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On the size of H II regions around high-redshift quasars

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ABSTRACT
We investigate the possibility of constraining the ionization state of the intergalactic medium (IGM) close to the end of reionization ($z \approx 6$) by measuring the size of H II regions in high-z quasar spectra. We perform a combination of multiphase smoothed particle hydrodynamics and three-dimensional radiative transfer simulations to predict reliably the properties of typical high-z quasar H II regions, embedded in a partly neutral IGM ($x_{\rm H\,I} = 0.1$). In this work we assume a fixed configuration for the quasar lifetime and luminosity, i.e. $t_{\rm Q} = 10^7$ yr and $\dot{N}_\gamma = 5.2 \times 10^{56}$ s^{-1}. From the analysis of mock spectra along lines of sight through the simulated QSO environment, we find that the H II region size derived from quasar spectra is on average 30 per cent smaller than the physical one. Additional maximum likelihood analysis shows that this offset induces an overestimate of the neutral hydrogen fraction, $x_{\rm H\,I}$, by a factor of ≈3. By applying the same statistical method to a sample of observed QSOs, our study favours a mostly ionized ($x_{\rm H\,I} < 0.06$) universe at $z = 6.1$.

Key words: radiative transfer – methods: numerical – intergalactic medium – cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

Studies of the Gunn–Peterson (Gunn & Peterson 1965, GP) test have now firmly established that at $z \gtrsim 6$ the mean volume (mass) weighted neutral hydrogen fraction, $x_{\rm H\,I}$, is higher than 10^{-3} (10^{-4}) (e.g. Fan et al. 2006a); a more precise determination is hampered by the strong sensitivity of Lyα photon resonant scattering to even tiny amounts of H I. Nevertheless, assessing the exact value of $x_{\rm H\,I}$ at $z \sim 6$ would provide crucial information to discriminate between different reionization scenarios and identify the nature and distribution of ionizing sources.

The size of H II regions around high-z luminous quasars prior to complete reionization is strongly dependent on $x_{\rm H\,I}$. Previous studies (e.g. Wyithe & Loeb 2004; Wyithe, Loeb & Carilli 2005) have tried to use this observable to improve upon the above GP constraints.

In a simple (pre-overlap) reionization picture in which isolated quasar H II regions expand into an intergalactic medium (IGM) with mean neutral hydrogen fraction $x_{\rm H\,I}$, the (physical) size of the H II region is

$$R_{\rm d} \approx \left(\frac{3\dot{N}_\gamma t_{\rm Q}}{4\pi n_{\rm H} x_{\rm H\,I}} \right)^{1/3}, \qquad (1)$$

where $\dot{N}_\gamma$ and $t_{\rm Q}$ are the ionizing photon emission rate and the quasar lifetime, respectively, and $n_{\rm H}$ is the hydrogen number density. This expression does not account for the recombinations occurring in the ionized gas, which can be neglected as typical quasar lifetimes ($t_{\rm Q} = 10^{6-8}$ yr) are much smaller than the IGM recombination time-scales at these epochs. Also note that equation (1) does not hold during the relativistic phase of the H II region expansion, which lasts as long as $\dot{N}_\gamma t \gtrsim (4\pi/3) n_{\rm H} x_{\rm H\,I} (ct)^3$. For typical high-$z$ quasar luminosity this phase could last as long as $t_{\rm Q}$. However (see e.g. White et al. 2003), the size of the *observed* H II region along the line of sight (LOS) to the quasar turns out to have exactly the same time evolution as deduced by assuming an infinite speed of light. In summary, equation (1) can be safely used to describe the size of the quasar H II region observed in quasar absorption spectra and to put constraints on $x_{\rm H\,I}$.

In practice, this procedure is affected by a number of possible sources of error, which range from uncertainties in the quasar properties (lifetime, luminosity, highly biased environment) to intrinsic problems in the practical definition of the H II region size from the observed spectra. Despite all these problems, the available set of quasar spectra with a transmission region inside the GP trough have been exploited to constrain $x_{\rm H\,I}$. Wyithe & Loeb (2004) and Wyithe et al. (2005) performed a statistical comparison between observed and predicted radii, and derived a mean $x_{\rm H\,I} \gtrsim 0.1$ at $z \approx 6$. An independent study by (hereafter MH04 Mesinger & Haiman 2004), not involving a direct measurement of the H II region size, found $x_{\rm H\,I} \gtrsim 0.2$ by modelling observed properties of the Lyα and Lyβ regions of the spectrum of the $z = 6.28$ quasar SDSS J1030+0524. These results seem to indicate a largely neutral IGM at those epochs, somewhat at odds with recent interpretations of the 3-yr *Wilkinson*

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Microwave Anisotropy Probe (*WMAP*) data (Choudhury & Ferrara 2006; Gnedin & Fan 2006). Independent analysis of the same QSO spectra by Yu & Lu (2005) and Fan et al. (2006a) finds much lower values for $x_{\rm H\,I}$. Furthermore, a recent theoretical analysis by Bolton & Haehnelt (2007) argues that current observed spectra are consistent with a very broad range of $x_{\rm H\,I}$ values.

In this work we move a step further: by performing a detailed statistical analysis of mock quasar spectra extracted from combined smoothed particle hydrodynamics and three-dimensional radiative transfer simulations, we quantify the confidence level at which $x_{\rm H\,I}$ can be constrained by inverting equation (1).

2 SIMULATIONS AND RESULTS

We have performed a combination of multiphase smoothed particle hydrodynamics (SPH) and three-dimensional (3D) radiative transfer (RT) simulations, in order to predict reliably the geometrical shape of the H II region around a typical quasar observed at $z \gtrsim 6$. High-z luminous QSOs reside in rare overdense regions where the IGM physical properties are highly biased (Yu & Lu 2005). This bias has been taken into account by using a snapshot centred at $z_{\rm Q} = 6.1$ of the G5 simulation described in Springel & Herquist (2003). With its large computational volume ($100\,h^{-1}$ comoving Mpc on a side) and a particle resolution of 2×324^3, G5 allows us to follow the quasar H II region volume properly at a sufficiently high resolution. The density field is centred on the most massive halo, $M_{\rm halo} \approx 2.9 \times 10^{12}$ M$_{\odot}$. This mass is consistent with that expected for haloes hosting high-z luminous quasars (Wyithe & Loeb 2004).

The SPH density field has been mapped on a Cartesian grid with 128^3 cells, in order to perform full 3D RT simulations. We have used the RT code CRASH (Ciardi et al. 2001; Maselli, Ferrara & Ciardi 2003), which follows the time evolution of gas ionization state and temperature. Our spatial resolution does not allow us to resolve the IGM clumpiness at scales above ≈ 0.76 Mpc comoving; this prevents us from correctly accounting for non-linear clumps like haloes, as well as for their effects on recombination rate and on shielding.

The quasar is embedded in the most massive halo and we have adopted a Telfer template (Telfer et al. 2002) in the energy range 13.6–42 eV as its ultraviolet spectrum. For computational economy, He physics was not included in the simulations. This might lead to a underestimate of the ionized gas temperature and could affect the recombination rate. A higher temperature might increase the inner (resonant) Lyα opacity, essentially because of thermal broadening of the line. In order to quantify this effect, we have re-calculated some examples of mock spectra along the same LOS as used in the analysis presented in the paper, increasing the temperature by 30 per cent.[1] Since recombinations are negligible and we have checked that the corresponding thermal broadening is not affecting the Lyα resonant opacity in a sensible way, He inclusion would not change the results shown here.

The quasar radiation is sampled by emitting $N_{\rm p} = 10^8$ photon packets. We have tested numerical convergence by running lower resolution simulations with $0.5N_{\rm p}$ and $0.25N_{\rm p}$. Furthermore we assume $t_{\rm Q} = 10^7$ yr and $\dot{N}_\gamma = 5.2 \times 10^{56}$ s^{-1}; the latter values have been adopted after MH04's analysis of SDSS J1030+0524. It is fair to note, though, that the estimates of $t_{\rm Q}$ and $\dot{N}_\gamma$ should be considered as tentative because of (i) the degeneracy among $\dot{N}_\gamma$, $t_{\rm Q}$, $x_{\rm H\,I}$ and the adopted spectral template, (ii) uncertainties in the quasar redshift, (iii) possible lensing effects, and (iv) IGM clumpiness [see e.g. White et al. (2003) for a discussion]. However, these difficulties have little impact on our study as our main aim is to estimate the confidence level at which $x_{\rm H\,I}$ can be extracted from observations.

Initially, the IGM is in photoionization equilibrium with an uniform ionizing background (produced by sources other than the considered quasar) with a mean photoionization rate $\Gamma_{12} = 0.015$ s^{-1}, yielding $\langle x_{\rm H\,I}\rangle = 0.1$. This value corresponds to the lower limit found by previous works (MH04; Wyithe & Loeb 2004); in a forthcoming paper we will present results relative to other initial $x_{\rm H\,I}$ values. In this work we do not account for spatial fluctuations in the ultraviolet background, which are believed to be in place even in a highly ionized IGM at this epoch (Maselli & Ferrara 2005). The main implication of the assumption of a uniform background is a possible underestimate of the deviation from spherical symmetry of the physical H II region. Accounting for a possible inhomogeneous background is expected to increase the dispersion of the physical radius of the H II region along different LOSs.

Fig. 1 shows the $x_{\rm H\,I}$ distribution across the quasar location at $t = t_{\rm Q}$, the end of the RT simulation. The H II region does not exhibit strong deviations from spherical symmetry. This result is not unexpected: the radiative energy density inside the H II region during the early phases of the evolution is so large that clumps possibly responsible for flux anisotropies are completely ionized and made transparent. RT effects are instead apparent in the jagged ionization front (IF), causing the size of the H II region to fluctuate along different LOSs. We define the radius of the H II region, $R_{\rm d}$, along a given LOS as the distance from the quasar at which $x_{\rm H\,I} > 10^{-3}$, marking

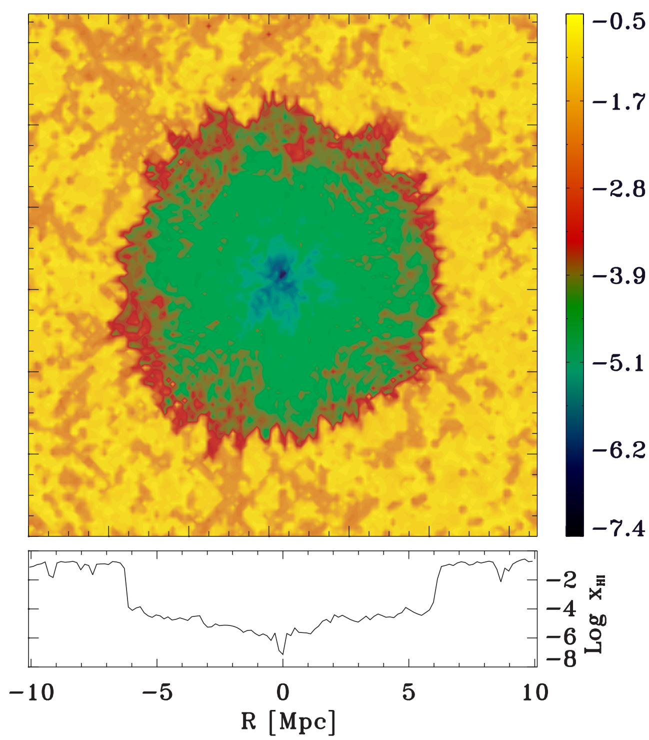

Figure 1. Map (upper panel) and cut (lower panel) of the simulated $x_{\rm H\,I}$ (logarithmic scale) across the quasar (located at the centre of the box). The quasar H II region is clearly identified.

[1] In a previous work (Maselli et al. 2003), we found that including He typically increases the temperature by 30 per cent for a mean density of 1 cm^{-3}. As in this work the densities are much lower, we can safely consider 30 per cent as an upper limit for the temperature increase due to helium photoheating.

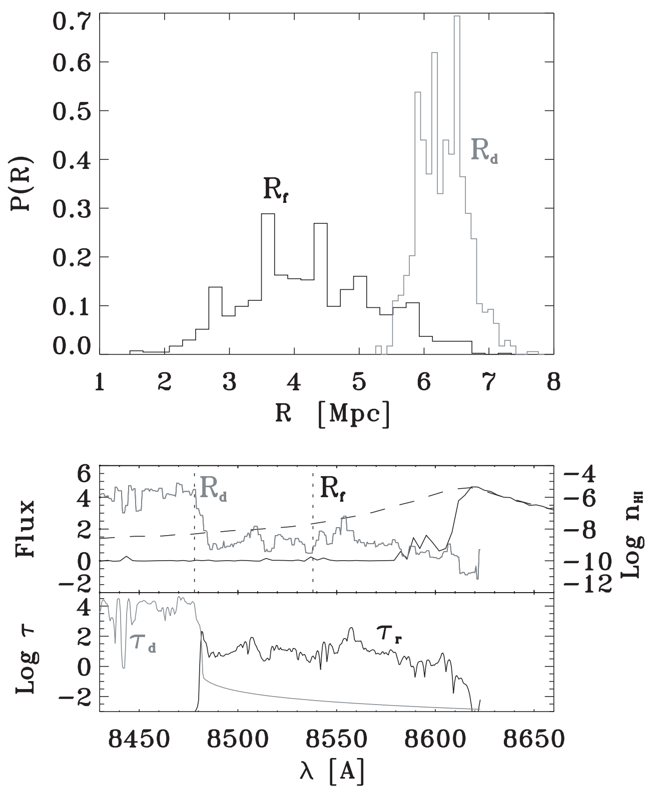

Figure 2. Upper panel: PDF for R_d and R_f (physical units) using 1000 LOSs through the simulation box. The offset between the two distributions quantifies the apparent shrinking (see text). Middle panel: illustrative template (dashed line) and absorbed (solid dark) spectra, along with the $n_{\rm H\,I}$ density distribution (light grey) as a function of observed wavelength, for a representative LOS. Bottom panel: contributions to the total GP optical depth, τ, from neutral hydrogen within (τ_r, dark) and outside (τ_d, light grey) the H II region for the same as LOS shown in the middle panel.

the IF. The RT-induced scatter in the radius of the H II region is seen in Fig. 2, via the probability distribution function (PDF) of R_d resulting from a sample of 1000 LOSs piercing the box through the quasar position. The mean value, $\langle R_d \rangle = 6.29 \pm 0.37\,(1\sigma)$, matches quite well the one derived from equation (1). In addition, the uncertainty on R_d induced by RT effects is likely to be smaller than the experimental error on the z_Q determination.[2]

Next, we derive 1000 mock quasar absorption spectra along the same set of LOSs as used for R_d. The details of the adopted technique are given in Gallerani, Choudhury & Ferrara (2006): in brief, each spectrum is characterized by a spectral resolution $\mathcal{R} = \lambda/\Delta\lambda \sim 8000$. To enable comparison with data each spectrum has been smoothed to $\mathcal{R} = 4500$ and Gaussian noise has been added, yielding a signal-to-noise ratio S/N = 50 (see e.g. Fan et al. 2006b).

From these spectra we aim to derive the observed H II region radius, R_f. In general, $R_f \neq R_d$ owing to possible effects of the Lyα damping wing absorption arising from H I located outside the H II region, and to resonant absorption from H I inside it. The definition of R_f is somewhat arbitrary, as the transmissivity of the IGM at $z \approx 6$ is a mixture of dark gaps and transmission peaks (Fan et al. 2006a). As a consequence, the edge of the H II region cannot be simply identified with the first point at which the transmitted flux drops to zero. Two different methods have been used so far in the literature: (i) R_f corresponds to the red side of the GP trough;[3] (ii) R_f is identified by the redshift at which the transmitted flux is >0.1, when the spectrum is rebinned to $\Delta\lambda = 20$ Å (Fan et al. 2006a). We have applied both methods to derive R_f from our synthetic spectra and found only marginal discrepancies. The R_f PDF obtained from method (i) is shown in Fig. 2 (top panel). From the figure, a large offset between $\langle R_d \rangle = 6.29$ Mpc and $\langle R_f \rangle = 4.25$ Mpc is seen: i.e. the size of the H II region extracted from the spectra is systematically underestimated. We refer to this effect as 'apparent shrinking'. Also shown in Fig. 2 (middle panel) are the template and absorbed spectra, along with the $n_{\rm H\,I}$ density distribution as a function of observed wavelength, for a representative LOS.

The total GP optical depth τ, responsible for the apparent shrinking, is the sum of two contributions: the damping wing absorption τ_d arising from H I outside the H II region,[4] and the resonant one, τ_r, from residual H I inside it. A detailed analysis of the mock spectra shows that, for $x_{\rm H\,I} = 0.1$, the τ_r contribution to τ is dominant. This can be appreciated from the lower panel of Fig. 2, where τ_r and τ_d are plotted separately along the same representative LOS.[5] The substantial contribution of resonant absorption results from the increase of the average $x_{\rm H\,I}$ with physical distance from the quasar due to flux geometrical dilution and attenuation. Close to the edge we find $\tau_r \approx 400$, on average.

The apparent shrinking introduces a mean systematic underestimate of the physical H II region size, R_d, by $\Delta R = \langle (R_d - R_f)/R_d \rangle = 0.32$. Note that the amplitude of ΔR is well above errors induced by RT effects and uncertainties in the quasar parameters. ΔR has a considerable dispersion around the mean value above, mostly due to the large fluctuation of R_f along different LOSs (see Fig. 2). In addition, there is no specific correlation between R_d and R_f along different LOSs. Both of these effects can be understood from the fact that R_f depends on the Lyα optical depth, which in turn is much more sensitive than the ionizing continuum opacity to tiny fluctuations of $x_{\rm H\,I}$ inside the bubble. In order to predict such fluctuations properly, it is very important to perform accurate RT calculations.

In practice, though, the lack of correlation between R_d and R_f makes it very difficult to derive R_d precisely from the observed spectrum, making it crucial to quantify the reliability of $x_{\rm H\,I}$ constraints obtained from R_f measurements in high-z quasar spectra. To address this issue, we have performed a maximum likelihood analysis of our results. Given the R_f distribution derived from our mock spectra by applying method (ii), we have calculated its likelihood function (LF) to match a Gaussian distribution, $\mathcal{G}(R)$, of mean value given by equation (1), and r.m.s. equal to that of R_f. We impose the following priors: $\dot{N}_\gamma = 5.2 \times 10^{56}$ s^{-1}, $x_{\rm H\,I} \in [0, 1]$, $t_Q \in [10^6, 10^8]$ yr. For the quasar lifetime t_Q we assume an a lognormal prior with mean

[2] Accurate quasar redshift determinations are compromised by the systematic velocity offset between emission lines of highly ionized elements (i.e. C IV and Si IV) and narrow lines probing the host galaxy directly (i.e. O III or CO molecular lines). Fan, Carilli & Keating (2006b) quantify an induced mean error in the measured redshift of $\Delta z \approx 0.02$, corresponding to a proper distance of ~1.2 Mpc at $z \sim 6$.

[3] As far as this work is concerned, we are taking into account the Lyα GP trough. The analysis of the Lyβ region could provide a larger R_f size. See Section 3 for further discussion.

[4] If the quasar H II region is embedded in a partially neutral IGM, the H I outside the H II region can produce significant absorption at wavelengths that in physical space correspond to the ionized region (see e.g. Madau & Rees 2000).

[5] As τ_d is $\propto x_{\rm H\,I}$ outside the H II region, it only becomes comparable to τ_r when $x_{\rm H\,I} \gtrsim 0.5$, and overcomes τ_r in the case of a complete neutral IGM.

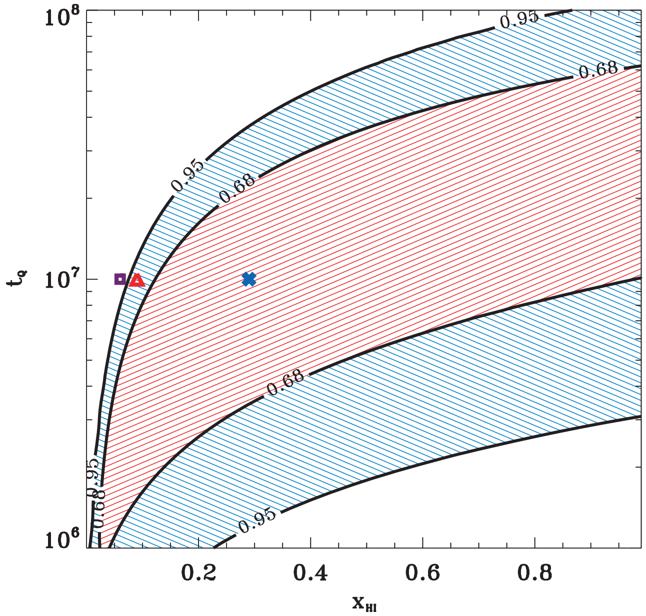

Figure 3. Likelihood contours (68 and 95 per cent confidence limit)for the $R_{\rm f}$ distribution in the ($x_{\rm H\,I}$, $t_{\rm Q}$) plane. The cross and triangle indicate the most likely values ($\hat{x}_{\rm H\,I}$, $\hat{t}_{\rm Q}$) for the $R_{\rm f}$ and $R_{\rm d}$ distributions, respectively. The square is the same quantity derived from the observed quasar sample.

equal to $t_{\rm Q} = 10^7$ yr; uncertainties in $\dot{N}_\gamma$ can be absorbed into $t_{\rm Q}$ variations. We have rejected ($x_{\rm H\,I}$, $t_{\rm Q}$) pairs yielding $R_{\rm d}$ values outside the extent of the $R_{\rm f}$ distribution. The LF maximum, ($\hat{x}_{\rm H\,I}$, $\hat{t}_{\rm Q}$), identifies the most likely values inferred from a given sample of observed quasar spectra. We find ($\hat{x}_{\rm H\,I}$, $\hat{t}_{\rm Q}$) = (0.34, 10^7 yr) – the point marked by a cross in Fig. 3: however, these values lay 2σ away from the actual values used in the simulation, ($x_{\rm H\,I}$, $t_{\rm Q}$) = (0.1, 10^7 yr), i.e. those that we would like to recover. As a sanity check we have repeated our maximum likelihood analysis on the $R_{\rm d}$ distribution, and indeed we find that the maximum coincides with the simulation values (triangle in Fig. 3). We conclude that the apparent shrinking effect induces an overestimate of the $x_{\rm H\,I}$ by a factor[6] $\approx$3. It is worth noting that, although our $R_{\rm f}$ distribution has been drawn from a simulation of a single quasar with fixed intrinsic properties ($\dot{N}_\gamma$, $t_{\rm Q}$, $z_{\rm Q}$), the range of acceptable $x_{\rm H\,I}$ values is quite large. For example, fixing $t_{\rm Q}$ to the simulation value, 10^7 yr, still allows $x_{\rm H\,I} >$ 0.1, 0.07 to a 1σ, 2σ confidence level. Possible dispersions in the intrinsic properties of the quasar sample are likely to make the $x_{\rm H\,I}$ determination even more difficult.

We have also applied the maximum likelihood analysis to a sample of six observed QSOs, having $z_{\rm Q} \in [6.0, 6.2]$, the spectra of which have been studied by Fan et al. (2006a). Such authors give the measured radii scaled to a reference common quasar absolute magnitude $M_{1450} = -27$. We have then calculated the LF of the sample of observed spectra to match a Gaussian with mean value given by equation (1) where $\dot{N}_\gamma$ is scaled to[7] to $M_{1450} = -27$, retaining the observed sample luminosity dispersion.

In this case the LF maximum is ($\hat{x}_{\rm H\,I}$, $\hat{t}_{\rm Q}$) = (0.06, 10^7), shown as a square in Fig. 3. By taking into account the overestimate of this result due to the apparent shrinking effect, and the uncertainties on the location of the maximum induced by the wide range of acceptable $x_{\rm H\,I}$ values, we find that our study slightly favours a mostly ionized universe at $z \approx 6.1$. By varying $\dot{N}_\gamma$ in the range [2.7, 7.7] $\times 10^{56}$ suggested by MH04, and scaling the theoretical radii accordingly, our results are always consistent with $x_{\rm H\,I} < 0.2$.

3 SUMMARY AND DISCUSSION

In this Letter we have discussed the robustness of constraints on the IGM H I fraction inferred from the extent of H II regions around high-redshift quasars by means of their absorption spectra.

We have performed a combination of state-of-the-art multiphase SPH and 3D RT simulations to predict reliably the properties of a typical high-z quasar H II region (e.g. extent, geometrical shape, inner opacity). We have found that RT effects do not induce strong deviations from spherical symmetry. The RT-induced dispersion in the H II region size along different LOSs is in fact of the order of roughly 6 per cent of the mean radius which is likely to be smaller than the typical error induced on $R_{\rm d}$ estimates by uncertainties in the quasar redshift determinations.

By deriving and analysing mock spectra through the simulated quasar environment, we have found that the H II region size deduced from quasar spectra, $R_{\rm f}$, typically underestimates the physical one by 30 per cent.

The fact that the observed H II region sizes can substantially underestimate the size of the region impacted by the ionizing radiation of the quasar was already noted by Bolton & Haehnelt (2007), but our results give the first quantitative estimate of the above underestimation, which we refer to as apparent shrinking. It is worthwhile to note here that the resolution of our simulations is not sufficient to resolve properly the IGM clumping which could result in an underestimate of RT effects. In this respect, our results can be considered a lower limit for the amplitude of the apparent shrinking effect: resolving the high-density clumps would result in fact in a higher inner opacity which is expected to be the origin of a stronger apparent shrinking.

The amplitude of the apparent shrinking effect could be decreased by defining $R_{\rm f}$ as the red side of the GP trough in the Lyβ region ($R_{\rm f}^\beta$). A clear offset between the red side of the GP trough in Lyα and Lyβ has been observed in the spectrum of the QSO SDSS J1030+0524 (White et al. 2003; MH04). Recently, Bolton & Haehnelt (2007) have argued that the ratio $R_{\rm f}^\beta/R_{\rm f}$ has a well-defined trend with $x_{\rm H\,I}$, which could be exploited to constrain such a quantity. However, the robustness of this result is uncertain given the small number of LOSs used in the study, and further investigation is needed.

In addition our analysis shows that, in the case of $x_{\rm H\,I} = 0.1$, the apparent shrinking is almost completely due to resonant absorption of residual H I inside the ionized bubble. This contribution is highly fluctuating along different LOSs, resulting in a large dispersion of the observed distribution of radii, which is mainly due to the fact that $R_{\rm f}$ depends on the inner Lyα opacity which is highly sensitive to $n_{\rm H\,I}$ fluctuations. This result implies that in order to account for these fluctuations properly, detailed RT calculations are required; the present work is unique in this respect, being the only study that accounts for the detailed effects of 3D RT.

The maximum likelihood analysis that we have performed on a sample of 1000 mock spectra shows that the apparent shrinking effect induces an overestimate of the $x_{\rm H\,I}$ by a factor of $\approx$3, if the IGM is only partially ionized ($x_{\rm H\,I} = 0.1$). Moreover, by applying the above analysis to a sample of observed QSOs, we conclude that our study favours a mostly ionized universe at $z \sim 6.1$ ($x_{\rm H\,I} \lesssim 0.06$), a

[6] This factor could depend slightly on $x_{\rm H\,I}$ and, to a lesser extent, on $\dot{N}_\gamma$.
[7] We use the scaling $\dot{N}_\gamma = \dot{N}_{\gamma,0} \times 10^{(-27-M_{1450,0})/2.5}$. We use $\dot{N}_{\gamma,0}$ = $(5.2 \pm 2.5) \times 10^{56}$ s^{-1} and $M_{1450,0} = -27.2$ which are the estimated values for the $z = 6.28$ QSO SDSS J1030+0524.

somewhat different conclusion with respect to the previous results given in MH04, Wyithe & Loeb (2004) and Wyithe et al. (2005), which found a lower limit for $x_{\rm H\,I}$ of 0.2, 0.1 and 0.1 respectively. Nevertheless, Bolton & Haehnelt (2007) found as an outcome of their analysis that the sizes of the H II regions in the higher redshift quasar spectra are consistent with a significantly neutral surrounding IGM, as well as with a highly ionized IGM. Furthermore, as mentioned in the Introduction, independent studies by Yu & Lu (2005) and Fan et al. (2006a) give evidence for much lower values of $x_{\rm H\,I}$. It is worth noting that our measurement agrees with the independent determination by Fan et al. (2006a) who found $x_{\rm H\,I} \simeq 1.3 \times 10^{-3}$, based on an analysis of the H II region size evolution.

Uncertainties remain owing to the fact that the range of acceptable $x_{\rm H\,I}$ values associated with a given sample of measured radii is quite large, e.g. our $R_{\rm f}$ sample, having as its most probable value $x_{\rm H\,I} = 0.34$, still allows $x_{\rm H\,I} > 0.1$, 0.07 to a 1σ, 2σ confidence level. This suggests that measurements of the H II size in quasar spectra can only provide rough constraints on $x_{\rm H\,I}$, as long as the knowledge of intrinsic properties of observed QSOs remains incomplete.

It is worth noting that quasar spectra could contain additional useful information on $x_{\rm H\,I}$. MH04 constrained $x_{\rm H\,I}$ from an analysis of the SDSS J1030+0524 absorption spectrum, which is completely independent from constraining $x_{\rm H\,I}$ using the radius extent. In this case the $x_{\rm H\,I}$ determination is based on the difference between the Lyα and Lyβ optical depths and it is free from the uncertainties related to the apparent shrinking effect. It is interesting to note that, differently from our results, MH04 find that the slightly smaller size of the H II region extent inferred from the Lyα and Lyβ opacity is due to the presence of a damping wing absorption produced by the H I in the IGM unperturbed by the quasar. This difference can be due to the fact that MH04 compute the neutral hydrogen fraction at the equilibrium with the quasar ionizing flux which is diluted geometrically, without taking into account the further attenuation due to the inner opacity. As a consequence the inner opacity itself is underestimated. We found instead that the inner opacity, the correct determination of which requires detailed 3D RT calculations, must be taken into account for a reliable determination of the apparent shrinking effect. A significant underestimate of the inner (resonant) opacity would require a higher $x_{\rm H\,I}$ outside the H II region in order to reproduce the observed difference between the apparent shrinking in the Lyα and Lyβ flux. In any case, though, it is necessary to await a larger sample of observed quasars and simulate a larger parameter space ($x_{\rm H\,I}$, $t_{\rm Q}$, $\dot{N}_\gamma$) before being able directly to compare results coming from different approaches in the modelling as well as in the statistical analysis technique adopted.

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