Astronomy Letters, Vol. 31, No. 6, 2005, pp. 365–374. Translated from Pis'ma v Astronomicheskiĭ Zhurnal, Vol. 31, No. 6, 2005, pp. 410–421.

Analysis of the Spatial Distribution of Gamma-Ray Bursts in Their Host Galaxies

S. I. Blinnikov[1], K. A. Postnov[2], D. I. Kosenko[1,2*], and O. S. Bartunov[2]

[1] *Institute for Theoretical and Experimental Physics, ul. Bol'shaya Cheremushkinskaya 25, Moscow, 117218 Russia and MPA, Garching, D-85741, Germany*

[2] *Sternberg Astronomical Institute, Universitetskiĭ pr. 13, Moscow, 119992 Russia*

Received January 26, 2005

Abstract—We compare the radial distributions of known localized gamma-ray bursts (GRBs) relative to the centers of their host galaxies with the distributions of known objects in nearby galaxies (supernovae of various types, X-ray binaries), the hypothetical dark-matter profiles, and the distribution of luminous matter in galaxies in the model of an exponential disk. By comparing the moments of empirical distributions, we show that the radial distribution of GRBs in galaxies differs significantly from that of other sources. We suggest a new statistical method for comparing empirical samples that is based on estimating the number of objects within a given radius. The exponential disk profile was found to be in best agreement with the radial distribution of GRBs. The distribution of GRBs relative to the centers of their host galaxies also agrees with the dark matter profile at certain model parameters. © *2005 Pleiades Publishing, Inc.*

Key words: *gamma-ray bursts, supernovae, dark matter.*

INTRODUCTION

At present, gamma-ray bursts (GRBs) and their afterglows are intensively studied over the entire electromagnetic spectrum (from gamma rays to radio waves) by all available astronomical methods. Detailed studies of the spectra and light curves provide insights into what properties the gamma-ray generator must have and give an idea of what physical conditions of the medium must be near a GRB. The rich phenomenology of GRBs and their possible astrophysical models have been discussed in many reviews (see, e.g., Postnov 1999; Blinnikov 2000; Zhang and Meszaros 2004). That the afterglows of GRBs are associated with the synchrotron radiation of the ultrarelativistic shock waves produced by them in the interstellar medium surrounding the GRB source may be considered to have been established (see the review by Piran (2004) and references therein).

The situation with the GRBs proper is not so good. The belief that GRBs could be directly associated with the explosions of supernovae of a special type, an energetic subclass of collapsing Type-Ibc supernovae with kinetic explosion energies above 10^{51} erg (the so-called hypernovae), has been strengthened in recent years. The association of GRBs with supernovae has received strong observational confirmation after GRB 030329, when spectral features typical of Type-Ibc supernovae were detected in the spectra of its optical afterglow (Hjorth *et al.* 2003; Stanek *et al.* 2003; Matheson *et al.* 2003). However, a recent analysis of the latest observations of GRBs and their accompanying supernovae (Postnov 2004) leads us to conclude that only relatively weak GRBs (like GRB 980425 and GRB 031203) could be associated with bright supernovae. Thus, at present, we cannot unequivocally associate each GRB with the collapse of a massive star accompanied by a hypernova explosion and reject other GRB formation hypotheses, including those outside the scope of the standard model of modern physics. Note, in particular, the interesting possibility of the association of GRBs with the poorly studied dark matter in galaxies (Gurevich *et al.* 1997; Blinnikov 2000).

Thus, the association of GRBs with known astrophysical objects in galaxies (in particular, with Type-Ibc supernovae) should be verified by independent methods. The presence of objects that are spatially distributed in galaxies in the same way as GRBs could be evidence for their relationship. This is the main motivation for our study.

A comparison of the locations of GRBs in galaxies with known populations of astrophysical sources is not new. Previously, this problem was tackled by Tsvetkov *et al.* (2001) and Bloom *et al.* (2002). Tsvetkov *et al.* (2001) showed that the surface-density distribution of the then known GRBs in

*E-mail: lisett@xray.sai.msu.ru

galaxies is similar to the surface brightness distribution of spiral galaxies (the Kolmogorov–Smirnov test yields a probability of $P_{KS} = 68\%$). The distribution of GRBs is similar to the surface brightness distribution of elliptical galaxies with a lower probability ($P_{KS} = 40\%$). The authors found no statistically significant correlation between the distributions of GRBs and OB associations ($P_{KS} = 4\%$) as well as between GRBs and Type-Ib and Ic supernovae ($P_{KS} = 9\%$).

Bloom *et al.* (2002) collected detailed statistics on the angular distances of GRB sources from the centers of their host galaxies and took into account the errors in the GRB localization and in the estimation of the galaxy radii. They compared the spatial distribution of GRBs with the distribution of merging binary neutron stars and black holes (which was theoretically derived by the population-synthesis method). The Kolmogorov–Smirnov probability was $P_{KS} \lesssim 2 \times 10^{-3}\%$ in the latter case and $P_{KS} = 45\%$ when comparing the distributions of GRBs and regions of massive star formation (in the model of a galactic disk with an exponential surface brightness distribution). This result is commonly cited as an indirect confirmation of the collapsar hypothesis, according to which GRBs originate from the evolution of massive stars (Woosley 1993).

In this paper, we analyze more complete data on the localization of GRBs in their host galaxies (mid-2004) and compare their distribution with the radial distributions of Type-Ibc and Ia supernovae and (high-mass and low-mass) X-ray binaries in nearby galaxies as well as with the hypothetical dark matter profiles. The Kolmogorov–Smirnov method, which is commonly used to compare the various empirical distributions, is inapplicable due to the occasionally significant localization errors of GRBs in galaxies. Therefore, in contrast to previous studies, we use a method for comparing the moments of the derived distributions by taking into account the localization errors of the sources. We also suggest a new method for comparing the fractions of sources within the optical radius of the galaxy.

All of the methods used yield the following main result: the statistically radial distribution of GRBs in galaxies does not coincide with that for any class of (thermonuclear and collapsing) supernovae and X-ray binaries. We also show that the radial distribution of GRBs in distant galaxies is similar to the distribution of luminous matter in the model of an exponential disk (which confirms the results by Tsvetkov *et al.* (2001) and Bloom *et al.* (2002)) and resembles the dark matter profile with the parameters that correspond to the optical radius of the GRB host galaxies.

In this paper, we present the observational data and briefly describe the methods for comparing the statistics and their application to specific samples: GRBs, Type-Ibc and Ia supernovae, X-ray binaries, and dark matter. Subsequently, we discuss the results obtained and give our conclusions.

OBSERVATIONAL DATA

We analyzed the following groups of objects: GRBs, Type-Ia and Ibc supernovae, (high-mass and low-mass) X-ray binaries, and the hypothetical dark matter distributions in galaxies. Each group (except dark matter) is a set of data on the galactocentric distances of the objects. All distances were normalized to the characteristic radius of the host galaxy. In general, this is either the optical radius of the galaxy r_{opt} (within which 50% of the galaxy's B-band luminosity is contained), or the radius calculated from the 25-mag. R band isophote, or the mean of these two values (for the GRB hosts, the latter two values can differ by several factors). For the GRBs, we additionally took into account their localization errors recalculated in units of the host radii.

GRBs. Data on the GRBs localized before 2002 were taken from Bloom *et al.* (2002). The locations of the GRBs in their host galaxies localized after 2002 were determined from original reports (see Table 1).

The degree of reliability of the GRB association with its host galaxy was discussed in detail by Bloom *et al.* (2002). In general, it is assumed everywhere that the galaxy closest to (within 1 arcsec of) the GRB is the host and that the misidenification probability is negligible. The (approximate) geometrical center, which is defined as the image centroid (Bloom *et al.* 2002) or half the maximum size of the galaxy image, is taken as the center of the observed host galaxy. The galaxy radius is calculated at half light or from the empirical magnitude–radius relation

$$r_{\text{half light}} = 0\farcs6 \times 10^{-0.075(m-21)},$$

where $m = R_{\text{host}}$ is the R magnitude of the host galaxy (Odewhan *et al.* 1996; Bloom *et al.* 2002). The accuracy of the characteristic sizes of the galaxy is estimated to be $\sim$30%. To take into account the projection effect of the galaxies onto the plane of the sky, the distances in Table 1 were multiplied by a projection factor of 1.15 (since the distances decrease, on average, by $\approx$13% when the spatial distribution of the sources is projected onto the plane of the sky; Bloom *et al.* 2002).

Supernovae. The radial distributions of various types of supernovae in galaxies were analyzed by Bartunov *et al.* (1992, 1994). To construct the radial distribution of supernovae, we used an updatable database of the Sternberg Astronomical Institute catalog of supernovae (http://virtual.sai.msu.ru/

~pavlyuk/distrib/radial.html). The centers of the host galaxies for supernovae are determined with an error of about 10% (Tsvetkov and Pavlyuk 2004).

X-ray binaries. We used data from Grimm *et al.* (2002) to construct the radial distribution of Galactic (high-mass and low-mass) X-ray binaries.

Dark matter. Since there is no generally recognized law for the dark matter density in galaxies at present, we used two model distributions. The Burkert (B) model (Burkert 1995; Gentile *et al.* 2004) without a central density peak explains satisfactorily the observed rotation curves of nearby galaxies. The spatial density distribution of dark matter in this model is described by the formula

$$\rho_{\mathrm{B}}(r) \propto \frac{1}{(r/r_{\mathrm{core}}+1)[(r/r_{\mathrm{core}})^2+1]}, \qquad (1)$$

where the scale parameter $r_{\mathrm{core}} \simeq 15$ kpc for current galaxies (see, e.g., Gentile *et al.* 2004).

In addition, we considered the Navarro–Frenk–White (NFW) theoretical model (Navarro *et al.* 1997; Wechsler *et al.* 2002; Gentile *et al.* 2004) with a central density peak, in which the density distribution of dark matter obeys the law

$$\rho_{\mathrm{NFW}}(r) \propto \frac{1}{(r/r_{\mathrm{s}})(1+r/r_{\mathrm{s}})^2}. \qquad (2)$$

Observational data for nearby galaxies yield $r_{\mathrm{s}} \simeq$ 30 kpc (Gentile *et al.* 2004).

To properly compare the radial distributions of objects in spiral galaxies with the dark matter profile, we assumed that the observed GRBs (if they are associated with dark matter) isolate a disklike region in a spherically symmetric cloud of dark matter, since the presence of a dense interstellar gas is a necessary condition for bright optical afterglows of GRBs. The characteristic galactic disk thickness is less than one kiloparsec, and the dark matter density changes only slightly across the disk on such scales (see Stoehr *et al.* 2003; Hayashi *et al.* 2004). Therefore, the dark matter surface density $\Sigma_{\mathrm{DM}}(r)$ has the same radial dependence as the volume density: $\Sigma_{\mathrm{DM}}(r) \propto \rho_{\mathrm{DM}}(r)$. Since the surface density should be multiplied by r to be compared with the one-dimensional density of the radial distribution of objects $f(r)$ derived from observations, we used the relation $f(r)[\mathrm{DM}] = r\rho_{\mathrm{DM}}(r)$ to analyze the dark matter.

The parameter r_{core} for the Burkert model is related to the optical radius of the galaxy by an empirical relation obtained by Donato *et al.* (2004):

$$\log(r_{\mathrm{core}}) = (1.05 \pm 0.11)\log r_{\mathrm{d}} + (0.33 \pm 0.04), \qquad (3)$$

where $r_{\mathrm{d}} = 0.59 r_{\mathrm{opt}}$ is a parameter in the model of an exponential galactic disk (since Donato *et al.* (2004) used a different definition for r_{opt}, the relationship to r_{d} is different). The parameter of the NFW model was taken to be $r_{\mathrm{s}} = 2r_{\mathrm{core}}$. Note that Stoehr *et al.* (2003) and Hayashi *et al.* (2004) provided evidence for the presence of a plateau in the spatial density distribution up to distances of 1 kpc in the NFW model, so the Burkert dark matter distribution may be considered as a fit to the NFW distribution without a central peak.

Table 1. New data on the localization of GRBs relative to the centers of their associated host galaxies

GRB	z	r_0	σ_r	References
000131	4.5	1.09	0.30	[1]
000210	0.846	1.50	0.99	[2]
000911	1.058	0.23	0.95	[3]
000926	2.038	0.13	0.01	[4]
010222	1.477	0.77	0.92	[5]
010921	0.45	1.28	0.44	[6]
011121	0.36	0.86	0.12	[7]
011211	2.14	1.15	0.38	[8]
020405	0.69	2.21	0.62	[9]
021004	2.3	0.00	0.94	[10]
021211	1.01	2.32	1.07	[11]
031203	0.1055	0.03	0.015	[12]
040701X	0.2146	0.00	0.86	[13]
040924	0.859	0.00	1.5	[14]
041006	0.0716	0.33	0.13	[15]

Note: z is the redshift of the galaxy, r_0 is the estimated distance between the centers of the GRB error region and the host galaxy (in units of the optical radius of this galaxy), and σ_r is the radius of the GRB error region. The data sources are: [1] Bloom *et al.* (GCN notice #1133); [2] Piro *et al.* (2002); [3] Price *et al.* (2002); [4] Castro *et al.* (2001); [5] Frail *et al.* (2001); [6] J.S. Bloom *et al.* (GCN notice #1135); [7] J.S. Bloom (GCN notice #1260); [8] D.W. Fox *et al.* (GCN notice #1311); [9] N. Masetti *et al.* (GCN notice #1375); [10] A. Fruchter *et al.* http://www-int.stsci.edu/~fruchter/GRB/021004/; [11] A. Levan *et al.* (GCN notice #1758); [12] A. Gal-Yam *et al.* (astro-ph/0403608); [13] E. Pian *et al.* (GCN notice #2638); [14] A. Henden (GCN notice #2811); [15] J.P.U. Fynbo *et al.* (GCN notice #2802).

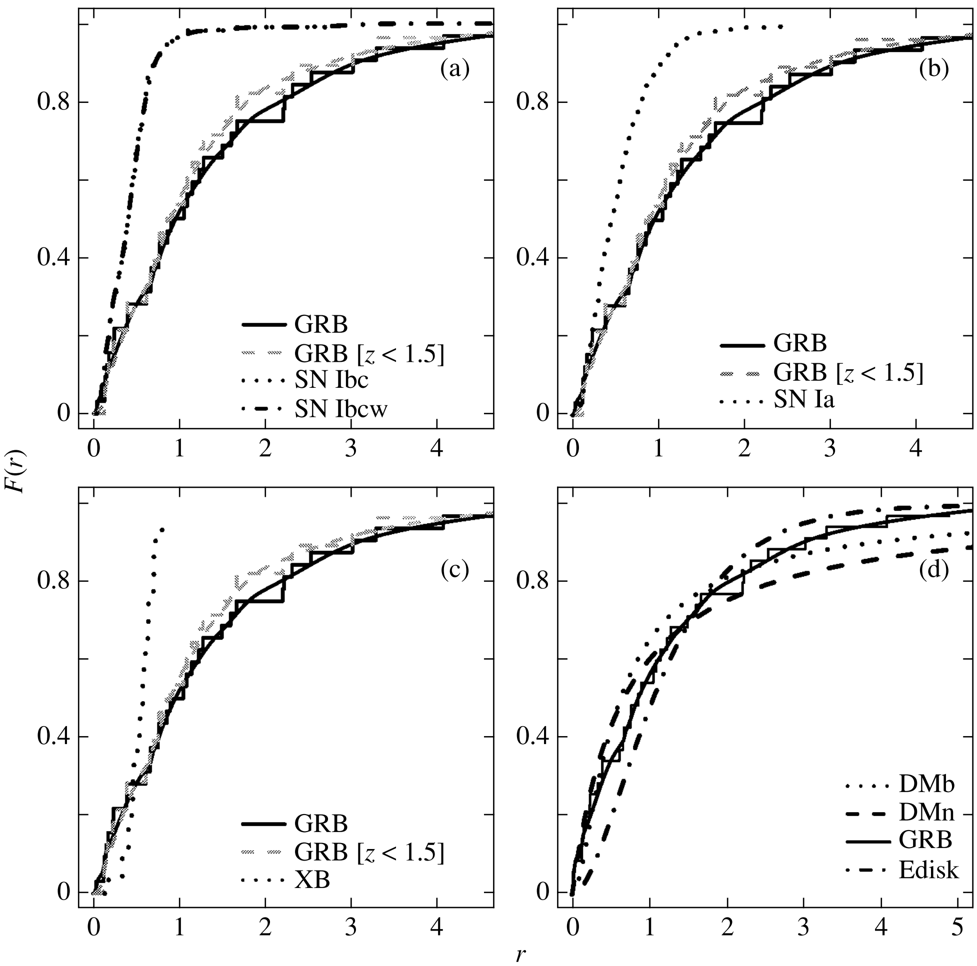

Fig. 1. Radial distributions of objects $F(r) = \int_0^r f(x)dx$, where $f(x)$ is the density of the distribution. We present data on GRBs (GRB), Type-Ibc (SN Ibc) and Ia (SN Ia) supernovae, X-ray binaries (XB), dark matter (DMb is the Burkert profile (1995), $r_{\rm core} = 0.83 r_{\rm opt}$; DMn is the NFW profile (Navarro *et al.* 1997), $r_{\rm s} = 1.67\, r_{\rm opt}$ and the exponential disk (Edisk). The step distributions were constructed for the GRBs without smoothing. The smooth curves represent the smoothed (with localization errors) cumulative distributions of GRBs (GRBw), supernovae (SN Ibcw), and GRBs at $z < 1.5$ (GRB [$z < 1.5$]).

We also used the surface-density profile of luminous matter in galaxies in the model of an exponential disk:

$$\Sigma_{\exp}(r) \propto \exp(-r/r_{\rm d}), \qquad (4)$$

for which the radial distribution is

$$f(r)[\exp] = r\exp(-r/r_{\rm d}). \qquad (5)$$

Radial Distributions of the Objects under Study

We begin by constructing the radial distributions of the objects under study. We use the dimensionless galactocentric distances (i.e., each galactocentric distance was normalized to the optical radius of the corresponding host galaxy). On average, $\langle r_{\rm opt}\rangle \simeq 2.5$ kpc for the distant galaxies in which GRBs were observed (Bloom *et al.* 2002) and $\langle r_{\rm opt}\rangle \simeq 15$ kpc for the nearby galaxies in which the supernovae under study are located. The radial distribution of the measured GRBs (without localization errors) as a function of the galactocentric distance is indicated in Fig. 1 by the step line for all the GRBs under consideration (black line) and GRBs with redshifts less than 1.5 (gray line).

Due to the GRB localization errors, the probability density of finding the source at a given radius, $p(r)$, should be taken in place of the galactocentric distance r. The probability density $f(r)$ for N sources can be calculated by adding the individual probability densities: $f(r) = \sum_{i=1}^N p_i(r)$. The localization errors are taken into account as follows. If r_0 is the measured galactocentric distance of the center of the error region for an individual GRB and σ_r is the error in the GRB location (i.e., the error region is assumed to be

Table 2. Central moments of the radial distributions of GRBs, supernovae, and X-ray binaries calculated without localization errors

$\mu^i(r), i = 1, 2$	$\mu^1(r) \pm \sigma_{\mu^1(r)}$	$\mu^2(r) \pm \sigma_{\mu^2(r)}$	$r^{\rm med} \pm \sigma_{r^{\rm med}}$
GRBs	1.18 ± 0.21	1.45 ± 0.42	0.82 ± 0.20
Nearby GRBs, $z < 1.5$	1.00 ± 0.17	1.03 ± 0.33	0.76 ± 0.18
Type-Ibc supernovae (SN Ibc)	0.43 ± 0.03	0.12 ± 0.06	0.40 ± 0.03
Type-Ia supernovae (SN Ia)	0.54 ± 0.02	0.14 ± 0.02	0.47 ± 0.03
X-ray binaries (LMXB and HMXB)	0.54 ± 0.03	0.029 ± 0.01	0.58 ± 0.04

Note: $\mu^1(r)$ is the mean, $\mu^2(r)$ is the variance, and $r^{\rm med}$ is the median. The rms deviations were calculated by the bootstrap method.

circular in shape, which is not always the case), then the probability density of finding the source at distance r from the center of the host galaxy is described by the Rice distribution (Bloom *et al.* 2002)

$$p(r; r_0, \sigma_r)dr = \frac{r}{\sigma_r^2} e^{-\frac{r^2+r_0^2}{2\sigma_r^2}} I_0\left(\frac{r\, r_0}{\sigma_r^2}\right) dr, \qquad (6)$$

where I_0 is a modified Bessel function of the zeroth order. In the case of a small localization error, the probability distribution of the source's location is close to the $\delta(r - r_0)$-function. A large localization error leads to a probability density with broad wings. Thus, more accurate data are more significant, and the weight (significance) of each observation is taken into account.

In analyzing the radial distributions of supernovae, the error with which the center of the galaxy image is determined rather than the localization error of the supernova itself makes a major contribution to the error in their galactocentric distances. Since analysis indicates (Tsvetkov and Pavlyuk 2004) that this error does not exceed 10% of the optical radius, we took $\sigma_r = 0.1r$ for supernovae.

The profiles of the GRB distribution function $F(r) = \int_0^r f(x)dx$ are represented in Fig. 1 by the smooth curves. For comparison, this figure also shows the smoothed radial distributions of supernovae (Figs. 1a and 1b) and X-ray binaries (Fig. 1c). In addition to the distributions of GRBs with and without localization errors, Fig. 1d shows the theoretical dark matter profiles $F(r)[\mathrm{DM}] = \int_0^r \rho_{\rm DM}(x)xdx$ in models (1) and (2) for the parameters calculated using relation (3), $r_{\rm core} = 0.83\, r_{\rm opt}$ and $r_{\rm s} = 1.67\, r_{\rm opt}$, respectively. This figure also shows the radial distribution of galactic luminous matter in the model of an exponential disk (4) for $r_{\rm d} = 0.59 r_{\rm opt}$, $F(r)[\exp] = \int_0^r \rho_{\exp}(x)xdx$.

THE METHODS FOR COMPARING THE POPULATIONS OF OBJECTS

We used the following methods to compare the empirical distributions of the objects under study in galaxies: (1) estimating the moments of the empirical distributions and (2) counting the number of objects within a given radius. These methods were applied to the observed distributions of objects in galaxies with and without localization errors.

Let us consider these methods successively.

Estimates of the Distribution Moments

The moments of the probability distribution functions are important in various astrophysical problems (see, e.g., the review by Blinnikov and Moessner 1998). Let us consider the estimates for the first two moments (mean and variance) of the distributions of the objects under study. The mean relative galactocentric distances (the first moment) in the distributions of various objects provide information about the characteristic localization (centroid) of the population under study. The variance (the second central moment) is indicative of the degree of scatter about the mean.

The mean (the first moment) and variance (the second central moment) of the empirical distributions were estimated using standard formulas (see, e.g., G. Korn and T. Korn 1968):

$$\mu^1(r) \equiv \langle r \rangle \simeq \frac{1}{N}\sum_{i=1}^{N} r_i, \qquad (7)$$

$$\mu^2(r) \simeq \frac{1}{N-1}\sum_{i=1}^{N}(r_i - \langle r \rangle)^2.$$

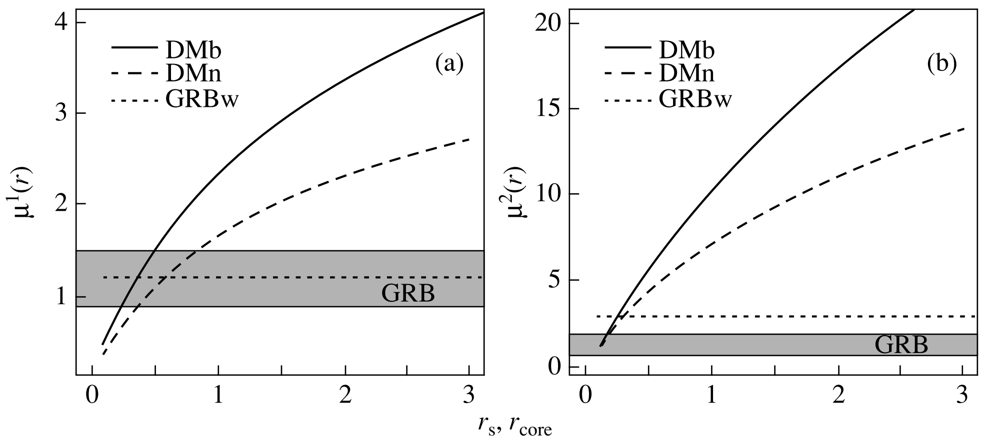

Fig. 2. First (a) and second (b) moments of the dark matter distribution versus Burkert and NFW model parameters.

Since the estimate of the mean for small samples with outliers can be strongly biased, in addition to the first two moments, the medians are considered. The median can be estimated from the condition $F(r_{\rm med}) \simeq 1/2$ (F is the empirical distribution function of the random variable r); i.e., a value of r above and below which there is half of all data is found.

We estimated the errors in the moments of the empirical distributions using the bootstrap method (Éfron 1988; see the Appendix). Table 2 gives our estimates for the first two moments of the distributions and the medians with the rms deviations estimated by the bootstrap method for gamma-ray bursts (GRB), Type-Ibc supernovae (SN Ibc), Type-Ia supernovae (SN Ia), and X-ray binaries (XB). We emphasize that the localization errors of the sources themselves are ignored in this analysis.

Including the localization errors of the sources allows us to directly consider the probability densities $f(r)$ rather than the distribution functions $F(r)$. If the probability density is known, the moments of the distributions can be calculated using the formulas

$$\bar{\mu}^i(r) = \frac{\int_0^\infty x^i f(x)dx}{\int_0^\infty f(x)dx}. \quad (8)$$

Table 3 gives the calculated moments of the distributions, $\bar{\mu}^1$ and $\bar{\mu}^2$, for GRBs with localization errors (GRBw) and Type-Ibc supernovae with localization errors (SN Ibcw). In Fig. 2, the first and second moments are plotted against the parameters r_s and $r_{\rm core}$ for the dark-matter models under consideration. The first moments for GRBs taken from Tables 2 and 3 are shown for comparison. We emphasize that, in contrast to the moments calculated using formulas (7), moments (8) are not central, but ordinary; therefore, only the first moments can be compared; the second moments μ^2 and $\bar{\mu}^2$ cannot be compared.

Table 3. Moments of the observed radial distributions $f(r)$ of GRBs and Type-Ibc supernovae with localization errors, moments of the theoretical radial dark matter distributions for the two models, and moments of the radial luminous-matter distribution in the exponential disk model

$\mu^i(r), i = 1, 2$	$\bar{\mu}^1(r)$	$\bar{\mu}^2(r)$
Gamma-ray bursts (GRBw)	1.22	2.94
Nearby gamma-ray bursts, $z < 1.5$ (GRBw)	1.07	2.35
Type-Ibc supernovae (SN Ibcw)	0.43	0.30
Dark matter		
Burkert (DMb: $r_{\rm core} = 0.83 r_{\rm opt}$)	2.11	8.61
NFW (DMn: $r_s = 1.67 r_{\rm opt}$)	2.13	9.70
Exponential disk ($r_d = 0.59 r_{\rm opt}$)	1.18	2.09

Counting the Objects within $r_{\rm opt}$

Consider another quantitative comparison test for empirical samples, which allows the localization errors of the sources to be taken into account.

Let us calculate the number of objects within $r_{\rm opt}$ of the galaxy for each sample. To take into account the localization errors, we proceed as follows. We specify the brightness profile of the galaxy in the form of the function

$$K(x) \simeq \begin{cases} 1, r < r_{\rm opt} \\ 0, r > r_{\rm opt}. \end{cases} \quad (9)$$

The approximate equality implies that we assume the error in the radius to be 30% and fit the edges

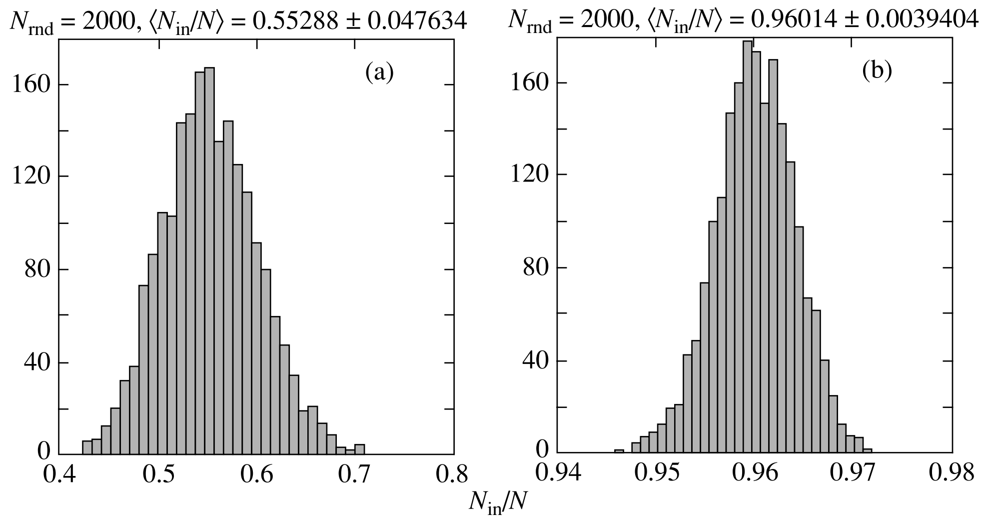

Fig. 3. Histograms of $N_{\rm in}/N$ obtained through numerical simulations for (a) GRBs and (b) Type-Ibc supernovae.

of the galaxy by a smooth monotonic function. For definiteness, the wings of the galaxy are assumed to be described by a normal (cumulative) distribution function with a dispersion of $\sigma = 0.3$.

Let the location of the object relative to the galactic center r_i and its (presumably normal) error σ_i be known. To count the fraction of objects within $r_{\rm opt}$, we can use the random variable

$$\frac{N_{\rm in}}{N} = \frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{\infty} f^i(x)K(x)dx, \qquad (10)$$

where $f^i(x) \equiv f(x, r_i, \sigma_i)$ is the probability density of the normal distribution with mean r_i and dispersion σ_i (x varies within the range $-\infty$ to ∞).

Let us find the distribution of the random variable $N_{\rm in}/N$. For this purpose, we draw a new random variable x for each source r_i in accordance with the initial distribution function $f^i(x)$ with known mean and dispersion. We repeat the procedure n times for each value of r_i. The estimate of the number of objects within $r_{\rm opt}$ is then

$$\left(\frac{N_{\rm in}}{N}\right)_k \simeq \frac{1}{N}\sum_{i=1}^{N} K(x_{i,k}), \quad k = 1, n. \qquad (11)$$

In the limit of a large number of trials, a normally distribution random variable $(N_{\rm in}/N)_k$ must be obtained, and its rms error can be estimated. The result of our numerical simulations for 2000 trials is given in Table 4 (column 2). Figure 3 shows the histograms of $N_{\rm in}/N$ for GRBs (Fig. 3a) and Type-Ibc supernovae (Fig. 3b) derived from numerical simulations.

For sources with a given radial distribution function (e.g., for dark matter and an exponential distribution of luminous matter in galactic disks), $N_{\rm in}/N$ can be estimated using the formula

$$\frac{N_{\rm in}}{N} \simeq \frac{\sum\limits_{i=1}^{\infty} K(x_i)f(x_i)}{\sum\limits_{i=1}^{\infty} f(x_i)}, \qquad (12)$$

where f is specified by relations (1), (2), and (5). The values of $N_{\rm in}/N$ are given in Table 5; $N_{\rm in}/N$ is plotted against $r_{\rm s}$ and $r_{\rm core}$ for dark matter in Fig. 4.

For discrete empirical samples, there is a different method for counting the number of objects within a given radius. Let us define the random variable

$$y_i = \begin{cases} 1, & |r_i| \le 1 \\ 0, & |r_i| > 1, \end{cases} \qquad (13)$$

which has a discrete probability distribution. The probability of an object being within the optical radius of the host galaxy ($y_i = 1$) is

$$p_i = \int_{-1}^{1} f^i(x)dx, \qquad (14)$$

while the probability $P(y_i = 0) = (1 - p_i)$ for the ith object under study. The estimates of the mean and variance of y_i (see G. Korn and T. Korn 1968) are

$$\mu^1(y_i) = p_i, \quad \mu^2(y_i) = p_i(1 - p_i). \qquad (15)$$

The sum of the means y_i for all objects of a given class

Table 4. Fractions of the sources within the galaxy radius, $N_{\rm in}/N$, for various types of objects

Tested sample	$N_{\rm in}/N$	$[N_{\rm in}/N]$
Gamma-ray bursts (GRB)	0.553 ± 0.048	0.5602 ± 0.054
Type-Ibc supernovae (SN Ibc)	0.960 ± 0.004	0.966 ± 0.007

Note: The second and third columns give the results of our numerical simulations for 2000 trials using formulas (11) and (16), respectively.

yields an estimate of $[N_{\rm in}/N]$ and its dispersion:

$$[N_{\rm in}/N] = \sum_{i=1}^{N} \mu^1(y_i)/N, \qquad \sigma_{[N_{\rm in}/N]} = \sqrt{\sum_{i=1}^{N} \mu^2(y_i)/N}. \quad (16)$$

The values of $[N_{\rm in}/N]$ and $\sigma_{[N_{\rm in}/N]}$ for GRBs and Type-Ibc supernovae calculated by this method are listed in the third column of Table 4; these are in close agreement with the values obtained by our numerical simulations using the first method.

DISCUSSION

Analyzing the moments of the empirical distributions of the objects under study (Tables 2 and 3) and counting the number of sources within the optical radius of the galaxy (Tables 4 and 5) led us to conclude that the radial distribution of GRBs in galaxies differs significantly from the distributions of supernovae of various types and X-ray binaries. GRBs are, on average, farther from the galactic centers and are distributed more widely than other objects. This conclusion is in conflict with the hypothesis that all GRBs are associated with the corresponding supernovae, but it is consistent with the assumption that only nearby GRBs with relatively small energy release are associated with bright hypernovae, while more energetic GRBs seen at high redshifts are not accompanied by bright supernovae.

Table 5. Fractions of the sources within the galaxy radius, $N_{\rm in}/N$, for the theoretical dark-matter and luminous-matter profiles in the exponential disk model calculated using formula (12)

Tested distributions	$N_{\rm in}/N$
Burkert dark matter (DMb: $r_{\rm core} = 0.83 r_{\rm opt}$)	0.38
NFW dark matter (DMn: $r_{\rm s} = 1.67 r_{\rm opt}$)	0.425
Exponential disk ($r_{\rm d} = 0.59 r_{\rm opt}$)	0.50

All GRBs are distant objects, and the typical redshift of the galaxies under consideration is $\sim$1; at such large distances, the morphology of the galaxies may differ from their current morphology. The typical optical radii of the GRB host galaxies and the galaxies in the closest neighborhood are $\sim$2–3 and 10–15 kpc, respectively. Such a large difference in the radii cannot be explained in terms of cosmological effects, which cause an effective decrease in the optical radius by only a few percent. Spectroscopic observations of the host galaxies of GRBs (Sokolov *et al.* 2001) lead one to conclude that these are galaxies with an enhanced star formation rate and significant internal extinction, although, in general, the properties of these galaxies (Hurley *et al.* 2003) correspond to those of late-type field galaxies at the corresponding redshifts. A small optical radius could be indicative of the actual small size (compact galaxies) or the existence of a significant part of the gaseous galactic disk unaffected by star formation (Begum *et al.* 2005).

If we associate GRBs with the evolution of massive stars, then the assumption about a nonstandard pattern of star formation in the GRB host galaxies, which causes the initial mass function of young stars to change in favor of a larger number of very massive stars, could be a possible way out of the disagreement between our radial distributions of Type-Ibc supernovae and GRBs in galaxies. This could explain why GRBs are associated mostly with late-type galaxies, because the (secondary) star formation in such galaxies proceeds under conditions of slow differential rotation. A detailed determination of the chemical composition of the gas in GRB host galaxies can serve as a test of this hypothesis.

Since no data are available for high-redshift Type-Ibc supernovae, a comparison is made using supernovae in nearby galaxies. We could attempt to attribute the difference between the derived radial distributions of GRBs and supernovae in galaxies to different radial distributions of massive stars in galaxies at high redshifts. However, this explanation is in conflict with the satisfactory description of the GRB distribution by the model of luminous matter with an exponential surface density.

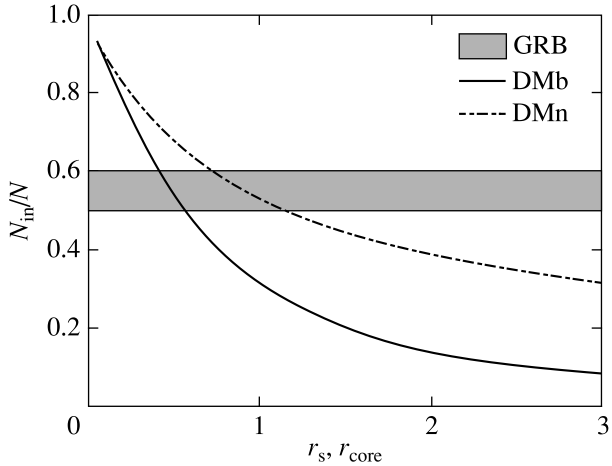

Fig. 4. N_{in}/N versus r_s and r_{core} for the dark matter models.

A comparison of Tables 4 and 5 shows that the observed fraction of GRBs within the optical radius differs only slightly from the fraction of dark matter in the disk. One would think that the behavior of the second moment for dark matter (see, in particular, Fig. 2) rejects the association of GRBs with dark matter. However, it should be kept in mind that the second (noncentral!) moment increases sharply the weight of the distant galactic regions. We calculated this moment directly for the dark matter density; of course, the GRB localization observations were selected by the presence of an ordinary gas, which also has a quasi-exponential radial distribution in the galaxy. Including this truncation changes only slightly the numbers in Tables 4 and 5 and the first moment, but reduces greatly the second moment for dark matter.

CONCLUSIONS

Our analysis shows that the distribution of luminous matter in galaxies in the model of an exponential disk with the parameter $r_d = 0.59 r_{opt}$ is in best agreement with the spatial distribution of GRBs in their host galaxies. This is confirmed by previous studies (Tsvetkov *et al.* 2001; Bloom *et al.* 2002). However, this result cannot be unequivocally interpreted as evidence that GRBs originate from young massive stars, since the disk brightness follows the distribution of all (including low-mass) population-I stars and gas.

It can also be asserted that the radial distribution of GRBs agrees with the distributions of dark matter in galaxies at quite realistic parameters r_s and r_{core} under the assumption that the decrease in the scale parameter of the spatial dark matter distribution is proportional to the optical radius of the GRB host galaxies.

ACKNOWLEDGMENTS

This work was supported in part by the Russian Foundation for Basic Research (project nos. 05-02-17480 and 03-02-16110). We wish to thank O.K. Sil'chenko and A.V. Zasov for helpful discussions.

APPENDIX

The Bootstrap Method

The main idea behind the method is to approximate the unknown distribution function of a random variable by an empirical distribution. This approximate distribution function is used to estimate the moments of the random variable (mean and variance).

In accordance with this empirical distribution, various realizations (samples) of the random variable are modeled by the Monte Carlo method. Obtaining a sample each time, we determine the moment (mean and dispersion) of the distribution of interest. Modeling the sample of the random variable many times, we obtain the mean value of the moment and its distribution function. According to the central limit theorem, the derived distribution function of the parameters approaches a normal law. Thus, the confidence intervals for the moments of the empirical distribution can be estimated.

REFERENCES

1. O. S. Bartunov, I. N. Makarova, and D. Yu. Tsvetkov, Astron. Astrophys. **264**, 428 (1992).
2. O. S. Bartunov, D. Yu. Tsvetkov, and I. V. Filimonova, Publ. Astron. Soc. Pac. **264**, 1276 (1994).
3. A. Begum, J. N. Chendalur, and I. D. Karachentsev, Astron. Astrophys. (2005, in press); astro-ph/0502307.
4. S. I. Blinnikov, Surv. High Energy Phys. **15**, 37 (2000).
5. S. Blinnikov and R. Moessner, Astron. Astrophys., Suppl. Ser. **130**, 193 (1998).
6. J. S. Bloom, S. R. Kulkarni, and S. G. Djorgovski, Astron. J. **123**, 1111 (2002).
7. A. Burkert, Astrophys. J. **447**, L25 (1995).
8. S. Castro, T. J. Galama, F. A. Harrison, *et al.*, astro-ph/0110566 (2001).
9. F. Donato, G. Gentile, and P. Salucci, Mon. Not. R. Astron. Soc. **353**, L17 (2004).
10. B. Éfron, *Non-traditional Methods of Multivariate Statistical Analysis* (Finansy i Statistika, Moscow, 1988) [in Russian].
11. D. A. Frail, F. Bertoldi, G. H. Moriarty-Schieven, *et al.*, astro-ph/0108436 (2001).
12. G. Gentile, P. Salucci, U. Klein, *et al.*, Mon. Not. R. Astron. Soc. **351**, 903 (2004).
13. H.-J. Grimm, M. Gilfanov, and R. Sunyaev, Astron. Astrophys. **391**, 923 (2002).

14. A. V. Gurevich, K. P. Zybin, and V. A. Sirota, Usp. Fiz. Nauk **167**, 913 (1997) [Phys. Usp. **40**, 869 (1997)].
15. E. Hayashi, J. F. Navarro, C. Power, *et al.*, Mon. Not. R. Astron. Soc. **355**, 794 (2004).
16. J. Hjorth, J. Sollerman, P. Meller, *et al.*, Nature **423**, 847 (2003).
17. K. Hurley, R. Sari, and S. G. Djorgovki, *Compact Stellar X-ray Sources*, Ed. by W. Lewin and M. van der Klis (Cambridge Univ. Press, Cambridge, 2003); astro-ph/0211620.
18. G. A. Korn and T. M. Korn, *Mathematical Handbook for Scientists and Engineers* (McGraw-Hill, New York, 1968; Nauka, Moscow, 1984).
19. T. Matheson, P. M. Garnavich, K. Z. Stanek, *et al.*, Astrophys. J. **599**, 394 (2003).
20. J. F. Navarro, C. S. Frenk, and S. D. M. White, Astrophys. J. **490**, 493 (1997).
21. S. C. Odewhan, R. A. Windhorst, S. P. Driver, and W. C. Kee, Astrophys. J. **472**, L13 (1996).
22. T. Piran, astro-ph/0405503 (2004).
23. L. Piro, D. A. Frail, J. Gorosabel, *et al.*, Astrophys. J. **577**, 680 (2002).
24. K. A. Postnov, Usp. Fiz. Nauk **169**, 545 (1999) [Phys. Usp. **42**, 469 (1999)].
25. K. A. Postnov, in *Proceedings of the International Workshop QUARKS-2004*, Ed. by D. V. Levkov, V. A. Matveev, and V. A. Rubakov; astro-ph/0409755 (2004).
26. P. A. Price, E. Berger, S. R. Kulkarni, *et al.*, Astrophys. J. **573**, 85 (2002).
27. V. V. Sokolov, T. A. Atkhullin, A. J. Castro-Tirado, *et al.*, Astron. Astrophys. **372**, 438 (2001).
28. K. Z. Stanek, T. Matheson, P. M. Garnavich, *et al.*, Astrophys. J. **591**, L17 (2003).
29. F. Stoehr, S. D. M. White, V. Springel, *et al.*, Mon. Not. R. Astron. Soc. **345**, 1313 (2003).
30. D. Yu. Tsvetkov and N. N. Pavlyuk, private communication (2004).
31. D. Yu. Tsvetkov, S. I. Blinnikov, and N. N. Pavlyuk, Pis'ma Astron. Zh. **27**, 483 (2001) [Astron. Lett. **27**, 411 (2001)].
32. R. H. Wechsler, J. S. Bullock, J. R. Primack, *et al.*, Astrophys. J. **568**, 52 (2002).
33. S. A. Woosley, Astrophys. J. **405**, 273 (1993).
34. B. Zhang and P. Meszaros, Int. J. Mod. Phys. A **19**, 2385 (2004).

Translated by V. Astakhov