An ideal mass assignment scheme for measuring the Power Spectrum with FFTs

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ABSTRACT

In measuring the power spectrum of the distribution of large numbers of dark matter particles in simulations, or galaxies in observations, one has to use Fast Fourier Transforms (FFT) for calculational efficiency. However, because of the required mass assignment onto grid points in this method, the measured power spectrum $\langle|\delta^f(k)|^2\rangle$ obtained with an FFT is not the true power spectrum $P(k)$ but instead one that is convolved with a window function $|W(\mathbf{k})|^2$ in Fourier space. In a recent paper, Jing (2005) proposed an elegant algorithm to deconvolve the sampling effects of the window function and to extract the true power spectrum, and tests using N-body simulations show that this algorithm works very well for the three most commonly used mass assignment functions, i.e., the Nearest Grid Point (NGP), the Cloud In Cell (CIC) and the Triangular Shaped Cloud (TSC) methods. In this paper, rather than trying to deconvolve the sampling effects of the window function, we propose to select a particular function in performing the mass assignment that can minimize these effects. An ideal window function should fulfill the following criteria: (i) compact top-hat like support in Fourier space to minimize the sampling effects; (ii) compact support in real space to allow a fast and computationally feasible mass assignment onto grids. We find that the scale functions of Daubechies wavelet transformations are good candidates for such a purpose. Our tests using data from the Millennium Simulation show that the true power spectrum of dark matter can be accurately measured at a level better than 2% up to $k = 0.7k_N$, without applying any deconvolution processes. The new scheme is especially valuable for measurements of higher order statistics, e.g. the bi-spectrum, where it can render the mass assignment effects negligible up to comparatively high k.

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1. INTRODUCTION

In studies of the cosmic large-scale structure, a number of different statistical methods are routinely used to extract various information of interest (e.g., regarding the cosmology, the initial perturbation, etc) that is embedded in the distribution of the dark matter particles (in the case of simulations) or the galaxies (in observations). The power spectrum $P(k)$ is one of the most powerful and basic statistical measures that describes the distribution of mass and light in the Universe, and one of the most throughly investigated quantities in modelling the structure formation process. The initial primordial power spectrum of the mass fluctuations is usually assumed to follow a power law, $P_0(k) = Ak^n$. The linearly processed power spectrum $P_{\rm lin}(k)$ can be well predicted by codes such as CMBFAST (Seljak & Zaldarriaga 1996), or approximated by various fitting formula (e.g. Bardeen et al. 1986; Efstathiou, Bond & White 1992; Eisenstein & Hu 1998) for different matter and energy content. Using N-body simulations, the non-linear power spectrum $P_{\rm NL}(k)$ has been modelled by various authors (e.g. Peacock & Dodds 1996; Ma & Fry 2000; Smith et al. 2003). Apart from these theoretical models, direct measurement of the power spectrum from observations plays an extremely important role both in cosmology and galaxy formation theories. Although there are different biases relative to the mass power spectrum, one can roughly say that $P(k)$ on very large scales measures the primordial density fluctuations, which is closely connected with the cosmology models (e.g., Spergel et al. 2007), while $P(k)$ on small scales characterizes the later non-linear evolution (e.g., Peacock & Dodds 1996).

As an essential statistical measure for the distribution of galaxies, the power spectrum $P(k)$ has been estimated and modeled from most of the redshift surveys. Recent investigations along this direction include the CfA and Perseus-Pisces redshift surveys (Baumgart & Fry 1991), the radio galaxy survey (Peacock & Nicholson 1991), the IRAS QDOT survey (Kaiser 1991), the 2Jy IRAS survey (Jing & Valdarnini 1993), the 1.2Jy IRAS survey (Fisher et al. 1993), the Las Campanas Redshift Survey (Lin et al. 1996; Yang et al. 2001), the 2dF Galaxy Redshift Survey (Percival et al., 2001; Tegmark, Hamilton & Xu 2002; Sánchez et al. 2006) and the Sloan Digital Sky Survey (Tegmark et al. 2004; Percival et al., 2007). Among these works, the galaxy power spectra are measured either using the Fast Fourier Transform (FFT) technique or direct summation, or other advanced techniques (e.g., Yang et al. 2001; Tegmark et al. 2004).

Apart from these observational probes, the power spectrum is also widely measured from N-body simulations (e.g. Davis et al. 1985). For these measurements, one has to use FFTs since there are too many particles in the simulations to apply direct summation. Before performing the FFT, one therefore needs to assign the particle distribution $\rho(\mathbf{r})$ onto grids $\rho(\mathbf{r}_g)$ (usually onto 2^{3i} grid cells, where i is an integer). As pointed out in a recent paper by Jing (2005), such an assignment process is equivalent to a convolution of the real density field with a given assignment window function $W(\mathbf{r})$, and sampling the convolved density field at the 2^{3i} grid points. Thus the power spectrum based on the FFT of $\rho(\mathbf{r}_g)$ is not equal to that based on the Fourier transform (FT) of $\rho(\mathbf{r})$. In order to obtain the true power spectrum to an accuracy of a few percent, the sampling effects should be carefully corrected (Jing 2005; and references therein).

To this end, Jing (2005) proposed an elegant algorithm to iteratively deconvolve the power spectrum measurement for the impact of the mass assignment and to extract the true power spectrum. Tests using N-body simulations show that their algorithm works extremely well for the three commonly used mass assignment functions, i.e., the Nearest Grid Point (NGP), the Cloud In Cell (CIC) and the Triangular Shaped Cloud (TSC) methods.

In this paper, rather than trying to correct for the influence of the window function, we seek to minimize the effects of the mass assignment by selecting special window functions. An ideal window function should fulfill the following criteria: (i) compact top-hat like support in Fourier space to avoid the sampling effects; (ii) compact support in real space to allow computationally efficient mass assignment onto grids. We find that the scale functions of the Daubechies wavelet transformations are good candidates for simultaneously matching both requirements. In fact, as we will demonstrate they allow an accurate measurement of the power spectrum with FFTs without the need for a deconvolution procedure. This is of great help especially for accurate measurements of higher order spectra, like the bi-spectrum, where FFTs are needed but the de-aliasing methods are not available yet. We will discuss this application to accurate measurements and modelling of the bi-spectrum in a subsequent paper.

This paper is organized as follows. In Section 2 we give a brief description of the methodology for measuring the power spectrum from the discrete distribution of dark matter particles. In Section 3 we first present the commonly used window functions in both real and Fourier spaces, and then and introduce our new mass assignment scheme. In Section 4 we compare the power spectra extracted from the Millennium Run using different methods. Finally, we summarize our results in Section 5.

2. Measuring the power spectrum

In this section we outline the methods used to measure the power spectrum from the distribution of dark matter particles (Peebles 1980). Unless stated otherwise we shall follow Jing (2005), and we refer readers who are interested in a more detailed and complete set of formulae to this paper and references therein. We start from the definition of the power spectrum. Let $\rho(\mathbf{r})$ be the cosmic density field and $\overline{\rho}$ the mean density. Then the density contrast $\delta(\mathbf{r})$ can be expressed as,

$$\delta(\mathbf{r}) = \frac{\rho(\mathbf{r}) - \overline{\rho}}{\overline{\rho}}. \tag{1}$$

Based on the cosmological principle, we assume that $\rho(\mathbf{r})$ in a very large volume V_μ fairly represents the overall cosmic density field, and that it can be taken to be periodic. The FT of $\delta(\mathbf{r})$ can be defined as:

$$\delta(\mathbf{k}) = \frac{1}{V_\mu} \int_{V_\mu} \delta(\mathbf{r}) e^{i\mathbf{r}\cdot\mathbf{k}} \mathrm{d}\mathbf{r} \,. \tag{2}$$

And by definition, its power spectrum $P(k)$ is simply related to $\delta(\mathbf{k})$ as

$$P(k) \equiv \langle \mid \delta(\mathbf{k}) \mid^2 \rangle \,, \tag{3}$$

where $\langle \cdots \rangle$ means the ensemble average.

However, in practice, the cosmic density field is usually traced by the distribution of galaxies or dark matter particles. In these cases, the density field $\rho(\mathbf{r})$ is replaced by the number density distribution of objects $n(\mathbf{r}) = \sum_j \delta^D(\mathbf{r} - \mathbf{r}_j)$, where $\mathbf{r}_j$ is the coordinate of object j and $\delta^D(\mathbf{r})$ is the Dirac-δ function. And the FT of the related number density contrast $\delta(\mathbf{r})$ can be expressed as,

$$\delta^d(\mathbf{k}) = \frac{1}{V_\mu \overline{n}} \int_{V_\mu} n(\mathbf{r}) e^{i\mathbf{r}\cdot\mathbf{k}} \mathrm{d}\mathbf{r} - \delta^K_{\mathbf{k},\mathbf{0}} \,, \tag{4}$$

where $\overline{n}$ is the mean number density, the superscript d represents the *discrete* case of $\rho(\mathbf{r})$, and δ^K is the Kronecker delta. If we divide the volume V_μ into infinitesimal elements $\{\mathrm{d}V_i\}$ within which there are either 0 or 1 objects, then the above equation can be written as:

$$\delta^d(\mathbf{k}) = \frac{1}{N} \sum_i n_i e^{i\mathbf{r}_i\cdot\mathbf{k}} - \delta^K_{\mathbf{k},\mathbf{0}} \,, \tag{5}$$

where N is the total number of objects in V_μ and n_i is either 0 or 1. After a bit of algebra, it is seen that the true power spectrum can be measured via

$$P(k) \equiv \langle \mid \delta(\mathbf{k}) \mid^2 \rangle \ = \langle \mid \delta^d(\mathbf{k}) \mid^2 \rangle - \frac{1}{N} \,. \tag{6}$$

Obviously, when the FT is directly applied to the distribution of the galaxies or dark matter particles, one needs to correct for the discreteness (or shot noise) effect, which introduces an additional term $1/N$ to the power spectrum $\langle| \delta^d(\mathbf{k}) |^2\rangle$.

The above method of using a direct summation in the FT can be used to measure the power spectrum from the distribution of galaxies, when the number of objects is not very large. However, because of the huge number of particles involved in N-body simulations, it is almost impossible to be applied to the dark matter particles of cosmological density fields. Instead, a computationally attractive approach is to use an FFT. The density contrast in Fourier space using a FFT is,

$$\delta^f(\mathbf{k}) = \frac{1}{N}\sum_{\mathbf{g}} n^f(\mathbf{r}_g)e^{i\mathbf{r}_g\cdot\mathbf{k}} - \delta^K_{\mathbf{k},\mathbf{0}}\,, \tag{7}$$

where the superscript f represents the FFT. $n^f(\mathbf{r}_g)$ is the convolved density value on the $\mathbf{g}$-th grid point $\mathbf{r}_g = \mathbf{g}H$ (where $\mathbf{g}$ is an integer vector; H is the grid spacing),

$$n^f(\mathbf{r}_g) = \int n(\mathbf{r})W(\mathbf{r}-\mathbf{r}_g)\,\mathrm{d}\mathbf{r}\,, \tag{8}$$

where $W(\mathbf{r})$ is the mass assignment function. Note that Eqs. (5) and (7) are different in that the summations carried out in the former equation is over the objects and the latter over space (the grid points). After several steps (see also Hockney and Eastwood 1981), Jing (2005) derived the following power spectrum estimator,

$$\begin{aligned}\langle|\delta^f(\mathbf{k})|^2\rangle &= \sum_{\mathbf{n}}|W(\mathbf{k}+2k_N\mathbf{n})|^2P(\mathbf{k}+2k_N\mathbf{n}) \\ &+ \frac{1}{N}\sum_{\mathbf{n}}|W(\mathbf{k}+2k_N\mathbf{n})|^2\,,\end{aligned} \tag{9}$$

where $W(\mathbf{k})$ is the FT of the window function $W(\mathbf{r})$, $k_N = \pi/H$ is the Nyquist wavenumber, and the summation is over all 3D integer vectors $\mathbf{n}$. According to equation (9), one can easily identify the impact of the mass assignment onto the measured power spectrum. First, the mass assignment introduces the factor $W^2(\mathbf{k})$ to both the true power spectrum and the shot noise ($1/N$) terms. Second, the quantity $\langle|\delta^f(\mathbf{k})|^2\rangle$ is a measure for a *convolved* power spectrum (i.e., the sums over $\mathbf{n}$) which suffers from *sampling* effects. As pointed out in Jing (2005) and will be shown in Section 3, the sampling effects are significant near the Nyquist wavenumber k_N and should be carefully corrected in an accurate measure of the power spectrum.

3. The role of the mass assignment function

As shown by equation (9), the mass assignment window function plays an important role in measuring the power spectrum using an FFT. We separate its impact into two parts: one on the shot noise term (second term of the r.h.s. of Eq. 9) and the other on the true power spectrum term (first term of the r.h.s. of Eq. 9). Hereafter we refer to the impact on the true power spectrum term as the *sampling effects*. Usually, the impact on the shot noise term can be handled analytical according to the FT of the window function. However, because of the convolution with the true power spectrum, the sampling effects can not be corrected easily.

There are basically two strategies for handling the sampling effects. One can either try to correct for them by deconvolving the impact of the window function (which is carried out in Jing 2005) or try to use an optimal window function that minimizes the sampling effects from the outset (the purpose of this work). Below we discuss a few commonly used window functions as well as the particular mass assignment proposed here both in real and Fourier spaces, and then discuss their impact on measuring the true power spectrum with an FFT in detail.

3.1. Traditional mass assignment functions

The most popular mass assignment functions used in measuring the power spectrum are the NGP, CIC and TSC methods. Their forms in real space can be described by $W(\mathbf{x}) = \Pi_i W(x_i)$, with

$$W(x_i) = \begin{cases} 1 & |x_i| < 0.5 \\ 0 & \text{else} \end{cases} \qquad \text{NGP,} \tag{10}$$

$$W(x_i) = \begin{cases} 1 - |x_i| & |x_i| < 1 \\ 0 & \text{else} \end{cases} \qquad \text{CIC,} \tag{11}$$

and

$$W(x_i) = \begin{cases} 0.75 - x_i^2 & |x_i| < 0.5 \\ \frac{(1.5-|x_i|)^2}{2} & 0.5 < |x_i| < 1.5 \\ 0 & \text{else} \end{cases} \qquad \text{TSC,} \tag{12}$$

where x_i $(i = 1, 2, 3)$ is the i-th component of $\mathbf{x}$. In the left panel of Fig. 1, we show these window functions in real space, with solid, dotted and dashed lines corresponding to the NGP, CIC and TSC methods, respectively. Their impact on the measurement of the power spectrum using an FFT (Eq. 9) can be understood most easily based on their Fourier space behavior. According to Hockney & Eastwood (1981), these three mass assignment window functions can be described in Fourier space by $W(\mathbf{k}) = \Pi_i W(k_i)$, with

$$W(k_i) = \Big[\frac{\sin(\frac{\pi k_i}{2k_N})}{(\frac{\pi k_i}{2k_N})}\Big]^p , \tag{13}$$

where k_i ($i = 1, 2, 3$) is the i-th component of $\mathbf{k}$, and $p = 1$ for NGP, $p = 2$ for CIC, and $p = 3$ for TSC. We show in the right panel of Fig. 1 (the square of) the related window functions in Fourier space. These window functions peak at $k = 0$ with $W^2(k) = 1$ and decrease sharply with $k \gtrsim 0$, especially for the CIC and TSC kernels. According to Eq. (9), the impact of the window functions can be separated into two parts, one on the shot noise and one on the true power spectrum.

It is quite easy to model and correct the impact on the shot noise term,

$$D^2(\mathbf{k}) \equiv \frac{1}{N} \sum_{\mathbf{n}} W^2(\mathbf{k} + 2k_N\mathbf{n}) \, . \tag{14}$$

For the NGP, CIC, and TSC assignments, the shot noise term can be expressed as,

$$D^2(\mathbf{k}) = \frac{1}{N} \begin{cases} 1, & \text{NGP,} \\ \Pi_i[1 - \frac{2}{3}\sin^2(\frac{\pi k_i}{2k_N})], & \text{CIC,} \\ \Pi_i[1 - \sin^2(\frac{\pi k_i}{2k_N}) + \frac{2}{15}\sin^4(\frac{\pi k_i}{2k_N})]. & \text{TSC.} \end{cases} \tag{15}$$

In practice, for the latter two cases, one often uses the following isotropic approximation to model the shot noise term,

$$D^2(\mathbf{k}) \approx \frac{1}{N} \begin{cases} [1 - \frac{2}{3}\sin^2(\frac{\pi k}{2k_N})], & \text{CIC,} \\ [1 - \sin^2(\frac{\pi k}{2k_N}) + \frac{2}{15}\sin^4(\frac{\pi k}{2k_N})], & \text{TSC,} \end{cases} \tag{16}$$

where $k = |\mathbf{k}|$. As has been shown in Jing (2005), this approximation works very well for $k \le 0.7k_N$, however, it can underestimate the true value by about 40% at $k \sim k_N$. Nevertheless, compared to the power spectrum term we are trying to measure in a CDM cosmology, this error in the shot noise term is usually negligible.

Now we turn to the impact of the window functions on the first term of the r.h.s of Eq. (9), the sampling effects. There are three aspects that need to be taken into account in measuring the true power spectrum if an accuracy of a few *percent* is required.

- Smearing effect

  In the summation of the true power spectrum over $\mathbf{n}$, only the term $\mathbf{n} = 0$ is what we intend to measure. However, according to the results shown in the right panel of Fig. 1, the $W^2(k)$ term decreases sharply from $W^2(0) = 1$ at $k \gtrsim 0$, especially for the CIC and TSC methods. Thus the contribution from the *related* true power spectrum $P(\mathbf{k})$ is greatly suppressed. This effect is the so-called smearing or smoothing effect, which has been discussed in the literature (e.g., Baumgart & Fry 1991; Jing & Valdarnini 1993; Scoccimarro et al. 1998)

- Anisotropy effect

  In pratice, one may use the average of the $\langle|\delta^f(\mathbf{k})|^2\rangle$ over different directions for a given k to estimate the power spectrum $P(k)$. However, the window function $W^2(\mathbf{k})$ is not isotropic for different directions for a given k, that is, $W^2(\mathbf{k})$ is different, e.g., for $\mathbf{k} = k(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$, $\mathbf{k} = k(1/\sqrt{2}, 1/\sqrt{2}, 0)$, $\mathbf{k} = k(1, 0, 0)$, etc. This effect is small for the NGP method, but quite significant for the CIC and TSC methods, especially at $k \sim k_N$ (eg. Baumgart & Fry 1991, Jing 2005).

- Aliasing effect

  The power spectrum estimator $\langle|\delta^f(\mathbf{k})|^2\rangle$ contains not only the contribution from $P(\mathbf{k})$ where $\mathbf{n} = 0$ but also from $P(2k_N\mathbf{n} + \mathbf{k})$ where $\mathbf{n} \neq 0$. The latter contribution, which is called the alias effect, prevents us from obtaining the true power spectrum $P(\mathbf{k})$ straightforwardly. This effect, which is prominent near the Nyquist wavenumber $k_N = 0.5 \times (2\pi/H)$ (significant for NGP method and less significant for TSC method), has been discussed and handled using an iterative correction method in Jing (2005).

The smearing and anisotropy effects are easy to be corrected. For instance, one can directly normalize the density contrast in Fourier space, $\delta^f(\mathbf{k})$, with the window function $W(\mathbf{k})$ (e.g., Baumgart & Fry 1991). Thus, the corrected density contrast $\delta^f(\mathbf{k})/W(\mathbf{k})$ obviously no longer suffer from these two effects at $k \leq k_N$, however at the price of a much enhanced aliasing effect (i.e., the $\mathbf{n} \neq 0$ terms in Eq. 9). Because of the aliasing effect the power spectrum measured at $k = k_N$ can become a factor of 2 larger than the true value. Such kind of aliasing effects also exist in radio imaging analyses based on FFTs, and various perticular mass assignment schemes have been discussed in order to minimize their impact (e.g. Briggs et al. 1999).

Using an elegant iterative correction methods, Jing (2005) has properly corrected the impact of the aliasing effect, and illustrated its success in obtaining the true power spectrum. On the other hand, his method can only be applied to the estimation of the power spectrum. For measurements of higher order spectra, e.g. the bi-spectrum, there is so far no straightforward method that can correct the aliasing effect. In what follows, rather than trying to correct the above three kinds of effects, we attemp to find a mass assignment window function that does not or only to very small degree suffer from these effects.

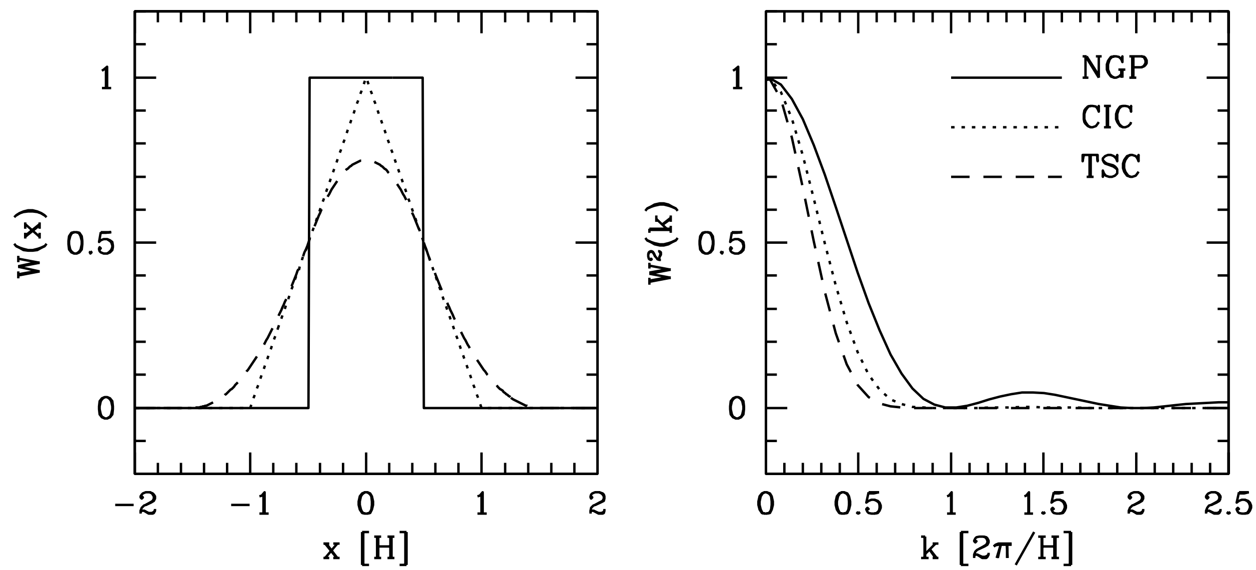

Fig. 1.— Left panel: the three commonly used mass assignment window functions, NGP, CIC, and TSC, as indicated. Right panel: the square of the window functions in Fourier space.

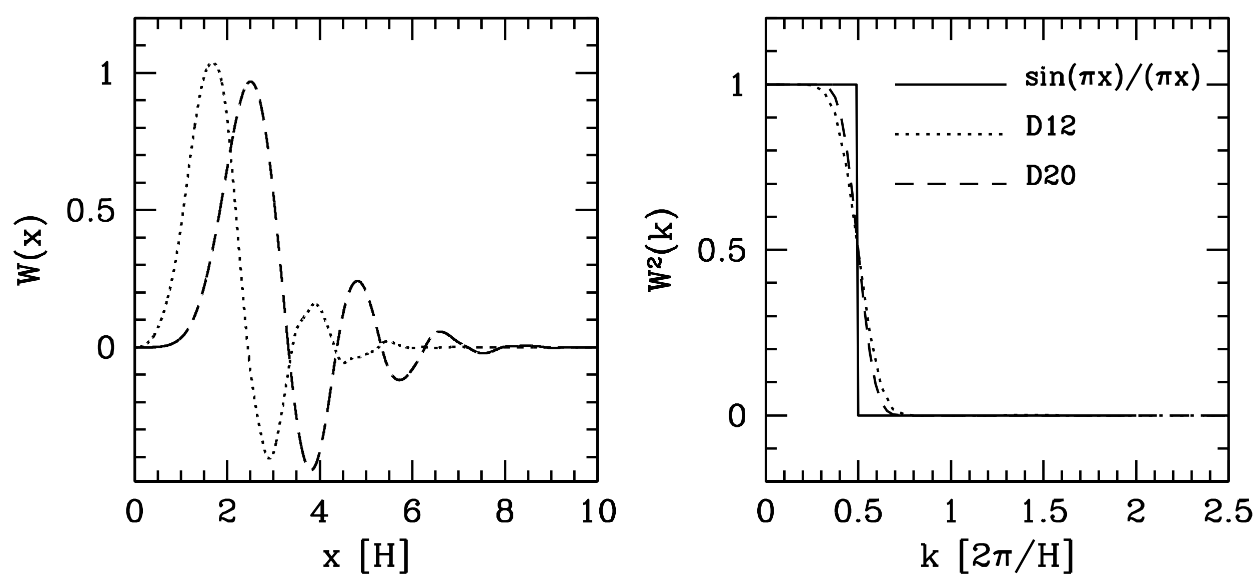

Fig. 2.— Left panel: the scaling functions 12 (D12) and 20 (D20) of Daubechies, used here as mass assignment window functions. Right panel: the square of these two scaling functions in Fourier space. For comparison, we also show a top hat window function in Fourier space, which corresponds to $W(x) = \sin(\pi x)/(\pi x)$ in real space.

3.2. Daubechies window functions

An ideal window function that does not suffer from the sampling effects mentioned above is obviously a top-hat in Fourier space. Using such a window function, the power spectrum measurement Eq. (9) can be reduced to Eq. (6). However such a mass assignment function, $W(x) = \sin(\pi x)/(\pi x)$, is not a compact localized function in real space. In the mass assignment onto the grid, one may then have to distribute each particle's mass to too many grid cells. In fact, if we want to maintain an accuracy of 1% in the mass assignment, the mass of each particle should be distributed to 60^3 grid cells! Such an assignment scheme may eliminate most if not all of the computational advantage that an FFT can bring us.

Thus, a suitable mass assignment window function should be localized both in real and Fourier space. A good candidate that features these properties is the scale function of the wavelet transformation. The wavelet transformation has been previously introduced to astrophysical studies and has been applied successfully in the analysis of various astrophysical observations (c.f., Fang & Thews 1998), e.g., on the distributions of galaxies (e.g. Martinez et al. 1993; Fang & Feng 2000; Yang et al., 2001; 2002a,b; Feng & Fang 2004), on the properties of Lyα absorption lines (e.g. Pando & Fang 1997; Meiksin 2000), on the galaxy clusters (e.g., Slezak et al. 1994; Grebenev et al. 1995; Gambera et al. 1997; Schäfer et al. 2005), etc. Here, we introduce the scale function $\phi(x)$ of the Daubechies wavelet transformation for use in power spectrum measurements, which has the following properties (e.g., Daubechies 1992),

$$\int \phi(x)\ \mathrm{d}x \equiv 1\,, \tag{17}$$

$$\sum_n \phi(x+n) \equiv 1\,, \tag{18}$$

and its Fourier transform, $\phi(k)$, satisfies

$$\int \phi^2(k)\ \mathrm{d}k \equiv 1\,, \tag{19}$$

$$\sum_n \phi^2(k+2\pi n) \equiv 1\,. \tag{20}$$

In this paper, we use the Daubechies D12 and D20 scale functions (Daubechies 1988, 1992) as our new mass assignment window functions, $W(x) = \phi(x)$, which are shown in the left panel of Fig. 2. In the right panel of Fig. 2, the squares of these two window functions in Fourier space are shown as dotted and dashed lines, as indicated. For comparison, we also show in the right panel the ideal case of a top-hat Fourier window function as the solid line. The D12 and D20 window functions in Fourier space $W^2(k)$ resemble the ideal case very well, especially in the D20 case whose deviation from the ideal case at $k = 0.35$ (i.e.,

$0.7k_N$) is smaller than 2%. Note that these particular mass assignment window functions are different from the traditional schemes, e.g. NGP, CIC and TSC in that: (1) they are not symmetric; (2) they are not positive definite. However these two features will not induce any undesirable consequences in our application. First, since the overall shifting of the window function will not impact the amplitude of $\delta(k)$, the window function $\phi(x)$ shown in the left panel of Fig. 2 can be treated as symmetric components centered at $x \sim 1.75$ and $x \sim 2.5$, respectively, with additional fluctuations at nearby grid cells. Second, the window function needs not necessarily be positive definite, as we are measuring the density contrast $\delta(x)$, and even the ideal window function $W(x) = \sin(\pi x)/(\pi x)$ is not positive definite.

Before we turn to a discussion of their impact on measuring the true power spectrum, let us consider the computational cost for the mass assignment using the D12 and D20 scale functions. According to their real space behavior, at much better than 0.5% accuracy, each mass particle should be distributed onto 6^3 (D12) or 8^3 (D20) grid cells, respectively, which is a factor of 10 or 20 times more than the TSC method with 3^3 grid cells. However, we argue that this cost is worthwhile given the following positive features of the Daubechies assignment.

First, according to Eq. (20), the shot noise term in Eq. (9) for these mass assignment algorithms is,

$$D^2(k) \equiv 1/N \,. \tag{21}$$

Second, by comparing the Fourier-space behaviors of the D12 and D20 functions with those of the traditional mass assignment methods, NGP, CIC and TSC, it becomes clear that the three sampling effects of smearing, alias and anisotropy are greatly suppressed. Moreover, for the D20 window function, if we only measure the power spectrum up to $k = 0.7k_N$, we do not need to take into account any of those three kinds of effects explicitly, because the true power spectrum is recovered with better than 2% accuracy!

Another very important aspect is that such a mass assignment scheme can be fruitfully applied to the measurement of the higher-order spectra using an FFT. For instance, in measuring the bi-spectrum using an FFT with the D20 mass assignment, we do not need to consider the sampling effects up to $k = 0.7k_N$ at all, since here the bi-spectrum can be measured directly with an accuracy level better than 3%. Note that so far there is no other approach known to accurately correct for the sampling effects in measuring the bi-spectrum with an FFT. We defer an application of our new technique and a theoretical modeling of the higher order spectra to a forthcoming paper.

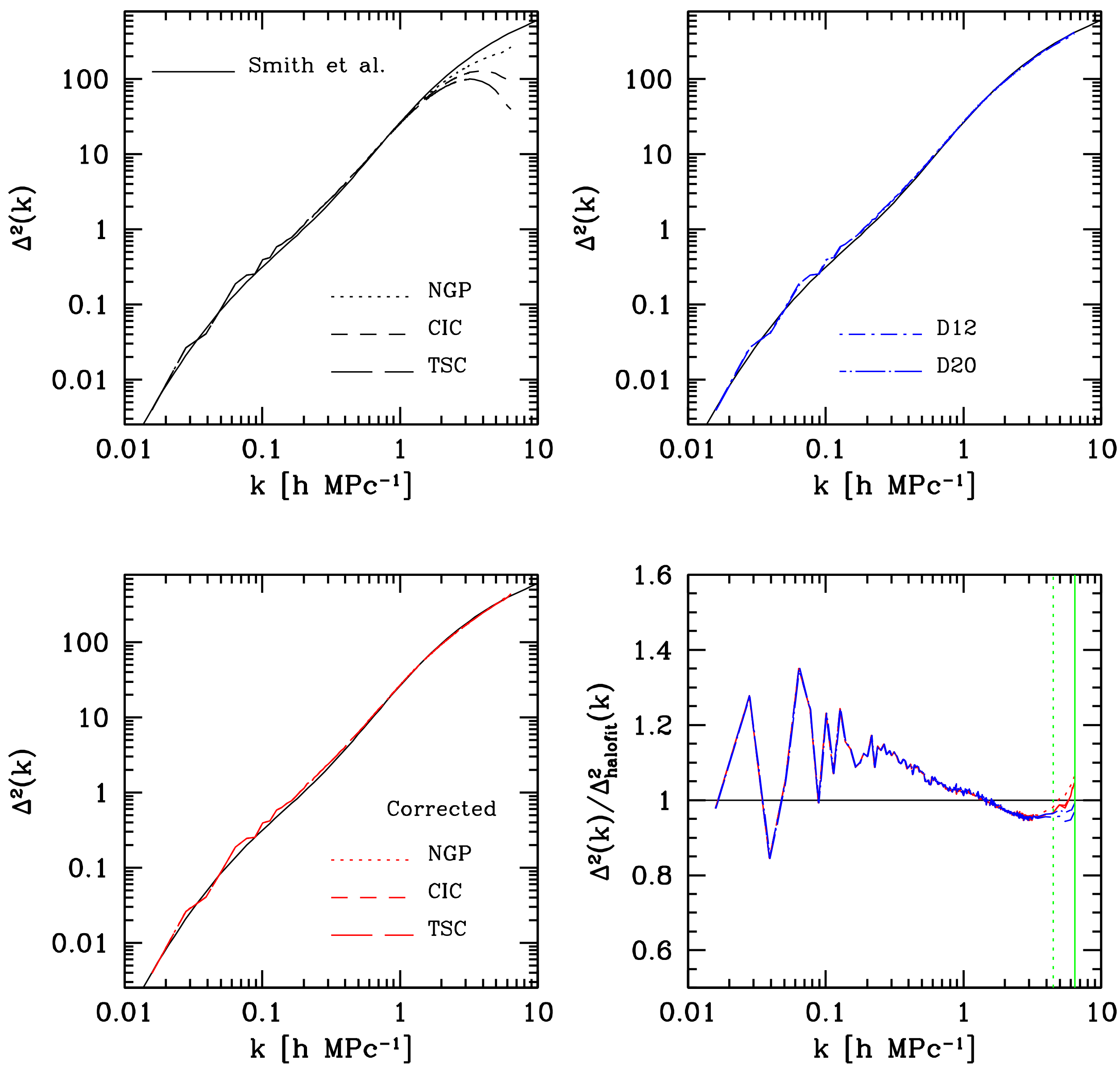

Fig. 3.— Upper-Left panel: the FFT power spectra measured using the commonly employed mass assignment functions: NGP, CIC and TSC, as indicated. In this panel, only the shot noise term has been corrected. Lower-left panel: the same measurements as shown in the upper-left panel, but now corrected for the sampling effects using the iterative correcting method proposed by Jing (2005). Upper-right panel: the FFT power spectra measured using the Daubechies scale functions D12 and D20 as mass assignment functions. Only the short noise term $1/N$ has been corrected. In these three panels, for reference we also plot the power spectrum prediction by Smith et al. (2003) using the Millennium Run's cosmological parameters. Lower-right panel: a comparison of the ratios between the measured power spectra and the 'halofit' prediction by Smith et al. (2003), for the NGP, CIC, TSC, D12 and D20 mass assignment window functions. The vertical lines mark the locations of $k = k_N$ and $k = 0.7k_N$, respectively.

4. Tests using N-body simulations

Having discussed the impact of the mass assignment window functions on the measurement of the true power spectrum using an FFT, and having introduced the D12 and D20 scale functions, we proceed to demonstrate their performance when applied to the measurement of the mass power spectrum of a large N-body simulation. Here we briefly describe the simulation, the Millennium Run, used for this project. The Millennium Run is a very large dark matter simulation of the concordance ΛCDM cosmology with $2160^3 \simeq 1.0078 \times 10^{10}$ particles in a periodic box of $500\,h^{-1}$Mpc on a side (Springel et al. 2005). The simulation was carried out by the Virgo Consortium using a customized version of the GADGET2 code. The cosmological parameters used in this simulation are $\Omega_{\rm m} = \Omega_{\rm dm} + \Omega_{\rm b} = 0.25$, $\Omega_{\rm b} = 0.045$, $h = 0.73$, $\Omega_\Lambda = 0.75$, $n = 1$, and $\sigma_8 = 0.9$. For our test investigation, we randomly select 10% of the dark matter particles (because of practical limits in computer memory) and measure their power spectra using the different window functions we described in the previous section.

To measure the power spectrum, we employ an FFT of the density distribution of dark matter particles assigned to a grid with 1024^3 cells using the mass assignment algorithms discussed in Section 3. Thus the corresponding Nyquist wavenumber is $k_N = 1024\pi/500\,h\,{\rm Mpc}^{-1}$. In the upper-left panel of Fig 3, we show the FFT power spectra measured using the traditional mass assignment functions, NGP, CIC and TSC, where only the shot noise term has been subtracted. In this figure, the power spectrum is presented in terms of $\Delta^2(k) \equiv 2\pi k^3 P(k)$. For comparison, we show the theoretical prediction of the non-linear power spectrum by Smith et al. (2003) as the solid line, based on the Millennium Run's cosmological parameters. Obviously, because of the sampling effects we discussed in Section 3.1, the power spectra are quite different at $k \gtrsim 1\,h{\rm Mpc}^{-1}$ ($\sim 0.2k_N$). The power spectra measured without correcting the sampling effects, especially for the TSC method, significantly under-predict the true power spectrum. Using the methods proposed by Jing (2005), we can iteratively correct for the sampling effects and extract estimates of the true power spectrum. The corrected power spectra for the NGP, CIC and TSC mass assignment methods are shown in the lower-left panel of Fig. 3. Comparing these power spectra among themselves and with the 'halofit' prediction of Smith et al. (2003), we are convinced that the true power spectrum is well recovered at all scales $k \le k_N$, and is roughly consistent with the prediction by Smith et al.

Now we turn to use the Daubechies scale functions D12 and D20 as our mass assignment window functions. The resulting power spectra after correcting for the shot noise term $1/N$ are shown in the upper-right panel of Fig. 3, as indicated. Without any correction for the sampling effects, the measured power spectra look very nice and match the theoretical predictions by Smith et al. (2003) on all scales up to $k \leq k_N$. This is very different from the results shown in the upper-left panel of Fig. 3 for the classical assignment functions. In fact, at a low resolution view, there is no visible difference between these results and the corrected measurements shown in the lower-left panel of Fig. 3.

Finally, we take more accurate comparisons between the power spectra measured with these different methods by showing their ratios with respect to the 'halofit' prediction of Smith et al. (2003). The de-convolved power spectra based on the NGP, CIC and TSC mass assignments, the directly measured power spectra using D12 and D20 mass assignments are ploted together for comparison in the bottom right panel of Fig. 3. Here are a few observations that can be made: (1) the three de-convolved power spectra are very well consistent with each other at a level better than 2% at $k \leq 0.7k_N$, and at a level of about 5% at $k \sim 1.0k_N$; (2) the directly measured power spectra based on the D12 and D20 (the latter slightly better) functions have an accuracy of better than 2% at $k \leq 0.7k_N$ and at a level of about 10% at $k \sim 1.0k_N$; (3) there is about 20% under-prediction on large scales (with $k < 1\ h\mathrm{Mpc}^{-1}$) and 5% over-prediction on small scales by Smith et al. (2003) for the power spectrum of the Millennium Simulation. According to these findings, we may conclude that both the deconvolution method and the direct measurement based on the Daubechies scale functions, especially for D20, can recover the *true* power spectrum with better than 2% accuracy at $k \leq 0.7k_N$. Moreover, as a conservative prediction, the bi-spectrum can be measured at a level better than 3% at $k \leq 0.7k_N$ if the D20 window function is used in the mass assignment for the FFT. This should be very useful for accurate studies of the bi-spectrum.

5. SUMMARY

To quantify the large-scale structure in the distributions of a large population of dark matter particles or galaxies, one may measure their power (or higher order) spectra using a FFT. However, the required mass assignment onto the points of the FFT-grid can introduce sampling effects in the measured power spectra. Most of these effects have been noticed and discussed in the literature before (e.g., Baumgart & Fry 1991; Jing & Valdarnini 1993; Jing 2005). Among these, Jing (2005) was the first to use an iterative correction method to compensate for all of these sampling effects, especially the alias effect.

In this paper, we follow Jing (2005) and discuss the impact of the mass assignment on measuring the power spectrum with an FFT. There are two components that the employed window function can impact: one is the shot noise term and the other is the term involving the true power spectrum. With respect to the influence on the true power spectrum term, there are three different sampling effects that need to be considered: the smearing effect, the aliasing effect and the anisotropy effect.

Rather than trying to deconvolve for the sampling effects, we propose to use a special window function: the Daubechies wavelet scale function that can minimize these sampling effects. In particular, the D12 and D20 scale functions considered here are compact in real space, which allows a fast mass assignment onto the grid cells, while at the same time their top-hat like shape in Fourier space leads only to very small sampling effects.

According to the Fourier transform $W^2(k)$ of the D20 function, at $k \leq 0.7k_N$, all the sampling effects induced by the mass assignment can only affect the measured power spectrum at less than a level of 2%. This is confirmed by the tests we carried out with the Millennium Run simulation. More importantly, as a conservative prediction, the new method proposed here can measure the bi-spectrum of dark matter particles at better than 3% accuracy for $k \leq 0.7k_N$, without the need to apply any correction for the sampling effects, apart from a simple substraction of the shot noise term.

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