Mon. Not. R. Astron. Soc. **000**, 1–5 (2006) Printed 24 April 2007 (MN LaTeX style file v2.2)

Variation of the Amati Relation with the Cosmological Redshift: a Selection Effect or an Evolution Effect?

Li-Xin Li*
Max-Planck-Institut für Astrophysik, 85741 Garching, Germany

Accepted 2007 April 23. Received 2007 April 21; in original form 2007 April 12

ABSTRACT
Because of the limit in the number of gamma-ray bursts (GRBs) with available redshifts and spectra, all current investigations on the correlation among GRB variables use burst samples with redshifts that span a very large range. The evolution and selection effects have thus been ignored, which might have important influence on the results. In this Letter, we divide the 48 long-duration GRBs in Amati (2006, 2007) into four groups with redshift from low to high, each group contains 12 GRBs. Then we fit each group with the Amati relation $\log E_{\rm iso} = a + b \log E_{\rm peak}$, and check if the parameters a and b evolve with the GRB redshift. We find that a and b vary with the mean redshift of the GRBs in each group systematically and significantly. Monte-Carlo simulations show that there is only ~ 4 percent of chance that the variation is caused by the selection effect arising from the fluence limit. Hence, our results may indicate that GRBs evolve strongly with the cosmological redshift.

Key words:
cosmology: theory – gamma-rays: bursts – gamma-rays: observations.

1 INTRODUCTION

A remarkable progress in the observation of gamma-ray bursts (GRBs) has been the identification of several very good correlations among the GRB observables (see Schaefer 2007 for a review). Based on several of those correlations, some people have eagerly proposed to use GRBs as standard candles to probe the cosmological Hubble diagram to very high redshift (Schaefer 2003; Dai, Liang & Xu 2004; Ghirlanda et al. 2004b; Lamb et al. 2005; Firmani et al. 2006; Schaefer 2007, and references therein). Enlightening comments and criticism on GRBs as standard candles can be found in Bloom, Frail & Kulkarni (2003) and Friedman & Bloom (2005).

All the GRB correlations have been obtained by fitting a hybrid GRB sample without discriminating the redshift. Indeed, the redshift in the sample usually spans a very large range: from $z \sim 0.1$ up to $z \sim 6$. This is of course caused by the fact that we do not have an enough number of GRBs with measured redshifts limited in a small range. Then, inevitably, the effect of the GRB evolution with the redshift, and the selection effects, have been ignored. This raises an important question about whether the relations that people have found reflect the true physics of GRBs or they are just superficial. [See Band & Preece (2005) for a nice discussion on the selection effect and the correlation between the GRB peak spectral energy and the isotropic-equivalent/jet collimated energy.]

For objects distributing from $z \sim 0.1$ to $z \sim 6$, it is hard to believe that they do not evolve. There are cumulative evidences suggesting that long-duration GRBs prefer to occur in low-metallicity galaxies (Fynbo et al. 2003; Hjorth et al. 2003; Le Floc'h et al. 2003; Sollerman et al. 2005; Fruchter et al. 2006; Stanek et al. 2006). With a sample of five nearby GRBs, Stanek et al. (2006) have found that the isotropic energy of GRBs is anti-correlated with the metallicity in the host galaxy (see, however, Wolf & Podsiadlowski 2007). It is well known that metallicities evolve strongly with the cosmological redshift (Kewley & Kobulnicky 2005; Savaglio et al. 2005). Hence, the evolution of GRBs with the redshift is naturally expected (see, e.g., Langer & Norman 2006).

In this Letter, we use the Amati relation as an example to test the cosmic evolution of GRBs. The Amati relation is a correlation between the isotropic-equivalent energy of long-duration GRBs and the peak energy of their integrated spectra in the GRB frame (Amati et al. 2002)

$$\log E_{\rm iso} = a + b \log E_{\rm peak} \ . \tag{1}$$

The isotropic-equivalent energy $E_{\rm iso}$ is defined in the 1–10000 keV band in the GRB frame.

With a sample of 41 long GRBs with firmly determined redshifts and peak spectral energy, Amati (2006) has obtained that $a = -3.35$ and $b = 1.75$ with the least squares method ($E_{\rm peak}$ in keV and $E_{\rm iso}$ in 10^{52} erg); and $a = -4.04$

* E-mail: lxl@mpa-garching.mpg.de

and $b = 2.04$ with the maximum likelihood method with an intrinsic dispersion in the relation (1) being included. Long GRBs detected by *Swift* and having measured redshifts and $E_{\rm peak}$ are found to be consistent with the Amati relation (Amati 2007).

The difference in the values of the parameters obtained with the two methods can be explained as follows. The maximum likelihood method directly probes the intrinsic relation between the two variables, $x = \log E_{\rm iso}$ and $y = \log E_{\rm peak}$ (D'Agostini 2005). However, roughly speaking, the least squares method estimates the average value of x at a given y, $\langle x\rangle = a' + b'y$. Teerikorpi (1984) has shown that, when x has a Gaussian distribution with a dispersion σ_x, and the relation $x = a + by$ has an intrinsic dispersion $\sigma_y^{\rm i}$ in y, b' is related to b by

$$b' = b\left(1 + \frac{b^2 {\sigma_y^{\rm i}}^2}{\sigma_x^2}\right)^{-1} . \quad (2)$$

Amati (2006) has found that $\sigma_x \approx 0.9$, $b \approx 2.04$, and $\sigma_y^{\rm i} \approx 0.15$. Then by equation (2) we have $b' \approx 1.83$, which is close to the value of 1.75 obtained by the least squares method.

To test if the Amati relation varies with the cosmological redshift, in this Letter we separate a sample of 48 long GRBs [consisting of the long 41 GRBs from Amati (2006) and seven additional *Swift* long GRBs from Amati (2007)] into four groups by the GRB redshift. That is, we sort the GRBs by their redshifts, and divide them into four groups with redshifts distributing from low values to high values. Each group contains 12 GRBs (for details see Section 2). We then fit each group by equation (1) and calculate the mean redshift, and check if the values of a and b evolve with the redshift.

As we will see that, the values of a and b strongly vary with the redshift. The variation is not likely to arise from the selection effect and hence may indicate that GRBs evolve strongly with the cosmological redshift.

Throughout the Letter, we follow Amati (2006) to adopt a cosmology with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, and $H_0 = 70$ km s^{-1} Mpc^{-1}.

2 VARIATION OF THE AMATI RELATION WITH THE COSMOLOGICAL REDSHIFT

To test if the Amati relation (1) evolves with the redshift, we separate a sample of 48 long GRBs into four groups according to the redshift of the GRBs, then fit each group with equation (1).

The sample contains 41 long GRBs from Amati (2006, Table 1), and seven additional *Swift* long GRBs from Amati (2007). The additional seven *Swift* GRBs are 060115, 060124, 060206, 060418, 060707, 060927, and 061007. Since a GRB sample with $z \lesssim 0.1$ is very incomplete, we select only GRBs with $z > 0.1$ and hence GRB 060218 ($z = 0.0331$) is not included. GRB 060614 is also excluded from our sample because of the very large uncertainty in its $E_{\rm peak}$ (Amati et al. 2007).

A least squares fit to the 48 GRBs as a single sample with equation (1) leads to $a = -3.42$, $b = 1.78$, with $\chi_r^2 = 5.9$. The χ_r^2 is the reduced χ^2, i.e., the χ^2 of the fit divided by the degree of freedom. A maximum likelihood fit, which includes an intrinsic dispersion $\sigma_{\rm i}$ in $\log E_{\rm peak}$ in the relation (1), leads to $a = -4.08$, $b = 2.04$, and $\sigma_{\rm i} = 0.14$. These results are consistent with that obtained with 41 GRBs by Amati (2006).

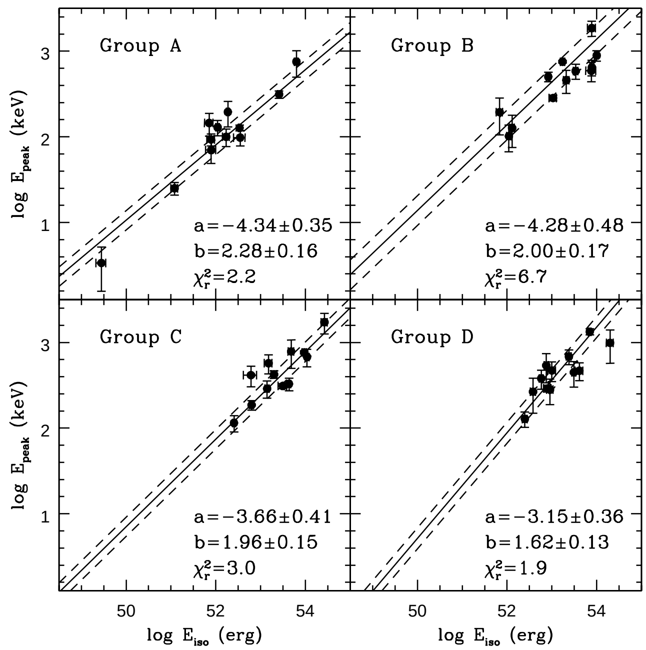

Figure 1. Least squares fit to each of the four groups of GRBs (data points with error bars; see text) by equation (1) (solid line). The two dashed lines mark the 1-σ deviation of the fit.

The redshift of the 48 GRBs spans a range of 0.17–5.6. GRB 030329 has the minimum redshift ($z = 0.17$). GRB 060927 has the maximum redshift ($z = 5.6$). The mean redshift is $\langle z\rangle = 1.685$. We separate the 48 GRBs into four groups with redshifts from low to high, each group contains 12 GRBs:

Group A—12 GRBs, $0.1 < z < 0.84$, $\langle z\rangle = 0.56$;
Group B—12 GRBs, $0.84 \leqslant z < 1.3$, $\langle z\rangle = 1.02$;
Group C—12 GRBs, $1.3 \leqslant z < 2.3$, $\langle z\rangle = 1.76$;
Group D—12 GRBs, $2.3 \leqslant z \leqslant 5.6$, $\langle z\rangle = 3.40$.

The least squares fit to each group of GRBs by equation (1), taking into account the errors in both $E_{\rm peak}$ and $E_{\rm iso}$, is shown in Fig. 1. Immediately one can see that, except Group B, the χ_r^2 for each group is smaller than that obtained by fitting the whole sample of GRBs. This fact indicates that treating the GRBs at different redshifts as a single sample may increase the data dispersion (see Fig. 3 below).

We find that the values of a and b vary with the mean redshift of the GRBs monotonically.[1] In Fig. 2 we plot a and b against $\langle z\rangle$. Clearly, a and b are correlated/anti-correlated with $\langle z\rangle$. The Pearson linear correlation coefficient between a and $\langle z\rangle$ is $r(a,\langle z\rangle) = 0.975$, corresponding to a probability $P = 0.025$ for a zero correlation. The correlation coefficient between b and $\langle z\rangle$ is $r(b,\langle z\rangle) = -0.960$, corresponding to a probability $P = 0.040$ for a zero correlation.

A least squares linear fit to a–$\langle z\rangle$ (the solid line in the

[1] Generally, the variations of a and b are not independent, see e.g., Li & Paczyński 2006

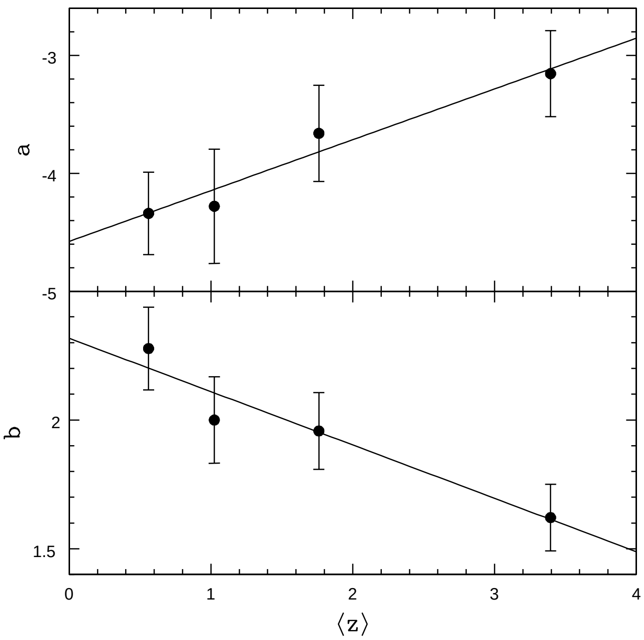

Figure 2. The fitted values of a and b against the mean redshift of GRBs. Each data point with error bars represents a group of GRBs (A, B, C, and D). The solid line is a least squares linear fit to a–$\langle z\rangle$ and b–$\langle z\rangle$.

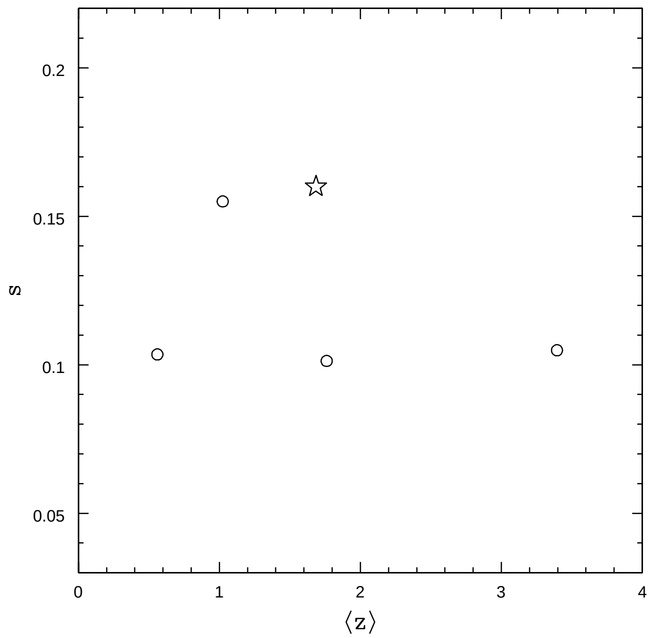

Figure 3. The deviation of fit. Each circle corresponds to a group of GRBs. The star represents the result obtained by fitting the whole sample (48 GRBs), which is $s = 0.16$.

upper panel of Fig. 2) leads to

$$a = -4.58(\pm 0.36) + 0.43(\pm 0.17)\, z \ , \qquad (3)$$

with $\chi_r^2 = 0.13$. A least squares linear fit to b–$\langle z\rangle$ (the solid line in the lower panel of Fig. 2) leads to

$$b = 2.32(\pm 0.15) - 0.207(\pm 0.066)\, z \ , \qquad (4)$$

with $\chi_r^2 = 0.31$.

The results indicate that a and b strongly evolve with the cosmological redshift.

In Fig. 3 we plot the deviation of fit (s; see Bevington & Robinson 1992, Li & Paczyński 2006) against the mean redshift of GRBs. There is not a clear trend for s to vary with $\langle z\rangle$. But it appears that the deviation of fit of each group is smaller than that of the whole sample.

3 IS THE VARIATION CAUSED BY THE SELECTION EFFECT?

To check if the variation of a and b with the cosmological redshift is caused by the selection effect, we use Monte-Carlo simulations to generate a sample of GRBs according to a pre-assumed Amati relation (1) and with a limit in the observed GRB fluence. Then, we divide the sample into four groups by the GRB redshift and fit each group by equation (1), just as we did in Section 2.

The lower limit in the bolometric fluence, $F_{\rm bol,lim}$, leads to a lower limit in the isotropic-equivalent energy of a detectable burst at redshift z

$$E_{\rm iso,lim} = 4\pi D_{\rm com}^2 (1+z) F_{\rm bol,lim} \ , \qquad (5)$$

where $D_{\rm com}$ is the comoving distance to the burst.

In Fig. 4 upper panel, we plot the isotropic energy of the 48 GRBs in the sample of Amati (2006, 2007) against their redshifts. The isotropic energy is clearly correlated with the redshift, with a Pearson linear correlation coefficient $r = 0.437$ and a probability $P = 0.0019$ for a zero correlation. The dashed line in the figure is the limit given by equation (5) with $F_{\rm bol,lim} = 1.2 \times 10^{-6}$ erg cm^{-2}, which reasonably represents the selection effect.

The distribution of the redshifts of the GRBs in the sample is plotted in Fig. 4 lower panel. It can be fitted by a log-normal distribution, with a mean $\mu = 0.151$ and a dispersion $\sigma = 0.332$ in $\log z$. The χ_r^2 of the fit is 0.25. Then, the frequency distribution in $\log z$ is

$$f_1(\log z) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(\log z - \mu)^2}{2\sigma^2}\right] \ , \qquad (6)$$

whose integration over $\log z$ (from $-\infty$ to ∞) is unity.

The distribution of the isotropic-equivalent energy is also described by a log-normal distribution, with a mean $= 1.09$ and a dispersion $= 0.85$ in $\log E_{\rm iso}$ ($E_{\rm iso}$ in 10^{52} erg).

Define $x \equiv \log E_{\rm iso}$ and $y \equiv \log E_{\rm peak}$, where $E_{\rm iso}$ is in 10^{52} erg, and $E_{\rm peak}$ is in keV. Assuming that the Amati relation is valid and independent of the redshift, and for a given x we have $y = mx + p$ with an intrinsic dispersion $\sigma_{\rm i}$ in y. Then, for a given x, the Gaussian distribution of y is given by

$$f_2(y) = \frac{1}{\sqrt{2\pi}\sigma_{\rm i}} \exp\left[-\frac{(y - mx - p)^2}{2\sigma_{\rm i}^2}\right] \ . \qquad (7)$$

By our maximum likelihood fit results in Section 2, we take $m = 0.49$, $p = 2.00$, and $\sigma_{\rm i} = 0.14$.

The Monte-Carlo simulation is done as follows. First, we generate N redshifts with the distribution in equation (6). Then, at each redshift, we generate an isotropic-equivalent energy with a log-normal distribution (mean of $\log E_{\rm iso} = 1.09$, dispersion $= 0.85$) and satisfying $E_{\rm iso} > E_{\rm iso,lim}$. Fi-

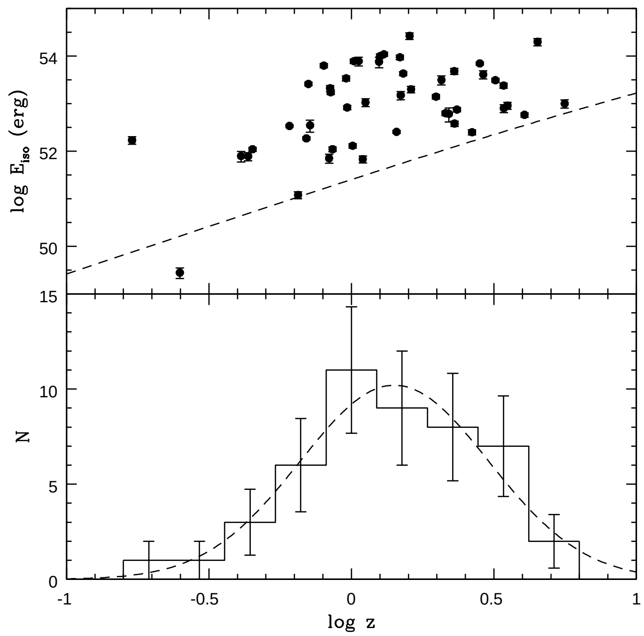

Figure 4. Upper panel: The isotropic-equivalent energy versus the redshift for the 48 GRBs in the sample. The dashed line is the limit given by equation (5) with $F_{\rm bol,lim} = 1.2 \times 10^{-6}$ erg cm^{-2}. Lower panel: The redshift distribution for the GRBs (histogram). The vertical error bar represents the Poisson fluctuation. The dashed curve is a fit to the N–$\log z$ relation by a Gaussian function. The bin size in $\log z$ is 0.1778.

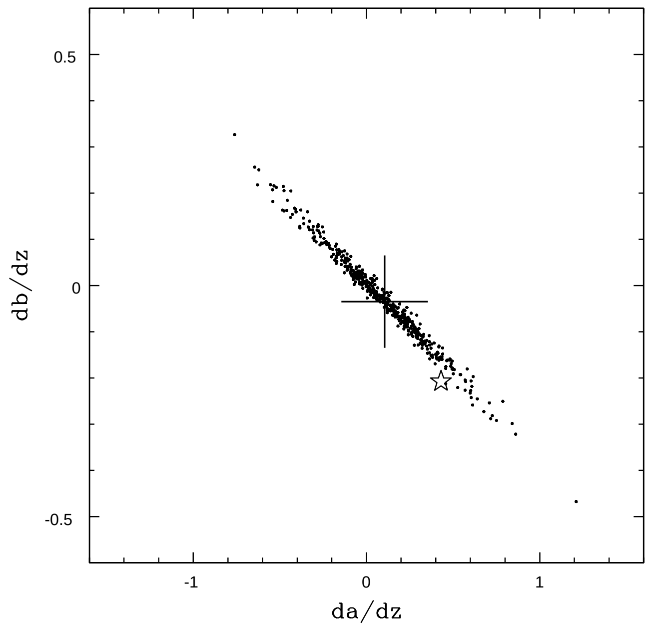

Figure 5. The slopes da/dz and db/dz obtained from Monte-Carlo simulations (points; 500 runs of 48 GRBs). The cross marks the average values of $da/dz = 0.105$ and $db/dz = -0.0347$, obtained with 10000 runs of 4000 GRBs (see text). The star is the value obtained with the 48 observed GRBs (eqs. 3 and 4).

nally, for any pair of $(z, E_{\rm iso})$, we generate a peak energy $E_{\rm peak}$ according to the distribution in equation (7). Then we have a sample of N GRBs, each GRB has a redshift, a peak spectral energy, and an isotropic-equivalent energy. These GRBs satisfy the distributions described above, and the select condition defined by equation (5) (with $F_{\rm bol,lim} = 1.2 \times 10^{-6}$ erg cm^{-2}).

With the above approach, we generated $N = 4000$ GRBs. We divided them into four groups by redshift, and each group contains 1000 GRBs. Then, we fitted each group of GRBs by equation (1) and got the values of a and b, and checked the evolution of a and b with the mean redshift $\langle z\rangle$. We repeated the process 10000 times, each time with a different sample of 4000 GRBs. We found that a and b indeed varied with $\langle z\rangle$

$$a = -3.57 + 0.105\, z\,, \qquad b = 1.84 - 0.0347\, z\,. \qquad (8)$$

This variation was caused by the selection effect, i.e. the limit in equation (5). If we turned off the limit, we found that a and b did not evolve with $\langle z\rangle$. However, comparing equation (8) to equations (3) and (4), we found that the selection effect is not likely the cause for the evolution in equations (3) and (4), since the a and b in equation (8) evolve too slowly with z. Even if we increased the value of $F_{\rm bol,lim}$ to 10^{-5} erg cm^{-2}, we got $da/dz = 0.22$ and $db/dz = -0.065$, whose values are still too small to explain the evolution in equations (3) and (4).

Of course, equation (8) only describes the average evolution of a and b for the 10000 runs. For each run, the evolution may deviate from equation (8). To obtain the chance probability for the evolution in equations (3) and (4) to arise from the selection effect, we used the Monte-Carlo simulation described above to generate $N = 48$ GRBs and repeated the process 500 times. For each 48 GRBs obtained in each run, we separate them into four groups and calculate da/dz and db/dz just as we did to the 10000 runs of 4000 GRBs. The results are shown in Fig. 5.

Based on our simulations (500 runs of 48 GRBs), we found that the probability for getting a pair of $(da/dz, db/dz)$ with $da/dz > 0.43$ and $db/dz < -0.207$ is 0.04. Hence, we have only ~ 4 percent of chance that the variation presented in Section 2 is caused by the selection effect.

4 CONCLUSIONS

If GRBs do not evolve with the redshift and the selection effects are not important, we would expect that the Amati relation does not change with the redshift. Hence, from the variation of the Amati relation with the redshift we may get some clues on the cosmic evolution of GRBs.

By dividing the 48 GRBs in Amati (2006, 2007) into four groups by their redshifts and fitting each group separately, we have found that the isotropic-equivalent energy and the peak spectral energy of GRBs remain being correlated in each group, even with a smaller dispersion than that for the whole sample. However, the parameters a and b in the Amati relation (1) evolve strongly with the redshift (eqs. 3 and 4).

Although the selection effect arising from the limit in the GRB fluence may cause a similar variation of a and b (eq. 8), generally the variation is too slow to explain what we have found for the observed GRBs (eqs. 3 and 4). With Monte-Carlo simulations we have shown that there is only ~ 4 percent of chance that the observed variation is caused by the selection effect. Hence, the variation of the Amati relation with the redshift that we have discovered may reflect the cosmic evolution of GRBs and indicates that GRBs are not standard candles.

Our results are limited by the small number of GRBs in the sample: we have 48 GRBs in total, and only 12 GRBs in each group. To get a more reliable conclusion, the number of GRBs with well determined redshifts and spectra need be significantly expanded. Since the launch of *Swift*, the fraction of GRBs with measured redshifts has increased rapidly. However, unfortunately, due to the narrow energy range of the Burst Alert Telescope (BAT) on *Swift*, the fraction of bursts that have accurately determined peak/isotropic energy has not increased proportionally. The Gamma-ray Large Area Space Telescope (*GLAST*) scheduled for launch in late 2007 will provide us with more promise for this purpose (Omodei 2006).

We must also stress that our treatment on the selection effect has been greatly simplified. The GRBs in the sample were detected and measured by different instruments, hence the selection effect is much more complicated. A more careful consideration of the various selection biases is required to determine if the observed evolution of the Amati relation reflects the cosmic evolution of GRBs.

No matter what the conclusion will be (the variation of parameters is caused by the GRB evolution effect or by the selection effect), our results suggest that it is a great risk to use GRBs with redshifts spanning a large range as a single sample to draw physics by statistically analyzing the correlations among observables. Although we have only tested the Amati relation, it would be surprising if any of the other relations (e.g., the Ghirlanda relation; Ghirlanda, Ghisellini & Lazzati 2004a) does not change with the redshift.

ACKNOWLEDGMENTS

The author thanks the referee Dr. P. O'Brien for a very helpful report. The Letter was based on a presentation by the author at the debate on "Through GRBs to Ω and Λ?" during the conference "070228: The Next Decade of GRB Afterglows" held in Amsterdam, 19–23 March 2007. The author acknowledges all the attendants at the debate for exciting and inspiring discussions.

REFERENCES

Amati L., 2006, MNRAS, 372, 233
Amati L., 2007, Il Nuovo Cimento C, in press (arXiv:astro-ph/0611189v2)
Amati L., Della Valle M., Frontera F., Malesani D., Guidorzi C., Montanari E., Pian E., 2007, A&A, 463, 913
Amati L. et al., 2002, A&A, 390, 81
Band D., Preece R. D., 2005, ApJ, 627, 319
Bevington P. R., Robinson D. K., 1992, Data Reduction and Error Analysis for the Physical Sciences. McGraw-Hill, New York
Bloom J. S., Frail D. A., Kulkarni S. R., 2003, ApJ, 594, 674
D'Agostini G., 2005, arXiv:physics/0511182v1
Dai Z. G., Liang E. W., Xu D., 2004, ApJ, 612, L101
Firmani C., Avila-Reese V., Ghisellini G., Ghirlanda G., 2006, MNRAS, 372, L28
Friedman A. S., Bloom J. S., 2005, ApJ, 627, 1
Fruchter A. S. et al., 2006, Nat, 441, 463
Fynbo J. P. U. et al., 2003, A&A, 406, L63
Ghirlanda G., Ghisellini G., Lazzati D., 2004a, ApJ, 616, 331
Ghirlanda G., Ghisellini G., Lazzati D., Firmani C., 2004b, ApJ, 613, L13
Hjorth J. et al., 2003, ApJ, 597, 699
Kewley L., Kobulnicky H. A., 2005, in de Grijs R., Gonzalez Delgado R. M., eds, Starbursts: From 30 Doradus to Lyman Break Galaxies. Springer-Verlag, Berlin, p. 307
Lamb D. Q. et al., 2005, white paper submitted to the Dark Energy Task Force (arXiv:astro-ph/0507362v1)
Langer N., Norman C. A., 2006, ApJ, 638, L63
Le Floc'h E. et al., 2003, A&A, 400, 499
Li L.-X., Paczyński B., 2006, MNRAS, 366, 219
Omodei N., 2006, in Holt S. S., Gehrels N., Nousek J. A., eds, Gamma-Ray Bursts in the Swift Era. American Institute of Physics, NY, p. 642
Savaglio S. et al., 2005, ApJ, 635, 260
Schaefer B. E., 2003, ApJ, 583, L67
Schaefer B. E., 2007, ApJ, in press (arXiv:astro-ph/0612285v1)
Sollerman J., Östlin G., Fynbo J. P. U., Hjorth J., Fruchter A., Pedersen K., 2005, New. Astron., 11, 103
Stanek K. Z. et al., 2006, Acta Astron., 56, 333
Teerikorpi P., 1984, A&A, 141, 407
Wolf C., Podsiadlowski P., 2007, MNRAS, 375, 1049

This paper has been typeset from a TEX/ LATEX file prepared by the author.