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A new test for the stable clustering hypothesis

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ABSTRACT

The stable clustering hypothesis is a fundamental assumption about the nonlinear clustering of matter in cosmology. It states that the mean physical separation of particles is a constant on sufficiently small scales. While many authors have attempted to test the hypothesis with cosmological N-body simulations, no consensus has been reached on whether and where the hypothesis is valid, because of the limited dynamical range this type of simulations can achieve. In this *Letter*, we propose to test the hypothesis with high resolution halo simulations, since the individual halo simulations can resolve much better the fine structures of the halos and since almost all pairs of particles with small separations are presumed to be inside virialized halos. We calculated the mean pair velocity for 14 high resolution halos of ~ 1 million particles in a low-density flat cold dark matter model. The result agrees very well with the stable clustering prediction within the measurement uncertainty $\sim 30\%$ over a large range of scales where the overdensity is 10^3 to 10^6. The accuracy of the test can be improved to $\sim 10\%$ if some 100 halos with a similar resolution are analyzed.

Subject headings: galaxies: clusters: general – cosmology

1. Introduction

The stable clustering hypothesis is a basic assumption in cosmology about the nonlinear clustering of cosmic matter (Peebles 1980). It states that the mean physical separations r

of particles in highly non-linear regions do not change with the expansion of the Universe. In terms of the scale factor a and the comoving coordinates $\mathbf{x}$, the hypothesis equivalently states that the relative pair velocity in the comoving coordinates $v = a\dot{x}$ cancels out the Hubble flow $\dot{a}x$, i.e.

$$v = -\dot{a}x = -Hr\,. \tag{1}$$

Based on this assumption, it is predicted that 1) the power-law index of the two-point correlation function $\xi(r)$ on sufficiently small scales is related to the power-law index n of the linear power spectrum $P(k) \propto k^n$ as $\xi(r) \propto r^{-3(3+n)/(5+n)}$, and 2) the N-point correlation functions ζ_N scale with ξ as $\zeta_N \propto \xi^{N-1}$ (Peebles 1974, Davis & Peebles 1977, Gott & Rees 1975, Peebles 1980, Suginohara et al. 1991, Suto 1993, Jain 1997). Based on the first prediction for clustering on small scale and the linear theory prediction for large scale, a phenomenological fitting formula has been found for the nonlinear power spectrum (Hamilton et al. 1991, Jain, Mo, & White 1995, Peacock & Dodds 1996, Ma 1998, Suginohara et al. 2001). This fitting formula has been applied to study a wide range of non-linear clustering problems, notably the gravitational lensing and the non-linear clustering of galaxies.

There have been many attempts to verify the stable clustering hypothesis with numerical N-body simulations. In their seminal work, Efstathiou et al. (1988) tested the hypothesis [Eq.(1)] and its two predictions with a set of scale-free N-body simulations of 32^3 particles. They concluded that their simulation results are consistent with the hypothesis, but it is not difficult to imagine that the accuracy they could achieve is quite limited for the limited resolution of this first generation of simulations. Subsequently, many improved simulations with 64^3 to 128^3 particles have been used to solve this problem (e.g. Suginohara et al. 1991, Suto 1993, Colombi et al. 1996, Munshi et al. 1998, and Jain 1997 and references therein). As shown by Jain (1997; his Fig.2, Fig.3 and Fig.8), his simulation results of the mean pair velocity have just approached the stable clustering condition [Eq.(1)] at the smallest scale that his simulations of $\sim 128^3$ particles can resolve. Thus, it remains an open question if the stable clustering holds for a range of scales (Munshi et al. 1998). One point clear from these previous studies is that only in high overdensity regions $\Delta\rho/\rho \gg 100$ is the stable clustering assumption possibly valid. One needs simulations which have much higher resolutions than those Jain analyzed, in order to convincingly test the stable clustering.

It is well known that close pairs of particles reside inside virialized halos (e.g. McClelland & Silk 1977, Peebles 1980). This is the reason why many people have recently constructed successful models for non-linear statistics based on the simulation result of the halo density profile and the halo mass function (e.g. Mo, Jing, & Börner 1997, Sheth & Jain 1997, Ma & Fry 2000a,b,c, Seljak 2000, Cooray, Hu, & Miralda-Escude 2000, Peacock

& Smith 2000, Scoccimarro et al. 2000, Sheth et al. 2000). In particular, Ma & Fry (2000b) checked the stable clustering assumption using such an approach. They derived the mean pair velocity by inserting the halo model two-point correlation function into the pair conservation equation (e.g. Peebles 1980), and found that the stable clustering is valid only when the halo mass function and the halo density profile obey specific relations. Considering the fact that both models of the halo density profile and the halo mass function have considerable uncertainties and scatters (Jing 2000, Jing & Suto 2000, Sheth & Tormen G. 1999, Jenkins etal. 2000), they concluded that it is yet unclear if the stable clustering can be realized in practice.

In this *Letter*, we propose to use high-resolution halo simulations to test the stable clustering hypothesis by examining the mean pair velocity within virialized halos. The validity of this approach underlies the above-mentioned fact that almost all close pairs of galaxies reside inside virialized halos. If the mean pair velocity within halos agrees with the stable clustering hypothesis, one can sufficiently conclude that the stable clustering is established, since the mean pair velocity in general is a $N^h_{pair}(r, M)n(M)$ weighted average of the mean pair velocity within halos, where $N^h_{pair}(r, M)$ is the mean pair number of separation r inside a halo of mass M, and $n(M)$ is the halo number density per unit halo mass. The advantage of this approach over the conventional approach which uses cosmological N-body simulations (e.g. Jain 1997) is that the individual halos can be simulated with a much better resolution. Here we use fourteen high-resolution halos to make such an analysis. Twelve of these halos were used for studying density profiles of dark matter halos (Jing & Suto 2000) and the other two were simulated after the work of Jing & Suto (2000).

2. Simulation data

Fourteen dark matter halos in total were selected from our one cosmological P^{3}M N-body simulation of 256^3 particles in a $(100h^{-1}\mathrm{Mpc})^3$ cube (Jing & Suto 1998, Jing 1998). The model for this simulation has the density parameter $\Omega_0 = 0.3$, the cosmological constant $\lambda_0 = 0.7$, the Hubble constant $h = 0.7$ and the power spectrum normalization $\sigma_8 = 1.0$. These model parameters are consistent with the current observations of the cluster abundance, of the cosmic background radiation anisotropies, of high-redshift supernova luminosity, and of many others. Of the fourteen halos, five, five, and four have mass scales of clusters, groups and galaxies respectively.

These halos are re-simulated with high resolution using the multiple mass method and our nested-grid P^{3}M code (Jing & Suto 2000). To minimize the contamination of the coarse particles on the halo properties within the virial radius at $z = 0$, $r_{\rm vir}$, we trace back the particles within $3r_{\rm vir}$ of each halo to their initial conditions at redshift $z = 72$. The virial radius $r_{\rm vir}$ is defined such that the spherical overdensity inside is $\sim 18\pi^2\Omega_0^{0.4} \sim 110$ times the critical density $\rho_{\rm crit}(z = 0)$ (Kitayama & Suto 1997, Bryan & Norman 1998). Typically 1.5×10^6 fine particles are placed in the high resolution region, and 0.7×10^6 coarse particles, whose mass increases monotonically with the distance from the high-resolution region, are placed outside the high resolution region to model the tidal force effect. The initial conditions for these particles are produced with the same random initial field as the cosmological N-body simulation but with the shorter wavelength perturbations added that the cosmological N-body simulation missed. The simulations are evolved by our nested-grid $\rm P^3M$ code which was designed to simulate high-resolution halos. The force resolution is typically $0.004r_{vir}$. At last about $(0.5 - 1) \times 10^6$ particles end up within $r_{\rm vir}$ of each halo. For more details about the simulation, we refer readers to Jing & Suto (2000).

3. The mean pair velocity

We calculate the mean pair velocity $v_r(r)$ by averaging the relative velocity of all pairs of particles within the virial radius. Separation bins are taken from $0.01r_{vir}$ to r_{vir} with $\Delta \lg r = 0.05$. Since one halo has $(0.5 - 1) \times 10^6$ particles, there are more than 10^{11} pairs in total in each halo, and more than 10^6 pairs within the single innermost bin. The high number of pairs is very crucial for investigating $v_r(r)$ at the smallest separation. This is because relative to the stable clustering prediction $v_r = -0.01H_0r_{vir}$ at $r = 0.01r_{vir}$, the random motion of particles (i.e. the noise for determining v_r) is very large which, in terms of the one-dimensional velocity dispersion, is about $5H_0r_{vir}$. In order to suppress the noise, more than $500^2 = 2.5 \times 10^5$ particle pairs are needed for the innermost bin. Only high-resolution halo simulations like those used in this paper can easily meet this requirement. In contrast, cosmological N-body simulations would need many more than 128^3 particles to study $v_r(r)$ at a similar scale.

We find that the mean pair velocity $v_r(r)$ varies significantly from one halo to another. In Figure 1, we show $v_r(r)/H_0r$ for the five cluster halos as a function of r. The halos are labeled in the same way as Jing & Suto (2000) with the newly added halo labelled as CL5. Although the measured values of $v_r(r)/H_0r$ are around the stable clustering prediction -1, there is significant difference in this quantity between one halo and another. Some halo has $v_r(r)/H_0r$ systematically higher than -1 (CL3), and some lower than -1 (CL1 & CL2). We have examined whether $v_r(r)/H_0r$ is correlated with the dynamical state and the density profiles of halos. From Jing & Suto (2000), we know that CL1 has the smallest concentration parameter and CL2 has the largest among the first four halos, while they have similar $v_r(r)/H_0 r$. Therefore there seems to be no apparent correlation between the density profiles and $v_r(r)/H_0 r$, which is also supported by the results of galaxy and group halos (Figure 2). The large variation of $v_r(r)/H_0 r$ from halo to halo might reflect the fact that the merging and relaxation processes are still taking place within the halos, which is not unexpected in the hierarchical clustering scenario, and the value of $v_r(r)/H_0 r$ depends on a fine balance of the both processes within a halo.

For testing the stable clustering hypothesis, the important quantity is not the $v_r(r)/H_0 r$ of an individual halo, but the averaged value of $v_r(r)/H_0 r$ over halos of a given mass. Figure 2 shows the averaged $v_r(r)/H_0 r$ for the cluster (CL), group (GR) and galaxy (GX) halos (note that the vertical axis is different from that of Figure 1). The measurement has a typical uncertainty $0.6 \sim 1.0$ (For clarity, the error bars are plotted for the cluster halos only). As the figure shows, the result of each set of halos is consistent with the stable clustering hypothesis within the measurement uncertainty.

Similarities have been found for the halos of different mass after the length (and correspondingly other quantities) is scaled to the halo size. Navarro et al. (1996) proposed a universal density profile for halos, and Klypin et al (1999) and Moore et al. (1999) showed that the number distributions of subhalos (after scaling to the circular velocity) are very similar between galaxy and cluster halos. Although there is some systematic difference between halos of different mass even after scaling (e.g. the density profiles, Jing & Suto 2000), the small difference might not be important for the current work. This is reflected in Figure 2 which shows no dependence of $v_r(r)/H_0 r$ on the halo mass. As a further step, we average $v_r(r)/H_0 r$ over all 14 halos at each r/r_{vir}-bin, and the results are presented in Figure 3. It shows that the averaged result is well consistent with the stable clustering prediction within a typical uncertainty $\sim 30\%$. According to the density profiles, the typical density contrast at $r = (0.01 \sim 0.1) r_{vir}$ is 10^3 to 10^6.

4. Conclusion and future work

We have made a first attempt to investigate the validity of the stable clustering hypothesis using high-resolution halo simulations. We have obtained the following conclusions.

- The mean pair velocity within a single halo fluctuates strongly around the stable clustering prediction. This may indicate that the merging and relaxation processes inside halos have not fully completed, as should be expected for hierarchical clustering scenarios.

- The mean pair velocity averaged over four or five halos of a similar mass is consistent with the stable clustering prediction with an uncertainty of $\sim 60\%$. No dependence on halo mass has been seen for the quantity $v_r(r)/H_0 r$.
- Based on the similarities exhibited by halos of different mass, we averaged the $v_r(r)/H_0 r$ over the fourteen halos at each r/r_{vir}-bin. The result is in very good agreement with the stable clustering assumption with an uncertainty of $\sim 30\%$ over a large range of the scales r, thus lending strong support for the stable clustering hypothesis.

It is important to emphasize again that the stable clustering condition (1) is, contrary to early expectations (e.g. Peebles 1980), NOT generally satisfied within one single virialized halo, but it can be achieved through averaging over many halos. Although there is clear evidence for the similarities among halos of different mass, we are still concerned of any dependence of $v_r(r)/H_0 r$ on the halo mass. We have started a program to simulate 50 halos at each mass range of clusters, groups, and galaxies, 150 halos in total. This will reduce the uncertainty of $v_r(r)/H_0 r$ to $\sim 15\%$ for each mass range and will show the dependence of $v_r(r)/H_0 r$ on the halo mass, if any.

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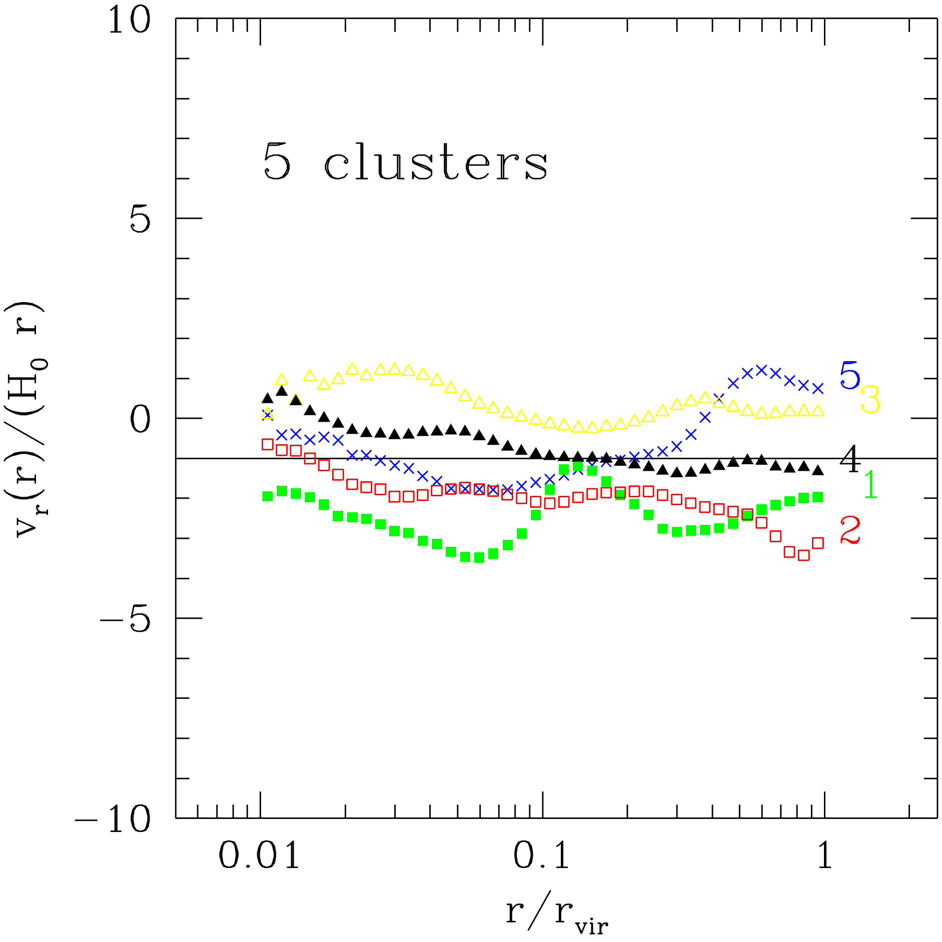

Fig. 1.— The mean peculiar velocity, in units of $H_0 r$, of particles within cluster halos.

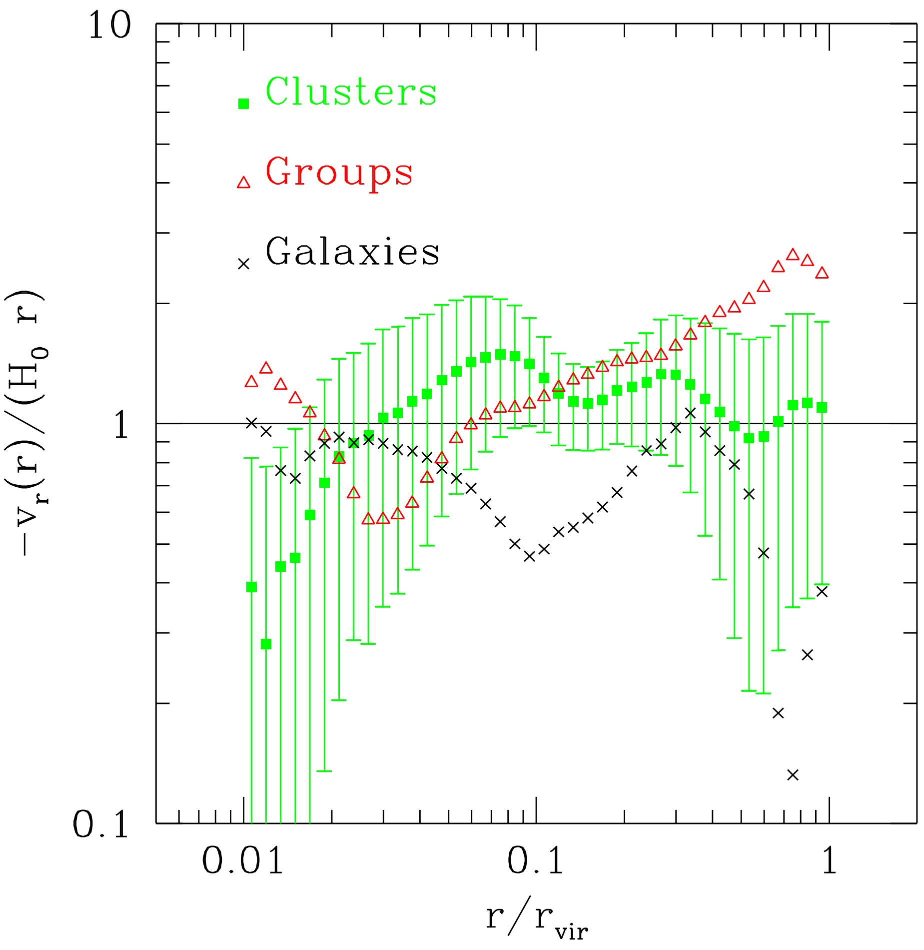

Fig. 2.— The mean peculiar velocity, in units of $H_0 r$, of particles averaged over four to five halos at cluster, group or galaxy masses. For clarity, errorbars are plotted for the cluster halos only.

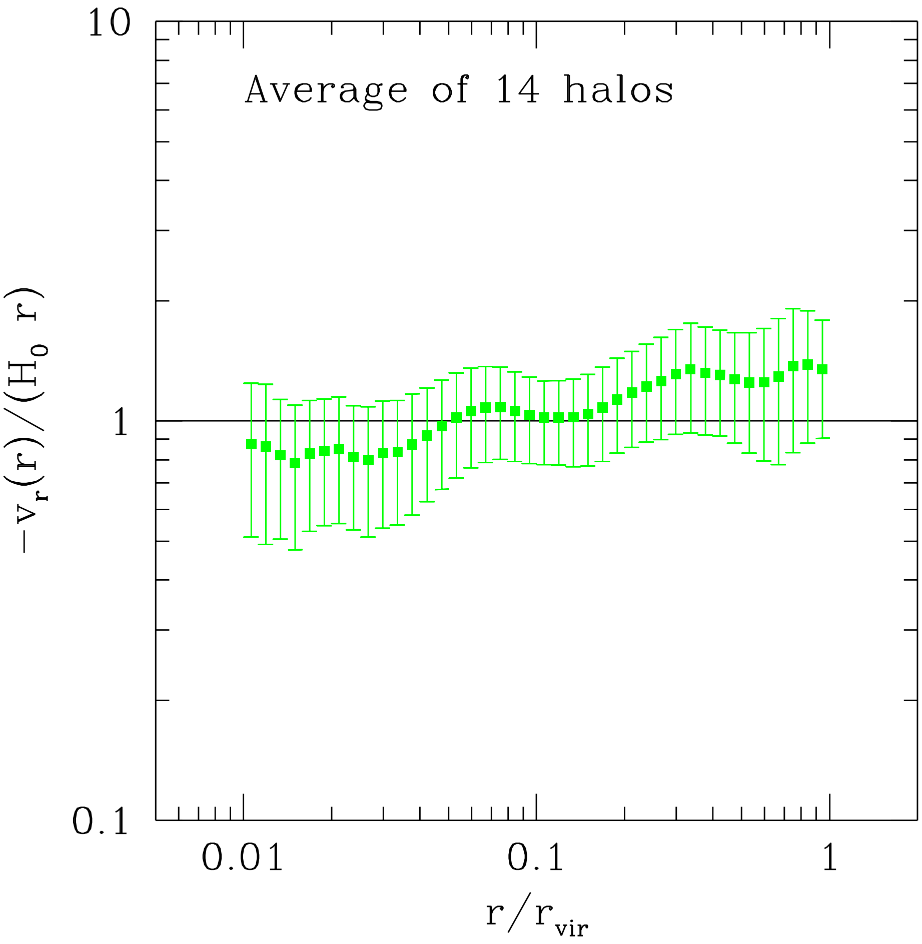

Fig. 3.— The mean peculiar velocity, in units of $H_0 r$, of particles averaged over the 14 halos.