Mon. Not. R. Astron. Soc. **372,** 1357–1365 (2006) doi:10.1111/j.1365-2966.2006.10943.x

Correlation between the peak spectral energy of gamma-ray bursts and the peak luminosity of the underlying supernovae: implication for the nature of the gamma-ray burst–supernova connection

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Accepted 2006 August 15. Received 2006 August 1; in original form 2006 May 1

ABSTRACT

In this paper, we present a correlation between the peak spectral energy of gamma-ray bursts (GRBs) and the peak bolometric luminosity of the underlying supernovae (SNe), based on a sample of four pairs of GRBs–SNe with a spectroscopically confirmed connection. Combining it with the well-known relation between the peak spectral energy and the isotropic equivalent energy of GRBs, we obtain an upper limit on the isotropic energy of GRBs, which is $\approx 10^{52}\,\mathrm{erg}(L_{\mathrm{SN,peak}}/10^{43}\,\mathrm{erg\,s^{-1}})^{10}$, where $L_{\mathrm{SN,peak}}$ is the peak bolometric luminosity of the SNe. Our results suggest that the critical parameter determining the GRB–SN connection is the peak luminosity of SNe, rather than the feature of the SN spectra and/or the SN explosion energy as commonly hypothesized. Because it is generally believed that the peak luminosity of SNe powered by radioactive decays is related to the amount of ^{56}Ni produced in the SN explosion, the mass of ^{56}Ni may be a key physical factor for understanding the nature of GRBs and their connection with SNe. Application of our relation to Type Ibc SNe with normal peak luminosities indicates that, if those normal SNe have GRBs accompanying them, the GRBs would be extremely soft and subenergetic in gamma-rays and, hence, easier to detect with X-ray or UV detectors than with gamma-ray detectors.

Key words: supernovae: general – gamma-rays: bursts.

1 INTRODUCTION

The discovery of SN 1998bw within the error box of GRB 980425 (Galama et al. 1998) inspired a lot of consideration of the connection between gamma-ray bursts (GRBs) and supernovae (SNe). Since then, three more pairs of GRBs and SNe with a spectroscopically confirmed connection have been found, namely GRB 030329/SN 2003dh (Hjorth et al. 2003; Stanek et al. 2003), GRB 031203/SN 2003lw (Malesani et al. 2004; Sazonov, Lutovinov & Sunyaev 2004), and the most recent one discovered by *Swift*, GRB 060218/SN 2006aj (Campana et al. 2006; Cobb et al. 2006; Masetti et al. 2006; Mirabal et al. 2006; Modjaz et al. 2006; Pian et al. 2006; Sollerman et al. 2006). Interestingly, all of the four SNe are among a special class of Type Ic SNe, called the broad-lined SNe, which are characterized by smooth and featureless spectra indicating a very large expansion velocity (Della Valle 2006; Woosley & Heger 2006b, and references therein). Modelling of the SN light curves reveals that the SNe with a GRB connection have a very large explosion energy and mass production of ^{56}Ni compared with normal Type Ibc SNe (Iwamoto et al. 1998; Nakamura et al. 2001; Deng et al. 2005; Mazzali et al. 2006b), except SN 2006aj, which requires an explosion energy that is comparable to that of normal Type Ibc SNe (Mazzali et al. 2006a). These facts have motivated people to invent the term 'hypernovae' for this special and much more powerful class of SNe (Iwamoto et al. 1998; see also Paczyński 1998a,b).

A less direct way of identifying the GRB–SN connection is observing the rebrightening and/or flattening (called 'red bumps') in the late GRB afterglows, which can be interpreted as the emergence of the underlying SN light curves (Bloom et al. 1999; Zeh, Klose & Hartmann 2004; Soderberg et al. 2005; Bersier et al. 2006, and references therein). Although alternative explanations with dust echoes (Esin & Blandford 2000) and dust sublimation (Waxman & Draine 2000) have been proposed, several groups have successfully fitted SN 1998bw templates to explain the late-time bumps (Bloom et al. 2002; Garnavich et al. 2003; Greiner et al. 2003; Stanek et al. 2005; Bersier et al. 2006). A systematic study on the GRB afterglows with this approach by Zeh et al. (2004) suggests that all long-duration GRBs are associated with SNe.

Despite the exciting developments in the past 8 years in the detection and observation of the GRB–SN connection, to date no quantitative relation between the parameters of GRBs and those of SNe has been found (see, e.g. Zeh et al. 2004; Ferrero et al. 2006), although

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Table 1. Gamma-ray bursts and supernovae with spectroscopically confirmed connection.†

GRB/SN	z^{a}	$E_{\gamma,\rm peak}{}^{b}$	$E_{\gamma,\rm iso}{}^{c}$	$M_{\rm SN,peak}{}^{d}$	$E_{\rm K}{}^{e}$	$M_{\rm ej}{}^{f}$	$M_{\rm Nickel}{}^{g}$
980425/1998bw	0.0085	55 ± 21	$0.000\,09 \pm 0.000\,02$	-18.65 ± 0.20	5.0 ± 0.5	10 ± 1	0.38–0.48
030329/2003dh	0.1687	79 ± 3	1.7 ± 0.2	-18.79 ± 0.23	4.0 ± 1.0	8 ± 2	0.25–0.45
031203/2003lw	0.1055	159 ± 51	0.009 ± 0.004	-18.92 ± 0.20	6.0 ± 1.0	13 ± 2	0.45–0.65
060218/2006aj	0.0335	4.9 ± 0.4	0.0059 ± 0.0003	-18.16 ± 0.20	0.2 ± 0.02	2 ± 0.2	0.2 ± 0.04

†Following Mazzali et al. (2006b), we assume $H_0 = 72\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_{\rm m} = 0.28$ and $\Omega_\Lambda = 0.72$. Quantities calculated in a different cosmology are converted to this cosmology.
[a]Cosmic redshift: from Galama et al. (1998; GRB 980425/SN 1998bw), Stanek et al. (2003; GRB 030329/SN 2003dh), Prochaska et al. (2004; GRB 031203/SN 2003lw), Pian et al. (2006) and Mirabal et al. (2006; GRB 060218/SN 2006aj).
[b]Peak energy of the integrated GRB spectrum in units of keV, measured in the GRB frame: from Yamazaki, Yonetoku & Nakamura (2003; GRB 980425), Sakamoto et al. (2005; GRB 030329), Ulanov et al. (2005; GRB 031203), and Campana et al. (2006; GRB 060218).
[c]Isotropic equivalent energy of the GRB in units of 10^{52} erg, defined in the 1–10 000 keV energy band in the GRB frame: from Campana et al. (2006; GRB 060218), and Amati (2006; other bursts).
[d]Peak bolometric magnitude of the SN, defined in the 3000–24 000 Å wavelength band in the SN frame: from Mazzali et al. (2006b; SNe 1998bw, 2003lw), Pian et al. (2006; SN 2006aj), and Deng et al. (2005; SN 2003dh; and Appendix A of this paper).
[e]Explosion kinetic energy of the SN in units of 10^{52} erg: from Mazzali et al. (2006a; SN 2006aj, error of 10 per cent added), and Mazzali et al. (2006b; other SNe).
[f]Ejected mass in the SN explosion in units of $\mathrm{M_\odot}$: fom Mazzali et al. (2006a; SN 2006aj, error of 10 per cent added), and Mazzali et al. (2006b; other SNe).
[g]Mass of ^{56}Ni produced by the SN explosion in units of $\mathrm{M_\odot}$: from Mazzali et al. (2006; SN 2006aj, the error corresponds to a 0.2-mag error in magnitude), and Mazzali et al. (2006b; other SNe).

it is commonly conceived that only very bright SNe can produce GRBs, based on the fact that all SNe with a confirmed GRB connection are much brighter than average and that the rate of GRBs and 'hypernovae' is several orders of magnitude lower than the rate of core-collapse SNe (Podsiadlowski et al. 2004). The lack of a quantitative relation between GRBs and SNe has frustrated the advance in understanding the nature of GRBs, although many people believe that long-duration GRBs are produced by the core collapse of massive stars (MacFadyen, Woosley & Heger 2001; Woosley & Heger 2006a).

In this paper, we present the discovery of a quantitative relation between GRBs and underlying SNe, based on the observational data of the four pairs of GRBs–SNe with a spectroscopically confirmed connection (Table 1). We show that the peak spectral energy of the GRB is strongly correlated with the peak bolometric luminosity of the SN. Then, combining with the relation between the peak spectral energy and the isotropic equivalent energy of GRBs found by Amati et al. (2002), we explore the implications of the correlation that we have found for the GRB–SN connection and for the nature of GRBs.

2 THE PEAK SPECTRAL ENERGY OF GRBs VERSUS THE PEAK BOLOMETRIC LUMINOSITY OF SUPERNOVAE

Information on the four pairs of GRBs and SNe with a spectroscopically confirmed connection is summarized in Table 1, including their cosmic redshift, the peak spectral energy and isotropic equivalent energy of the GRBs, and the peak bolometric magnitude, explosion energy, ejected mass and nickel yield of the SNe. Following Mazzali et al. (2006b), we assume a cosmology with $H_0 = 72\,\mathrm{km\,s^{-1}\,Mpc^{-1}}, \Omega_{\rm m} = 0.28$ and $\Omega_\Lambda = 0.72$. All quantities calculated in a different cosmology are converted to the above cosmology.

Among the four bursts, GRB 030329 is the brightest one in terms of the isotropic equivalent energy (or the peak luminosity). However, the SN associated with it, SN 2003dh, is not the most powerful. Its total explosion kinetic energy is exceeded by that of SNe 1998bw and 2003lw, associated with GRBs 980425 and 031203, respectively. In terms of the bolometric luminosity, SN 2003dh is also fainter than SN 2003lw. Although GRB 030329 is very bright and energetic compared with the other three GRBs, it is significantly weaker than average long-duration GRBs.

GRB 980425, the nearest burst with measured redshift to date and the first GRB that has been discovered to be connected to an SN, is the least energetic in terms of the isotropic equivalent energy. However, the SN associated with it, SN 1998bw, is very powerful and very bright. GRB 031203, associated with SN 2003lw, is analogous to GRB 980425 in many aspects (Soderberg et al. 2004b). It is also underluminous and has a very bright and powerful SN. However, as can be seen from Table 1, GRB 031203 has a much harder spectrum than GRB 980425, as indicated by its much larger peak spectral energy.

GRB 060218, recently discovered and the second nearest one, is a very peculiar burst (Campana et al. 2006; Pian et al. 2006). It has an extremely long duration (≈2000 s) and an extremely soft spectrum (with a peak spectral energy $E_{\gamma,\rm peak} \approx 4.9$ keV in the GRB frame). It is also subenergetic, has an isotropic equivalent energy similar to that of GRB 031203. The SN associated with it, SN 2006aj, is the faintest and the least powerful one among the four GRB-connected SNe. The modelling by Mazzali et al. (2006a) reveals that it has an explosion kinetic energy $E_{\rm K} \approx 2 \times 10^{51}$ erg, ejected mass $M_{\rm ej} \approx 2\,\mathrm{M_\odot}$, and ejected $^{56}\mathrm{Ni} \approx 0.2\,\mathrm{M_\odot}$. Although SN 2006aj is much brighter than average Type Ic SNe and has a much smoother spectrum, its explosion appears to be less powerful than other GRB-connected SNe but closer to normal Type Ic SNe.

Despite the very narrow distribution in the peak bolometric magnitudes of the four SNe, from −18.16 to −18.92 mag [corresponding to a factor of 2 variation in the peak bolometric luminosity, $(0.559\text{–}1.13) \times 10^{43}\,\mathrm{erg\,s^{-1}}$], the distribution in the isotropic energy of the GRBs is extremely wide, $(0.0001\text{–}1.7) \times 10^{52}$ erg. It appears that there does not exist a relation between the isotropic equivalent energy of the bursts and the explosion energy of the SNe, as can be seen from Fig. 1.

However, there appears to be a very good correlation between the peak spectral energy of the GRB (defined in the GRB frame) and the peak bolometric magnitude (luminosity) of the SN, as shown in Fig. 2. Despite the large systematic errors in the peak bolometric magnitude relative to its narrow distribution, a correlation between

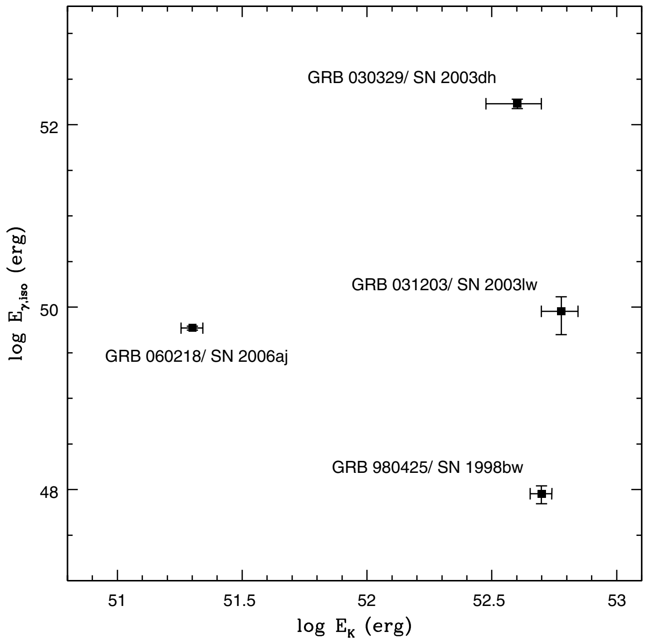

Figure 1. The isotropic equivalent energy of GRBs versus the explosion kinetic energy of the underlying SNe. Clearly, there is no correlation between them.

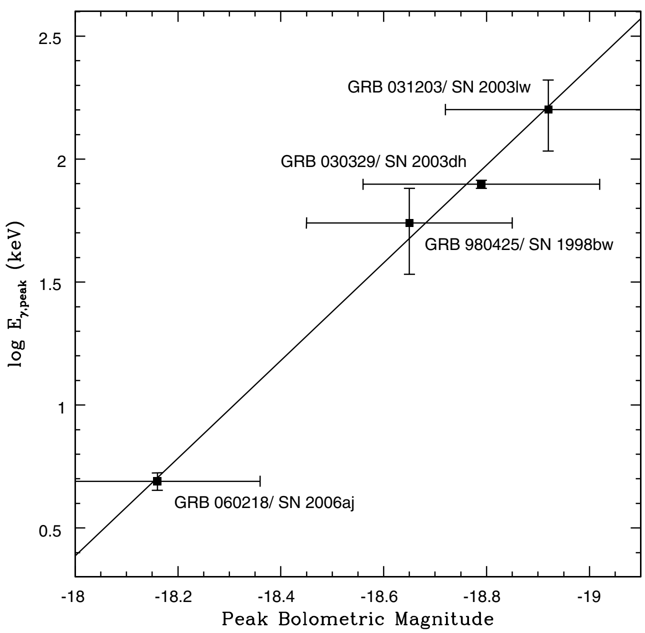

Figure 2. The peak spectral energy of GRBs versus the peak bolometric magnitude of the underlying SNe. The straight line is a least-χ^2 fit to the data (equation 1).

$M_{\mathrm{SN,peak}}$ (the peak bolometric magnitude of the SN) and $E_{\gamma,\mathrm{peak}}$ (the peak spectral energy of the GRB) is remarkable. The Pearson linear correlation coefficient between $-M_{\mathrm{SN,peak}}$ and $\log E_{\gamma,\mathrm{peak}}$ is calculated to be $r = 0.997$, corresponding to a probability $P = 0.003$ for zero correlation. This indicates that $-M_{\mathrm{SN,peak}}$ and $E_{\gamma,\mathrm{peak}}$ are strongly correlated. (For comparison, the Pearson linear correlation coefficient between the log explosion energy and the log isotropic energy in Fig. 1 is $r = 0.019$, corresponding to a probability $P = 0.981$ for zero correlation.)

A least-χ^2 linear fit to $M_{\mathrm{SN,peak}}$–$\log E_{\gamma,\mathrm{peak}}$, taking into account the errors in both variables, gives

$$\log E_{\gamma,\mathrm{peak}} = -35.38 - 1.987\, M_{\mathrm{SN,peak}} \qquad (1)$$

with χ^2/degrees of freedom (d.o.f.) = 0.02, where $E_{\gamma,\mathrm{peak}}$ is in keV. This relation is equivalent to

$$E_{\gamma,\mathrm{peak}} = 90.2\,\mathrm{keV} \left(\frac{L_{\mathrm{SN,peak}}}{10^{43}\,\mathrm{erg\,s^{-1}}} \right)^{4.97}, \qquad (2)$$

where $L_{\mathrm{SN,peak}}$ is the peak bolometric luminosity of the SN defined in the 3000–24 000 Å wavelength band in the SN frame.

It is well known that the peak luminosity of SNe powered by radioactive decays is related to the mass of ^{56}Ni generated in the SN ejecta (Arnett 1982; Maeda et al. 2003; Nomoto et al. 2004). Approximately, the maximum luminosity is proportional to the mass of ^{56}Ni. However, it also depends on the diffusion time of the photons generated by the deposition of the gamma-rays emitted by the decay of freshly synthesized ^{56}Ni to ^{56}Co and hence to stable ^{56}Fe (Mazzali et al. 2006a). To check the relation between the peak spectral energy of GRBs and the mass of ^{56}Ni produced by the SNe, in Fig. 3 we plot $E_{\gamma,\mathrm{peak}}$ against M_{Nickel}, the mass of ^{56}Ni. Not surprisingly, $E_{\gamma,\mathrm{peak}}$ is also correlated with M_{Nickel}, although the correlation is not as tight as that in $E_{\gamma,\mathrm{peak}}$–$M_{\mathrm{SN,peak}}$ in Fig. 2. The Pearson linear correlation coefficient between $\log E_{\gamma,\mathrm{peak}}$ and $\log M_{\mathrm{Nickel}}$ is $r = 0.95$, corresponding to a probability $P = 0.05$ for zero correlation.

Although the mass of ^{56}Ni is a parameter that is more physical than the peak luminosity, in this paper we focus on the relation between the peak spectral energy of GRBs and the peak luminosity of SNe because the peak luminosity is a directly measurable quantity. Unlike the mass of ^{56}Ni, the peak luminosity does not depend on the SN model and hence does not suffer the errors from the model assumptions.

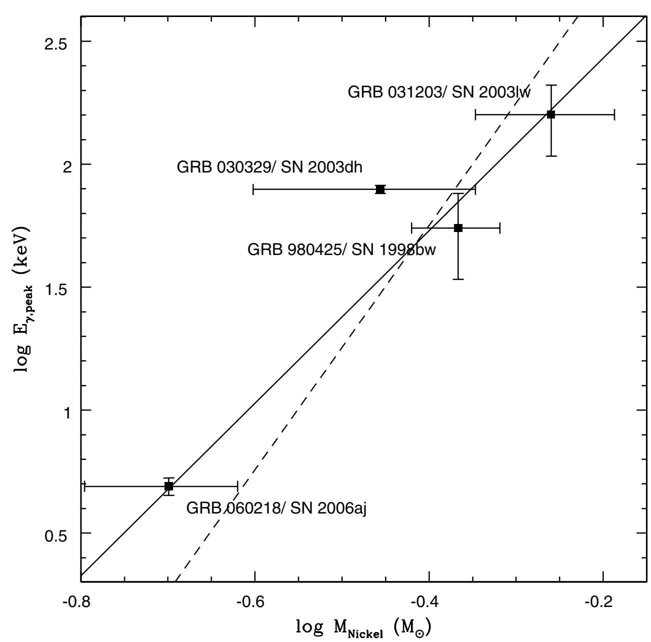

Figure 3. The peak spectral energy of GRBs versus the mass of ^{56}Ni generated in the underlying SNe. For SNe 1998bw, 2003dh and 2003lw, the value of M_{Nickel} is taken to be the mean of the upper and lower limits in Table 1. The solid straight line is a least-χ^2 fit to the data, $\log E_{\gamma,\mathrm{peak}} = 3.13 + 3.51 \log M_{\mathrm{Nickel}}$ with χ^2/d.o.f. = 0.4. If the slope is fixed at 4.97 (i.e. assuming that equation 2 holds and the nickel yield is proportional to the peak luminosity of the SNe), a least-χ^2 fit leads to $\log E_{\gamma,\mathrm{peak}} = 3.74 + 4.97 \log M_{\mathrm{Nickel}}$ with χ^2/d.o.f. = 1.03 (the dashed line).

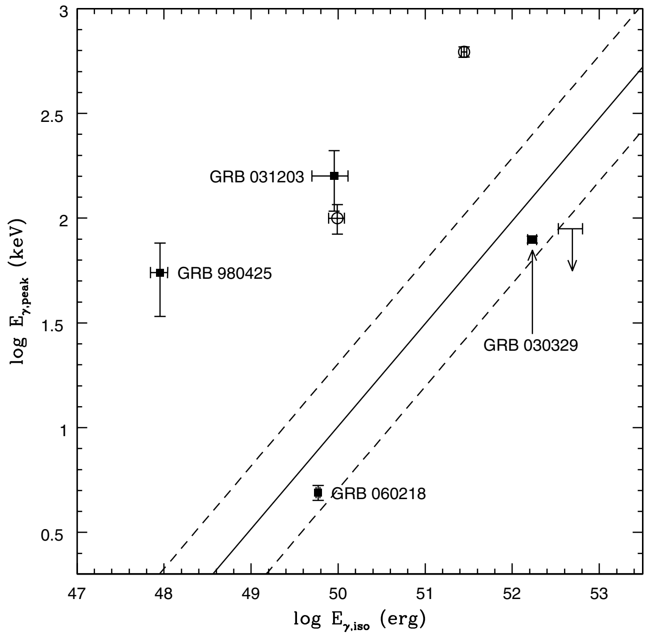

Figure 4. The peak spectral energy versus the isotropic equivalent energy, for the four GRBs with an SN connection. The solid line is the relation in equation (3), the best power-law fit to 41 long-duration GRBs (Amati 2006). The two dashed lines delineate the region of a logarithmic deviation of 0.3 (2σ) in $E_{\gamma,\rm peak}$. The two open circles are short-duration GRBs 050709 and 051221a. The downward arrow on the right shows the upper bound on the $E_{\gamma,\rm peak}$ of GRB 050315 (see the text).

3 THE ENERGETIC NATURE OF GRBs ASSOCIATED WITH SUPERNOVAE

It has been found that the isotropic equivalent energy of long-duration GRBs, defined in the 1–10 000 keV band in the GRB frame, is correlated with the peak energy of the integrated spectra of GRBs, with only a few outliers (Amati et al. 2002; Amati 2006). The correlation is even better when the correction to the GRB energy from jet collimation is included (Ghirlanda, Ghisellini & Lazzati 2004). A recent study with an updated GRB sample consisting of 41 long-duration GRBs by Amati (2006) gives, when normalized to the cosmology adopted in this paper and outliers are excluded,

$$E_{\gamma,\rm peak} = 97\,{\rm keV}\left(\frac{E_{\gamma,\rm iso}}{10^{52}\,{\rm erg}}\right)^{0.49}. \tag{3}$$

GRBs 030329 and 060218 are consistent with the relation in equation (3), but 980425 and 031203 are not (see Fig. 4). Among well-studied long-duration GRBs, 980425 and 031203 are indeed the only known outliers to the $E_{\gamma,\rm peak}$–$E_{\gamma,\rm iso}$ relation (Amati et al. 2006).

It appears that all GRBs that violate the $E_{\gamma,\rm peak}$–$E_{\gamma,\rm iso}$ relation stay on the side of having smaller isotropic energy than predicted by the relation (see Fig. 4). However, there is one possible exception: GRB 050315 at redshift 1.949, a bright long burst discovered by *Swift*. Vaughan et al. (2006) estimated that, for this burst, the peak spectral energy is $\lesssim$30 keV in the observer frame, i.e. $E_{\gamma,\rm peak} \lesssim$ 89 keV in the GRB frame. This low value of $E_{\gamma,\rm peak}$ makes GRB 050315 marginally violate the $E_{\gamma,\rm peak}$–$E_{\gamma,\rm iso}$ relation by having a slightly larger isotropic energy (Fig. 4). However, in obtaining their result, Vaughan et al. (2006) have assumed a too large absolute value for the photon index of low energy, $\alpha = -1.88$. By taking $\alpha = -1.3$, they obtained a larger upper bound for the peak spectral energy ($\lesssim$43 keV in the observer frame), making GRB 050315 closer to the $E_{\gamma,\rm peak}$–$E_{\gamma,\rm iso}$ relation. The most likely value of $-\alpha$ for GRBs observed by BATSE (on the *Compton Gamma-Ray Observatory*, *CGRO*) was 1 (Preece et al. 2000), much smaller than the value that was assumed by Vaughan et al. Hence, because of the fact that the low-energy photon index of GRB 050315 cannot be determined with the Burst Alert Telescope/*Swift* data alone, we would not consider GRB 050315 as a serious case that violates the $E_{\gamma,\rm peak}$–$E_{\gamma,\rm iso}$ relation.

Then, based on the data of GRBs that have accurately determined peak spectral energy and isotropic equivalent energy, we can say with fair confidence that the $E_{\gamma,\rm peak}$–$E_{\gamma,\rm iso}$ relation, i.e. equation (3), gives a fairly accurate estimate of the isotropic energy for normal GRBs and an upper bound on the isotropic energy for subenergetic GRBs.[1] Then, the combination of equations (2) and (3) leads to

$$E_{\gamma,\rm iso} \lesssim 0.86 \times 10^{52}\,{\rm erg}\left(\frac{L_{\rm SN,peak}}{10^{43}\,{\rm erg\,s^{-1}}}\right)^{10}. \tag{4}$$

Equation (4) provides a strong constraint on the isotropic equivalent energy of GRBs associated with SNe. Because of the very steep slope in $\log E_{\gamma,\rm iso}$–$\log L_{\rm SN,peak}$, equation (4) describes the fact that the isotropic energy of GRBs is distributed in an extremely wide range while the peak luminosity of the underlying SNe has an extremely narrow distribution.

4 THE MILDLY RELATIVISTIC NATURE OF GRBs WITH SOFT SPECTRA

A common feature of the four SN-connected GRBs is that all of them are soft, characterized by their small peak spectral energy compared with normal cosmological GRBs. An analysis by Amati (2006) on 45 GRBs with well-determined peak spectral energy shows that $E_{\gamma,\rm peak}$ can be described by a log-normal distribution with a mean $\sim$350 keV and a logarithmic dispersion of $\sim$0.45. The hardest one in the four SN-connected GRBs, 031203, has a peak spectral energy $\approx$159 keV, smaller than the mean of the distribution but still within 1σ, while the softest one, GRB 060218, has a peak spectral energy as small as $\approx$4.9 keV, deviating from the mean by 4σ.

The peak spectral energy of GRBs is anticorrelated with the jet opening angle (Lamb, Donaghy & Graziani 2005). In Fig. 5, we plot the jet opening angle at the time of jet break, versus the peak spectral energy of the burst for 26 GRBs (see the figure caption for the sources of data). All the opening angles were calculated from the time of jet break in the afterglows, except that of GRB 030329 (the only SN-connected GRB included in the plot and marked by a star), which was obtained less directly by modelling the radio afterglow. With GRB 030329 and those bursts with only limits on opening angles being excluded (then leaving 17 GRBs), we obtained a maximum-likelihood fit to the data

$$\log\theta_{\rm jet} = 3.84 - 1.17\,\log E_{\gamma,\rm peak}, \tag{5}$$

where $E_{\gamma,\rm peak}$ is in keV and $\theta_{\rm jet}$ is in degrees. This relation is not sensitive to the assumed cosmology, because the jet opening angle weakly depends on the luminosity distance (Sari, Piran & Halpern 1999; Frail et al. 2001; Bloom, Frail & Kulkarni 2003).

Thus, a smaller value of the peak spectral energy indicates a larger jet opening angle and, hence, a smaller Lorentz factor Γ because $\Gamma \sim \theta_{\rm jet}^{-1}$ at the time of jet break. For a GRB with a very small peak spectral energy, the Lorentz factor of its outflow must be very

[1] Nakar & Piran (2005) showed that at least 25 per cent of the BATSE GRBs are outliers to the $E_{\gamma,\rm peak}$–$E_{\gamma,\rm iso}$ relation and suggested that equation (3) should be considered as an upper bound on the isotropic energy of GRBs.

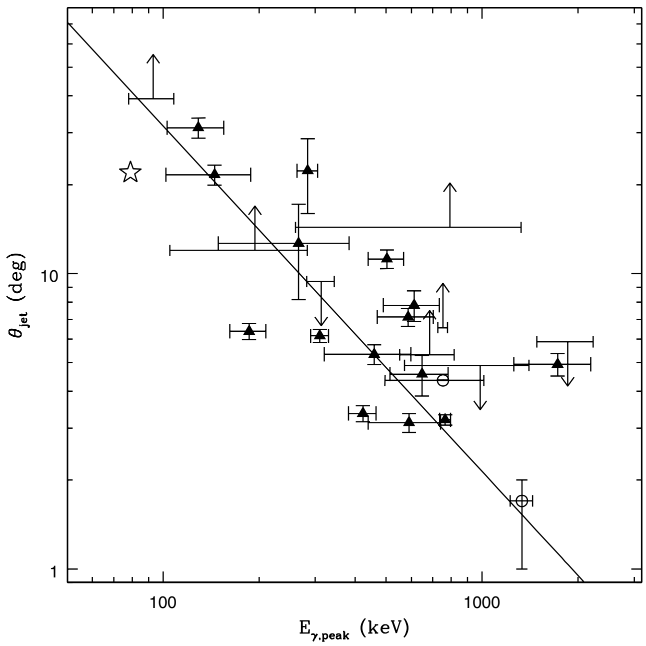

Figure 5. The jet opening angle of GRBs versus the peak energy of their spectra measured in the GRB frame. The filled triangles are 15 GRBs that have accurately determined jet opening angles, taken from Bloom et al. (2003; excluding GRB 000301c whose peak spectral energy is not available). The upward (downward) arrows are lower (upper) limits on the opening angles for eight bursts from the same paper. Two open circles are GRB 050603 (Berger & Becker 2005; Grupe et al. 2006) and GRB 051022 (without an error bar in $\theta_{\rm jet}$; Racusin et al. 2005). The peak spectral energy of all GRBs was taken from Amati (2006). Whenever they are available, error bars are indicated. The straight line is a maximum-likelihood fit to the data (equation 5), excluding the eight GRBs with only limits. [The star is GRB 030329 (not included in the fit), whose jet opening angle is $\sim 22°$ as inferred from radio observation (van der Horst et al. 2005)].

small compared with typical GRBs whose Lorentz factors have been argued to be $\gtrsim 300$ based on the fact of the presence of MeV photons in their spectra (Piran 2004). Hence, GRBs with soft spectra must be mildly relativistic, where by 'mildly relativistic' we mean that the Lorentz factor $\Gamma < 100$.

Given that $E_{\gamma,\rm peak} \approx 159$ keV for GRB 031203, from Fig. 5 its jet opening angle would be in the range of 10°–30°, while for GRB 980425 with $E_{\gamma,\rm peak} \approx 55$ keV, relation (5) predicts a jet opening angle $\sim 60°$. For GRB 060218, who has the smallest peak energy $E_{\gamma,\rm peak} \approx 4.9$ keV, the relation (5) predicts that its jet opening angle would be 1000°! Hence, the anticorrelation between the peak spectral energy and the jet opening angle indicates that GRB 060218 is almost perfectly spherical.

Equation (5) suggests that all GRBs (at least those of long duration) with peak spectral energy $\lesssim 40$ keV (in the GRB frame) are spherical and only mildly relativistic because then the predicted jet opening angle $\gtrsim 90°$.

A popular explanation for subenergetic GRBs has been that they are normal GRBs viewed away from their jet axes (Waxman 2004; Ramirez-Ruiz et al. 2005, and references therein), but our results in this section suggest that subenergetic GRBs are spherical and hence intrinsically faint. Our view is supported by the radio observations on GRBs 980425, 031203 and 060218 (Soderberg, Frail & Wieringa 2004a; Soderberg et al. 2004b, 2006b). For example, the radio afterglow light curve of GRB 060218 did not show a signature of jet break after 22 d of the burst, indicating that the jet opening angle $\theta_{\rm jet}$ > 1.4 rad $\approx 80°$ (Soderberg et al. 2006b). The fact that the rate of low-luminosity GRBs exceeds that expected from off-axis models by at least a factor of 10 also suggests that low-luminosity GRBs are intrinsically subenergetic (Cobb et al. 2006; Liang, Zhang & Dai 2006; Soderberg et al. 2006b).

5 IMPLICATION FOR THE NATURE OF THE GRB–SN CONNECTION

Although it is always a risk to extend a relation beyond the range from which the relation was derived, we cannot resist applying the relations derived in previous sections (equations 2 and 4) to normal Type Ibc SNe and to cosmological GRBs to see where the relations lead us and if the results contradict observations.

In fig. 1 of Pian et al. (2006), the brightest SN next to SN 2006aj is the 'standard' Type Ic SN 1994I in the spiral galaxy M51 with a distance 8.4 ± 0.6 Mpc from us (Feldmeier, Ciardullo & Jacoby 1997). The peak bolometric luminosity of SN 1994I is $\approx 2.34 \times 10^{42}$ erg s^{-1} (Sauer et al. 2006), fainter than SN 1998bw by 1.4 mag. By equation (4), if there was a GRB associated with SN 1994I, its isotropic energy would be $\lesssim 4 \times 10^{45}$ erg, smaller than that of GRB 980425 by 2 orders of magnitude. Although SN 1994I is 4 times closer to us than SN 1998bw/GRB 980425, the burst related to SN 1994I would still be 10 times fainter than GRB 980425 in gamma-rays if it had a similar duration. The peak spectral energy of the burst inferred from equation (2) is ≈ 0.07 keV, in the soft X-ray and extreme UV band.

Applying equations (2) and (4) to SN 1997ef (Iwamoto et al. 2000; Mazzali, Iwamoto & Nomoto 2000; Mazzali et al. 2004; Pian et al. 2006) and SN 2002ap (Mazzali et al. 2002; Tomita et al. 2006), which have been classified as 'hypernovae' by the similarity of their spectra to that of SN 1998bw and their large explosion energy, we obtain $E_{\gamma,\rm peak} \approx 0.017$ keV, $E_{\gamma,\rm iso} \lesssim 2.7 \times 10^{44}$ erg for SN 1997ef, and $E_{\gamma,\rm peak} \approx 0.016$ keV, $E_{\gamma,\rm iso} \lesssim 2.3 \times 10^{44}$ erg for SN 2002ap. The peak spectral energy of the potential bursts is in the UV band, and the isotropic gamma-ray energy is smaller than that of GRB 980425 by more than 3 orders of magnitude.

SN 1997ef, which occurred in UGC 4107, has a mass of ^{56}Ni that is about twice that in other SNe with similar brightness because of its very late peak (Iwamoto et al. 2000; Mazzali et al. 2000, 2004). Converted to the cosmology adopted in this paper, its $M_{\rm Nickel} \approx 0.13\,{\rm M}_{\odot}$. Then, the $E_{\gamma,\rm peak}$–$M_{\rm Nickel}$ relation found in Section 2 (the solid straight line in Fig. 3) gives $E_{\gamma,\rm peak} \approx 1$ keV, and hence $E_{\gamma,\rm iso} \lesssim 0.9 \times 10^{48}$ erg by equation (3). That is, the upper limit of the isotropic energy of the burst associated with SN 1997ef suggested by the $E_{\gamma,\rm peak}$–$M_{\rm Nickel}$ relation is comparable to that of GRB 980425. The $E_{\gamma,\rm peak}$–$M_{\rm Nickel}$ relation leads to larger values of $E_{\gamma,\rm peak}$ and $E_{\gamma,\rm iso}$ than the $E_{\gamma,\rm peak}$–$M_{\rm SN,peak}$ relation, resulting from the smaller slope in $\log E_{\gamma,\rm peak}$–$\log M_{\rm Nickel}$ (Fig. 3, the solid line versus the dashed line).

SN 1997ef has been suggested to be associated with GRB 971115 by the fact that the two may be compatible with each other in position and time of occurrence (Wang & Wheeler 1998). However, the correlation is much weaker than that in the case of SN 1998bw/GRB 980425. SN 1997ef was slightly outside the 2σ error box of GRB 971115, and the angular separation between them was as large as 25°. The temporal association was also weak: the maximum of the optical light curve of SN 1997ef was delayed from GRB 971115 by about 20 d, in contrast to 9–17 d for the four spectroscopically confirmed SNe–GRBs. The explosion date of SN 1997ef was estimated to be November 20 ± 1 d (Mazzali et al. 2000), delayed from GRB

971115 by 5 ± 1 d, which is much longer than typical SN–GRB time lags (Della Valle 2006).

Like SN 1994I, SN 2002ap is another nearby SN, discovered in M74 with a distance $7.8^{+0.4}_{-0.7}$ Mpc. An intensive search of all available gamma-ray data obtained between 2002 January 21 and 29 for bursts that could be localized by the Interplanetary Network (IPN) found no GRB associated with SN 2002ap (Hurley et al. 2002; see, however, Gal-Yam, Ofek & Shemmer 2002). The peak bolometric luminosity of SN 2002ap is $\approx 1.75 \times 10^{42}$ erg s^{-1}, fainter than SN 1998bw by 1.75 mag (Tomita et al. 2006). Despite its closer distance, our relation predicts that the burst associated with SN 2002ap would look ∼190 times fainter than GRB 980425 in gamma-rays.

SN 2004aw is one of the most well observed Type Ic SNe, discovered in a tidal tail of a barred spiral galaxy NGC 3997 at redshift $z = 0.0163$ (Taubenberger et al. 2006). It is intrinsically slightly brighter than SN 1994I, but fainter than SN 1998bw by 1.3 mag. The optical spectrum of SN 2004aw bridges a normal Type Ic SN like SN 1994I and the group of broad-lined Type Ic SNe. No GRB has been found to be associated with SN 2004aw (Taubenberger et al. 2006). Submitting the peak bolometric luminosity of SN 2004aw ($\approx 2.63 \times 10^{42}$ erg s^{-1}, Taubenberger et al. 2006) into equations (2) and (4), we get $E_{\gamma,\rm peak} \approx 0.12$ keV and $E_{\gamma,\rm iso} \lesssim 1.4 \times 10^{46}$ erg. Given its distance of 68.2 Mpc, the potential GRB associated with SN 2004aw would look at least 200 times fainter than GRB 980425.

Therefore, if normal Type Ibc SNe are accompanied by GRBs, the GRBs should be extremely underluminous in the gamma-ray band despite their close distances. Their peak spectral energy is expected to be in the soft X-ray and UV band, so they may be easier to detect with an X-ray or UV detector than with a gamma-ray detector. We note that, in terms of both total energy and photon energy, the bursts are similar to the shock breakout flashes predicted for Type Ibc SNe (Blinnikov et al. 2002; Li 2006). Flashes from shock breakout in SNe were first predicted by Colgate (1968) almost 40 yr ago, originally proposed for GRBs that had not been discovered yet. However, they have never been unambiguously detected in SN observations because of their short duration compared with SNe (Calzavara & Matzner 2004).

Given its very soft spectrum and underenergetic nature, and the fact that the SN associated with it appears to have a moderate explosion energy and ejected mass, we speculate that GRB 060218 is a marginal GRB because it appears to be close to the bottom line of the GRB–SN connection. This consideration is best illustrated in Fig. 6, which shows the four GRB-connected SNe and six Type Ibc SNe with no detected GRB-connection in the ejected mass–explosion energy and the nickel mass–explosion energy plane. Clearly, SN 2006aj is closer to normal Type Ibc SNe than to the other three GRB-connected SNe.

SN 2003jd, discovered in MCG–01-59-021 at redshift $z = 0.01886$, has been argued to be evidence of an aspherical explosion viewed from a direction near the equatorial plane, based on the observation of its double-peaked nebular lines of neutral oxygen and magnesium (Mazzali et al. 2005). This Type Ic SN is only slightly less luminous than SN 1998bw but brighter than SN 2006aj, thus it has been anticipated that a GRB could have accompanied it but has not been seen because of the off-axis nature. However, a radio observation on it taken ∼1.6 yr after the explosion has detected no emission from an off-axis jet, which has been used to argue against a GRB connection for SN 2003jd (Soderberg et al. 2006a). Thus, SN 2003jd might be a violator of our equations (2) and (4). However, our equation (4) only gives an upper bound on the isotropic energy of the GRB. Another point is that Soderberg et al. (2006a) have only tested a single model for off-axis GRBs and their no-detection result may have just ruled out that specific model (Mazzali, private communication).

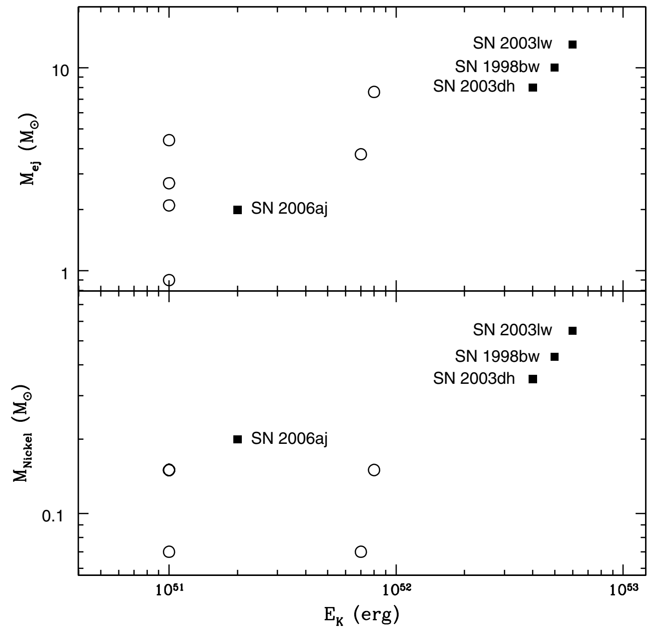

Figure 6. The ejected mass (upper panel) and nickel yield (lower panel) of core-collapse SNe, versus the SN explosion energy. The filled squares are the four SNe with a confirmed GRB connection. The open circles are six Type Ibc SNe without an observational signature for association with GRBs, taken from Hamuy (2004). The two circles with explosion energy near 10^{52} erg are the 'hypernovae' 1997ef (upper circle) and 2002ap (lower circle). The circle with the lowest ejected mass and nickel yield at $E_{\rm K} = 10^{51}$ erg is the 'standard' Type Ic SN 1994I. In the lower panel, the upper circle at $E_{\rm K} = 10^{51}$ erg represents three SNe as they have identical explosion energy and nickel yield. (For SNe 1998bw, 2003dh and 2003lw, the value of $M_{\rm Nickel}$ is taken to be the mean of the upper and lower limits in Table 1.)

If equation (2) is extended to cosmological GRBs, a limit on the peak luminosity of the underlying SNe can be calculated. Applying to GRB 990123, which is at redshift 1.6 and has the maximum determined intrinsic peak spectral energy $E_{\gamma,\rm peak} \approx 2000$ keV (and $E_{\gamma,\rm iso} = 2.66 \times 10^{54}$ erg; Amati et al. 2002; Amati 2006), we get $L_{\rm SN,peak} \approx 1.87 \times 10^{43}$ erg s^{-1}, only 2 times brighter than SN 1998bw (while GRB 990123 is brighter than GRB 980425 by 6 orders of magnitude!).

If cosmological GRBs are also associated with SNe (Zeh et al. 2004), it would be interesting to know out to what redshift would the SNe be detectable. This is a question that is not easy to answer because, as the redshift increases, the luminosity of the SN would be easily overshined by the afterglow of the GRB if the afterglow is bright. For an SN that is twice as luminous as SN 1998bw, the Ultra-Violet/Optical Telescope (UVOT) on board *Swift* would be able to detect it to a redshift of ≈0.7 according to the sensitivity of UVOT $m_{\rm B} = 24.0$ in white light in 1000 s (i.e. able to detect a 24th magnitude B-star in 1000 s; Roming et al. 2005). Because the luminosity of SN 1998bw is comparable to that of Type Ia SNe, it can be expected that the upcoming space observatory *SuperNova Acceleration Probe* (*SNAP*) would be able to detect GRB-connected SNe to redshift ∼1.7 (Aldering 2005) under favourable conditions (i.e. the afterglow of the GRB does not overshine the SN but the GRB is still detectable as in the lucky case of GRB 980425/SN 1998bw).

6 SUMMARY AND CONCLUSIONS

We have found a strong correlation between the peak spectral energy of GRBs and the peak bolometric magnitude (i.e. the peak luminosity) of their underlying SNe, based on the observational data of the four pairs of GRBs and SNe with a spectroscopically confirmed connection (Fig. 2, equations 1 and 2). The Pearson linear correlation coefficient between $\log E_{\gamma,\mathrm{peak}}$ (the peak spectral energy of GRBs) and $-M_{\mathrm{SN,peak}}$ (the peak bolometric magnitude of SNe) is 0.997, corresponding to a probability $P = 0.003$ for zero correlation. Although the sample is limited by the small number of GRBs–SNe, we consider the result to be very suggestive because of the large correlation coefficient.

Combined with the relation between the peak spectral energy and the isotropic equivalent energy of GRBs (Amati 2006), the correlation that we have found leads to a relation between the isotropic energy of a GRB and the peak bolometric luminosity of the underlying SN (equation 4). If a GRB is among the normal cosmological class (i.e. it has a normal total gamma-ray energy) and is indeed associated with an SN, then equation (4) would take the $\approx$ sign. If a GRB is subenergetic, like some of the SN-connected GRBs, equation (4) gives an upper bound on the isotropic gamma-ray energy.

The slope of $\log E_{\gamma,\mathrm{iso}}$–$\log L_{\mathrm{SN,peak}}$ is extremely steep, which is $\approx$10 by equation (4). This naturally describes the observational fact that GRBs have a very large diversity in properties (e.g. the isotropic equivalent energy) compared with SNe.

Applying the relations that we have obtained (equations 2 and 4) to normal Type Ibc SNe that are not as luminous as SN 1998bw, we found that the prompt emission from the potential GRBs associated with them peaks in the soft X-ray and UV band, and the total gamma-ray energy of the bursts is extremely small. Hence, the bursts associated with normal Type Ibc SNe would be more appropriately qualified as soft X-ray transients, which might be easier to detect with X-ray or UV detectors than with gamma-ray detectors.

Despite the fact that cosmological GRBs are typically more luminous than the subenergetic GRB 980425 by 5 orders of magnitude or more, the potential SNe associated with them are expected to be brighter than SN 1998bw only by a factor of $\sim$2. Although it is hard to predict in a general case up to what distance an SN associated with a cosmological GRB can be observed, under favourable conditions a GRB-connected SN that is as luminous as a Type Ia SN should be observable up to redshift $\sim$1.7 with the upcoming *SNAP* space observatory.

Our results suggest that the critical parameter characterizing the GRB–SN connection is the large peak luminosity of the SNe, rather than the broad-lined spectra (or, equivalently, the large expansion velocity) and/or the huge explosion energy as commonly hypothesized (Nomoto et al. 2004; Della Valle 2006; Woosley & Heger 2006b, and references therein). Given the general Ansatz that the SN luminosity at the peak equals the power generated by the decay of ^{56}Ni (Arnett 1982; Maeda et al. 2003; Nomoto et al. 2004; Mazzali et al. 2006a; Pian et al. 2006), our results may indicate that the mass of ^{56}Ni produced in the SN explosion is a key physical factor for understanding the nature of the GRB–SN connection as well as the nature of GRBs (Fig. 3). Although a physical relation between the peak spectral energy of GRBs and the mass of ^{56}Ni of SNe cannot be established based only on the results in this paper, the following consideration may provide us a clue. Popular models of GRBs involve an aspherical explosion of massive stars, where gamma-ray emission is produced along the axis of the explosion via a jet or a shock (Woosley 1993; Paczyński 1998a; MacFadyen et al. 2001; Woosley & Heger 2006a). Investigation of nucleosynthesis in aspherical SN explosions indicates that ^{56}Ni is distributed also preferentially in the direction along the jet axis where the ejecta carry more kinetic energy and the shock is stronger (Maeda et al. 2002).

Finally, we remark that an SN has been claimed to be detected in the afterglow of GRB 020903, an extremely soft burst at redshift 0.251 (Soderberg et al. 2005; Bersier et al. 2006). The peak spectral energy of GRB 020903 is 3.37 ± 1.79 keV. The isotropic gamma-ray energy is $(2.8 \pm 0.7) \times 10^{49}$ erg, which makes GRB 020903 consistent with the $E_{\gamma,\mathrm{peak}}$–$E_{\gamma,\mathrm{iso}}$ relation (Amati 2006). Equation (1) then predicts that the SN associated with GRB 020903 has a peak bolometric magnitude ≈ -18.06, fainter than SN 1998bw by $\sim$0.6 mag. Fitting the SN 1998bw template to the bump in the afterglow light curve of GRB 020903, Bersier et al. (2006) found that the SN is fainter than SN 1998bw by 0.8 ± 0.1 mag at the peak in the *R* band, consistent with the 0.6 ± 0.5 mag reported by Soderberg et al. (2005) earlier. Considering the fact that the spectrum of the SN of GRB 020903 38.6 d after the burst was redder than SN 1998bw, the difference in the bolometric magnitudes is likely to be less than 0.8 mag (Bersier, private communication). It appears that GRB 020903 and its SN are consistent with equations (2) and (4), although a conclusion cannot be made without the availability of data in other filters.

ACKNOWLEDGMENTS

The author thanks D. Bersier, S. Campana, J. Deng, P. Mazzali, F. Patat, E. Pian and S. Taubenberger for useful communications and sharing data. He also thanks the referee (P. Mazzali) for a wonderful report, which has led to significant improvements to the paper.

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APPENDIX A: THE PEAK BOLOMETRIC MAGNITUDE OF SN 2003DH

The peak of the light curve of SN 2003dh was not captured (Deng et al. 2005). To obtain the peak magnitude of SN 2003dh, we fit the available light-curve data with a model-independent empirical approach.

The striking similarity between the spectrum of SN 2003dh and that of SN 1998bw (Stanek et al. 2003) enables us to use the bolometric light curve of SN 1998bw, which has been well sampled and studied (Nakamura et al. 2001; Patat et al. 2001; Mazzali et al. 2006b; Pian et al. 2006), as a template to fit the available light-curve data of SN 2003dh. This has been a standard approach in searching for SNe in the optical afterglows of GRBs (Zeh et al. 2004; Soderberg et al. 2005; Bersier et al. 2006).

First, we fit the rest-frame bolometric light curve of SN 1998bw with a polynomial. The data (the same as those used in Mazzali et al. 2006b and Pian et al. 2006) are kindly provided by E. Pian and differ from that in Patat et al. (2001) by a constant scaling factor in the bolometric luminosity. The bolometric luminosity of SN 1998bw in Mazzali et al. (2006b) is smaller than that in Patat et al. (2001) by a factor $\approx$0.83 (i.e. 0.2 mag fainter), resulting from the fact that a different cosmology and different reddening/extinction have been adopted in Patat et al. (2001). In order to construct the template light curve, we use the light-curve data from 2.5 to 187 d after GRB 980425 in the rest frame, which consist of a total of 96 data points and span a time interval that is large enough for the purpose here. We find that the light curve of SN 1998bw in the above time range is best fitted by a 9th-order polynomial (Fig. A1), with $\chi^2/\mathrm{d.o.f.} = 0.04$ (d.o.f. = 86).

Then, we take the smooth curve defined by the 9th-order polynomial (the solid curve in Fig. A1) as a template and fit it to the bolometric light curve of SN 2003dh. In doing so, we stretch the template light curve, and shift it in magnitude and time. That is, if we denote the template light curve in magnitude by $M_{\mathrm{template}}(t)$, we fit the light curve of SN 2003dh with a magnitude function

$$M(t) = M_{\mathrm{template}}(\alpha t + \beta) + M_0, \tag{A1}$$

where t is time, and α, β and M_0 are parameters to be determined.

The bolometric light-curve data of SN 2003dh are taken from Deng et al. (2005), rescaled to the cosmology adopted in Mazzali et al. (2006b) and this paper. In Deng et al. (2005), the luminosity distance of SN 2003dh was taken to be 809 Mpc (i.e. distance modulus = 39.54), while in our cosmology, the luminosity distance of SN 2003dh is 791 Mpc. Thus, the luminosity of SN 2003dh is reduced by a factor of 0.956 (i.e. 0.05 mag fainter), adopting the same reddening/extinction.

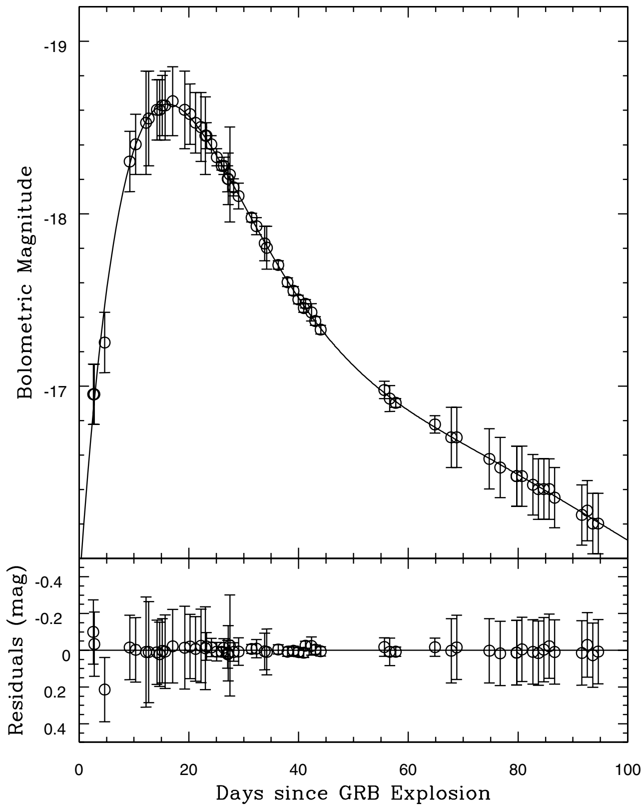

Figure A1. Upper panel: fitting the rest-frame bolometric light curve of SN 1998bw with a 9th-order polynomial. The best fit (the solid curve) has χ^2/d.o.f. = 0.04, with d.o.f. = 86. Lower panel: residuals of the fit.

The results of fitting the template to the data of SN 2003dh are as follows: $\alpha = 1.32, \beta = -1.6$ and $M_0 = -0.16$ (Fig. A2). The χ^2/d.o.f. = 0.3, where d.o.f. = 7. We estimate the peak magnitude of SN 2003dh by the minimum of the $M(t)$ (the peak of the solid curve in Fig. A2), which is -18.79 ± 0.23. The peak occurs 13.4 d after GRB 030329 in the rest frame, consistent with the 10–13 d estimated by Hjorth et al. (2003).

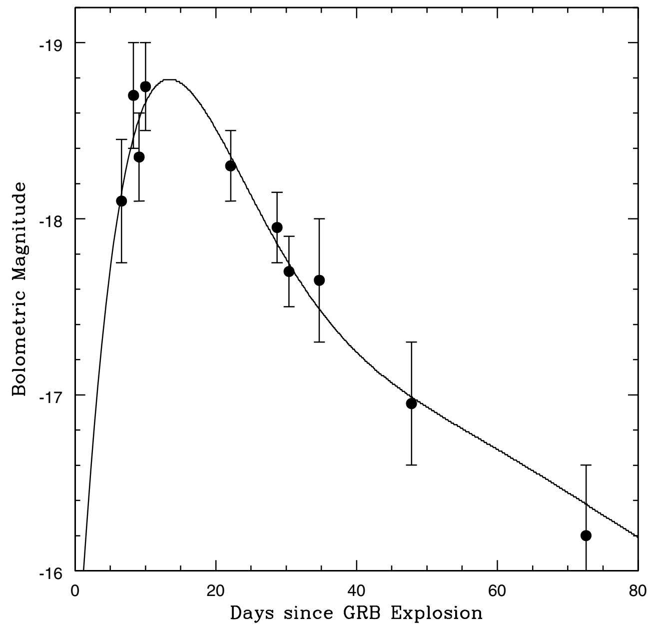

Figure A2. Fitting the rest-frame bolometric light curve of SN 2003dh with a template of SN 1998bw defined by the solid curve in Fig. A1, by stretching, rescaling the template light curve, and shifting its time origin (equation A1). The best fit (the solid curve) has χ^2/d.o.f. = 0.3, with d.o.f. = 7. The peak magnitude given by the solid curve is −18.79, occurring 13.4 d after GRB 030329.

Applying the procedure to SNe 2003lw and 2006aj, we obtain results that are consistent with the numbers listed in Table 1.

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