The mechanism of core-collapse supernovae and the ejection of heavy elements

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We present here the first results of two-dimensional hydrodynamical simulations of the neutrino-heating phase in the collapsed core of a 15 $M_\odot$ star, where the neutrino transport is treated with a variable Eddington factor method for solving the Boltzmann transport equation, and the neutrino interactions include nucleon-nucleon bremsstrahlung, nucleon recoils and correlations, and weak-magnetism effects as well as direct interactions between neutrinos of different flavors. With the given input physics (neutrino reactions and nuclear equation of state), our best simulations do not develop strong convection in the neutrino-heating layer behind the shock and do not yield explosions. With about 30% higher neutrino-energy deposition behind the shock, however, an explosion occurs on a timescale of 150 ms after core bounce. It leaves behind a neutron star with an initial baryonic mass of 1.4 $M_\odot$ and ejects the $N = 50$ isotopes of Sr, Y and Zr in amounts consistent with Galactic abundances.

1. INTRODUCTION

Understanding the explosion mechanism of massive stars is one of the most important problems in stellar evolution theory. What is the mechanism which inverts the catastrophic collapse of the iron core of a massive star to the powerful ejection of matter with bulk velocities up to ten percent of the speed of light? This is not only a fundamentally interesting question and a challenging physics problem which combines relativistic fluid dynamics with nuclear physics and neutrino physics. It is also the theoretical link between models of progenitor stars and observable properties of supernovae and supernova remnants, e.g., their explosion energies, radioactive products of nucleosynthesis and neutron star and black hole masses. Many supernovae show global asymmetries or large-scale mixing, and young pulsars have been measured with mean space velocities of several hundred kilometers per second [1]. Both observations most likely also point to events happening at the onset of the explosion or shortly afterwards.

Although the historical discovery of neutrinos from Supernova (SN) 1987A has basically confirmed our picture of stellar collapse and neutron star formation, the two dozen events were too sparse to yield insight into the cause of the explosion. Knowledge about the latter is therefore based on numerical simulations and analytic arguments, but we are still far from a satisfactory understanding. It seems to be clear that the explosion is powered by the energy that is released when the stellar core collapses to a compact remnant, either to a neutron star or a black hole. But what fraction of this energy is transferred to the ejecta, by which physical process and on what timescale?

One possible answer to these questions, which has been investigated by numerical simulations in much detail, is the direct hydrodynamical bounce-shock mechanism, i.e., the transfer of mechanical energy from the re-expanding inner region of the collapsing core to the forming shock on a dynamical timescale. Propagating through the still infalling core matter, this prompt shock, however, turns out to be severely damped by energy losses due to the photodisintegration of iron-group nuclei. It finally stalls when large amounts of electron neutrinos are emitted at the moment of breakout through the neutrinosphere.

The current standard paradigm for the mechanism is based on the fact that at later times high-energy neutrinos, which are emitted from the nascent neutron star in large fluxes, begin to deposit a small fraction of their energy behind the standing shock. If strong enough, this heating can revive the shock and cause a supernova explosion [2, 3]. Since the transfer of energy from the settling proto-neutron star to the postshock matter through neutrinos works on a secular timescale, this mechanism is called "delayed".

Neutrinos undoubtedly play a crucial role in the supernova core. Their production and escape does not only determine the collapse dynamics of the stellar iron core and the propagation of the bounce-shock. Since they interact with matter only via the weak force, they can carry away up to 99% of the gravitational binding energy of the nascent neutron star and thus dominate the supernova event energetically. Depositing energy and lepton number in the stellar material adjacent to the newly formed neutron star they also determine the conditions for nucleosynthesis reactions in the ejecta that come from the region just around the mass cut. Analytic estimates [for reviews, see 4, 5, 6] and, in particular, hydrodynamical simulations show that neutrinos from the nascent neutron star can indeed drive the explosion, if their luminosities and mean energies are large enough. The heating rate behind the shock depends sensitively on the neutrino spectra and on the neutrino energy density, which scales with the neutrino flux.

Wilson and collaborators ([7]–[10]) have routinely obtained explosions for more than ten years in one-dimensional simulations by *assuming* that neutron-finger convective instabilities boost the neutrino luminosities from the hot neutron star. Moreover, Mayle et. al. [9] used a special nuclear equation of state and invoke a high abundance of pions in the nuclear matter to reproduce the explosion energy of SN 1987A. Both assumptions are not generally accepted. Therfore investigations should be continued with making less controversial postulats about the involved physics.

Forced by observations of large-scale anisotropies and mixing in SN 1987A and directed by the fact that the neutrino-heating layer is convectively unstable [4], supernova modelers began to perform multi-dimensional simulations of the post-bounce supernova evolution and discovered that violent motions set in on a timescale relevant for the delayed mechanism ([11]–[15]). In fact, increasing the postshock pressure by rising, outward pushing neutrino-heated matter and allowing for simultaneous accretion (thus maintaining high accretion luminosities) and shock expansion (thus enlarging the layer of neutrino-energy deposition), this postshock convection turned out to have a very helpful influence on the explosion: Models which failed to blow up in spherical symmetry were found to succeed in two dimensions.

Making use of this effect and employing an advanced version of the Herant et. al. code

[12], Fryer has started to explore a variety of progenitors to determine whether neutron stars or black holes are left behind as compact remnants [16]. Fryer and Heger expanded the study to rotating stars [17], and most recently Fryer and Warren have finished the first full supernova simulations in three dimensions, finding good agreement with results that were obtained in the two-dimensional case [18].

Though these simulations are undoubtedly pioneering first steps, they suffer from a major uncertainty: The transport of neutrinos in the supernova core is described in a much simplified way, using grey flux-limited diffusion and ignoring Doppler-shift and gravitational redshift effects on the neutrino spectrum. This approximation is too crude to allow for serious conclusions on the explosion mechanism. The importance of an accurate numerical description of the neutrino transport has repeatedly been pointed out by Bruenn and by Mezzacappa and collaborators (e.g., [19]–[21]).

For this reason, a new generation of supernova codes has been developed in the past years with the aim to replace the widely used (multi-group) flux-limited diffusion treatments (e.g., [22]–[26]) by a direct solution of the Boltzmann transport equation ([27]–[30],[19, 20]). Progress was also made for including the effects of general relativity (GR) in a rigorous way in neutrino-hydrodynamics codes in spherical symmetry ([31]–[33]). Although improved flux limiters for the neutrino transport [34] or two-moment closure schemes [35, and references therein] have been suggested and may work better than previously used flux limiting prescriptions (for an apparently successful new development in this direction, see the results by Bruenn shown in Ref. [33]), any such approximation has ultimately to be compared with Boltzmann solvers applied in dynamical supernova calculations.

In this conference contribution we shall summarize our efforts to combine a Boltzmann solver for the neutrino transport with a multi-dimensional hydrodynamics code, using a state-of-the-art description of neutrino-matter interactions and an approximation of GR which is (hopefully) sufficiently accurate to model supernova explosions and neutron star formation. First results of two-dimensional simulations with this new code named MuDBaTH (**Mu**lti-**D**imensional **B**oltzm**a**nn **T**ransport and **H**ydrodynamics) will also be presented.

2. MuDBaTH: A NEW RADIATION-HYDRODYNAMICS CODE FOR NEUTRINOS

For the integration of the equations of hydrodynamics we employ the Newtonian finite-volume code PROMETHEUS [36], which was supplemented by additional problem specific features by Keil [37]. PROMETHEUS is a direct Eulerian, time-explicit implementation of the Piecewise Parabolic Method (PPM) of Colella and Woodward [38]. As a second-order Godunov scheme employing a Riemann solver it is particularly well suited for following discontinuities in the fluid flow like shocks or boundaries between layers of different chemical composition. A notable advantage in the present context is its capability of tackling multi-dimensional problems with high computational efficiency and numerical accuracy. Our current code contains a simplified version of the "Consistent Multifluid Advection (CMA)" method [39] for ensuring accurate advection of different chemical components in the fluid, and switches from the original PPM method to the more diffusive HLLE solver of Einfeldt [40] in the vicinity of strong shocks to avoid spurious oscillations (the so-called "odd-even decoupling" phenomenon) when such shocks are aligned with one of the coordinate lines in multidimensional simulations ([41]–[43]).

The neutrino transport is done with the Boltzmann solver scheme that is described in much detail in Ref. [44]. The integro-differential character of the Boltzmann equation is tamed by applying a variable Eddington factor closure to the neutrino energy and momentum equations (and the simultaneously integrated first and second order moment equations for neutrino number). For this purpose the variable Eddington factor is determined from the solution of the Boltzmann equation and the system of Boltzmann equation and its moment equations is iterated until convergence is achieved. Employing this scheme in multi-dimensional simulations in spherical coordinates, we solve the moment equations on the different angular bins of the numerical grid but calculate the variable Eddington factor only once on an angularly averaged stellar background. This approximation is good only for situations without significant global deformations. Since the iteration of the Boltzmann equation has to be done only once per time step, appreciable amounts of computer time can be saved [44]. We point out here that it turned out to be necessary to go an important step beyond this simple "ray-by-ray" approach. Physical constraints, namely the conservation of lepton number and entropy within adiabatically moving fluid elements, and numerical requirements, i.e., the stability of regions which should not develop convection according to a mechanical stability analysis, make it necessary to take into account the coupling of neighbouring rays at least by lateral advection terms and neutrino pressure gradients.

General relativistic effects are treated only approximately in our code [44]. The current version contains a modification of the gravitational potential by including correction terms due to pressure and energy of the stellar medium and neutrinos, which are deduced from a comparison of the Newtonian and relativistic equations of motion. The neutrino transport contains gravitational redshift and time dilation, but ignores the distinction between coordinate radius and proper radius. This simplification is necessary for coupling the transport code to our basically Newtonian hydrodynamics. Although a fully relativistic treatment would be preferable, tests showed that these approximations seem to work reasonably well as long as there are only moderate deviations ($\sim$10–20%) of the metric coefficients from unity and the infall velocities do not reach more than 10–20% of the speed of light.

As for the neutrino-matter interactions, we discriminate between two different sets of input physics. On the one hand we have calculated models with conventional ("standard") neutrino opacities, i.e., a description of the neutrino interactions which follows closely the one used by Bruenn and Mezzacappa and collaborators [23, 45, 46]. It assumes nucleons to be uncorrelated, infinitely massive scattering targets for neutrinos. In these reference runs we have usually also added neutrino pair creation and annihilation by nucleon-nucleon bremsstrahlung [47]. Details of our implementation of these neutrino processes can be found in Ref. [44].

A second set of models was computed with an improved description of neutrino-matter interactions. Besides including nucleon thermal motions and recoil, which means a detailed treatment of the reaction kinematics and allows for an accurate evaluation of nucleon phase-space blocking effects, we take into account nucleon-nucleon correlations [following

Refs. 48, 49], the reduction of the nucleon effective mass, and the possible quenching of the axial-vector coupling in nuclear matter [50]. In addition, we have implemented weak-magnetism corrections as described in Ref. [51]. The sample of neutrino processes was enlarged by also including scatterings of muon and tau neutrinos and antineutrinos off electron neutrinos and antineutrinos and pair annihilation reactions between neutrinos of different flavors (i.e., $\nu_{\mu,\tau} + \bar{\nu}_{\mu,\tau} \longleftrightarrow \nu_e + \bar{\nu}_e$; [52]).

Our current supernova models are calculated with the nuclear equation of state of Lattimer and Swesty [53]. For the density regime below $6 \times 10^7\,\mathrm{g/cm^3}$ we switch to an equation of state that considers electrons, positrons and photons, and nucleons and nuclei with an approximative treatment for composition changes due to nuclear burning and shifts of nuclear statistical equilibrium [44].

3. RESULTS: A NEW GENERATION OF 1D AND 2D SUPERNOVA SIMULATIONS

We have performed a number of core-collapse simulations in spherical symmetry and then followed the post-bounce evolution in one and two dimensions. All described calculations were started from a $15\,M_\odot$ progenitor star, Model s15s7b2, provided to us by S. Woosley. Adopting the naming, we label our models by s15N for Newtonian runs and s15G for runs with approximate treatment of general relativity, followed by letters "so" when "standard neutrino opacities" were used and by "io" in case of our state-of-the-art improvement of the description of neutrino-matter interactions. The model names have a suffix that discriminates between 1D ("_1d") and 2D simulations ("_2d").

We have varied yet another aspect in our simulations. Some of the models were computed with a version of the transport code where the velocity dependent (Doppler shift and aberration) terms in the neutrino momentum equation (and the corresponding terms in the Boltzmann equation for the antisymmetric average of the specific intensity; see Ref. [44]) were omitted. These terms are formally of order v/c and should be small for low velocities. This simplification of the neutrino transport, our so-called "Case A", was used in the Newtonian models and in those models with approximate GR which have names ending with the letter "a". Models with neutrino transport including all velocity dependent terms also in the neutrino momentum equation ("Case B") can be identified by the ending "b" of their names. Table 1 provides an overview of the computed models.

The shock trajectories of all models are displayed in Fig. 1. No explosions were obtained with the most complete implementation of the transport equations, Case B (see lines labeled with H1, H2, S1, B1, corresponding to Models s15Gio_1d.b, s15Gio_2d.b, s15Gso_1d.b and s15Gso_1d.b*, respectively). The shock trajectories of this sample of models form a cluster that is clearly separated from the models computed with the transport version of Case A, which generally show a larger shock radius and therefore more optimistic conditions for explosions. Both classes of calculations differ mainly in the neutrino energy densities around and outside of the neutrinosphere. In Case A these energy densities are larger, corresponding to somewhat smaller neutrino losses from the cooling layer and somewhat higher neutrino energy deposition in the heating layer behind the supernova shock. Although the differences are moderate (typically 10–30%, depending on the quantity, radial position and time) during the first 80 ms after bounce, the

Table 1
Input physics for our set of computed models. See text for details.

Model	Dimensions	Gravity	ν Reactions	Transport
s15Nso_1d	1-dim. (1D)	Newtonian	standard	Case A
s15Nso_2d	2-dim. (2D)	Newtonian	standard	Case A
s15Gso_1d.b	1-dim. (1D)	approx. GR	standard	Case B
s15Gso_1d.b*	1-dim. (1D)	approx. GR	standard†	Case B
s15Gio_1d.a	1-dim. (1D)	approx. GR	improved	Case A
s15Gio_2d.a	2-dim. (2D)	approx. GR	improved	Case A
s15Gio_1d.b	1-dim. (1D)	approx. GR	improved	Case B
s15Gio_2d.b	2-dim. (2D)	approx. GR	improved	Case B

† Calculation without neutrino-pair creation by nucleon-nucleon bremsstrahlung.

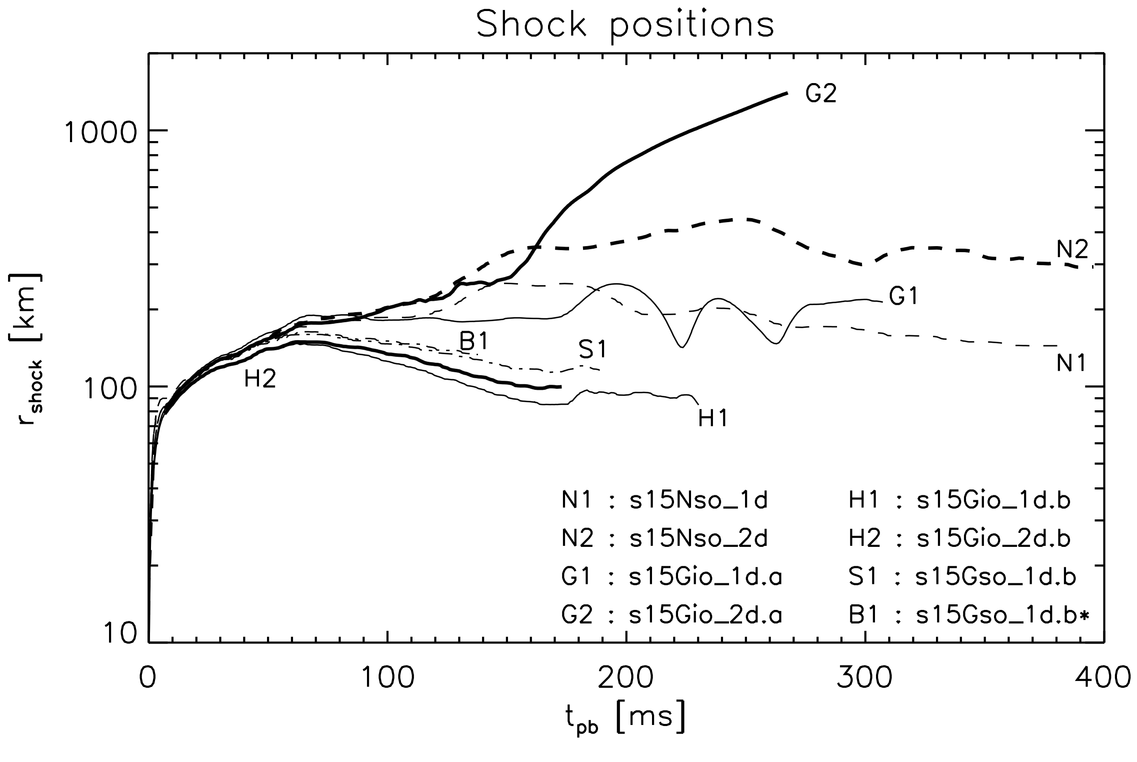

Figure 1. Shock trajectories for all models. Bold lines correspond to two-dimensional simulations.

accumulating effect clearly damps the shock expansion and leads to a dramatic shock recession after this initial phase in all models of Case B. This demonstrates the sensitivity of the long-time evolution of the collapsing stellar core to smaller "details" of the neutrino transport.

A sufficiently large shock radius for a sufficiently long time is also crucial for the growth

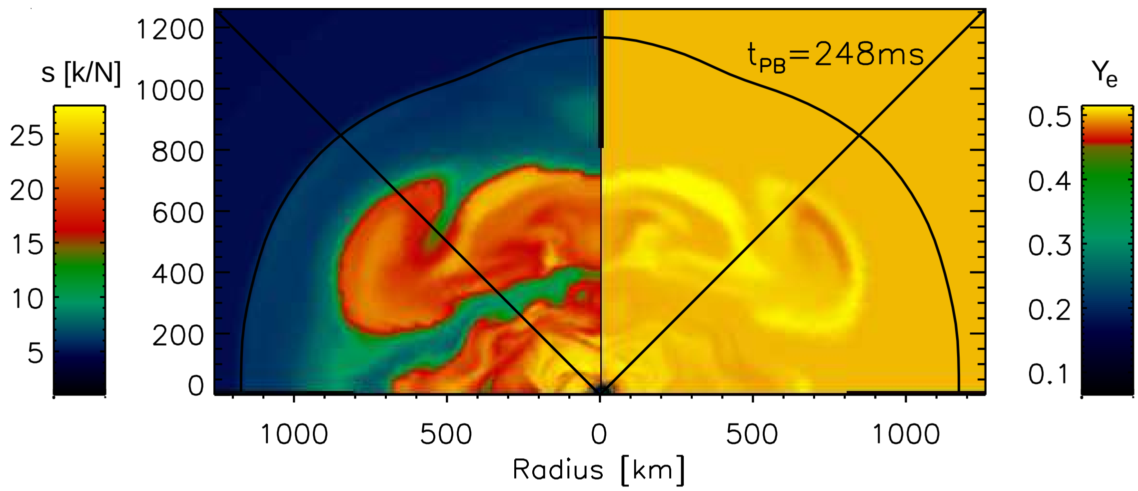

Figure 2. Entropy per nucleon, s (left), and proton-to-nucleon ratio, Y_e (right), for exploding Model s15Gio_2d.a at 248 ms after core bounce. The diagonal line indicates the equatorial plane of the computational grid.

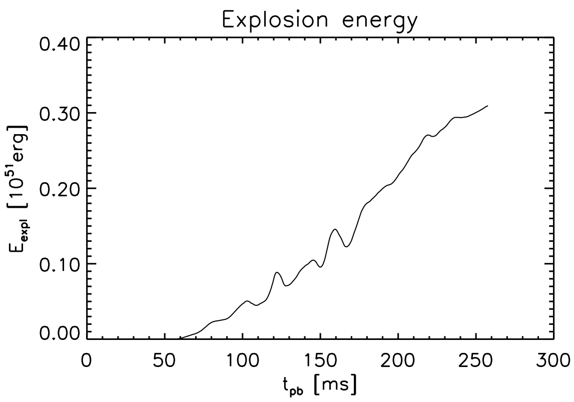

Figure 3. Explosion energy as function of time for Model s15Gio_2d.a.

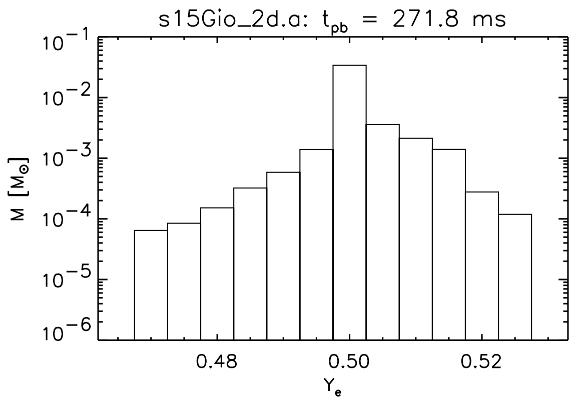

Figure 4. Ejecta mass vs. Y_e for Model s15Gio_2d.a at 272 ms after bounce. The total mass is about 0.041 $M_\odot$.

of convective instabilities in the neutrino-heating layer. This is visible from the two-dimensional Models s15Gio_2d.b and s15Gio_2d.a. The former model fails to explode because the convective activity behind the shock is suppressed when the shock retreats, despite of a high energy transfer rate to the postshock matter by neutrinos. In contrast, convective overturn behind the shock becomes strong and is crucial for getting an explosion in Model s15Gio_2d.a. This is obvious from a comparison with the corresponding one-dimensional simulation, Model s15Gio_1d.a. It is interesting that the Newtonian 2D run, Model s15Nso_2d (with standard opacities) does marginally not explode, whereas Model s15Gio_2d.a with relativistic corrections (and state-of-the-art neutrino reactions) succeeds[1]. The influence of the neutrino opacities can be directly seen by comparing Mod-

[1]At the time of the conference, we had just these two 2D runs and announced the possible success of

els s15Gso_1d.b and s15Gio_1d.b. It is not dramatic but sufficient to justify the inclusion of all the improvements described in Sect. 2. Neutrino-pair creation by bremsstrahlung makes a minor difference during the considered phases of evolution (Model s15Gso_1d.b* vs. Model s15Gso_1d.b).

The successfully exploding 2D model, Model s15Gio_2d.a, has very interesting properties. Figure 2 shows a snapshot at 248 ms after bounce, giving the distribution of entropy per baryon (s in units of Boltzmann's constant) and electron fraction behind the shock, which is located at a radius of nearly 1200 km and is expanding with about 10000 km/s. The explosion energy of this model might become rather low. It is only $\sim 3 \times 10^{50}$ erg at 260 ms after bounce, but still increasing (Fig. 3). The explosion starts so late (at ~ 150 ms post bounce) that the proto-neutron star has accreted enough matter to have attained an initial baryonic mass of $1.4\,M_{\odot}$. Therefore our simulation does not exhibit the problem of other successful models which produced rather small ($\sim 1.1\,M_{\odot}$) neutron stars. Also another problem of published explosion models [e.g., 12, 54, 55, 16] has disappeared: The ejecta mass with $Y_e \lesssim 0.47$ is less than $10^{-4}\,M_{\odot}$ (Fig. 4), thus fulfilling a constraint pointed out by Hoffman et. al. [56] for supernovae if they should not overproduce the $N = 50$ (closed neutron shell) nuclei, in particular ^{88}Sr, ^{89}Y and ^{90}Zr, relative to the Galactic abundances. Of course, we will have to follow the explosion for a longer time to make final statements about explosion energy and ejecta composition, and also the neutron star mass may grow by later fallback, especially when the explosion energy remains low.

4. SUMMARY AND CONCLUSIONS

We have presented a set of core collapse and supernova models, which were computed in spherical symmetry and in two dimensions, using a new Boltzmann solver for the neutrino transport and improved neutrino opacities compared to the standard treatment in current supernova codes. We did not find explosions, neither without nor with postshock convection, in our most complete simulations.

Omitting the velocity-dependent terms from the neutrino momentum equation, however, leads to sufficiently large changes in the neutrino quantities and neutrino energy loss or deposition (10–30% between neutrinosphere and shock) that very strong convective overturn in the neutrino-heating region develops and drives a successful explosion. This demonstrates that the models are rather close to the threshold for an explosion, causing an enormous sensitivity of the post-bounce evolution to an accurate treatment of the neutrino transport and hydrodynamics. It also confirms the importance of postshock convection, because the 1D model corresponding to our exploding 2D case does not explode. When the neutrino heating is too weak, and the shock recedes to a small radius quickly after its has reached its maximum radius, however, convective activity behind the shock is suppressed and has no crucial influence on the evolution.

We also found that Ledoux convection below the neutrinosphere, although setting in shortly after bounce and persistent until the end of our simulations, occurs so deep inside the neutron star that its effects on the ν_e and $\bar{\nu}_e$ luminosities and on the supernova

Model s15Gio_2d.a. A later 2D computation with the full neutrino moment equations (Model s15Gio_2d.b for Case B), however, then turned out to produce a dud.

dynamics are insignificant.

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