

STRONG AND WEAK LENSING BY GALAXY CLUSTERS

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ABSTRACT

I present an overview of strong and weak gravitational lensing by galaxy clusters. After briefly introducing the principles of gravitational lensing, I discuss the main lessons learned from lensing on the mass distribution in clusters and their relation to cosmology.

1. INTRODUCTION

Gravitational lensing phenomena due to galaxy clusters can naturally be split into two categories, strong and weak. Strong lensing was detected in 1986, when highly elongated, curved, long features of low surface brightness were found in two clusters, Abell 370 and Cl 2244 (Lynds & Petrosian 1986; Soucail et al. 1987a,b). Of the three possible explanations suggested, spectroscopy selected gravitational lensing when it turned out that these “giant arcs” had substantially higher redshifts than the clusters (Soucail et al. 1988). Weak lensing gives rise to much less spectacular distortions of background-galaxy images, termed “arclets” (Fort et al. 1988; Tyson et al. 1990). Since galaxies are not intrinsically symmetric, such distortions can only be quantified statistically by averaging over many images, commonly adopting the assumption that galaxy ellipticities average to zero in absence of lensing.

While arcs require compact, dense cluster cores and thus probe their central mass distribution, arclets can be found everywhere across clusters and allow their mass distribution to be mapped even at clustercentric distances comparable to the virial radius. Unlike other methods for quantifying the mass distribution in clusters, lensing has the advantage that it is sensitive only to the surface mass density projected along the line-of-sight, irrespective of its composition or physical state.

I review here the main lessons that have been learned from both weak and strong lensing by clusters. I first summarise the physical assumptions and principles underlying interpretations of lensing phenomena, keeping the formalism to the necessary minimum. I then turn to strong lensing and explain the key results and a number of open problems. After explaining the principle of weak-lensing techniques, I describe results obtained from weak lensing in clusters and conclude with an outlook and a summary.

2. BASIC PRINCIPLES OF GRAVITATIONAL LENSING

2.1. Assumptions, Fermat’s Principle

Gravitational lensing theory is based upon three key assumptions (see, e.g. Narayan & Bartelmann 1999 for a review). First, the Newtonian potential Φ of the lensing mass distribution is assumed to be small, $\Phi \ll c^2$. Second, the lenses are assumed to be slow, i.e. their bulk velocities and the velocities of their constituents are assumed to be small, $v \ll c$. Finally, individual lenses are taken to be thin, i.e. their typical size L has to be small, $L \ll c/H_0$, where c/H_0 is the Hubble radius.

Under these assumptions, individual lenses like galaxies or galaxy clusters can be treated as embedded into locally flat, or Minkowskian, space-time. According to the third assump-

tion, curvature effects of space-time become important only on scales much larger than the lens. Light rays propagating past the lens can then be approximated as geodesic lines of the background Friedmann metric between the observer and close to the lens, and from close to the lens to the source, with a connecting geodesic of the locally nearly flat space-time which is weakly perturbed by the lens.

The weakly perturbed Minkowskian metric implies an index of refraction $n = 1 - 2\Phi/c^2$. The potential is normalised such that it approaches zero at infinity, thus negative, and the index of refraction is larger than unity. The speed of light in a gravitational field, c' , is thus less than c , $c' = c/n$. Like in geometrical optics, Fermat’s principle can now be applied to find the light path. It asserts that the variation of the optical light path vanishes,

$$\delta \int_a^b n[\vec{x}(\lambda)] \left| \frac{d\vec{x}}{d\lambda} \right| d\lambda = 0, \quad (1)$$

where $\vec{x}(\lambda)$ is the light path parameterised by the curve parameter λ . Indices a and b mark the start and end points of the light path. Euler’s equation applied to (1) then implies that light is deflected by an angle

$$\vec{\alpha} = \frac{2}{c^2} \int \nabla_{\perp} \Phi(\vec{x}) dz, \quad (2)$$

with the integration formally proceeding along the light path, and the gradient taken perpendicular to it. However, according to the assumption that lenses are weak, the deflection angle is small, and it is permissible to integrate along the *unperturbed* light path, i.e. a straight line tangential to the incoming light ray. This is the Born approximation in the context of gravitational lensing.

2.2. Lens Equation

A gravitational lens system is characterised by three distances, $D_{d,ds,s}$, from the observer to the lens, from the lens to the source, and from the observer to the source, respectively. The centre of the lens is connected with the observer by the optical axis. A light ray leaving the observer under an angle $\vec{\theta}$ with respect to the optical axis is deflected by an angle $\vec{\alpha}$ and arrives at the source, which would appear an angle $\vec{\beta}$ away from the optical axis in absence of lensing. Simple ray-tracing shows that these angles are related by

$$D_s \vec{\beta}(\vec{\theta}) = D_s \vec{\theta} - D_{ds} \vec{\alpha}(\vec{\theta}). \quad (3)$$

Dividing by D_s , and introducing the *reduced* deflection angle $\vec{\alpha} = D_{ds}/D_s \vec{\alpha}$, we can write (3) in the simple form

$$\vec{\beta}(\vec{\theta}) = \vec{\theta} - \vec{\alpha}(\vec{\theta}). \quad (4)$$

This equation relates the angular positions of source and image on the sky. It is generally non-linear and can give rise to phenomena like multiple images, image magnifications and distortions. The distances $D_{d,ds,s}$ are angular diameter distances, which are defined such that simple ray-tracing can be done even in curved space-time. Equation (2) suggests introducing the lensing potential,

$$\psi(\vec{\theta}) = \frac{2D_{ds}}{D_d D_s} \int \frac{\Phi(D_d \vec{\theta}, z)}{c^2} dz, \quad (5)$$

such that the reduced deflection angle is the gradient of ψ ,

$$\vec{\alpha}(\vec{\theta}) = \nabla_{\theta} \psi(\vec{\theta}). \quad (6)$$

2.3. Local Lens Mapping

Typical angular scales in galaxy clusters are much larger than typical source galaxies. We can thus linearise the lensing equation (4) and search for its local imaging properties. The Jacobian matrix of (4) is

$$\mathcal{A}(\vec{\theta}) = \frac{\partial \vec{\beta}(\vec{\theta})}{\partial \vec{\theta}} = \left(\delta_{ij} - \frac{\partial \alpha_i(\vec{\theta})}{\partial \theta_j} \right) = (\delta_{ij} - \Psi_{ij}), \quad (7)$$

where we have used (6) and introduced the Hessian matrix of the lensing potential,

$$\Psi_{ij} = \frac{\partial^2 \psi(\vec{\theta})}{\partial \theta_i \partial \theta_j}. \quad (8)$$

The Jacobian matrix \mathcal{A} is real and symmetric, thus it has two real eigenvalues. Either of them can vanish, so there can be two *critical curves* $\vec{\theta}_{cr}$ where $\det \mathcal{A} = 0$. These curves are closed. Their images under the lens equation (4) are called *caustics*, $\vec{\beta}_{ca}$. Sources near caustics are highly distorted because of the singular Jacobian. They give rise to giant arcs. Sources further away from caustic curves are weakly distorted and give rise to arclets.

The eigenvalues of the Jacobian matrix \mathcal{A} can be written as $\lambda_{\pm} = 1 - \kappa \pm \gamma$. The *convergence* κ is proportional to the surface mass density Σ of the lens,

$$\kappa = \frac{1}{2} \nabla_{\theta}^2 \psi = \frac{4\pi G}{c^2} \frac{D_d D_{ds}}{D_s} \Sigma \equiv \frac{\Sigma}{\Sigma_{cr}}, \quad (9)$$

while the *shear* γ has two components,

$$\gamma_1 = \frac{1}{2} (\Psi_{11} - \Psi_{22}), \quad \gamma_2 = \Psi_{12}, \quad (10)$$

and $\gamma = (\gamma_1^2 + \gamma_2^2)^{1/2}$. The shear quantifies the gravitational tidal field of the lensing mass distribution and is responsible for image distortions, while the convergence causes isotropic image expansion or contraction. Images are magnified by $\mu = |\det \mathcal{A}|^{-1} = |\lambda_+ \lambda_-|^{-1} = |(1 - \kappa)^2 - \gamma^2|^{-1}$. It will be important in the following that both κ and γ are linear combinations of second derivatives of the lensing potential ψ . Typically, critical curves require κ to be of order unity or larger, which means that the surface mass density Σ needs to be comparable to or larger than the *critical surface mass density* Σ_{cr} defined in (9).

3. GIANT ARCS

3.1. Arc Morphology and Immediate Consequences

The morphologies of the giant arcs observed so far can be broadly characterised by four simple statements. First, large

arcs are generally thin, some are unresolved even on HST images, and some show detailed structure like bright spots or darker lanes. Second, giant arcs have curvature radii larger than the radii of cluster galaxies, and they lack bright and extended counter-arcs. Third, “straight” arcs have been observed (Pelló et al. 1991) i.e. structures in clusters which resemble arcs by their length and brightness, but lack curvature. Fourth, “radial” arcs exist (e.g. Fort et al. 1992; Hammer et al. 1997); these are features pointing radially away from cluster centres and generally appear very close to the central cluster galaxies¹. The basic relations of lensing theory summarised above allow several immediate consequences to be derived from these morphological characteristics and types of arcs. These are:

- Substantial amounts of dark matter are required in galaxy clusters which have to be much more smoothly distributed than the light. Otherwise, arcs would be much more curved, e.g. around individual galaxies (Hammer et al. 1989; Bergmann et al. 1990).
- Cluster density profiles need to be steep, because otherwise arcs would be thicker than observed (Hammer & Rigaut 1989). This will be explained below.
- Cluster cores need to be substantially smaller than the X-ray cores. Otherwise, no arcs would be formed at all, or radial arcs would appear further away from the cluster centres (Fort et al. 1992; Miralda-Escudé 1992).
- Cluster mass distributions need to be asymmetric, because bright arcs would have comparably bright counter-arcs otherwise (Grossman & Narayan 1988; Kovner 1989).
- Clusters need to have substructures which encompass an appreciable fraction of the total cluster mass, because otherwise straight arcs would not appear (Kassiola et al. 1992).

3.2. Cluster Masses and the Mass Discrepancy

In principle, cluster masses can easily be estimated from strong gravitational lensing. For axially symmetric lenses, it can be shown that the tangential critical curve encloses a mean convergence of unity. Since large arcs appear very close to such critical curves, their angular distance θ_{arc} to the cluster centre provides an estimate for the radius of the tangential critical curve. Thus, $\langle \kappa \rangle(\theta_{arc}) \approx 1$, and the cluster mass enclosed by a circle traced by a giant arc is

$$M(\theta_{arc}) \approx \pi \theta_{arc}^2 \Sigma_{cr}. \quad (11)$$

For this to work, the critical surface mass density Σ_{cr} is required, hence the redshifts of cluster and source need to be known in addition to estimates for the cosmological parameters. While the simple assumption of axial cluster symmetry is good for a rough mass estimate, cluster mass distributions are usually modelled in detail until they fit the observed images and the cluster light distribution well, and then masses are derived from these models.

While these masses are generally in good qualitative agreement with other mass measures, e.g. from X-rays or the kinematics of the cluster galaxies, there is a consistent tendency in many clusters for X-ray mass estimates to be systematically lower than strong-lensing mass estimates by a factor of two to three (Wu 1994; Smail et al. 1995; Miralda-Escudé & Babul 1995). Solutions to this problem were attempted from several sides. The

¹Lensing is rich in spectacular misnomers. The term “gravitational lens” itself is misleading because gravitational lenses are highly astigmatic, poor optical systems without a well-defined focal length. “Straight” and “radial arcs” are further examples for memorable oxymorons.

suggestion by Loeb & Mao (1994) that magnetic fields could provide some pressure support for the intracluster gas and thus allow the gas to be cooler than expected from purely thermal equilibrium is probably not feasible because intracluster fields are not likely to be strong enough (Dolag et al. 2001).

Bartelmann & Steinmetz (1996) used hydrodynamic cluster simulations to show that X-ray mass estimates can be systematically lower than strong-lensing mass estimates in merging clusters because most of the X-ray gas is still colder than expected from the total mass of the merging clusters (see also Wu & Fang 1996). In addition, strong-lensing masses derived from simple, symmetric mass models tend to be systematically too high because substructures increase the gravitational tidal field of the mass distribution, hence also the shear of the lens, and thus critical curves at a given distance from the cluster centre require lower mass density (Bartelmann 1995; Hattori et al. 1997; Ota et al. 1998).

Finally, Allen (1998) noted that the mass discrepancy occurs only in clusters without cooling flows, while X-ray and strong-lensing mass estimates agreed well in clusters with cooling flows. The straightforward interpretation of this observation is that mass estimates agree well in dynamically relaxed clusters which were unperturbed for sufficiently long time to develop a cooling flow. Therefore, it appears that the discrepancy between lensing and X-ray mass estimates is restricted to unrelaxed clusters and can fully be explained by systematic effects which cause X-ray mass estimates to be low and strong-lensing mass estimates to be high.

3.3. Mass Profiles

As mentioned before, thin arcs require steep density profiles. This can easily be understood as follows. First, arcs require tangential critical curves, for which $1 - \kappa - \gamma = 0$, or $\gamma = 1 - \kappa$. The radial magnification, which determines the width of the arcs, is $(1 - \kappa + \gamma)^{-1}$, or $[2(1 - \kappa)]^{-1}$ at the tangential critical curve. Thin arcs require radial magnifications of unity or less, which can be achieved if $\kappa \lesssim 0.5$ at the tangential critical curve. On the other hand, for axially symmetric lenses, the tangential critical curve encloses a mean κ of unity, which implies that κ has to drop steeply from the cluster centre to the critical curve (Hammer & Rigaut 1989; Wu & Hammer 1993; Grossman & Saha 1994). This argument can be alleviated for asymmetric clusters, which can have lower mean κ within the tangential critical curve because of the enhanced shear.

As a corollary, this consideration implies that cluster cores need to be small if they exist, in any case substantially smaller than the area enclosed by the tangential critical curve (Fort et al. 1992). This is also required by radial arcs, which have to be outside the cluster core, but are observed much closer to cluster centres than tangential arcs. However, radial arcs can also form in cuspy density profiles like that suggested by Navarro et al. (1996), provided the central cusp is not too steep (Bartelmann 1996).

It has been pointed out that the relative abundance of radial and tangential arcs is a sensitive measure for the central slope of the cluster density profile (Miralda-Escudé 1995). Molikawa & Hattori (2001) showed that changing the central profile slope can change the abundance ratios by orders of magnitude. While this provides in principle a highly sensitive diagnostic for density profiles in cluster centres, observations of radial arcs are difficult because they occur very close to central cluster galaxies and are thus hard to detect. Current statistics of radial arcs does not allow any firm conclusions to be drawn.

3.4. Arc Statistics

The ability of a cluster lens to produce giant arcs is commonly quantified by its arc cross section. This is defined as the area in the source plane in which a source has to lie in order to be imaged as a giant arc. Since giant arcs form close to tangential critical curves, arc sources have to be close to tangential caustics, thus arc cross sections can be pictured as narrow stripes covering the tangential caustic of a lens.

The first important thing to note is that axially symmetric lens models are entirely inadequate for reliable arc statistics. First, the tangential caustics of axially symmetric lenses degenerate to a point. Perturbing the lens by external shear or internal ellipticity of the lensing potential or the surface-mass density makes the tangential caustic rapidly expand to form a diamond-shaped curve. The arc cross section of lens models are therefore expected to be highly sensitive to deviations from axial symmetry. Second, asymmetries and substructures in the lenses increase the shear and allow lenses to form arcs at lower surface-mass density than in symmetric cases. The strong effect on arc cross sections of deviations from symmetry has been demonstrated using numerically simulated clusters as lenses (Bartelmann & Weiss 1994; Bartelmann et al. 1995). One could attempt to use analytic, elliptical lens models for arc statistics, taking cluster ellipticities from numerical simulations. However, direct comparison shows that even this approach is insufficient because simulated clusters have a much higher level of substructure and are embedded in an inhomogeneous environment which contributes to the gravitational tidal field. Although the qualitative features of arc statistics may be captured by elliptically distorted analytic models, quantitative results require numerical simulations (Meneghetti et al. 2002).

The second important thing to note is that the evolution of the cluster population depends sensitively on cosmological parameters. While clusters tend to form at low redshift in high-density universes, they form much earlier in low-density universes. For a cluster to be an efficient lens, it has to be approximately halfway between the observer and the source, typically at redshifts around 0.4. Depending mainly on the cosmic density, cluster evolution between redshifts zero and 0.4 can be so rapid that the number of clusters available for strong lensing can be very low. In other words, the population of cluster lenses is potentially a strong discriminant for the cosmic density.

We thus see that arc statistics depends crucially on two ingredients, detailed cluster structure and the cosmic evolution of the cluster population. Numerical simulations carried out to quantify the probability for giant arcs to be formed in different cosmologies led to the result that about two orders of magnitude more giant arcs are expected in an open CDM universe with $\Omega_0 = 0.3$ than in an Einstein-de Sitter CDM model with $\Omega_0 = 1.0$, and, perhaps surprisingly, that a flat, low-density CDM model with $\Omega_0 = 0.3$ and cosmological constant $\Omega_\Lambda = 0.7$ falls one order of magnitude below the open model (Bartelmann et al. 1998).

Comparisons with observed numbers of arcs are difficult and somewhat uncertain because only a very small fraction of the sky has been surveyed for arcs. However, extrapolating the arc abundance observed in X-ray selected cluster samples (Le Fèvre et al. 1994; Luppino et al. 1999) to the full sky and comparing with numerical simulations shows that the simulated comes near the observed arc abundance only in the open CDM model, while the other two models fall short by one to two orders of magnitude. This is in marked contrast with the results of other cosmological experiments, which consistently show that the universe is most probably spatially flat and has low matter density, $\Omega_0 \sim 0.3$. Extensive tests and refinements of the simulations have so far only confirmed these results (Meneghetti et al. 2000;

Flores et al. 2000), so the interesting problem persists as to how expected and observed arc abundances could be brought into agreement.

An interesting corollary to the importance of cluster asymmetries for arc cross sections is directly related to physical properties of dark matter particles. Mainly in order to resolve potential problems of CDM models in reproducing the measured density profiles of dwarf galaxies, several authors suggested that dark matter particles may interact with each other. Such self-interaction would tend to make clusters more symmetric and less compact because local inhomogeneities in the dark-matter distribution would be smoothed out (e.g. Miralda-Escudé 2002). Both these effects would lower the ability of clusters to form giant arcs, because high compactness and a high level of asymmetry are crucial as we saw before (Wyithe et al. 2001). Numerical simulations of strong lensing by clusters consisting of self-interacting dark matter demonstrate that even small interaction cross sections of order $1 \text{ cm}^2 \text{ g}^{-1}$ would almost entirely destroy the strong-lensing ability of clusters (Meneghetti et al. 2001). Thus, arc statistics puts strong constraints on models of dark-matter self-interaction.

4. WEAK LENSING BY CLUSTERS

4.1. Principles

Observations of weak lensing by galaxy clusters aim at reconstructing the cluster mass distribution from the appearance of arclets, i.e. weakly distorted images of faint background galaxies. The number density of the background sources down to currently accessible flux limits is as high as $\sim 30 \text{ arcmin}^{-2}$. Clusters are thus seen against a finely structured “wallpaper” of background sources. What is most commonly observed are the image ellipticities of these sources (see below for alternatives). Since the sources are not intrinsically circular, weak-lensing distortions cannot be inferred from individual images. Rather, several of them need to be averaged, assuming that their intrinsic ellipticities are randomly oriented and would thus average to zero in absence of lensing. The finite number density of the background sources implies a resolution limit for weak-lensing mass reconstructions. Suppose ten galaxy images need to be averaged to sufficiently suppress their intrinsic ellipticities. They cover a solid angle of $\sim 0.3 \text{ arcmin}^2 \sim (0.5 \text{ arcmin})^2$, hence structures smaller than $\sim 0.5'$ cannot be resolved that way.

The key problem of weak lensing is that what is observed are the image distortions due to the tidal field or the shear γ , but what is sought is the surface mass density or the convergence κ . The key idea of weak-lensing techniques is that both κ and γ are linear combinations of second-order derivatives of the lensing potential ψ , so they are related through the potential. The inversion of γ to find κ is most easily done in Fourier space and leads in real space to a convolution equation which can symbolically be written

$$\kappa(\vec{\theta}) = \Re \left[\int d^2\theta' \mathcal{D}(\vec{\theta} - \vec{\theta}') \gamma(\vec{\theta}') \right], \quad (12)$$

where the kernel \mathcal{D} and the shear γ are considered as complex quantities (Kaiser & Squires 1993). Typical image ellipticities induced by weak lensing are of order a few per cent. Their measurement is challenging, but analysis techniques are well developed (e.g. Kaiser et al. 1995).

4.2. Problems and Solutions

The straightforward weak-lensing inversion technique sketched above has several problems, the most important of which are as

follows.

By design, measurements of image distortions cannot distinguish whether they were caused by a Jacobian matrix \mathcal{A} , or by the matrix multiplied by a scalar, $(1 - \lambda)\mathcal{A}$, with $\lambda \neq 1$. In both cases, the size of the images will be different, but their ellipticities will be the same. Consequently, any weak-lensing technique based on observations of image ellipticities alone cannot distinguish a lens characterised by convergence κ and shear γ from another lens which has $(1 - \kappa') = (1 - \lambda)(1 - \kappa)$ and $\gamma' = (1 - \lambda)\gamma$. For $\lambda \ll 1$, this degeneracy transformation is approximated to first order by $\kappa' = \kappa + \lambda$. In that limit, it corresponds to adding a sheet of constant surface mass density to the lens, hence it has been called the “mass-sheet degeneracy”. It should be noted that this degeneracy cannot be broken in experiments measuring shear alone.

Strictly, image ellipticities do not measure the shear γ but rather the *reduced shear* $g = \gamma/(1 - \kappa)$. If lensing is truly weak, $\kappa \ll 1$ and $|\gamma| \ll 1$, and $g = \gamma$ to first order. If this approximation breaks down, one can insert g instead of γ into the convolution (12) to obtain an integral equation,

$$\kappa_{i+1}(\vec{\theta}) = \Re \left[\int d^2\theta' \mathcal{D}(\vec{\theta} - \vec{\theta}') \frac{\gamma}{1 - \kappa_i} \right], \quad (13)$$

which can be solved iteratively starting from $\kappa_0 = 0$ (Seitz & Schneider 1995).

Still, the convolution formally covers all of two-dimensional real space, while real data fields are of course finite. Adopting the kernel \mathcal{D} on a finite field introduces the unwanted bias that the total mass reconstructed in the entire field is zero. The problem can be alleviated on large fields by cutting away the edges, but alternative kernels have been constructed which are designed for finite fields (e.g. Kaiser 1995; Schneider 1995; Seitz & Schneider 1996; Squires & Kaiser 1996).

Finally, it is a crucial assumption underlying shear-based weak-lensing techniques that the intrinsic ellipticities of background galaxies average to zero. Galaxies, however, form on top of large-scale matter distributions and experience similar tidal fields from their surroundings, so there is no *a priori* reason to assume that the principal axes of neighbouring galaxies should not be aligned. Deep surveys observe background galaxies typically distributed over distance ranges which are much wider than galaxy autocorrelation scales, so any shape correlations should be washed out in projection. They can be a problem, however, for moderately deep or shallow surveys (Heavens et al. 2000; Croft & Metzler 2000; Crittenden et al. 2001).

4.3. Alternative techniques

While most cluster reconstructions undertaken to date are based on ellipticity measurements and the convolution equation (12), alternative techniques have been suggested and developed.

Lensing magnifies and distorts images, but the principal problem with image magnifications is that the size of the sources is generally unknown. A possible way to measure magnifications is through the magnification bias. Magnification due to lensing is caused by increasing the solid angle under which a source is seen, thus focussing a larger fraction of the source’s flux on the observer. In addition, the patch of sky around the source is stretched, thus reducing the number density of sources. Thus, fewer sources are seen per solid angle, but they appear brighter. The net effect on the observed source counts depends on the slope α of the source counts as a function of flux S . If α is large, i.e. if source counts decrease rapidly with increasing flux, many more sources are gained by flux magnification than lost by dilution. The number densities of lensed (observed) and unlensed

(intrinsic) sources are related by

$$m_{\text{lensed}}(S) = \mu^{\alpha-1} n_{\text{unlensed}}(S), \quad (14)$$

where μ is the magnification. Since $\alpha < 1$ for faint galaxies, magnification generally leads to source depletion near clusters, which has been observed by several groups (e.g. Broadhurst et al. 1995; Mayen & Soucail 2000; Rögvaldsson et al. 2001). A problem with this method is that the source counts fluctuate, and the unlensed source counts need to be known very accurately.

Less elegant than the convolution technique, but perhaps more flexible, are maximum-likelihood techniques (Bartelmann et al. 1996; Seitz et al. 1998). They aim at reconstructing the lensing potential ψ by minimising

$$\chi^2 = \sum_i \left[\frac{[\gamma(\psi) - \gamma_i]^2}{\sigma_\gamma^2} + \frac{[\mu(\psi) - \mu_i]^2}{\sigma_\mu^2} \right], \quad (15)$$

where $\gamma(\psi)$ and $\mu(\psi)$ are shear and magnification derived from the potential ψ , and γ_i and μ_i are measured from the data field at resolution element i . The uncertainties $\sigma_{\gamma,\mu}$ can be estimated from the data, and estimates for the magnification can be obtained comparing image sizes near clusters and in empty fields.

4.4. Mass maps and cluster masses

Weak-lensing techniques allow the surface density distribution of clusters to be mapped with an angular resolution determined by the number density of background galaxies. ‘‘Mass’’ maps have been produced so far for many galaxy clusters. While it is sometimes difficult to assess the significance of peaks found in these maps, the impressive signal-to-noise ratio in some of them is impressive.

One should bear in mind, however, that these maps are *not* mass maps, but maps of the lensing convergence, which are subject to the mass-sheet degeneracy. Even in absence of the latter, converting the convergence to a surface mass density requires the redshift of the cluster and the redshift distribution of the background galaxies to be known, and the cosmological model to be fixed. Having fixed the geometry of the lens system, the mass-sheet degeneracy in its lowest-order form allows adding a sheet of constant surface-mass density to the cluster. Although the projected distribution of dark matter can thus be well mapped, the determination of cluster masses requires further calibration.

Ignoring these principal uncertainties, cluster mass estimates from weak lensing outside cluster cores generally agree very well with those obtained from other techniques (e.g. Squires et al. 1996, 1997; Seitz et al. 1996; Fischer 1999 among many others). It is perhaps a surprising result that the mass-to-light ratios derived from weak lensing vary considerably from cluster to cluster. While it was unclear for a while whether this could be attributed to systematic effects in the data analysis techniques, it now seems that these variations are real. A good example is given by Gray et al. (2002), who reconstruct the mass distributions of three clusters found in one field and find that, while mass coincides well with light in two of them (Abell 901a and 902), they are significantly offset in Abell 901b, giving the latter cluster a substantially higher mass-to-light ratio.

4.5. Cluster Mass Profiles

A less ambitious, but better constrained problem is the determination of cluster mass profiles, rather than mass maps, from weak-lensing data. Despite the resolution limit of weak-lensing

techniques, it has been shown that it should be possible to constrain parameterised cluster profiles well (King & Schneider 2001; King et al. 2001), irrespective of whether power-law or NFW profiles are adopted. However, it is not possible yet to distinguish conclusively between NFW and isothermal profiles with weak-lensing data alone; the density profiles derived from weak lensing for many clusters are adequately fit by both profile types. If, however, the NFW profile is adopted, then the best-fitting parameters are in good agreement with expectations from numerical simulations (e.g. Allen et al. 2001; King et al. 2002). So far, weak lensing has not given a conclusive answer on cluster density profiles, but what has been found agrees well with theoretical predictions.

An independent method for constraining cluster density profiles was suggested by Bartelmann et al. (2001). Schneider (1996) suggested to search for dark-matter haloes by measuring the total tangential alignment of background-galaxy images in circular apertures of typically a few arc minutes radius. This *aperture mass* technique effectively determines a weighted integral of the lensing convergence within the aperture. It turns out that a significant measurement of the aperture mass requires less halo mass if the density profile is flatter in the centre. Specifically, NFW haloes are substantially more efficient in producing a significant weak-lensing signal than singular isothermal haloes with equal virial mass. Consequently, NFW haloes are detectable at lower mass, thus the number density of haloes detectable with the aperture-mass technique is expected to be about an order of magnitude larger if haloes have NFW rather than isothermal profiles. Wide-area weak-lensing surveys will allow this technique to be applied in the near future.

An example for the detection of a cluster in weak lensing was given by Wittman et al. (2001). While their cluster is clearly visible also in optical data, Erben et al. (2000) found in two independent data sets a significant weak-lensing signal 7' south of the cluster Abell 1942 which has no counterpart in the optical and near infrared, and perhaps a marginally significant signal in the X-rays. They estimate a mass of $\sim 10^{14} h^{-1} M_\odot$ within $0.5 h^{-1} \text{Mpc}$ at the redshift of Abell 1942, $z = 0.223$. Umetsu & Futamase (2000) found a weak-lensing signal at a similarly dark place near the cluster Cl 1604 at $z = 0.897$ which was also confirmed in independent data, and they estimate a mass of $\sim 10^{14} h^{-1} M_\odot$ within $\sim 140 h^{-1} \text{kpc}$. In an ‘‘empty’’ field of $50'' \times 50''$ in the STIS parallel data, Miralles et al. (2002) detected a strong tangential alignment of background-galaxy images typical for the weak-lensing signal produced by a massive galaxy cluster, and a multiple-image candidate, but no obvious galaxy concentration. Together with the considerable variation in cluster mass-to-light ratios derived from weak-lensing, these detections raise the exciting possibility of there being a population of very faint or completely dark clusters.

4.6. Massive clusters at high redshift

One of the surprises that came with weak-lensing measurements in cluster fields is the detection that X-ray clusters at redshifts as high as ~ 0.8 are already massive objects (see Clowe et al. 1998 for two examples, RXJ 1716 at $z = 0.81$ and MS 1137 at $z = 0.78$). The shear signal detected from these objects is highly significant, and the parameters of the density profiles are typical for well-known clusters at lower redshifts. Mass-to-light ratios in the V band are estimated between $200 - 250 h$ in solar units, and projected masses within $\sim 0.5 h^{-1} \text{Mpc}$ are of order $2 \times 10^{14} h^{-1} M_\odot$ (see also Luppino & Kaiser 1997; Hoekstra et al. 2000; Clowe et al. 2000). It is highly interesting for cosmology and structure formation that massive clusters were already in place when the Universe was half its present age.

4.7. Outlook: combination with other data

Cluster data of many different types are now becoming available. Besides optical data which determine the more traditional richness parameter and the kinematics of the cluster galaxies, cluster data are available in the X-rays, in the microwave regime where clusters appear through the thermal Sunyaev-Zel'dovich effect, and, as discussed here, in the gravitational-lensing domain. It is therefore a valid question how two or more of these types of data can be combined in order to draw a consistent picture of galaxy clusters. Three algorithms have so far been suggested for jointly analysing cluster data taken specifically in the X-rays, Sunyaev-Zel'dovich, and gravitational-lensing regimes, to "deproject" clusters and determine their structure along the line-of-sight (Zaroubi et al. 2001; Doré et al. 2001; Reblinsky 2000; Reblinsky & Bartelmann 2001). Being algorithmically different, these methods exploit the fact that X-ray, Sunyaev-Zel'dovich and lensing data can all be described as differently weighted projections of the cluster gravitational potential if the cluster gas can be assumed to be in thermal and hydrostatic equilibrium. For instance, Reblinsky & Bartelmann (2001) have developed an algorithm based on the Richardson-Lucy deprojection technique for reconstructing the three-dimensional cluster potential, assuming that it is axially symmetric. The algorithm was shown to perform well when applied to simulated galaxy clusters (Reblinsky 2000). It appears feasible that the structure of many galaxy clusters can soon be analysed in much more detail than so far.

5. SUMMARY

The main lessons learned from lensing by clusters can be summarised as follows:

- Clusters are dominated by dark matter. They are asymmetric, substructured, and highly concentrated, and their mass-to-light ratios in the optical are of order $M/L \sim 200 - 400 h$ in solar units.
- Masses determined from X-ray data and gravitational lensing tend to disagree in unrelaxed clusters, for which lensing masses are biased high, and X-ray masses biased low. In relaxed clusters, the different mass estimates tend to agree well.
- The statistics of giant arcs constrain cosmology. Detailed numerical simulations indicate that there is a disagreement with other cosmological experiments in that arc statistics prefers an open, low-density model *without* cosmological constant. Arc statistics can also place an upper bound on interaction cross sections for dark-matter particles.
- Weak gravitational lensing allows the projected distribution of the total cluster matter to be mapped. Cluster density profiles are compatible with numerical simulations. There are considerable fluctuations in cluster mass-to-light ratios.
- Clusters, more generally dark-matter haloes, can be detected through weak-lensing techniques. Several independent observations suggest that very faint and possibly dark clusters exist. Massive and compact clusters exist at redshifts as high as ~ 0.8 .
- Joint analyses of different types of cluster data (e.g. from gravitational lensing, X-ray, and thermal Sunyaev-Zel'dovich observations) allow clusters to be deprojected along the line-of-side.

Perhaps the most exciting issues in lensing-related cluster studies are the detection of dark-matter haloes irrespective of the radiation they emit or absorb, detailed cluster analyses jointly using different types of data, possible constraints on the nature of dark matter, and the relation between the statistics of giant arcs, cluster formation, and cosmology.

Due to lack of time, I was unable to touch on several additional exciting aspects of cluster lensing. To mention just one, clusters have been used highly successfully as gravitational telescopes for studying populations of faint sources at high redshift. For instance, the gravitational magnification by clusters was used to study high-redshift galaxies in the sub-millimetre regime (Smail et al. 1997) or spectroscopically in the optical (Pelló et al. 1998). While such work does not primarily target clusters, it shows how gravitational lensing by clusters can be used as a powerful astronomical tool.

REFERENCES

- Allen, S.W. 1998, MNRAS 296, 392
Allen, S.W., Ettori, S., Fabian, A.C. 2001, MNRAS 324, 877
Bartelmann, M. 1995, A&A 299, 11
Bartelmann, M. 1996, A&A 313, 697
Bartelmann, M., Weiss, A. 1994, A&A 287, 1
Bartelmann, M., Steinmetz, M. 1996, MNRAS 283, 431
Bartelmann, M., Steinmetz, M., Weiss, A. 1995, A&A 297, 1
Bartelmann, M., Narayan, R., Seitz, S., Schneider, P. 1996, ApJ 464, L115
Bartelmann, M., Huss, A., Colberg, J.M., Jenkins, A., Pearce, F.R. 1998, A&A 330, 1
Bartelmann, M., King, L.J., Schneider, P. 2001, A&A 378, 361
Bergmann, A.G., Petrosian, V., Lynds, R. 1990, ApJ 350, 23
Broadhurst, T.J., Taylor, A.N., Peacock, J.A. 1995, ApJ 438, 49
Clowe, D., Luppino, G.A., Kaiser, N., Henry, J.P., Gioia, I.M. 1998, ApJ 497, L61
Clowe, D., Luppino, G.A., Kaiser, N., Gioia, I.M. 2000, ApJ 539, 540
Crittenden, R.G., Natarajan, P., Pen, U.L., Theuns, T. 2001, ApJ 559, 552
Croft, R.A.C., Metzler, C.A. 2000, ApJ 545, 561
Dolag, K., Evrard, A.E., Bartelmann, M. 2001, A&A 369, 36
Doré, O., Bouchet, F.R., Mellier, Y., Teyssier, R. 2001, A&A 375, 14
Erben, T., Van Waerbeke, L., Mellier, Y., Schneider, P. et al. 2000, A&A 355, 23
Fischer, P. 1999, AJ 117, 2024
Flores, R.A., Maller, A.H., Primack, J.R. 2000, ApJ 535, 555
Fort, B., Prieur, J.L., Mathez, G., Mellier, Y., Soucail, G. 1988, A&A 200, L17
Fort, B., Le Fèvre, O., Hammer, F., Cailloux, M. 1992, ApJ 399, L125
Gray, M.E., Taylor, A.N., Meisenheimer, K., Dye, S. et al. 2002, ApJ 568, 141
Grossman, S.A., Narayan, R. 1988, ApJ 324, L37
Grossman, S.A., Saha, P. 1994, ApJ 431, 74
Hammer, F., Rigaut, F. 1989, A&A 226, 45
Hammer, F., Rigaut, F., Le Fèvre, O., Jones, J., Soucail, G. 1989, A&A 208, L7
Hammer, F., Gioia, I.M., Shaya, E.J., Teyssandier, P. et al. 1997, ApJ 491, 477
Hattori, M., Watanabe, K., Yamashita, K. 1997, A&A 319, 764
Heavens, A., Refregier, A., Heymans, C. 2000, MNRAS 319, 649
Hoekstra, H., Franx, M., Kuijken, K. 2000, ApJ 532, 88
Kaiser, N. 1995, ApJ 439, L1
Kaiser, N., Squires, G. 1993, ApJ 404, 441
Kaiser, N., Squires, G., Broadhurst, T. 1995, ApJ 449, 460
Kassiola, A., Kovner, I., Blandford, R.D. 1992, ApJ 396, 10
King, L.J., Schneider, P. 2001, A&A 369, 1
King, L.J., Schneider, P., Springel, V. 2001, A&A 378, 748
King, L.J., Clowe, D., Schneider, P. 2002, A&A 383, 118
Kovner, I. 1989, ApJ 337, 621
Le Fèvre, O., Hammer, F., Angonin, M.C., Gioia, I.M., Luppino, G.A. 1994, ApJ 422, L5
Loeb, A., Mao, S. 1994, ApJ 435, L109
Luppino, G.A., Kaiser, N. 1997, ApJ 475, 20
Luppino, G.A., Gioia, I.M., Hammer, F., Le Fèvre, O., Annis, J.A. 1999, A&AS 136, 117
Lynds, R., Petrosian, V. 1986, BAAS 18, R1014
Mayen, C., Soucail, G. 2000, A&A 361, 415
Meneghetti, M., Bolzonella, M., Bartelmann, M., Moscardini, L., Tormen, G. 2000, MNRAS 314, 338
Meneghetti, M., Yoshida, N., Bartelmann, M., Moscardini, L. et al. 2001, MNRAS 325, 435
Meneghetti, M., Bartelmann, M., Moscardini, L. 2002, A&A, submitted
Miralda-Escudé, J. 1992, ApJ 390, L65
Miralda-Escudé, J. 1995, ApJ 438, 514
Miralda-Escudé, J., Babul, A. 1995, ApJ 449, 18
Miralda-Escudé, J. 2002, ApJ 564, 60
Miralles, J.M., Erben, T., Hämmerle, H., Schneider, P. et al. 2002, A&A 388, 68
Molikawa, K., Hattori, M. 2001, ApJ 559, 544
Narayan, R., Bartelmann, M. 1999, in: Formation of Structure in the Universe, eds. A. Dekel, J.P. Ostriker. Cambridge: University Press
Navarro, J.F., Frenk, C.S., White, S.D.M. 1996, ApJ 462, 563

Ota, N., Mitsuda, K., Fukazawa, Y. 1998, ApJ 495, 170
Pelló, R., Sanahuja, B., Le Borgne, J.-F., Soucail, G., Mellier, Y. 1991, ApJ 366, 405
Pelló, R., Kneib, J.P., Bruzual, G. 1998, Ap&SS 263, 55
Rögnvaldsson, Ö.E., Greve, T.R. Hjorth, J., Gudmundsson, E. et al. 2001, MNRAS 322, 131
Reblinsky, K. 2000, A&A 364, 377
Reblinsky, K., Bartelmann, M. 2001, in: Gravitational Lensing: Recent Progress and Future Goals, ASP Conference Proceedings, Vol. 237, eds. T.G. Brainerd and C.S. Kochanek
Schneider, P. 1995, A&A 302, 639
Schneider, P. 1996, MNRAS 283, 837
Seitz, C., Schneider, P. 1995, A&A 294, 411
Seitz, S., Schneider, P. 1996, A&A 305, 383
Seitz, S., Schneider, P., Bartelmann, M. 1998, A&A 337, 325
Seitz, S., Kneib, J.P., Schneider, P., Seitz, C. 1996, A&A 314, 707
Smail, I., Hogg, D.W., Blandford, R., Cohen, J.G. et al. 1995, MNRAS 277, 1
Smail, I., Ivison, R.J., Blain, A.W. 1997, ApJ 490, L5
Soucail, G., Fort, B., Mellier, Y., Picat, J.P. 1987a, A&A 172, L14
Soucail, G., Mellier, Y., Fort, B., Mathez, G., Hammer, F. 1987b, A&A 184, L7
Soucail, G., Mellier, Y., Fort, B., Mathez, G., Cailloux, M. 1988, A&A 191, L19
Squires, G., Kaiser, N. 1996, ApJ 473, 65
Squires, G., Neumann, D.M., Kaiser, N., Arnaud, M. et al. 1997, ApJ 482, 648
Tyson, J.A., Wenk, R.A., Valdes, F. 1990, ApJ 349, L1
Umetsu, K., Futamase, T. 2000, ApJ 539, L5
Wittman, D., Tyson, J.A., Margoniner, V.E., Cohen, J.G., Dell'Antonio, I.P. 2001, ApJ 557, L89
Wu, X.P. 1994, ApJ 436, L115
Wu, X.P., Hammer, F. 1993, MNRAS 262, 187
Wu, X.P., Fang, L.Z. 1996, ApJ 467, L45
Wyithe, J.S.B., Turner, E.L., Spergel, D.N. 2001, ApJ 555, 504
Zaroubi, S., Squires, G., de Gasperis, G., Evrard, A.E. et al. 2001, ApJ 561, 600