the ICM behaviour. In particular, they ignored ICM flows as well as the effect of significant gravitational heating. In these two aspects, our model appears more general and realistic.

[3] We have verified that the constant coefficient in equation (4) of Qin & Wu (2001) was in error.

In most model calculations for clusters of galaxies, the DMP mass density distribution is fitted with the universal NFW mass profile obtained by numerical simulations (e.g., Navarro, Frenk & White 1995, 1996, 1997), namely

$$\rho_{\rm x}(r) = \rho_{\rm x0}(r/r_{\rm s})^{-1}(1+r/r_{\rm s})^{-2}\ , \tag{9}$$

where $r_{\rm s}$ is a radial scale and $\rho_{\rm x0}$ is a DMP mass density scale when $r \cong 0.48\, r_{\rm s}$. For $r \ll r_s$, DMP mass density $\rho_{\rm x}(r)$ scales as r^{-1}, while for $r \gg r_s$, $\rho_{\rm x}(r)$ scales as r^{-3}. There are also other possible dark matter mass density profiles $\rho_{\rm x}(r) \propto r^{-l}(r+r_s)^{l-q}$ with $1 \lesssim l \lesssim 1.5$ and $2.5 \lesssim q \lesssim 3$ for different combinations of inner and outer radial scalings of DMP mass density (e.g., Moore et al. 1998, 1999; Jing 2000; Rasia et al. 2004; Voit 2005).

In a spherical hydrostatic equilibrium with no mean streaming motions such that $\bar{v}_r = \bar{v}_\theta = \bar{v}_\phi = 0$, the velocity dispersion of DMPs in a galaxy cluster and the mass distribution is related by the Jeans equation (e.g., Binney & Tremaine 1987; Subramanian 2000)

$$\frac{\rm d}{{\rm d}r}(\rho_{\rm x} v_r^2) + \frac{2\beta_a}{r}\rho_{\rm x} v_r^2 + \rho_{\rm x}\frac{GM_r}{r^2} = 0\ . \tag{10}$$

Here, the velocity anisotropy parameter β_a is defined as

$$\beta_a = 1 - v_t^2(r)/[2v_r^2(r)]\ , \tag{11}$$

where $v_t^2 \equiv v_\theta^2 + v_\phi^2$ and v_r, v_θ, v_ϕ are the radial and angular velocity dispersions with respect to the mean velocity which is almost zero in a quasi-hydrostatic equilibrium. The formal solution of Jeans equation (10) with a constant β_a is

$$v_r^2(r) = \frac{r^{-2\beta_a}}{\rho_{\rm x}(r)}\int_r^\infty x^{2\beta_a-2}\rho_{\rm x}(x) G M_x {\rm d}x\ . \tag{12}$$

Numerical simulations indicate a variation of the anisotropy parameter β_a between $\beta_a = 0$ at the halo centre and $\beta_a = 0.6$ at the virial radius (e.g., Colín et al. 2000). For β_a as a function of r, Jeans equation (10) can be integrated numerically with additional approximations (e.g., Subramanian 2000). As the form of β_a cannot be determined directly by observations, we simply set $\beta_a = 0$ for an isotropic velocity dispersion in our model analysis of galaxy clusters. In fact, our calculations indicate that a nonzero β_a only affects v_r^2 at a level of $\lesssim 10\%$.

2.2 The upper limit of elastic collisional cross section constrained by X-ray observations

Observations have shown that the ICM temperature is comparable to the equivalent temperature of cluster galaxies (e.g., Jones & Forman 1984), implying that the ICM has not changed its temperature very much from the initial hydrostatic equilibrium state since the epoch of cluster formation. We thus consider a more general problem of ICM heating and cooling [see equations (2) and (3)] in a quasi-hydrostatic manner.

Integrating equation (3) from the centre $r = 0$ to the cooling radius $r_{\rm c}$ and using equation (2), we derive

$$\left(h + \phi + \frac{v^2}{2}\right)\Bigg|_0^{r_{\rm c}} = \int_0^{r_{\rm c}} \frac{\Lambda - H}{\dot{M}/(4\pi r^2)}{\rm d}r\ . \tag{13}$$

For 'cooling flow' galaxy clusters, we have $\phi_0 < \phi(r_{\rm c})$ and $T_0 \lesssim T(r_{\rm c})$ according to satellite X-ray observations. Near the cluster centre, $v \sim 10$ km s^{-1} is very much lower than the sound speed there. If we take either a quasi-hydrostatic or a quasi-magnetostatic solution (Lou & Wang 2006, 2007) for cluster evolution just before a complete virialization or equilibrium, then $v \sim 0$ at $r = 0$ and thus $0 < v^2(r_{\rm c})/2$. By the above consideration and estimates of three terms at both $r = r_{\rm c}$ and $r = 0$ separately, we show the left-hand side (LHS) of energy equation (13) is positive.[4] In fact, this requirement may be relaxed a bit. In fact, as long as the LHS of energy equation (13) is non-negative and irrespective of the relative magnitudes of each individual terms at both $r = r_{\rm c}$ and $r = 0$, we have for the baryonic ICM the following inequality for the net heating rate

$$\int (\Lambda - H) r^2 {\rm d}r > 0\ , \tag{14}$$

and the collisional cross section is limited from above by

$$\sigma_{\rm xp} < \int_0^{r_{\rm c}} \Lambda r^2 {\rm d}r \Bigg/ \int_0^{r_{\rm c}} H_1 r^2 {\rm d}r\ , \tag{15}$$

where $H_1 \equiv H/\sigma_{\rm xp}$ according to expression (5), or by

$$\sigma_0 V_0^{-a} < \int_0^{r_{\rm c}} \Lambda r^2 {\rm d}r \Bigg/ \int_0^{r_{\rm c}} H_2 r^2 {\rm d}r\ , \tag{16}$$

where $H_2 \equiv H V_0^a/\sigma_0$ according to expression (6). By the above analysis and reasoning for the roles of flow kinetic energy and magnetic energy and in reference to inequalities (14)$-$(16), our model calculations and X-ray data comparisons can be much simplified within a purely quasi-hydrostatic framework and that is what we do in the following.

We calculate the DMP-proton elastic collisional cross section limit using X-ray observations of a sample of five presumably relaxed galaxy clusters with apparent 'cooling' cores, namely, Abell 478 (e.g., Pointecouteau et al. 2004), Abell 1795 (e.g., Ikebe et al. 2004), Abell 1983 (e.g., Pratt & Arnaud 2003), Abell 1991 (e.g., Pratt & Arnaud 2005), and PKS 0745-191 (e.g., Chen et al. 2003; Pointecouteau et al. 2005). Physical parameters estimated for this galaxy cluster sample are summarized in Table 1. This sample of galaxy clusters covers wide ranges of redshift z $(0-0.1)$, cluster virial mass M_{200} ($[1-11]\times 10^{14}$ M$_\odot$), and ICM temperature T $(2-7$ keV), respectively. These chosen galaxy clusters are analysed with the deprojection technique based on the X-ray data taken from the *XMM-Newton* EPIC to reveal the real spectra of gaseous ICM in different spherical shells and to determine the deprojected temperature and the mass distribution of the gas in a galaxy cluster. The *XMM-Newton* EPIC is the most sensitive X-ray space telescope which also has high spatial and spectral resolutions, and thus meets all the requirements for a detailed spectral analysis. The statistical study on the comparison of cluster mass measurements using strong gravitational lensing and using X-ray observations shows an excellent mutual agreement for 'cooling flow' galaxy clusters, which are perceived as dynamically more relaxed systems and the quasi-spherical hydrostatic assumption used in X-ray mass determinations should be valid (Allen 1998; Wu 2000). On this ground, the dark matter mass density distribution used in this paper is

[4] Note that in reference to a large radius, $h+\phi+v^2/2$ should be negative for a gravitationally bound system, while the difference $(h+\phi+v^2/2)_{r_{\rm c}} - (h+\phi+v^2/2)_0$ can be positive as shown.

Table 1. Estimated physical parameters for a sample of five 'cooling flow' clusters of galaxies

Cluster	z	T (keV)	c_{200}	R_{200} (kpc)	M_{200} ($10^{14}\mathrm{M}_\odot$)	r_c(kpc)	$\dot{M}$ ($\mathrm{M}_\odot$ yr^{-1})^a
A478	0.0881	6.8	4.22 ± 0.39	2060 ± 110	10.8 ± 1.8	171^{+113}_{-115}	736^{+114}_{-434}
A1795	0.0616	5.7	4.47 ± 0.27	1760 ± 30	6.54 ± 0.35	129^{+80}_{-87}	321^{+168}_{-213}
A1983	0.0442	2.3	3.83 ± 0.71	1100 ± 140	1.59 ± 0.61	34^{+59}_{-34}	$6.0^{+10.8}_{-6.0}$
A1991	0.0586	2.6	5.78 ± 0.35	1106 ± 41	1.63 ± 0.18	52^{+67}_{-12}	37^{+36}_{-11}
PKS0745	0.1028	7.0	7.05 ± 0.28	1880 ± 130	8.34 ± 0.84	126^{+22}_{-31}	579^{+399}_{-215}

^{a} The dark matter mass deposition rates from 'cooling flows' (column 8) are taken from White et al. (1997). While their model calculations used cosmological parameters of $h = 0.5$ and $q_0 = 0.5$, the corresponding parameter values here are all comparable to theirs.

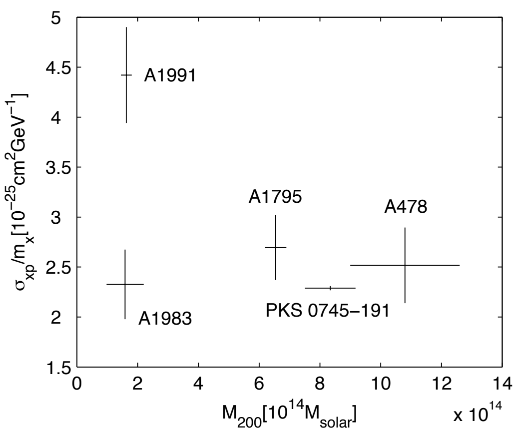

Figure 1. On the basis of data analysis and for the case of expression (5), the upper limits of specific cross section $\sigma_{\mathrm{xp}}/m_{\mathrm{x}}$ for the five selected 'cooling flow' galaxy clusters (namely, PKS 0745-191, Abell 478, Abell 1795, Abell 1991, and Abell 1983) are shown here. The value range of the estimated upper limit for $\sigma_{\mathrm{xp}}/m_{\mathrm{x}}$ is mainly due to uncertainties of the cooling radius r_c and the virial mass.

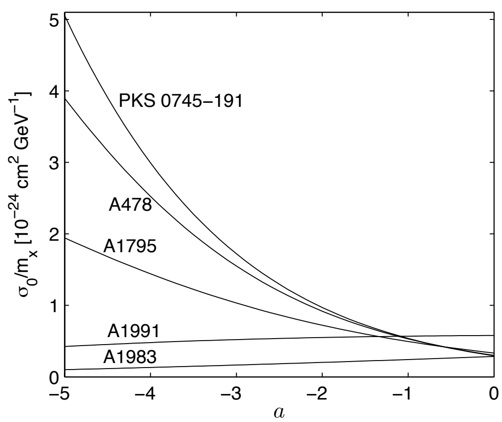

Figure 2. For the relative velocity V−dependent case of expression (6), the upper limits of cross section σ_0/m_{x} as functions of exponent a for the five selected 'cooling flow' galaxy clusters (namely, PKS 0745-191, Abell 478, Abell 1795, Abell 1991, and Abell 1983) are shown here by five separate solid curves with corresponding galaxy cluster names labelled explicitly.

regarded as reliable, especially for the lensing galaxy cluster PKS 0745-191 (e.g., Allen 1996).

For the velocity independent case as given by expression (5), the upper limits of the specific cross section $\sigma_{\mathrm{xp}}/m_{\mathrm{x}}$ for DMP-proton collisions are displayed in Figure 1. The limits of $\sigma_{\mathrm{xp}}/m_{\mathrm{x}}$ for the five selected sample galaxy clusters lie in the range of $[2.0,\ 4.9]\times 10^{-25}$ cm^2 GeV^{-1} and a common consistent value would be taken as $\sigma_{\mathrm{xp}}/m_{\mathrm{x}} \lesssim 2 \times 10^{-25}$ cm^2 GeV^{-1}. For the velocity dependent case as given by expression (6), the value of σ_0/m_{x} is displayed in Figure 2. Here, we take the mean value of cooling radius r_c of every cluster for simplicity. The upper limit of DMP specific cross section is $\sigma_{\mathrm{xp}}/m_{\mathrm{x}} < 2\times 10^{-25}(V/10^3\ \mathrm{km\ s^{-1}})^a$ cm^2 GeV^{-1}.

Note that $\sigma_{\mathrm{xp}}/m_{\mathrm{x}}$ is constant only for $m_{\mathrm{x}} \gg m_{\mathrm{p}}$ in expressions (5) and (6). Our estimated upper limit applies well to $m_{\mathrm{x}} > 10^3$ GeV. The cross-section limit for the lighter DMPs ($m_{\mathrm{x}} = 10 - 10^3$ GeV) has been strictly constrained by direct detection experiments (e.g., Wandelt et al. 2001).

3 DARK MATTER–PROTON COLLISIONS AS THE ICM HEATING SOURCE IN 'COOLING FLOW' CLUSTERS OF GALAXIES

The energy equilibrium equation of the ICM can be written as

$$\mathrm{d}\ln S/\mathrm{d}t = \mathrm{d}\ln\varepsilon/\mathrm{d}t = (H - \Lambda)/\varepsilon\ , \qquad (17)$$

where $S = k_{\mathrm{B}}T/(\mu m_{\mathrm{p}}\rho_{\mathrm{b}}^{2/3})$ is the ICM entropy (e.g., Voit 2005). We note that the ratio $v_{\mathrm{p}}/v_{\mathrm{x}}$ in two expressions (5) and (6) is almost a constant in dynamically and thermally well relaxed galaxy clusters (Hu & Lou 2007). Hence for a specific 'cooling flow' cluster, $H \propto \rho_{\mathrm{b}}T^0$ and $\Lambda \propto \rho_{\mathrm{b}}^2 T^b$, and equation (17) becomes

$$\mathrm{d}\ln(T/\rho_{\mathrm{b}}^{2/3})/\mathrm{d}t = AT^{-1} - B\rho_{\mathrm{b}}T^{b-1}\ , \qquad (18)$$

where A and B are two coefficients independent of ρ_{b} and T. A systematic comparison between the ICM temperatures T and velocity dispersion of DMP base on the X-ray observations indicates that T could only slightly change ($\Delta T \lesssim 1$ keV) in the cosmic age of clusters (Hu & Lou 2007), which is consistent with the numerical simulations of the cluster evolution (e.g., Bryan & Norman 1998). If the RHS of equation (18) maintains zero, i.e. cooling is fully compensated by heating, we have $\mathrm{d}\log\rho_\mathrm{b} = -b\mathrm{d}\log T$, where $|b| \lesssim 1$ for $T \sim 10^7 - 10^8$ K. Therefore ρ_b will be roughly a constant in such a case.

Now we fix the value of T and perform an isothermal stability analysis on the solution ρ_b of equation (18). Suppose the DMP-proton elastic collisional heating can compensate radiative cooling for certain values $T = T_0$ and $\rho_\mathrm{b} = \rho_0$, let us consider the stability of ρ_b in equation (17) at a constant temperature. A small baryon density variation $\delta\rho_\mathrm{b}$ obeys

$$\frac{2}{3\rho_\mathrm{b}}\frac{\mathrm{d}\delta\rho_\mathrm{b}}{\mathrm{d}t} = BT^{b-1}\delta\rho_\mathrm{b}\ . \tag{19}$$

A simple estimation indicates that $\delta\rho_\mathrm{b}$ will double in a timescale of

$$2^{-a}\times 10^9\mathrm{yr}\left(\frac{v_\mathrm{x}}{10^3\ \mathrm{km\ s^{-1}}}\right)^{-1-a} \times\left(\frac{\rho_\mathrm{x}}{0.02\ \mathrm{M_\odot\ pc^{-3}}}\frac{\sigma_0/m_\mathrm{x}}{10^{-25}\ \mathrm{cm^2\ GeV^{-1}}}\right)^{-1}\ . \tag{20}$$

Therefore, without other heating sources, the solution of equation (17) appears unstable for whatever values of a, at least in the case of a fixed ICM temperature profile.

4 DISCUSSION AND CONCLUSIONS

Based on two different elastic collision models and X-ray observations of galaxy clusters, we have re-examined effects of collisional interactions between heavy DMPs and protons in 'cooling flow' clusters of galaxies and estimated a more reliable upper limit for the specific cross section between DMPs and protons by solving the equation of energy conservation for five "cooling flow" galaxy clusters using available X-ray data. In the regime of $m_\mathrm{x} \gg m_\mathrm{p}$, the upper limit for $\sigma_\mathrm{xp} = \sigma_0(V/10^3\ \mathrm{km\ s^{-1}})^a$ with $a \le 0$ can be expressed as $\sigma_0/m_\mathrm{x} \lesssim 2\times 10^{-25}\ \mathrm{cm^2\ GeV^{-1}}$, which is fully consistent with the earlier results. Similar to other astrophysical constraints, this upper limit is independent of the underlying model for particle physics governing collisional interactions.

In our model calculations, we have assumed that the total gravitational potential ϕ remains invariable during the process of galaxy cluster evolution. The gravitational potential ϕ of a galaxy cluster is dominantly determined by the amount of DMPs. Besides the weak energy-momentum transfer between the DMPs and baryons, the density distribution of dark matter is only affected by the mild accumulation of cool gas in the central region of clusters. The exact solution of such complex co-evolution of DMPs and baryons should be treated with Fokker-Planck equation (e.g., Binney & Tremaine 1987). We will pursue the relevant problems in contexts of galaxy clusters in separate papers.

We have also demonstrated an intrinsic instability in explaining the problem of 'cooling flow' galaxy clusters via the heat transfer by heavy DMP-proton collisional scattering. According to expression (5), the DMP-proton collisional heating of ICM works only when $m_\mathrm{p}v_\mathrm{p}^2 < m_\mathrm{x}v_\mathrm{x}^2$. In 'cooling flow' galaxy clusters, $v_\mathrm{p}^2/v_\mathrm{x}^2 \sim \mu \simeq 0.6$ (Hu & Lou 2007) and thus $m_\mathrm{x} > 0.6$ GeV is required in this model. However, as the collisional cross section in the DMP mass range 0.5 GeV$< m_\mathrm{x} < 10^5$ GeV is less than 5×10^{-28} cm^2 (e.g., Wandelt et al. 2001), the light DMP collisional heating mechanism can then be completely ignored. Therefore, we rule out the possibility of DMP collisional scattering as a major stable non-gravitational heating sources of the ICM in 'cooling flow' galaxy clusters.

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APPENDIX A: X-RAY COOLING FUNCTION

The radiative energy loss from a hot plasma per unit volume and per unit time is a cooling function of temperature, density, and chemical abundances of various elements. The cooling functions for a hot plasma under equilibrium conditions have been calculated over a photon energy range of $0.001 - 30$ keV and a range of abundances for $0 - 1.0\ Z_\odot$ by Sutherland & Dopita (1993). They defined the normalized cooling function by $\Lambda_0 \equiv \Lambda/(n_{\rm e} n_{\rm i})$, where $n_{\rm e}$ and $n_{\rm i}$ are the electron and ion number densities, respectively. Tozzi & Norman (2001) used an approximate analytic expression to fit Λ_0 in the following polynomial form

$$\Lambda_0 = [C_1(k_{\rm B}T)^{-1.7} + C_2(k_{\rm B}T)^{0.5} + C_3] \times 10^{-22}\ {\rm erg\ cm^3\ s^{-1}}\ , \tag{A1}$$

where $k_{\rm B}T$ is in unit of keV. The three constant coefficients C_i $(i = 1, 2, 3)$ and $n_{\rm e}/n_{\rm H}$, $n_{\rm i}/n_{\rm H}$, μ depend on the metallicity Z. We have refitted these parameters and summarized the results in Table A1 for the convenience of reference. For an arbitrary metallicity Z, we take the parameters by the following polynomial interpolations:

Table A1. Parameters for the Normalized Cooling Function Λ_0

Z $(Z_\odot)$	C_1	C_2	C_3	$n_{\rm e}/n_{\rm H}$	$n_{\rm i}/n_{\rm H}$	μ
0	−0.003	0.0605	0.0204	1.128	1.064	0.58
0.1	0.0193	0.0632	0.0218	1.131	1.064	0.58
0.32	0.480	0.0658	0.0306	1.165	1.080	0.60
1.0	0.1434	0.0762	0.0355	1.209	1.099	0.62

$$C_1 = -0.003 + 0.26Z - 0.41Z^2 + 0.29Z^3\ , \tag{A2}$$
$$C_2 = 0.0605 + 0.034Z - 0.069Z^2 + 0.052Z^3\ , \tag{A3}$$
$$C_3 = 0.019 + 0.041Z - 0.025Z^2\ , \tag{A4}$$
$$n_{\rm e}/n_{\rm H} = 1.12 + 0.14Z - 0.05Z^2\ , \tag{A5}$$
$$n_{\rm i}/n_{\rm H} = 1.06 + 0.06Z - 0.02Z^2\ , \tag{A6}$$
$$\mu = 0.575 + 0.074Z - 0.030Z^2\ . \tag{A7}$$

Expression (A1) with parameters defined by equations (A2)−(A7) can grossly reproduce the X-ray cooling function of Sutherland & Dopita (1993) to within a few percent in the typical ICM thermal energy range of 1.0 keV $\lesssim k_{\rm B}T \lesssim$ 10 keV.

APPENDIX B: DERIVATION OF ENERGY CONSERVATION FOR ICM BARYONS

The energy conservation equation for ICM baryons (equation 3) is derived in this Appendix B. We start with the thermodynamic relation

$$dh_{\rm b} = \rho_{\rm b}^{-1}dp + dQ = \rho_{\rm b}^{-1}dp + \rho_{\rm b}^{-1}(H - \Lambda)dt\ , \tag{B1}$$

where $h_{\rm b}$ is the enthalpy per unit baryon mass, Q is the heating rate per unit baryon mass, p is the ICM (baryon) gas pressure, and Λ and H are cooling and heating rates per unit volume, respectively. Differentiating along the moving direction of a bulk baryon flow, equation (B1) can be written as

$$\mathbf{v} \cdot (\rho_{\rm b}^{-1}\nabla p) = \mathbf{v} \cdot \nabla h_{\rm b} + \rho_{\rm b}^{-1}(\Lambda - H)\ . \tag{B2}$$

Using the Euler equation (e.g., Landau & Lifshitz 1959)

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\rho_{\rm b}^{-1}\nabla p - \nabla\phi\ , \tag{B3}$$

where ϕ is the total gravitational potential (including that of a dark matter halo), and a vector identity

$$(\mathbf{v} \cdot \nabla)\mathbf{v} = \nabla\left(\frac{v^2}{2}\right) - \mathbf{v} \times (\nabla \times \mathbf{v})\ , \tag{B4}$$

equation (B2) appears as

$$\mathbf{v} \cdot \frac{\partial \mathbf{v}}{\partial t} = -\mathbf{v} \cdot \nabla\left(\frac{v^2}{2} + \phi + h_{\rm b}\right) + \frac{(H - \Lambda)}{\rho_{\rm b}}\ . \tag{B5}$$

Here we assume that the moving direction of a cooling flow is radially inward (i.e., negative), such that

$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial r}\left(\frac{v^2}{2} + \phi + h_{\rm b}\right) + \frac{(H-\Lambda)}{\rho_{\rm b} v} \ . \qquad \text{(B6)}$$

We now estimate and compare the magnitudes of terms in equation (B6). For example,

$$\frac{\partial v}{\partial t} \sim \frac{v}{t_{\rm H}} \sim \frac{10\ {\rm km\ s^{-1}}}{10^{10}\ {\rm yr}} \sim 3\times 10^{-14}{\rm m\ s^{-2}}\ , \qquad \text{(B7)}$$

$$\frac{\partial \phi}{\partial r} = \frac{GM_r}{r^2} \gtrsim \frac{GM_r}{r^2}\Big|_{r=R_{200}} \sim \frac{10^{14}\ {\rm M_\odot G}}{(1\ {\rm Mpc})^2} \sim 10^{-11}{\rm m\ s^{-2}}\ , \qquad \text{(B8)}$$

$$\frac{\partial}{\partial r}\left(\frac{v^2}{2}\right) \sim \frac{v^2}{2r} \sim \frac{(10{\rm km/s})^2}{1{\rm Mpc}} \sim 3\times 10^{-15}{\rm m\ s^{-2}}\ , \qquad \text{(B9)}$$

$$\frac{\partial h_b}{\partial r} \sim \frac{h_b}{r} \sim \frac{5\ (1{\rm keV})}{2\mu m_p\ 1{\rm Mpc}} \sim 10^{-11}{\rm m\ s^{-2}}\ , \qquad \text{(B10)}$$

where GM_r/r^2 is a decreasing function with increasing r for NFW mass profiles obtained through numerical simulations. The value of $(H-\Lambda)/(\rho_b v)$ term is to be determined, yet it should be much larger than terms $\partial(v^2/2)/\partial r$ and $\partial v/\partial t$ in equation (B6). Therefore the LHS and $\partial(v^2/2)/\partial r$ term on the RHS of equation (B6) may be neglected and we finally derive equation (3) as

$$\rho_{\rm b} v \frac{\partial}{\partial r}\left(\frac{v^2}{2} + \phi + h_{\rm b}\right) = \Lambda - H\ . \qquad \text{(B11)}$$