Quintessence cosmologies with a growing matter component

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We investigate coupled quintessence cosmologies with a matter consisting of particles with an increasing mass. While negligible in early cosmology, the appearance of a growing matter component has stopped the evolution of the cosmon field at a redshift around six. In turn, this has triggered the accelerated expansion of the Universe. We propose to associate growing matter with neutrinos. Then the presently observed dark energy density and its equation of state are determined by the neutrino mass.

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Growing observational evidence indicates a homogeneous, at most slowly evolving dark energy density that drives an accelerated expansion of the universe since about 6×10^9 years [1,2]. The origin of dark energy is unknown, be it a cosmological constant [3], a dynamical dark energy due to a scalar field (quintessence) [4,5], a modification of gravity [6], or something still unexpected. A pressing question arises: why has the cosmological acceleration set in only in the rather recent cosmological past? Within quintessence models we need to explain a transition from the matter dominated Universe to a scalar field dominated Universe at a redshift $z \simeq 0.5$.

A similar crossover from radiation to matter domination occurs earlier in the cosmological history. It is bound to happen at some time since the dilution of the energy density with increasing scale factor a obeys $\rho_r \propto a^{-4}$ for radiation and $\rho_c \propto a^{-3}$ for cold dark matter. At some moment matter must win. We suggest in this paper that the presently observed crossover to a dark energy dominated Universe is of a similar type.

In addition to the usual cold dark matter we propose "Growing Matter," an unusual form of matter whose energy density decreases slower than the one of the usual cold dark matter, or even increases:

$$\rho_g \propto a^{3(\gamma-1)}, \qquad \gamma > 0. \tag{1}$$

This may be realized by particles whose mass increases with time. In presence of both cold dark matter and growing matter a crossover to a new epoch is then necessary at some moment. In our model this transition is witnessed now. Similar as for the radiation-matter transition the time for the crossover is set by the mass and abundance of the growing matter particles. In this respect the crossover resembles the matter-radiation transition—we have again two fluids with a different rate of dilution. We will also find important differences, however, related to the crucial role played by a cosmological scalar field.

We suggest that growing matter consists of neutrinos. In this case the abundance is computable. In our model the crossover time and the present dark energy density $\rho_h(t_0)$ are then determined by the present (average) value of the neutrino mass $m_\nu(t_0)$:

$$[\rho_h(t_0)]^{1/4} = 1.07\left(\frac{\gamma m_\nu(t_0)}{\text{eV}}\right)^{1/4} 10^{-3}\ \text{eV}. \tag{2}$$

Here γ corresponds to a ratio of dimensionless coupling constants which determines the present ratio between dark energy and neutrino components, $\Omega_h(t_0)/\Omega_\nu(t_0) \approx \gamma$. Also the present equation of state is given by the neutrino mass:

$$w = -1 + \frac{m_\nu(t_0)}{12\ \text{eV}}. \tag{3}$$

(Eqs. (2) and (3) are derived below after Eq. (13)). The time when dark energy becomes of similar size as dark and baryonic matter is directly related to the time when the neutrinos become nonrelativistic. In view of the known lower and upper bounds on $m_\nu(t_0)$ this happens in the recent past. Therefore our scenario provides for a qualitative explanation of the coincidence problem. For arbitrary values of γ of the order one the matter dominated period ends and dark energy becomes important slightly after the neutrinos become nonrelativistic and trigger the transition to a new regime. Except for $m_\nu(t_0)$ and γ there are no further parameters determining the timing of the crossover. For a quantitatively accurate cosmology γ has to be mildly adjusted according to $m_\nu(t_0)$ (see Eq. (2)).

Quite generally, the appearance of a substantial growing matter component strongly influences the dynamical behavior of the scalar field responsible for quintessence, the cosmon. Indeed, a time evolution of the mass of growing matter particles requires a time evolution of the cosmon field ϕ. We assume

$$m_g(\phi) = \bar{m}_g e^{-\beta(\phi/M)}, \tag{4}$$

with $M \equiv 1/\sqrt{8\pi G_N}$ the reduced Planck mass and $\bar{m}_g$ a constant. For $\beta < 0$ an increase in ϕ will induce an increasing mass.

In turn, the growing matter energy density ρ_g influences the evolution of the cosmon. Our approach is a model of "Coupled Quintessence" [7–9]. For a homogeneous cosmon field the field equation [10]:

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi} + \frac{\beta}{M}\rho_g, \tag{5}$$

contains a "force" $\propto \rho_g$ that will counteract an increase of ϕ once $\beta\rho_g$ is comparable to $\partial V/\partial\phi$. In our model, this effect will dramatically slow down a further evolution of the cosmon. For an almost static $\phi(t)$ the cosmon potential $V(\phi)$ will then act similar to a cosmological constant. The expansion of the Universe therefore accelerates soon after ϕ stops to move. The coupling between the growing matter and the scalar field ties the time of onset of the accelerated expansion to the crossover time when $\beta\rho_g$ becomes important. The mechanism we propose shows similarities to the one presented in Ref. [11]. Here we suggest to identify the coupled matter component with the neutrinos and we discuss the key role played by the growing matter mass. This yields the key relations (2) and (3) and solves the "why now" problem.

Let us specify our model. Besides gravity and the cosmon field, for which we assume an exponential potential:

$$V(\phi) = M^4 e^{-\alpha(\phi/M)}, \tag{6}$$

cosmology is determined by cold dark matter, growing matter, baryons, and radiation. We denote the fraction of homogeneous dark energy by Ω_h, and similarly for cold dark matter and growing matter by Ω_c and Ω_g. The cosmologically relevant parameters of our model are the dimensionless couplings α and β (Eqs. (4) and (6)), as well as the energy density of growing matter at some initial time, e.g. $\rho_g(t_{eq})$. The initial density of cold dark matter, $\rho_c(t_{eq})$, can be translated to the present value of the Hubble parameter H_0. We assume a flat Universe. The cosmological equations are the standard ones, except for the modified energy-momentum conservation for growing matter [10]:

$$\dot{\rho}_g + 3H\rho_g + \frac{\beta}{M}\rho_g\dot{\phi} = 0, \tag{7}$$

which accounts for the exchange of energy between growing matter and the cosmon [7,10]. In case of neutrino growing matter, Eqs. (5) and (7) are modified by pressure terms in early cosmology.

For the radiation and the matter dominated epochs in early cosmology the cosmon field follows a "tracker solution" or "cosmic attractor" with a constant fraction of early dark energy [4]:

$$\Omega_{h,e} = \frac{n}{\alpha^2}, \tag{8}$$

where $n = 3(4)$ for matter (radiation). This intermediate attractor guarantees that the initial conditions for the scalar field are not fine-tuned. Observations require that α is large, typically $\alpha \geq 10$ [12]. In this "scaling regime" one has

$$\begin{aligned} &\phi = \phi_0 + \frac{2M}{\alpha}\ln\left(\frac{t}{t_0}\right), \qquad V \sim \dot{\phi}^2 \sim \rho_c \sim t^{-2}, \\ &m_g \sim \Omega_g \sim t^{2\gamma}, \qquad \rho_g \sim t^{2(\gamma-1)}, \qquad \gamma = -\frac{\beta}{\alpha}. \end{aligned} \tag{9}$$

The growing matter plays no role yet. Its relative weight Ω_g grows, however, for $\gamma > 0$ or $\beta < 0$ such that growing matter corresponds to an unstable direction. The scaling regime ends once $\gamma\Omega_g$ has reached a value of order one.

The future of our Universe is described by a different attractor [7,8], where the scalar field and the growing matter dominate, while baryons and cold dark matter become negligible. For this future attractor the expansion of the Universe accelerates according to ($\gamma > 1/2$):

$$\begin{aligned} &H(t) = \frac{2(\gamma+1)}{3}t^{-1}, \qquad w = -1 + \frac{1}{(\gamma+1)}, \\ &\Omega_h = 1 - \Omega_g = 1 - \frac{1}{(\gamma+1)} + \frac{3}{\alpha^2(\gamma+1)^2}. \end{aligned} \tag{10}$$

For large γ the total matter content of the Universe, $\Omega_M = \Omega_c + \Omega_b + \Omega_g$, will be quite small in the future, $\Omega_M \approx \Omega_g \approx 1/\gamma$. The presently observed value $\Omega_M \approx 0.25$ indicates then that we are now in the middle of the transition from matter domination ($\Omega_M \approx 1 - 3/\alpha^2$) to a scalar field dominated cosmology ($\Omega_M \approx 1/\gamma$). The energy-momentum tensor for combined quintessence and growing matter is conserved and defines the equation of state (EOS) in the nonrelativistic regime as $w = p_h/(\rho_h + \rho_g)$.

The limiting case $\gamma \gg 1$ admits a particularly simple description. In this case we encounter a sudden transition between the two cosmic attractors at the time t_c when the two terms on the right-hand side. of Eq. (5) have equal size, namely, for $\alpha V = -\beta\rho_g$ or $\Omega_g = \Omega_h/\gamma$. While the cosmon was evolving before this time, it suddenly stopped at a value $\phi_c \equiv \phi(t)$ at t_c. Thus, for $t \geq t_c$ and large γ the cosmology is almost the same as for a cosmological constant with value $V(\phi_c)$. On the other hand, before t_c standard cold dark matter cosmology is only mildly modified by the presence of an early dark energy component (8). For large enough α this ensures compatibility with observations of cosmic microwave background (CMB) anisotropies and structure formation. The redshift of the transition z_c may be estimated by equating the potential V at the end of the scaling solution (9) to its present value. It is given by $H(z_c)/H_0^2 = 2\Omega_{h,0}\alpha^2/3$ and can be approximated as

$$1 + z_c \approx [2\Omega_{h,0}\alpha^2/(3 - 3\Omega_{h,0})]^{1/3}, \tag{11}$$

with the present dark energy fraction $\Omega_{h,0} \approx 0.75$. In the numerical examples below we will assume $\alpha = 10$ and either $\gamma = 5.2$ or $\gamma = 39$. Then we obtain numerically $z_c \approx 6(5)$ for $\gamma = 5.2(39)$. Thus z_c is large enough not to affect the present supernovae observations. We plot the time evolution of the different cosmic components and the effective equation of state for the combined cosmon and growing matter components in Fig. 1. For not too large α and γ our model differs from ΛCDM, and we will come back below to the interesting possibilities of observing these deviations.

So far we have made no assumptions about the constituents of the growing matter component. It could be a heavy or superheavy massive particle, say with a mass 1 TeV or 10^{16} TeV. Then growing matter is nonrelativistic at all epochs where it plays a role in cosmology. In this case the initial value $\rho_g(t_{eq})$ has to be chosen such that the crossover occurs in the present cosmological epoch. Even more interesting, growing matter could be associated with neutrinos. In this case our model shares certain aspects with the "Mass Varying Neutrinos" scenario [13], although being much closer to "standard" Coupled Quintessence [8]. Neutrino growing matter offers the interesting perspective that no new particles (besides the cosmon and cold dark matter) need to be introduced. Furthermore, the present value of ρ_g can be computed from the relic neutrino abundance and the present (average) neutrino mass $m_\nu(t_0)$ (assuming $h = 0.72$):

$$\Omega_g(t_0) = \frac{m_\nu(t_0)}{16\ \text{eV}}. \tag{12}$$

For large $|\beta|$ the neutrino mass becomes rapidly very small in the past such that neutrinos cannot affect the early structure formation. The standard cosmological bounds on the neutrino mass [2] do not apply.

For a given neutrino mass $m_\nu(t_0)$ our model has only two parameters, α and γ. They will determine the present matter density $\Omega_M(t_0)$ as in Eq. (10). Replacing γ by $\Omega_M(t_0)$, our model has then only one more parameter, α, as compared to the ΛCDM model. For an analytical estimate of the relation between $\Omega_M(t_0) = 1 - \Omega_h(t_0)$ and $m_\nu(t_0)$ we use the observation that the ratio Ω_g/Ω_h (averaged) has already reached today its asymptotic value (10) :

$$\Omega_h(t_0) = \left[\frac{\gamma+1}{1-\frac{3}{\alpha^2(\gamma+1)}} - 1\right]\frac{m_\nu(t_0)}{30.8h^2\ \text{eV}} \approx \frac{\gamma m_\nu(t_0)}{16\ \text{eV}}. \tag{13}$$

This important relation determines the present dark energy density by the neutrino mass and γ according to Eq. (2). The value of $\rho_h(t_0)$ will change very slowly in the future since the value $\gamma = 5.22(800)$ for the maximal (minimal) neutrino mass (no sterile neutrinos) $m_\nu(t_0) = 2.3$ eV(0.015 eV) must indeed be large and w is therefore close to -1. The late dark energy density is essentially determined as the neutrino energy density times γ. Its actual value is given by the value of the scalar potential at the crossing time t_c, i.e. $9M^2H^2(t_c)/2\alpha^2$. Since the equation of state is today already near the asymptotic value of eq.(10), cf. Fig. 1, we can relate it to the neutrino mass ($\Omega_{h,0} \approx 3/4$) by Eqs. (10) and (13), yielding Eq. (3). This expression is remarkable since it directly relates the present dark energy equation of state to the present value of the neutrino mass. It yields $m_\nu(t_0) < 2.4$ eV for $w < -0.8$.

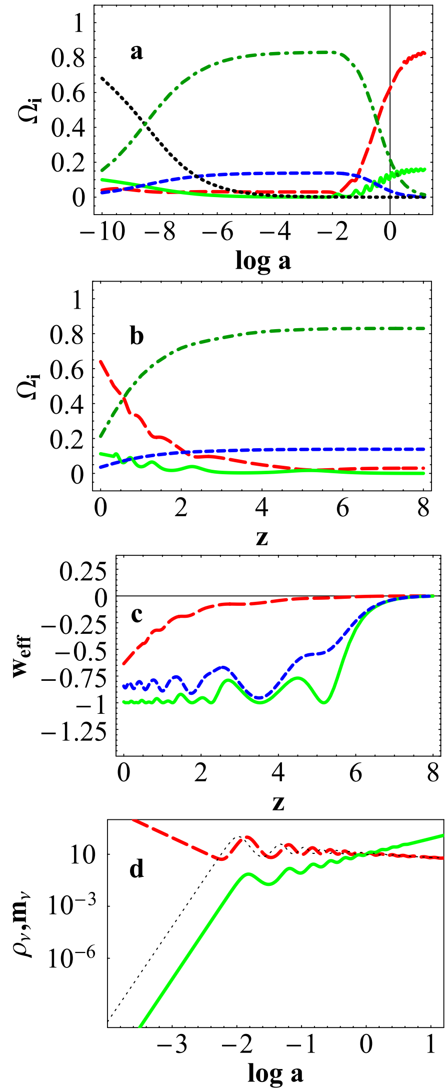

FIG. 1 (color online). Cosmological evolution for neutrino growing matter for $\alpha = 10$, $\gamma = 5.2$ and $m_{\nu,0} = 2.3$ eV. Panel a): density fractions for radiation Ω_{rad} (black, dots), cold dark matter Ω_c (dark green, dot-dashes), baryons Ω_b (blue, short dashes), dark energy Ω_h (red, long dashes) , and growing matter (neutrinos) Ω_g (light green, solid). Panel b): blow up of panel a) near the present time. Panel c): total equation of state $w_{\text{eff}} \equiv p_{\text{tot}}/\rho_{\text{tot}}$ (red, long dashes); combined EOS of cosmon and neutrinos (blue, short dashes); and EOS of cosmon alone (green, solid). Panels a)–c) remain almost identical for heavy growing matter. Panel d): neutrino energy density (red, long dashes), neutrino mass (green, solid) normalized to unity today. The dotted curve represents the energy density of always nonrelativistic heavy growing matter.

How can our model based on growing matter be tested and constrained? First, the presence of early dark energy manifests itself by the detailed peak location of the CMB anisotropies [14], the change in the linear growth of cosmic structures [15,16], and the abundance and properties of nonlinear structures [17]. Second, for not too large γ there would be a sizable fraction of growing matter today (for neutrino growing matter this would require rather large neutrino masses). Then the present matter density $\rho_M = \rho_c + \rho_b + \rho_g$ differs from the (rescaled) matter density in the early Universe $\rho_c + \rho_b$. This may affect the matching of the present values of Ω_M and Ω_b/Ω_M obtained from supernovae, baryon acoustic oscillations and clusters, with determinations from the CMB at high redshift, through the value of t_{eq} and the baryon content of the Universe at last scattering. This effect is small for large values of γ (small neutrino mass).

Third, growing matter can affect the formation of structures in the late stages. For very massive particles, growing matter would consist of relatively few particles which have presumably fallen into the cold dark matter structures formed in early cosmology. For scales smaller than the range of the cosmon interaction these particles feel a strong mutual attraction, enhanced by a factor $(2\beta^2 + 1)$ as compared to gravity. This force is mediated by the cosmon [7,18]. Thus, once a sufficient Ω_g is reached, the growing matter structures $\delta\rho_g$ grow rapidly. They will influence, in turn, the structures in baryons and cold dark matter once the gravitational potential of the growing matter structures becomes comparable to the one of the cold dark matter structures. This happens rather late, especially for large γ since growing matter constitutes only a small fraction of the present matter density in this case.

The condition for the onset of an enhanced growth of $\delta\rho_g$ requires that the average cosmon force $\sim 2\beta^2\Omega_g$ is comparable to the average gravitational force $\sim\Omega_M$. This happens first at a redshift z_{eg} somewhat larger than the crossover z_c. At this time the scaling solution is still valid, with $\Omega_M \approx 1 - 3/\alpha^2$ and

$$\Omega_g(z) = [(1 + z_c)/(1 + z)]^{3\gamma}\Omega_g(z_c), \tag{14}$$

$$\Omega_g(z_c) \approx \gamma\Omega_V(z_c) \approx 3\gamma/(2\alpha^2), \tag{15}$$

resulting in:

$$\frac{1 + z_{eg}}{1 + z_c} = \{3\gamma^2(\gamma + 1)\}^{1/3\gamma}. \tag{16}$$

For large γ one finds z_{eg} quite close to z_c such that the enhanced growth concerns only the very last growth epoch before the accelerated expansion reduces further linear growth in the dark matter component. For heavy growing matter this results in an enhancement of σ_8 as compared to the ΛCDM model, which may be compensated by a slower growth rate before z_c due to early dark energy [15,16].

An enhanced growth of $\delta\rho_g$ concerns only structures with size smaller than the range $l_\phi = M/(\alpha\sqrt{V})$ of the cosmon-mediated interaction. For the scaling solution this yields $l_\phi(t) = \sqrt{2}/(3H(t))$ [7], whereas l_ϕ remains essentially constant for $t > t_c$. A different regime of growth applies for $l > l_\phi$. A "window of adiabatic fluctuations" opens up in the range $l_\phi < l < H^{-1}$ where the fluctuations of the coupled cosmon fluid and growing matter can be approximated as a single fluid. In this regime the enhanced growth is suppressed by the small range of the cosmon interaction.

Neutrino growing matter was relativistic in earlier time, so that free streaming prevents clustering. For $\beta < 0$ neutrinos have actually remained relativistic much longer than neutrinos with constant mass. In the limit of large γ one can estimate that the neutrinos are relativistic at $a < a_R$ where

$$a_R \approx (m_\nu(t_0)/T_{\nu,0})^{-1/4} = 0.11\left(\frac{m_\nu(t_0)}{1\ \text{eV}}\right)^{-1/4}, \tag{17}$$

which corresponds to $z_R \in (2 - 10)$ for $m_{\nu,0} \in (0.015 - 2.3)$. The growth of neutrino structures only starts for $z < z_R$. Even then, neutrinos cannot cluster on scales smaller than their "free streaming scale" l_{fs}. This scale is given by the time when the neutrinos become nonrelativistic, Eq. (17), close to $H^{-1}(a_R) \approx 200(m_\nu/1\ \text{eV})^{-3/8}h^{-1}$ Mpc $\approx$ (150–1500) Mpc.

For scales within the window $l_{fs} < l < l_\phi$ the neutrino clustering is strongly enhanced (for $z < z_{eg}$) due to the additional attractive force mediated by cosmon exchange. This enhanced clustering starts first for scales close to l_{fs}. One may thus investigate the possible formation of lumps with a characteristic scale around l_{fs}. For the range $l_\phi < l < H^{-1}$ one expects again an adiabatic growth of the coupled neutrinos and cosmon fluctuations, approximated by a single fluid. In summary, on large scales $l > l_{fs}$ the neutrino fluctuations grow similar to the heavy growing matter fluctuations. The growth starts, however, only very late for $z > z_R$ and only from a low level given by the tiny fluctuations in a relativistic fluid at z_R.

It is interesting to realize that Ω_h during the matter dominated epoch depends only on α (Eq. (8)), while during the final accelerated phase depends on α, γ (Eq. (10)). The linear fluctuation growth during the two phases will also depend on the two parameters in a different way; an estimation of the growth rate during the two epochs will therefore constrain separately the two model parameters. Along with the comparison between the neutrino mass and the dark energy equation of state (Eq. (3)) this offers a direct way to test the growing matter scenario.

A close look at Fig. 1 shows oscillations of Ω_g, starting around z_c and being damped subsequently. Both the oscillation period and the damping time can be understood in terms of the eigenvalues of the stability matrix for small fluctuations around the future attractor solution [7,8]. We note, however, that the oscillations concern only the relative distribution between Ω_g and Ω_h, while the sum $\Omega_h + \Omega_g = 1 - \Omega_M$ remains quite smooth. A detection of the oscillations by investigations of the background evolution, like supernovae, seems extremely hard—the luminosity distance is a very smooth function of z.

We have explored here only the simplest possibility of constant α and β. It is well conceivable (and quite likely) that in a fundamental theory α and β are functions of ϕ. Slow changes will not affect our phenomenological discussion which only concerns a rather small range of ϕ/M. In contrast, extrapolations back to the Planck epoch or inflation could look completely different. Our scenario does not need a huge overall change of the mass of the growing matter particles. For neutrinos a growth of the mass by a factor 10^7, corresponding in the seesaw mechanism to a decrease of the right-handed neutrino mass from M to 10^{11} GeV , would largely be sufficient, provided a fast change happens during recent cosmology. We conclude that our solution of the why now problem by neutrino growing matter leads to realistic cosmologies for large enough α and γ. It will be a challenge to measure the deviations from the ΛCDM model or to falsify the growing matter scenario. For neutrino growing matter a determination of α and β would fix the neutrino mass, allowing for an independent test of this hypothesis by comparing with laboratory experiments.

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