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No *Higher Criticism* of the Bianchi-corrected *Wilkinson Microwave Anisotropy Probe* data

L. Cayón,[1⋆] A. J. Banday,[2] T. Jaffe,[2] H. K. Eriksen,[3,4,5,6] F. K. Hansen,[3,4] K. M. Gorski[5,6] and J. Jin[7]

[1]*Department of Physics, Purdue University, 525 Northwestern Avenue, West Lafayette, IN 47907-2036, USA*
[2]*Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Strasse 1, Postfach 1317, D-85741 Garching bei München, Germany*
[3]*Institute of Theoretical Astrophysics, University of Oslo, PO Box 1029 Blindern, N-0315 Oslo, Norway*
[4]*Centre of Mathematics for Applications, University of Oslo, PO Box 1053 Blindern, N-0316 Oslo, Norway*
[5]*JPL, M/S 169/327, 4800 Oak Grove Drive, Pasadena, CA 91109, USA*
[6]*California Institute of Technology, Pasadena, CA 91125, USA*
[7]*Department of Statistics, Purdue University, 150 N. University Street, West Lafayette, IN 47907-2067, USA*

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ABSTRACT

Higher Criticism (HC) has been proposed by Donoho & Jin as an effective statistic to detect deviations from Gaussianity. Motivated by the success of the Bianchi VII_h model in addressing many of the anomalies observed in the *Wilkinson Microwave Anisotropy Probe* (*WMAP*) data (Jaffe et al.), we present calculations in real and in wavelet space of the HC statistic of the Bianchi-corrected *WMAP* first-year data. At the wavelet scale of 5°, the HC of the *WMAP* map drops from a value above the 99 per cent confidence level (c.l.) to a value below the 68 per cent CL when corrected by the Bianchi template. An important property of the HC statistic is its ability to locate the pixels that account for the deviation from Gaussianity. The analysis of the uncorrected *WMAP* data pointed to a cold spot in the Southern hemisphere, centred at $l \sim 209°$, $b \sim -57°$. The HC of the Bianchi-corrected map indicates that this spot remains prominent, albeit at a level completely consistent with Gaussian statistics. Consequently, it is debatable how much emphasis should be placed on this residual feature, but we consider the effect of modestly increasing the scaling of the template. A factor of only 1.2 renders the spot indistinguishable from the background level, with no noticeable impact on the results published in Jaffe et al. for the low-*l* anomalies, large-scale power asymmetry or wavelet kurtosis. A trivial interpretation would be that the Bianchi template may require a small enhancement of power on scales corresponding to the wavelet scale of 5°.

Key words: methods: statistical – cosmic microwave background – cosmology: miscellaneous.

1 INTRODUCTION

The first year of observations of the *Wilkinson Microwave Anisotropy Probe* (*WMAP*) satellite has provided us with a data set of unprecedented accuracy (Bennett et al. 2003). It is exciting to see that we are still far from understanding all the information that is encoded in the data. Among the many papers that have been published based on analyses of the *WMAP* data, a large section of them point to possible deviations from the standard model. Different so-called anomalies have been detected and possible explanations have been offered by several authors. (1) Low value of the quadrupole, its alignment with the octopole and other multipole alignments (De Oliveira-Costa et al. 2004; Schwarz et al. 2004; Land & Magueijo 2005; Vale 2005), (2) asymmetries (Eriksen et al. 2004a,b; Hansen, Banday & Gorski 2004a; Hansen et al. 2004b; Larson & Wandelt 2004; Park 2004; Bielewicz et al. 2005; Donoghue & Donoghue 2005; Freeman et al. 2006; Tojeiro et al. 2006; Tomita 2005), and (3) deviations from Gaussianity, in particular, detections in wavelet space pointing to a cold spot as the source of non-Gaussianity (Vielva et al. 2004; Cayón, Jin & Treaster 2005; Cruz et al. 2005).

Several works have explored the possibility of accounting for some or all of the anomalies observed in the *WMAP* data by introducing some shear and vorticity in the universe (Jaffe et al. 2005, 2006; Land & Magueijo 2006; McEwen et al. 2006). Models introducing these anisotropic characteristics fall under the class of Bianchi VII_h models (Barrow, Juszkiewicz & Sonoda 1985). In this paper, we confirm the validity of the Bianchi model obtained as a

⋆E-mail: cayon@physics.purdue.edu

best fit to the *WMAP* data by Jaffe et al. (2006), as a possible explanation of all the observed anomalies. Following the same procedure as in Cayón et al. (2005), we show that the Higher Criticism (HC) statistic (Donoho & Jin 2004) of the Bianchi-corrected *WMAP* map is compatible with Gaussianity [at the 99 per cent confidence level (CL)] at all the wavelet scales.

This paper is organized as follows. We present the formalism in Section 1. Section 2 is dedicated to introduce the analyzed data and simulations. Results are presented in Section 3 and they are discussed in Section 4 (conclusions also included in this section).

2 FORMALISM

As indicated above we here follow the formalism presented in Cayón et al. (2005). The statistical study is based on the HC statistic proposed in Donoho & Jin (2004) and Jin (2004). For a set of n individual observations X_i from a certain distribution, HC is defined as follows. The X_i observed values are first converted into p-values: $p_i = P\{|N(0,1)| > |X_i|\}$. After sorting the p-values in ascending order $p_{(1)} < p_{(2)} < \cdots < p_{(n)}$, we define the HC at each pixel i by

$$\mathrm{HC}_{n,i} = \sqrt{n}\left|\frac{i/n - p_{(i)}}{\sqrt{p_{(i)}\left(1 - p_{(i)}\right)}}\right|.$$

Unusually large values of $HC_{n,i}$ imply deviations from Gaussianity. The fact that the statistic is calculated at every pixel will allow for the location of the source of any detected deviation. In order to quantify the statistical level of any detection, the maximum of the $\mathrm{HC}_{n,i}$ for a given map (generally denoted HC) will be compared to the distribution of HC values obtained from Gaussian simulations (see discussion in the next section). HC has been shown to capture the unusual behaviour of the few most-extreme observations as well as any unusually large number of moderately high value observations. One should note that this is a completely different test to one based on the kurtosis statistic. The kurtosis enhances any deviation from Gaussianity hidden in the fourth-order moment and it depends on the bulk of the data.

The HC of the Bianchi-corrected *WMAP* data is estimated in real and in wavelet space. Several groups have presented analyses of the *WMAP* data in wavelet space (Mukherjee & Wang 2004; Vielva et al. 2004; Cayón et al. 2005; Cruz et al. 2005; Liu & Zhang 2005; McEwen et al. 2005). In particular, the Spherical Mexican Hat (SMH) wavelet has been shown to be sensitive to the presence of non-Gaussianity in the *WMAP* data. Analyses based on kurtosis and on HC show detections above the 99 per cent CL at a scale of 5°. A cold spot in the Southern hemisphere seems to be accounting for these detections. Our aim in this paper is to study what is the effect on the HC results caused by the subtraction of a Bianchi-based template from the *WMAP* data. HC is therefore calculated not only for the Bianchi-corrected *WMAP* map, but also for the convolution of this map with the SMH wavelet at 15 scales ranging from 13.74 to 1050.0 arcmin.

3 BIANCHI-CORRECTED *WMAP* DATA AND SIMULATIONS

The first year of *WMAP* observations is available at the Legacy Archive for Microwave Background Data Analysis (LAMBDA) website.[1] For the purpose of cosmological studies, the Foreground Cleaned Intensity Maps at bands Q,V and W are used. A combined map (the so-called *WMAP* data/map in this work) is built following Komatsu et al. (2003). The weighted combination is given by

$$T(i) = \frac{\sum_{r=3}^{10} T_r(i) w_r(i)}{\sum_{r=3}^{10} w_r(i)}.$$

The temperature at pixel i, $T(i)$ results from the ratio of the weighted sum of temperatures at pixel i at each radiometer divided by the sum of the weights of each radiometer at pixel i. The radiometers Q1, Q2, V1, V2, W1, W2, W3 and W4 are sequentially numbered from 3 to 10. The weights at each pixel, for each radiometer $w_r(i)$, are the ratio of the number of observations $N_r(i)$ divided by the square of the receiver noise dispersion $\sigma_{\mathrm{o},r}$. The resulting map is downgraded from resolution $nside = 512$ to resolution $nside = 256$ (the total number of pixels being $12 \times nside^2$).

The Bianchi template that is used in this work is the best fit found by Jaffe et al. (2006). They considered Bianchi type $\mathrm{VII_h}$ models characterized by the values of Ω_o and $x = [h/(1 - \Omega_\mathrm{o})]^{1/2}$, where h is the scale on which the basis vectors change orientation. The best fit was obtained for $\Omega_\mathrm{o} = 0.5$ and $x = 0.62$. We present here an analysis of the Bianchi-corrected *WMAP* data set obtained after subtraction of this template (and multiples of it) from the original *WMAP* data. The final analysis is performed on a masked version of this map that includes the so-called *Kp*0 mask (released by the *WMAP* team and available at the LAMBDA website). An extension of this mask is applied to the wavelet-convolved maps as discussed in Vielva et al. (2004).

The CLs of the HC statistic corresponding to the Gaussian assumption are drawn from 5000 simulations, were the power spectrum is the one that best fits the *WMAP*, Cosmic Background Imager (CBI) and Arcminute Cosmology Bolometer Array Receiver (ACBAR) cosmic microwave background data, plus the Two-degree Field (2dF) and Lyman α data. The simulations also take into account the beam transfer functions, the number of

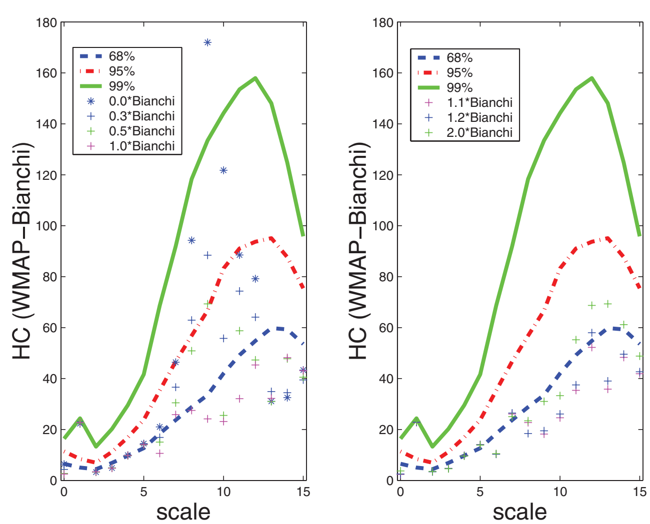

Figure 1. Values of the HC statistic for the *WMAP* (stars) and the Bianchi-corrected *WMAP* (crosses) data. The bands outlined by dashed, dotted-dashed and solid lines correspond to the 68, 95 and 99 per cent confidence regions, respectively. The Bianchi template is multiplied by different factors before subtraction. Factors 0.3, 0.5 and 1.0 are considered in the results presented in the left-hand panel (blue, green and magenta crosses, respectively). Results for factors 1.1, 1.2 and 2.0 are presented in the right-hand panel (magenta, blue and green crosses, respectively).

[1] http://lambda.gsfc.nasa.gov/.

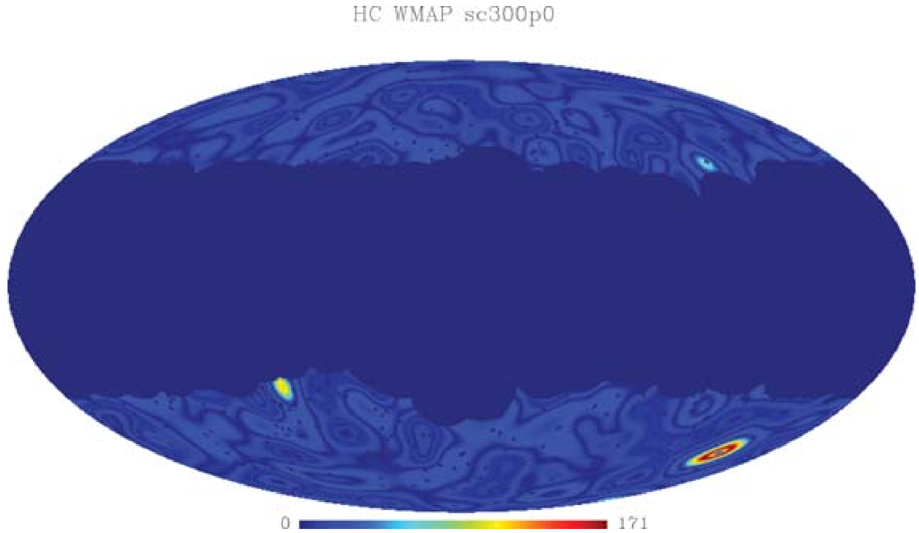

Figure 2. HC values of the *WMAP* data at the wavelet scale of 5°.

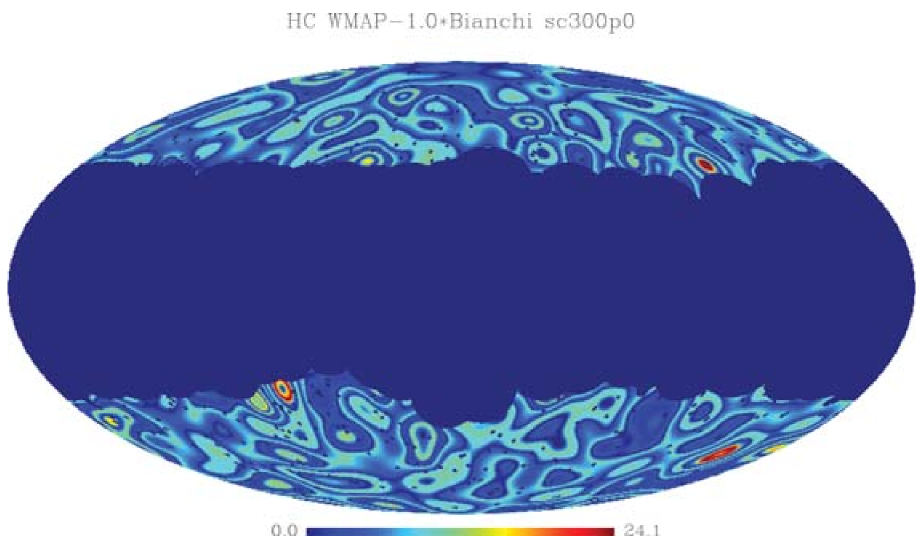

Figure 3. HC values of the Bianchi-corrected *WMAP* data at the wavelet scale of 5°.

observations and the noise dispersion for each receiver. All these are provided by the *WMAP* team through the LAMBDA website.

4 RESULTS

The values of the HC statistic for the *WMAP* and the Bianchi-corrected *WMAP* data are presented in Fig. 1. The solid, dot–dashed and dashed lines show the 99, 95 and 68 per cent CLs, respectively. These are obtained from the 5000 Gaussian simulations described above. Since the HC statistic of a map is a maximum value [$HC = max_i(HC_{n,i})$], the confidence regions are one-sided. As one can see, there is no detection of deviations from Gaussianity in the Bianchi-corrected *WMAP* data (factor of 1.0 in the figure). This corroborates the results obtained by Jaffe et al. (2006). Subtraction of the Bianchi template corrects most of the anomalies that have been observed in the first-year *WMAP* data.

The maps of HC values for the *WMAP* and the Bianchi-corrected *WMAP* data at a wavelet scale of 5° are presented in Figs 2 and 3. As one can immediately see, the detected non-Gaussianity in the *WMAP* data is dominated by the values of a ring of pixels at the spot centred at $l \sim 209°$, $b \sim -57°$ (note that the ring structure is likely to be caused by the convolution with the wavelet). Subtraction of the Bianchi template drops the values of the HC statistic at that spot below the 68 per cent CL (see Fig. 1). However, the spot still appears as one of the most prominent regions. Even if the relevance of this feature is debatable, we decided to see whether different normalizations of the subtracted template can influence its amplitude. A factor of 1.2 reduces the amplitude of the spot making it comparable with the surrounding values as can be seen in Fig. 4. We have also studied whether this normalization factor affects the other anomalies observed in the *WMAP* first-year data. The effect on the alignment between the quadrupole and the octopole and the planarity as defined in De Oliveira-Costa et al. (2004) is presented in Table 1. These large-scale anomalies are not very much affected by a change in the normalization factor of the Bianchi template. The ratio of power between two hemispheres for a certain range of l was calculated as in Eriksen et al. (2004a). The probability of having a maximum power asymmetry ratio larger than that of a given map

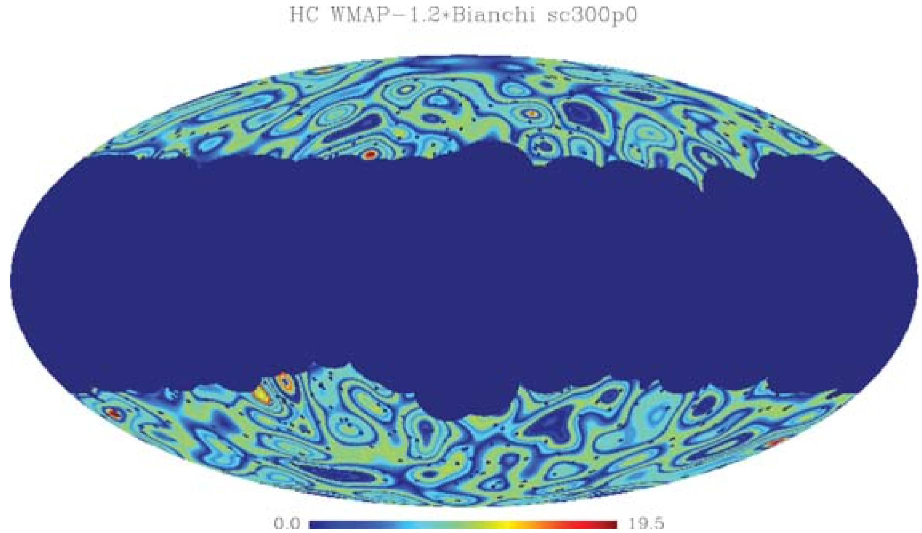

Figure 4. HC values of the *WMAP* data minus 1.2 times the Bianchi template, at the wavelet scale of 5°.

Table 1. Alignment and planarity of low-order multipoles. The normalization F of the Bianchi template is included in the first column. The second and third columns correspond to the direction of $l = 2$ (Galactic longitude and latitude in degrees). The direction of $l = 3$ is shown in the fourth and fifth columns (l and b in degrees). The angle α (degrees) corresponding to the alignment between the quadrupole and the octopole is shown in column 6. The probability of finding a weaker alignment is presented in column 7. The values in the eighth, ninth and tenth columns correspond to the probability of finding a more planar multipole for $l = 3$, 5 and 6, respectively.

F	l_2 (°)	b_2 (°)	l_3 (°)	b_3 (°)	α (°)	P	P_3	P_5	P_6
0.0	61.9	247.8	63.4	232.8	7.0	0.992	0.89	0.001	0.98
0.5	8.3	57.1	73.7	253.3	82.7	0.127	0.73	0.003	0.99
1.0	3.3	44.5	56.8	323.5	82.3	0.134	0.55	0.05	0.98
1.2	0.3	40.6	45.6	327.2	78.3	0.203	0.62	0.10	0.97
2.0	9.0	212.2	26.9	317.3	80.9	0.159	0.86	0.37	0.92

Table 2. Probability of the observed asymmetry in the power spectrum between the two hemispheres. The normalization factor F is given in column 1. The probability for $l = 2$, 20 is presented in column 2. Values in column 3 correspond to the probability for $l = 2$, 40.

F	$P(l = 2, 20)$ (per cent)	$P(l = 2, 40)$ (per cent)
0.0	1.0	0.7
0.5	9.8	6.0
1.0	17.0	22.0
1.2	15.9	28.9
2.0	4.3	13.6

(either the *WMAP* or the Bianchi-corrected *WMAP* maps) is shown in Table 2. This probability is slightly affected by the normalization factor. There is not much of a difference between the results for a factor of 1 and a factor of 1.2. However, above this, an increase in the normalization factor starts worsening the results. This agrees with the trend observed for the HC statistic as can be seen in Fig. 1.

5 DISCUSSION AND CONCLUSIONS

Several explanations have been discussed in the literature to account for the different anomalies observed in the first year of *WMAP* data (Schwarz et al. 2004; Freeman et al. 2006; Jaffe et al. 2005, 2006; Land & Magueijo 2006; Tomita 2005; Vale 2005). These anomalies include: (1) the low value of the quadrupole, its alignment with the octopole and other multipole alignments, (2) the asymmetries observed in the distribution of large-scale power, and (3) the non-Gaussianity detected in wavelet space which source seems to be centred at a cold spot in the Southern hemisphere. However, much of this work has concentrated on explaining the anomalous low-l structure. Jaffe et al., however, serendipitously discovered that at least part of the large angular scale structure of the *WMAP* data was described by a particular Bianchi VII_h model, and that correcting the data for this contribution provided a potential resolution for each of the above anomalies. We here confirm their results by estimating the effect of subtracting the proposed Bianchi template from the *WMAP* data, on the HC statistic. A detection of non-Gaussianity localized in a cold spot at $l \sim 209°$, $b \sim -57°$ was found by the estimation of the HC statistic of the *WMAP* data (Cayón et al. 2005). Subtraction of the Bianchi template renders the *WMAP* data statistically compatible with the expected levels based on Gaussian simulations.

The map of HC values for the Bianchi-corrected *WMAP* data (see Fig. 3) still shows the cold spot in the Southern hemisphere as one of the most prominent regions. Although no special consideration should now be given to the spot on a statistical basis, it remains interesting, given that the original detection of non-Gaussianity was intimately associated with the region. We have determined that a modest increase in the amplitude of the Bianchi template (by a factor of 1.2) renders the spot indistinguishable from the background level. Importantly, such a change in amplitude does not perturb the effect of the template in resolving issues related to the low-l multipole features and power asymmetry. There is no compelling statistical reason, therefore, to believe that the Bianchi template amplitude derived in Jaffe et al. should be considered underestimated. The HC results presented here, however, may be interpreted as implying that the Bianchi template may require a small enhancement of power on scales corresponding to the wavelet scale of 5°. One could create a Bianchi template with more power on small scales by reducing the matter density, but this would also affect the large-scale structure and therefore the significance of the fit to the data. The most pragmatic interpretation of Jaffe et al. – that the best-fitting Bianchi model provides a template temperature pattern which alternative models would need to reproduce in order to resolve the observed anomalous anisotropy structure – remains valid.

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