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On the limit-cycle instability in magnetized accretion discs

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ABSTRACT

Observational evidence accumulated over the past decade indicates that accretion discs in X-ray binaries are viscously stable unless they accrete very close to the Eddington limit. This is at odds with the most basic standard accretion disc theory, but could be explained by either having the discs to be much cooler whereby they are not radiation pressure dominated, or a more sophisticated viscosity law. Here we argue that the latter is taking place in practice, on the basis of a stability analysis that assumes that the magnetorotational instability (MRI) responsible for generating the turbulent stresses inside the discs is also the source for a magnetically dominated corona. We show that observations of stable discs in the high/soft states of black hole binaries, on the one hand, and of the strongly variable microquasar GRS 1915+105 on the other, can all be explained if the magnetic turbulent stresses inside the disc scale proportionally to the geometric mean of gas and total pressure with a constant of proportionality (viscosity parameter) having a value of a few times 10^{-2}. Implications for bright AGN are also briefly discussed.

Key words: accretion, accretion discs – black hole physics – X-rays: binaries.

1 INTRODUCTION

According to the widely adopted standard solutions (Shakura & Sunyaev 1973), luminous accretion discs close to the Eddington rate should be radiation pressure dominated and therefore unstable to perturbations of both mass flow (Lightman & Eardley 1974) and heating rate (Pringle, Rees & Pacholczyk 1973) for the commonly adopted assumption that viscous stresses within the disc are proportional to the total (gas plus radiation) pressure (α-viscosity prescription). Taking into account the stabilizing effect of radial advection near the Eddington rate, often modelled with the 'slim disc' solutions (Abramowicz et al. 1988), a limit-cycle-type behaviour should be expected, which has been confirmed by numerical simulations of time-dependent discs (Taam & Lin 1984; Honma, Matsumoto & Kato 1991; Szuszkiewicz & Miller 1997).

These instabilities may also operate in accretion discs of supermassive black holes (SMBHs). It is now believed that the main phase of growth of these black holes must have occurred in short-lived episodes of near-Eddington accretion (Yu & Tremaine 2002; Merloni 2004; Hopkins, Narayan & Hernquist 2006), most likely associated with bright quasar phases. The lifetimes and duty cycles of such luminous objects, crucial to understand their cosmological evolution, may be influenced by the radiation pressure instabilities (or lack thereof) of accretion discs.

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Galactic black holes in binary systems provide an important laboratory to study these instabilities on humanly observable timescales. Among all of them, only one – the famous microquasar GRS 1915+105 – displays a variability which can be explained by some kind of viscous instability (Belloni et al. 1997; Janiuk, Czerny & Siemiginowska 2000; Nayakshin, Rappaport & Melia 2000; for a recent review and a more comprehensive list of references on this enigmatic source, see Fender, & Belloni 2004). On the other hand, the vast majority of transient galactic black holes are observed to be stable in a disc-dominated state (the so-called high/soft state, or thermal-dominant state, see McClintock & Remillard 2006) at luminosities which are a few tenths of the Eddington one, which is hard to reconcile with the original viscosity prescription of stresses proportional to total pressure.

Two physically plausible ways to make luminous accretion discs stable are well known. First of all, the discs may be cooling much faster than the standard solution assumes due to an additional rapid energy transfer from the disc mid-plane into a corona, a jet or a wind, so that the radiation pressure simply never dominates in the disc (e.g. Svensson & Zdziarski 1994). Secondly, the anomalous viscosity of accretion discs, now understood to be due to magnetorotational instability (MRI), may not scale with the total disc pressure (e.g. Lightman & Eardley 1974). Numerical magnetohydrodynamics (MHD) simulations of turbulent accretion flows are the most promising tools for differentiating between these possibilities from first principles (see, in particular, Sano et al. 2004, and references therein). However, due to immense numerical challenges, one will have to wait until global 3D radiative simulations are performed over a large enough range of parameter space before a clear answer will emerge.

In this paper, we investigate the time-dependent evolution of magnetized accretion discs by means of semi-analytic techniques and discuss their relation to observations. In particular, we suggest plausible scalings for the fraction of power dissipated in the corona and the viscosity law, and we perform a local viscous disc stability analysis, later checked with time-dependent disc models. We find that the observed stability properties of galactic black hole discs are most likely explained by a modified viscosity law rather than by an extra coronal or jet cooling.

2 VISCOSITY LAW

The nature and extent of the posited limit-cycle instabilities at high accretion rates depend critically on the poorly understood prescription for the viscous torques: it is well known that assuming viscous stresses scale proportionally to the gas pressure results in accretion discs which are stable throughout, even at the highest accretion rates (Lightman & Eardley 1974; Stella & Rosner 1984). In fact, our ignorance of the physical mechanisms giving rise to the disc viscosity, and in particular of its exact scaling, has led many authors to consider the outcome of radiation pressure dominated discs obeying a more general prescription for the viscous stresses $t_{r\phi}$ (Taam & Lin 1984; Szuszkiewicz 1990; Honma et al. 1991; Watarai & Mineshige 2003):

$$t_{r\phi} = \alpha_0 P_{\rm tot}^{1-\mu/2} P_{\rm gas}^{\mu/2}, \tag{1}$$

with α_0 constant, where

$$P_{\rm tot} = P_{\rm gas} + P_{\rm rad} = 2\rho\, m_{\rm p}^{-1} kT + \frac{aT^4}{3} \tag{2}$$

is the sum of gas plus radiation pressure (for hydrogen-rich material), T is the mid-plane disc temperature and $m_{\rm p}$ is the proton mass. Within this approach, the parameter μ can take any value between 0 (stresses proportional to total pressure) and 2 (stresses proportional to gas pressure only).

2.1 Magnetorotationally unstable discs and the generation of the corona

Numerical studies of the last decade have shed new light on the nature of viscosity in accretion discs, by elucidating the crucial role of MHD turbulence for their enhanced transport properties. Since MRI (see Balbus & Hawley 1998, and reference therein) is the primary driver of the angular momentum transfer in the discs, the turbulent magnetic stresses scale with magnetic pressure, and therefore $t_{r\phi} \propto P_{\rm mag}$, where $P_{\rm mag} = |B_{\rm disc}|^2/8\pi$ is the magnetic pressure inside the disc.

One of the main open issues in the physics of black hole accretion discs is the relationship between the disc MRI-driven turbulent viscosity and the generation of the hot coronae that are usually postulated in order to explain the observed X-ray emission (Liang & Price 1977; Galeev, Rosner & Vaiana 1979; Blackman & Field 2000; Kuncic & Bicknell 2004). Phenomenological models usually assume that at each radius, a fraction f of the internally generated power is transferred vertically outside the disc, and powers a magnetically dominated corona (Haardt & Maraschi 1991; Svensson & Zdziarski 1994). As customary (see e.g. Svensson & Zdziarski 1994; Merloni 2003), we assume that in MRI-turbulent discs such a fraction f of the binding energy is transported from large to small depths by some form of collective mean electromagnetic action (Poynting flux). One should always keep in mind, however, that this is by no means the only way in which energy can be removed non-radiatively from the optically thick disc (for an alternative, see e.g. Tagger & Pellat 1999).

We can now estimate the vertical Poynting flux, $F_{\rm P}$, in the simplest way, assuming that $F_{\rm P} \simeq v_{\rm D}\, P_{\rm mag}$, where $v_{\rm D}$ is the upward drift velocity of a magnetic flux tube within the disc. In Merloni (2003), it was argued that $v_{\rm D}$ should in general be of the order of the Alfvén speed $v_{\rm A}$. This translates into the following expression for the fraction of power dissipated in the corona, uniquely relating this quantity to the magnetic disc viscosity parameter α_0 (Merloni 2003; Hirose, Krolik & Stone 2006):

$$f \simeq \frac{v_{\rm A}}{c_{\rm s}} = \sqrt{2\alpha_0 \beta^{\mu/2}}, \tag{3}$$

where $\beta = 1/(1+\xi)$, is the ratio of gas to total pressure, $\xi = P_{\rm rad}/P_{\rm gas}$ is the ratio of the radiation to the gas pressure. Note that in this approach f is an implicit function of radius, through the radial dependence of the pressures.

Recent progress in numerical studies of the disc–corona coupling has been made by simulating a gas pressure dominated local patch of an accretion disc (with vertical gravity included) in which heating by dissipation of the MHD turbulence is balanced by radiative cooling (Hirose et al. 2006; see also Miller & Stone 2000). In broad accordance with equation (3), it was found that the fraction of power released outside the disc main body was less than about 10 per cent for a measured stress parameter of $\alpha_0 \approx 0.02$. However, due to the increased magnetic pressure support in the upper disc layers, most of the Poynting flux emerging from the disc main body is dissipated *below* the photosphere, and therefore cannot be directly associated with the observed hot, optically thin X-ray emitting plasma. Obviously, global radiative simulations are needed to assess the role of long-wavelength Parker instability modes, and the scaling with the radiation pressure predicted by equation (3) for the generation of genuinely hot coronae from disc magentic fields.

3 STABILITY ANALYSIS

Let us consider now the general case of the viscosity prescription (1), with $0 < \mu < 2$. We can calculate analytically the value of the disc parameters at which the instability sets in by studying the stability properties of the stationary solution. In the one (vertical) zone limit, the equation for hydrostatic equilibrium in the vertical direction is

$$P_{\rm tot} = \frac{GM\Sigma H}{2R^3}, \tag{4}$$

while the angular momentum conservation equation reads:

$$P_{\rm gas}^{\mu/2} P_{\rm tot}^{(1-\mu/2)} = \frac{3\Omega_{\rm K}\dot{M} J(R)}{8\pi\alpha_0 H}, \tag{5}$$

where the function $J(R) = (1-\sqrt{R_{\rm in}/R})$, with $R_{\rm in} = 3R_{\rm S}$, describes the Newtonian approximation of the no-torque at the inner boundary condition for a disc around a Schwarzschild black hole. Finally, the energy balance equation is given by

$$F_{\rm rad} \simeq \frac{cP_{\rm rad}}{\tau_T} = \frac{3}{2}\sigma_{r\phi}\,\Omega H = \frac{3\Omega_{\rm K}^2 \dot{M} J(R)(1-f)}{8\pi}, \tag{6}$$

where $F_{\rm rad}$ is the vertical radiation flux and we have $f \propto \beta^{\mu/4}$, from equation (3).

By differentiating logarithmically the above expressions, together with the equation of state (2), with respect to $\dot{M}$, we obtain four equations in terms of the logarithmic derivatives of $P_{\rm tot}$, ρ, T and H, that

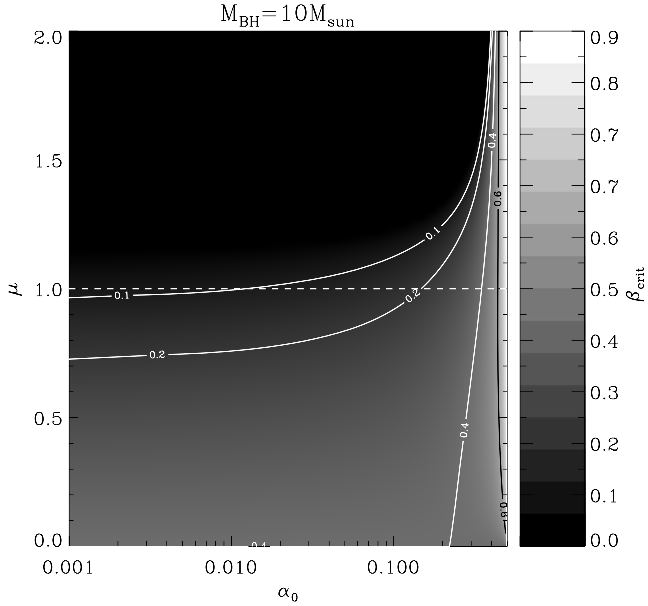

Figure 1. The critical value of the gas to total pressure ratio, $\beta_{\rm crit}$, below which the disc is unstable as a function of the viscosity parameter α_0 and index μ. Calculations are for the case of a black hole mass $M_{\rm BH} = 10\,{\rm M}_{\odot}$. The solid lines mark the values $\beta_{\rm crit} = 0.1$, 0.2, 0.4 and 0.8. The horizontal dashed line marks the $\mu = 1$ case.

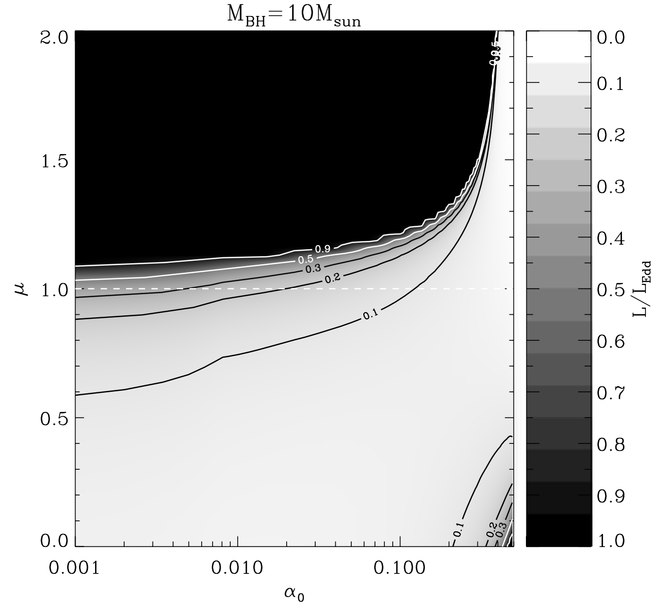

Figure 2. The critical accretion rate in units of Eddington, $\dot{m}_{\rm crit}$, calculated for a radiative efficiency of $\epsilon_0 = 1/12$, above which a disc is unstable is plotted as contours as a function of the viscosity parameter α_0 and index μ. Calculations are for a black hole mass $M_{\rm BH} = 10\,{\rm M}_{\odot}$. The solid lines mark the values $\dot{m}_{\rm crit} = 0.15$, 0.2, 0.3, 0.5 and 0.9. The horizontal dashed line marks the $\mu = 1$ case.

can be solved for $d\log\rho/{\rm d}\log\dot{M}$ and ${\rm d}\log H/{\rm d}\log\dot{M}$ as functions of β and f. The lower turning point of the $\dot{M}(\Sigma)$ curve, which indicates the local instability condition is then found by setting

$$\frac{{\rm d}\log\Sigma}{{\rm d}\log\dot{M}} = \frac{{\rm d}\log\rho}{{\rm d}\log\dot{M}} + \frac{{\rm d}\log H}{{\rm d}\log\dot{M}} = 0, \qquad (7)$$

which in turn gives

$$\beta_{\rm crit} = \left(\frac{P_{\rm gas}}{P_{\rm tot}}\right)_{\rm crit} = \frac{7\mu(2-3f) - 16(1-f)}{7\mu(2-3f) - 40(1-f)}. \qquad (8)$$

The instability sets in at a transition radius which can be found by solving the following algebraic equation:

$$\frac{R_{\rm tr}}{J(R_{\rm tr})^{16/21}} \simeq 350 R_{\rm S}\left(\frac{\beta_{\rm crit}}{1-\beta_{\rm crit}}\right)^{20/21} \times (\alpha_0 m)^{2/21}\dot{m}^{16/21}(1-f)^{6/7}, \qquad (9)$$

where we have defined $m \equiv M_{\rm BH}/{\rm M}_{\odot}$.

In the limit of $f = 0$, we obtain the known result (see Szuszkiewicz 1990) that, for the instability condition to be satisfied, the gas pressure needs to be smaller than 0.4 times the total pressure if $\mu = 0$, and just 1/13 times if $\mu = 1$. Thus, the larger is μ, the more difficult is to excite the instability. By using equation (3) to relate f and β, we can then explore the whole space spanned by the parameters α_0 and μ. In Fig. 1, where we plot as contours the surface of critical gas to total pressure ratio $\beta_{\rm crit}$, while in Fig. 2 we plot the corresponding critical values of the accretion rate, defined as $\dot{m} \equiv \epsilon\dot{M}c^2/L_{\rm Edd}$, where ϵ is the radiative efficiency.[1]

[1] In the following, for the sake of consistency with our Newtonian inner boundary condition, we will assume $\epsilon = \epsilon_0 \equiv 1/12$. However, it should be kept in mind that for a black hole of arbitrary spin, the true critical value of $L/L_{\rm Edd}$ should be rescaled by a factor ϵ/ϵ_0 with respect to what shown in Fig. 2.

First of all, as it is well known, there exists a region of parameter space for which the disc is always stable. Such a region corresponds to the upper left-hand corners of Figs 1 and 2, i.e. for large values of μ, implying stresses coupled more with the gas, rather than the total pressure, and small magnetic viscosity parameter α_0.

On the other hand, if viscous stresses are proportional to total pressure ($\mu = 0$), the presence of a corona as a sink of energy has a stabilizing effect on the disc (Svensson & Zdziarski 1994), and the critical accretion rate scales as $\dot{m}_{\rm crit} \propto (1-f)^{-9/8}$. However, if α_0 is large enough the disc can be unstable even for large values of μ (upper right-hand corner of Figs 1 and 2).

3.1 Observational constraints

It is an observational fact that only the microquasar GRS 1915+105, the most luminous transient black hole binary in our galaxy, displays strong variability on time-scales compatible with the limit-cycle behaviour expected from thermal instabilities (see Belloni et al. 1997; Janiuk et al. 2000; Nayakshin et al. 2000; Janiuk, Czerny & Siemiginowska 2002). Many other systems are known to be stable up to luminosities which are a few tens of per cent of the Eddington luminosity. This is clearly impossible for standard viscosity prescriptions ($\mu = 0$), unless the viscosity parameter α_0 (and thus the fraction of power released into the corona) is high enough. But, as Figs 1 and 2 show, there is only a very limited region of the parameter space for which this is possible, and within such a region, the fraction of power released outside the optically thick disc is always large, in contradiction with the observed spectral properties of high/soft state black hole binaries.

On the other hand, if $\mu \approx 1$, as argued, for example in Merloni & Fabian (2002) or Merloni (2003), we can have accretion discs which are stable at more than half of the Eddington luminosity, and this holds over a very large range in possible viscosity parameters α_0.

This can indeed explain the observed lack of unstable, disc-dominated galactic black hole binaries up to luminosities of about a half of the Eddington one (Gierliński & Done 2004), while at the same time allowing for *some* instability, taking place only in those systems which are constantly accreting very close to (or above) the Eddington rate, as supposedly is GRS 1915+105 (Done, Wardziński & Gierliński 2004).

We can extend this result to the case of SMBHs. It is well known that for any value of μ and α_0 the critical accretion rate scales with mass as $\dot{m}_{\rm cr} \propto M^{-1/8}$. Then for a 10^9-$M_{\odot}$ black hole the limit-cycle instability should set in at a luminosity, in units of Eddington, which is just one tenth of that of a binary black hole, both viscosity law and black hole spin being the same. If indeed most of the bright, high-redshift quasars that dominate the growth history of SMBH have $L/L_{\rm Edd} > 0.1$, as recently proposed (McLure & Dunlop 2004; Vestergaard 2004), then they should be undergoing limit-cycle instability.

4 TIME-DEPENDENT EVOLUTION

In order to test the above conclusions based on the stability analysis of *stationary* solutions, we have carried out a set of time-dependent simulations for different values of the parameters α_0 and $\dot{m}$. From the equation of conservation of mass and angular momentum, we can write (Pringle 1981; Livio & Pringle 1992):

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}(\nu \Sigma R^{1/2})\right]. \tag{10}$$

The time-dependent energy equation needed to close the set must take into account large radial gradients of temperature, and therefore includes a number of additional terms with respect to the standard heating and cooling terms of the stationary solution. The form of the energy equation that we use follows the formalism of Nayakshin et al. (2000):

$$P_{\rm tot} H \frac{4-3\beta}{\Gamma_3 - 1}\left[\left(\frac{\partial \ln T}{\partial t} + v_{\rm R}\frac{\partial \ln T}{\partial R}\right) - (\Gamma_3 - 1)\left(\frac{\partial \ln \Sigma}{\partial t} + v_{\rm R}\frac{\partial \ln \Sigma}{\partial R} - \frac{\partial H}{\partial t}\right)\right] = F^+ - F^- - \frac{2}{R}\frac{\partial (R F_{\rm R} H)}{\partial R} + J, \tag{11}$$

where γ is the ratio of specific heats ($\gamma = 5/3$) and Γ_3 is given in Abramowicz, Chen & Taam (1995). Here, the radial velocity $v_{\rm R}$ induced by viscous stresses is given by (equation 5.7 of Frank, King & Raine 1992):

$$v_{\rm R} = -\frac{3}{\Sigma R^{1/2}}\frac{\partial}{\partial R}(\nu \Sigma R^{1/2}). \tag{12}$$

The terms on the left-hand side of equation (11) represent the full time derivative (e.g. $\partial/\partial t + v_{\rm R}\partial/\partial R$) of the gas entropy, while the terms on the right-hand side are the viscous heating, the energy flux in the vertical direction, the diffusion of radiation in the radial direction and the viscous diffusion of thermal energy. Following Cannizzo (1993; and references therein), we take $J = 2\,c_{\rm p}\nu(\Sigma/R)$ $[\partial(R\partial T/\partial R)/\partial R]$ to be the radial energy flux carried by viscous thermal diffusion, where $c_{\rm p}$ is the specific heat at constant pressure. F^+ is the accretion disc heating rate per unit area, and is given by

$$F^+ = \left(\frac{9}{4}\right)\nu \Sigma \Omega_{\rm K}^2 (1-f). \tag{13}$$

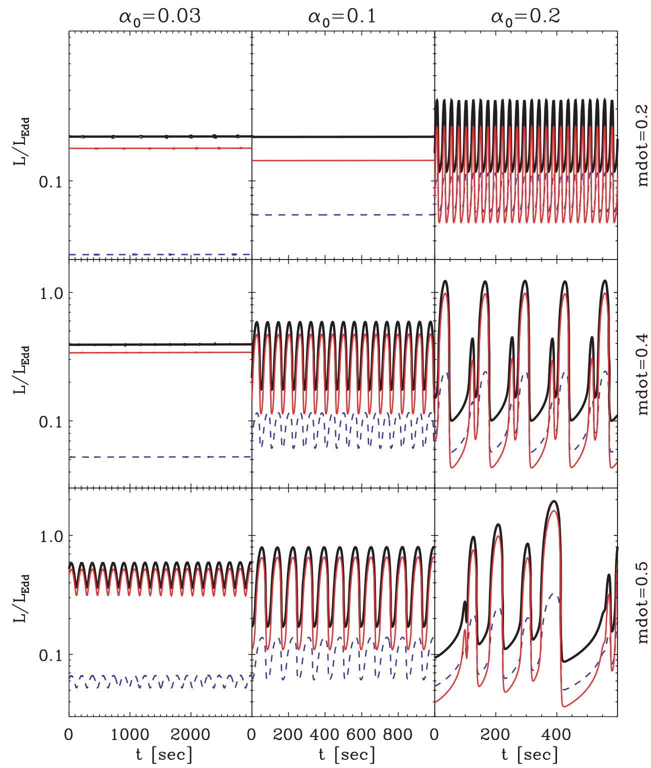

Figure 3. Set of light curves for magnetized discs with the viscosity law corresponding to $\mu = 1$, calculated for different accretion rates ($\dot{m} = 0.2$, 0.4 and 0.5) and viscosity parameters ($\alpha_0 = 0.03$, 0.1 and 0.2). Each panel shows one light curve over a time-scale approximately equal to one tenth of the viscous time at the outer boundary of our discs ($r = 200$). Thick black lines represent the total (disc plus corona) emission, thin red lines the optically thick disc emission and thin blue dashed ones the coronal (X-ray) emission. The black hole mass is fixed to $10\,M_{\odot}$.

The radiation flux in the radial direction is

$$F_{\rm R} = -2\frac{c P_{\rm rad}}{\tau_T} H \frac{\partial \ln T}{\partial R}, \tag{14}$$

where τ_T is the optical depth of the disc, $\tau_T \equiv \kappa \Sigma/2$, and κ is the radiative opacity (assumed here to be dominated by electron scattering opacity). Finally, the radiative cooling rate in the vertical direction is given by

$$F^- = \frac{c P_{\rm rad}}{\tau_T}. \tag{15}$$

The results of our time-dependent simulations for the case $\mu = 1$ and $M_{\rm BH} = 14\,M_{\odot}$ (as appropriate for GRS 1915+105) are shown in Fig. 3 as a set of light curves plotted over a time typically of the order of the viscous time at the outer domain boundary. Shown separately are the total disc (thin red solid lines) and coronal (blue dashed lines) emissions, together with their sum (thick black solid lines), representing the total dissipated energy in the disc–corona system. These results broadly confirm the analysis presented in Section 3, in that we observe stable discs at $\dot{m} > 0.4$ and 0.2 for $\alpha_0 = 0.03$ and 0.1, respectively (see Fig. 2).

We find in general a relationship between the amplitude of the instability and its duty cycle (i.e. the ratio of the 'burst duration' to the period of the oscillation), whereas small duty cycles are associated with large amplitude variability. The amplitude itself is much smaller than in standard α-discs, and grows with the ratio $\dot{m}/\dot{m}_{\rm crit}$.

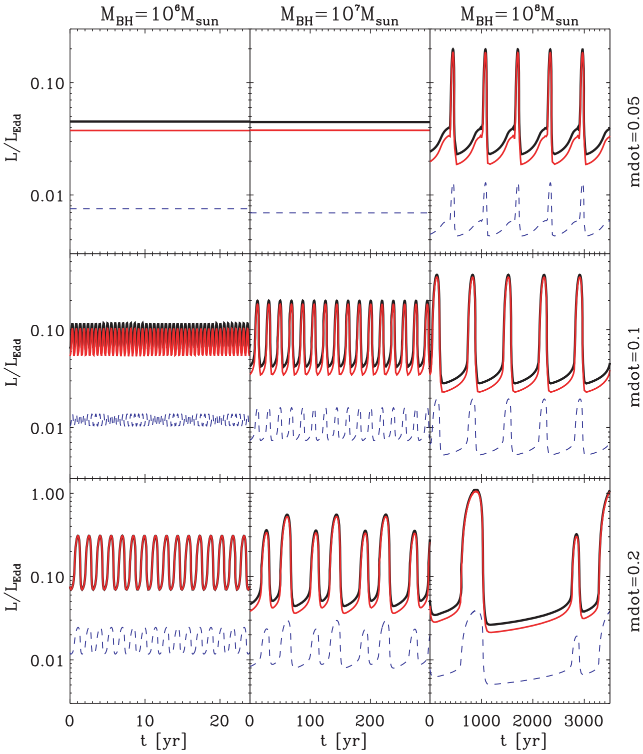

Figure 4. Set of light curves calculated with $\mu = 1, \alpha_0 = 0.03$, for different accretion rates ($\dot{m} = 0.05, 0.1$ and 0.2) and black hole masses ($M_{\rm BH} = 10^6$, 10^7 and 10^8 M$_\odot$). Each panel shows one light curve over a time-scale approximately equal to half of the viscous time at the outer boundary of our discs ($r = 200$). Line styles are as in Fig. 3.

For accretion rates just above the critical values, the large cycles imply a 'refilling' time of the inner disc faster than the viscous time at the transition radius. In fact, we found that the viscosity prescription, $\mu \approx 1$, reproduces many features of the phenomenological one introduced by Nayakshin et al. (2000) in order to explain the time variability of GRS 1915+105 (a task which is beyond the aim of this work).

Another general property of the simulated light curves is that, thanks to the nature of the magnetic viscosity law adopted, the fraction f of power dissipated outside the optically thick disc is higher when gas pressure dominates, i.e. when the inner disc is denser and cooler, thus making coronal X-ray emission more prominent during the low-luminosity parts of the instability cycles.

Fig. 4 shows a set of simulated light curves with $\alpha_0 = 0.03$ and $\mu = 1$ for SMBHs of different masses. As we noted in Section 3, for larger black hole masses the instability sets in at lower $L/L_{\rm Edd}$. As the duty cycle also gets smaller as the instability amplitude grows, then we must conclude that the higher the external accretion rate, $\dot{M} \propto M\dot{m}$, the more time will be spent by a source in the dense, cold, low-luminosity portion of the instability cycles, close to its critical luminosity.

5 DISCUSSION

The stability analysis we have presented here can be used to explain the observed properties of accreting stellar mass black holes in different states. First of all, a major constraint can be derived from the absence of any limit-cycle instability in disc-dominated spectra. Those can be effectively selected in two complementary ways: by detail modelling, choosing those spectra in which the power-law component contribute to less than a fixed amount (say, 10 per cent); and by studying the luminosity–temperature relation of the disc component, selecting those spectra for which $L_{\rm disc} \propto T^4$ as expected from an optically thick disc extending down to the innermost stable orbit (Kubota & Makishima 2004). Modulo the distance uncertainty, these studies unambiguously demonstrated that accretion discs can be viscously stable up to at least $L_{\rm disc}/L_{\rm Edd} \gtrsim 0.3$ (Gierliński & Done 2004).

From Fig. 2, then, we conclude that only two disjoint regions of parameter space are consistent with the observed phenomenology of black hole binaries. The first corresponds to $\alpha_0 \gtrsim 0.3$ and, within our theoretical framework, it corresponds to accretion discs sandwiched by powerful, stabilizing coronae as postulated by Svensson & Zdziarski (1994). The second acceptable region of the parameter space is roughly delimited by $\mu \gtrsim 0.8$ and $\alpha_0 \lesssim 0.15$ and would indicate that the stability properties of high/soft state black holes are dictated by a modified viscosity law.

The former constraint, however, implying a large fraction of power transported vertically by magnetic fields, seems inconsistent with the spectral properties of the observed black holes in the high/soft state, but see, however, Hirose et al. (2006) or Blaes et al. (2006). Moreover, for the only source that exhibits limit-cycle instabilities, the microquasar GRS 1915+105, one can associate the observed variability time-scales with the disc viscous time (Belloni et al. 1997; Janiuk et al. 2000; Nayakshin et al. 2000). These models consistently suggest that $\alpha_0 \sim$ a few per cent. If this is indeed a general properties of all accretion discs, a coherent picture emerges for black hole binaries in the high/soft state in which MRI-driven turbulence generates stresses which scale with the disc pressure as the geometric mean of gas and total pressure (Merloni 2003), i.e. $\mu \approx 1$, with a constant of proportionality α_0 of about a few per cent. This in turn implies quite low values of f and consequent small contribution from the power-law component to the observed X-ray spectra.

5.1 The low/hard state

The scenario we have outlined above applies only to the high/soft state of black hole binaries.[2] It seems unlikely that the soft-to-hard transition can be solely due to a decrease of the external accretion rate and a consequent increase of the gas pressure dominated part of the disc, accompanied by an increase of the strength of the corona (Merloni & Fabian 2002). In fact, low values of α_0 in the high/soft state also imply low values of $f \approx \sqrt{2\alpha_0}$ in the low state, if the disc physics is not dramatically altered by the $\dot{m}$ variation. Thus, there must be an additional physical mechanism, not included in the simple treatment of the coupled magnetized disc–corona system presented here, which is responsible for the observed state change. Obvious candidates are: a radial transition to an inner optically thin, radiatively inefficient flow, due to some kind of disc evaporation (Meyer & Meyer-Hofmeister 1994; Dullemond & Turolla 1998; Spruit & Deufel 2002), or a global rearrangement of a large-scale poloidal magnetic field, so that the accretion energy can be dissipated almost entirely into the bulk flow of a relativistic jet (Livio, Pringle & King 2003).

[2] For a general discussion of black hole binaries low/hard state, the reader is referred to Markoff (2005) and Narayan (2005).

5.2 Variability in AGN

Since the radiation pressure dominated regime sets in at smaller dimensionless accretion rates for higher black hole masses, large-scale variability due to radiation pressure driven viscous instabilities must be more widespread in AGN than in stellar mass accreting black holes. However, observing such variability in practice is only feasible for lower mass SMBHs. Indeed, the viscous time-scale, $t_{\rm visc} = (1/\Omega)(1/\alpha\beta^{\mu/2})(R/H)^2$, calculated at the transition radius (see equation 9) is approximately given by

$$t_{\rm visc}(r_{\rm tr}) \approx 40\alpha_0^{-2/3} m^{4/3} \dot{m}^{2/3} \beta_{\rm crit}^{10/3-\mu/2} (1-\beta_{\rm crit})^{10/3} \text{ s}. \quad (16)$$

This ranges from a few tens to a few hundreds of years if $m = 10^7$, consistent with estimates of switch on/off times for some 'changing look' AGN (Guainazzi et al. 2005), to 10^4–10^5 years for a 10^9-$M_\odot$ black hole. This, interestingly, coincides with the proposed intermittency time of radio galaxies, based on radio sources number counts and on the properties of radio galaxies' size distribution (Reynolds & Begelman 1997).

Another interesting aspect of the predicted variability for AGN concerns their feedback on galaxy formation. Indeed, in our model, black holes accreting at a time-averaged subEddington rate may spend a fraction of their time accreting at above-Eddington accretion rate. If an AGN feedback (e.g. in the form of powerful relativistic jets) is significant when $L \gtrsim L_{\rm Edd}$, as suggested by the phenomenology of microquasars (see Fender, Belloni & Gallo 2004), then such a source would produce feedback while on the 'hot' branch of the S-curve, whereas a completely stable source at the same time-averaged accretion rate would not.

6 CONCLUSIONS

In this paper, we discussed the implications of the observed long-term stability of the vast majority of black hole X-ray binaries in the so-called high/soft state (or thermal-dominant state). This fact, together with the properties of the light curves of GRS 1915+105, the most luminous galactic black hole and the only such system to display limit-cycle-type instabilities, can be used to put constraints on the nature of the viscosity law in accretion discs.

We performed a stability analysis for accretion flows with a flexible viscosity prescription under the hypothesis that the MRI responsible for generating the turbulent stresses inside the discs is also the source for a magnetically dominated corona. By varying the scaling index of the viscosity law, μ, and its overall normalization, α_0, we have identified those regions of the parameter space for which limit-cycle instability can develop as a function of the accretion rate. From this analysis, a coherent picture emerges for black hole binaries in the high/soft state in which MRI-driven turbulence generates stresses which scale with the disc pressure as the geometric mean of gas and total pressure, i.e. $\mu \approx 1$, with a constant of proportionality α_0 of about a few per cent. This in turn implies quite low values of f and consequent small contribution from the power-law component to the observed X-ray spectra. Provided that the viscosity law itself does not change dramatically at lower accretion rates, a consequence of this result is that the transition to the low/hard state must be caused by additional physics, such as the evaporation of, or a global large-scale magnetic field re-arrangement in the inner portions of the disc.

The scaling with black hole mass of the accretion equations is such that limit-cycle instabilities should play a role in all bright AGN and quasars. The typical time-scale for these oscillations, however, grows with black hole mass faster than linearly. Only smaller mass black holes (less than a few times 10^7 solar masses) may have limit-cycle instability time-scales of just a few years. Even considering the uncertainties in the absolute values of the predicted critical luminosity that come from uncertainties in the black hole mass and spin, it would be interesting to search for any evidence of the predicted behaviour by looking at distribution of observed disc luminosities in large samples of AGN with available estimates of the central black hole mass.

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