On the spin–up of neutron stars to millisecond pulsars in long–period binaries
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Hans Ritter and Andrew King

On the spin–up of neutron stars to millisecond pulsars in long–period binaries

Hans Ritter

Max–Planck–Institut für Astrophysik, Karl–Schwarzschild–Str. 1, D–85741 Garching bei München, Germany

Andrew King

Astronomy Group, University of Leicester, Leicester LE1 7RH, United Kingdom

Abstract. We study the accretion efficiency of neutron stars in long–period binaries ($P \gtrsim 20^{\rm d}$) which accrete from a giant companion. Using α–disc models and taking into account the effect of irradiation of the accretion disc by the central accretion light source we derive explicit expressions for the duty cycle d and the accretion efficiency η in terms of the parameters of the binary system and the disc instability limit cycle. We show that the absence of millisecond pulsars in wide binaries with circular orbits and periods $P \gtrsim 200^{\rm d}$ can be understood as a consequence of the disc instability if the duration of the quiescent phase between two subsequent outbursts is at least a few decades.

1. Introduction

Ever since the detection of the first millisecond pulsar (Backer et al. 1982) it has been clear that a ms-pulsar is a neutron star (NS) which has been spun up by accretion in a close binary system (for a review see Bhattacharya & van den Heuvel 1991). Because the amount of mass needed to spin up a NS to ms periods is rather small, i.e. $0.05{\rm M}_\odot \lesssim \Delta M_{\rm NS} \lesssim 0.1{\rm M}_\odot$ (Burderi et al. 1999) and the mass available from the donor is typically many times larger, i.e. typically of order $1{\rm M}_\odot$, it appeared as if there was no problem spinning up a NS to such short periods. Yet in recent years, evidence has accumulated which shows that NS are inefficient accretors. Thorsett & Chakrabarty (1999) noted that in all cases where the mass of a NS could be measured *with precision* it turned out to be in the very narrow interval $1.3{\rm M}_\odot \lesssim M_{\rm NS} \lesssim 1.4{\rm M}_\odot$. Even if one makes the extreme assumption that all NSs are born with exactly the same mass, this would imply that none of these NSs could have accreted more than $\sim 0.1{\rm M}_\odot$, despite the fact that some of these NSs have been spun up by accretion and the mass available from the donor star was probably much larger than $\sim 0.1{\rm M}_\odot$. In another context, King & Ritter (1999) have presented further evidence that NSs can be very inefficient accretors. The main reason for this low accretion efficiency is the frequent disparity between the mass transfer rate provided by the donor star and the maximum rate at which the NS can accrete. The latter is approximately the Eddington rate $\dot{M}_{\rm E} \approx 1.5\times 10^{-5}{\rm M}_{\odot}{\rm yr}^{-1}$, whereas, depending on circumstances, the former can be larger by many orders of magnitude.

One of the obvious places in which NSs can be spun up to ms–pulsars are wide binaries in which a NS accretes from a giant. Tauris & Savonije (1999) have recently devoted a detailed study to this formation channel. One of their predictions is that, in principle, ms–pulsars should form in binaries with final orbital periods of up to $P \simeq 1000^{\rm d}$, even when taking into account that the mass transfer rate in long–period systems ($P \gtrsim 20^{\rm d}$) exceeds the Eddington accretion rate. In addition, their calculations predict a correlation between the final orbital period and the mass of the ms–pulsar. Unfortunately, this cannot be tested at present. But their first prediction can. And here observations show, contrary to their prediction, that there are no ms–pulsars in wide binaries with a circular orbit and a period $P \gtrsim 200^{\rm d}$ (see e.g. table 1 in Taam, King & Ritter 2000). In this context, Li & Wang (1998) have already noted that the accretion efficiency of the NS in such a binary is far lower than has been assumed by Tauris & Savonije (1999) if one takes into account the fact that in all these wide binaries the accretion disc around the NS is thermally unstable and accretion is transient (King et al. 1997). This is because a) during an outburst the mass flow rate through the disc is much larger than the mass transfer rate from the donor, which, in turn, is already super–Eddington if $P \gtrsim 20^{\rm d}$, and b) because a NS cannot accrete in excess of the Eddington rate. In fact, Li & Wang (1998) found that if the duty cycle of the disc instability limit cycle is $d \lesssim 10^{-2}$ then in systems with a final orbital period $P_{\rm f} \gtrsim 100^{\rm d}$ the total mass accreted by the NS is $\Delta M_{\rm NS} \lesssim$ few $10^{-2}{\rm M}_{\odot}$, too little to spin up a NS to ms periods.

In this paper we elaborate on the idea of Li & Wang (1998) and derive explicit expressions for the duty cycle and the accretion efficiency of the NS in terms of the parameters of the binary system and the outburst cycle. Our calculations show that if the duration of the quiescent phase between two subsequent outbursts exceeds a few decades, the accretion efficiency becomes so small that in binaries with a final orbital period $P_{\rm f} \gtrsim 200^{\rm d}$, the NS cannot accrete enough mass to become a ms pulsar.

2. Basic model ingredients

The binary systems for which we wish to decribe the accretion processes are wide, long–period binaries with orbital periods of typically $P \gtrsim 20^{\rm d}$ and in which a neutron star primary of mass M_1 accretes from a giant secondary, of mass M_2. The basic model ingredients we use can be summarized as follows:

2.1. Mass transfer

We assume that mass transfer from the secondary is thermally (and adiabatically) stable. In this case, the mass transfer rate can be computed from a simple analytical approximation (e.g. Ritter 1999). Following King et al. (1997) and using the same model parameters we can write for the mass transfer rate

$$-\dot{M}_2 = 7.3\times 10^{-10}{\rm M}_{\odot}{\rm yr}^{-1}(\zeta_e-\zeta_R)^{-1}m_2^{1.7425}p^{0.9281}, \qquad (1)$$

where $m_2 = M_2/\mathrm{M}_\odot, p = P(\mathrm{d})$ is the orbital period in days, ζ_e the thermal equilibrium mass–radius exponent of the donor star, and ζ_R the mass–radius exponent of the donor's critical Roche radius. For (thermal) stability we require $\zeta_e - \zeta_R > 0$ (e.g. Ritter 1988).

2.2. Disc irradiation

Accretion on to a compact star can result in significant external irradiation of the accretion disc by the central accretion source (e.g. van Paradijs 1996). Here we follow King & Ritter (1998) and assume that the effective temperature of the disc is kept above the hydrogen ionization temperature $T_\mathrm{H} \simeq 6500K$ by irradiation from the central accretion source out to a radius

$$R_\mathrm{h} = (B_1 \dot{M}_c)^{1/2}, \tag{2}$$

where $\dot{M}_c$ is the central accretion rate, and, for a neutron star accretor, $B_1 = 3.9 \times 10^5$ cgs (King & Ritter 1998). Furthermore, we shall assume that the neutron star cannot accrete in excess of the Eddington accretion rate $\dot{M}_\mathrm{E} \simeq 1.5 \times 10^{-8}\mathrm{M}_\odot \mathrm{yr}^{-1}$, i.e. that $\dot{M}_c \lesssim \dot{M}_\mathrm{E}$. This means that the maximum radius out to which the disc can be kept hot by irradiation, i.e. at $T_\mathrm{eff} > T_\mathrm{H}$, is

$$R_\mathrm{crit} \simeq 7.1\mathrm{R}_\odot (b_1\, \dot{m}_\mathrm{E})^{1/2}. \tag{3}$$

Here $\dot{m}_\mathrm{E} = \dot{M}_\mathrm{E}/10^{-8}\mathrm{M}_\odot \mathrm{yr}^{-1}$ and $b_1 = B_1/3.9 \times 10^5$ cgs. At the same time our assumption that $\dot{M}_c \lesssim \dot{M}_\mathrm{E}$ means that whenever the mass flow rate through the disc is $-\dot{M}_\mathrm{d} > \dot{M}_\mathrm{E}$, the excess matter must be ejected from the disc.

2.3. The α–disc model

For describing the properties of the accretion disc around the neutron star we use the Shakura & Sunyaev (1973) α–disc model. Accordingly, if the vertical temperature stratification of the disc is dominated by viscous heating, the disc viscosity is (e.g. Frank, King & Raine 1992)

$$\nu_\mathrm{visc} = 2.7 \times 10^{15}\mathrm{cm}^2\mathrm{s}^{-1} \alpha^{4/5} (-\dot{m}_\mathrm{d})^{3/10} m_1^{-1/4} r^{3/4}, \tag{4}$$

where $-\dot{M}_\mathrm{d}$ is the mass flow rate through the disc, R the disc radius, $\dot{m}_\mathrm{d} = \dot{M}_\mathrm{d}/10^{-8}\mathrm{M}_\odot \mathrm{yr}^{-1}$, and $r = R/\mathrm{R}_\odot$. α is the viscosity parameter, here assumed to be constant with radius.

(4) holds only to the extent that the vertical temperature stratification in the disc is dominated by viscous heating, hence its dependence on $\dot{M}_\mathrm{d}$. If, on the other hand, the disc is strongly heated by external irradiation, the vertical temperature profile becomes nearly isothermal at the irradiation temperature T_irr. Taking this into account in the α–viscosity ansatz we find (see also King 1998)

$$\begin{aligned} \nu_\mathrm{visc} &= \alpha \frac{\mathcal{R} T_\mathrm{H}}{\mu} (B_1 \dot{M}_c)^{1/4} (G M_1)^{-1/2} R \\ &= 3.8 \times 10^{15}\mathrm{cm}^2\mathrm{s}^{-1} \alpha \left(\frac{\mu}{0.6}\right)^{-1} \left(\frac{T_H}{6500K}\right) (b_1 \dot{m}_c)^{1/4} m_1^{-1/4} r. \end{aligned} \tag{5}$$

Here $\mathcal{R}$ is the gas constant and μ the mean molecular weight.

The maximum suface density which a disc can reach in the cool state, i.e. before it becomes thermally unstable and must turn into the hot state, is conventionally denoted by Σ_B. Taking Σ_B from computations of Ludwig, Meyer–Hofmeister & Ritter (1995), the mass accumulated before an outburst sets in can be written as

$$M_d \simeq 7 \times 10^{-10} \mathrm{M}_\odot \ \alpha_c^{-4/5} m_1^{-0.37} r_\mathrm{d}^{3.10} \mathcal{F}. \tag{6}$$

Here $r_\mathrm{d} = R_\mathrm{d}/\mathrm{R}_\odot$, where R_d is the outer radius of those parts of the disc which are involved in the outburst, α_c the α–parameter in the cool state, and $\mathcal{F} < 1$ is the filling factor defined by this equation. $\mathcal{F}$ takes into acount that a the onset of an outburst $\Sigma = \Sigma_B$ at only one point in the disc and that $\Sigma < \Sigma_B$ elsewhere.

2.4. Assumptions about the disc instability limit cycle

We shall assume that the disc instability limit cycle in these wide, long–period binary systems with a neutron star accretor can be described as follows:

a) During an outburst the disc is kept in the hot state by irradiation out to the radius $R_\mathrm{h}(\dot M_c)$ given by (2). Because $\dot M_c \lesssim \dot M_\mathrm{E}, R_\mathrm{h}(\dot M_c) \lesssim R_\mathrm{h}(\dot M_\mathrm{E})$. If the physical outer radius of the disc $R_\mathrm{disc} > R_\mathrm{h}(\dot M_c)$, only the inner part with radius $R_\mathrm{d} = R_\mathrm{h}(\dot M_c)$ is hot, whereas the outer part with $R_\mathrm{h}(\dot M_c) < R < R_\mathrm{disc}$ remains in the cold state (e.g. King & Ritter 1998). Hereafter we shall call R_d the radius of the "active" disc. If, on the other hand, $R_\mathrm{h}(\dot M_c) > R_\mathrm{disc}, R_\mathrm{d} = R_\mathrm{disc}$. If the mass flow rate through the outer parts of the active disc is $-\dot M_\mathrm{d}(R_\mathrm{d}) > \dot M_\mathrm{E}$, then as long as $-\dot M_\mathrm{d}(R_\mathrm{d}) > \dot M_\mathrm{E}, \dot M_c = \dot M_\mathrm{E}$, and $R_\mathrm{h}(\dot M_c) = R_\mathrm{h}(\dot M_\mathrm{E}) = \mathrm{const}$.

b) During an outburst the active part of the disc (out to R_d) is essentially emptied, i.e. the mass remaining in the active part of the disc immediately after an outburst is insignificant compared to the mass M_d given by (6) immediately before the onset of an outburst.

c) During quiescence the mass of the active part of disc which was accreted/ ejected during the previous outburst is replenished at the mass transfer rate $-\dot M_2$. This is tantamount to assuming that if an inactive outer part of the disc exists, i.e if $R_\mathrm{h}(\dot M_c) < R_\mathrm{disc}$, this outer part is stationary, i.e. that $\dot M_\mathrm{d}(R) = \dot M_2$ for all $R_\mathrm{h}(\dot M_c) < R < R_\mathrm{disc}$.

d) We assume that the hot part of the disc (during an outburst) is characterized by a viscosity parameter α_h, whereas in the cool state (i.e. in the active part during quiescence and in the inactive part) the viscosity parameter is $\alpha_c < \alpha_\mathrm{h}$.

3. Disc accretion in long–period neutron star binaries

Here we are mainly interested in systems which end their nuclear time scale–driven mass transfer at very long orbital periods, i.e. $P \gtrsim 200^\mathrm{d}$. With the analytical solution (Ritter 1999) one can show that the corresponding initial periods at which stable mass transfer must have started are typically $P_{\rm i} \gtrsim 20^{\rm d}$, (the precise value depending on whether mass transfer is conservative or not, cf. Taam et al. 2000). It has been shown by King et al. (1997) and Li & Wang (1999) that at such long periods the mass transfer rate provided by nuclear evolution of the donor is always too small for stable disc accretion, i.e. the systems in question are transient, going through a disc instability limit cycle.

In addition, the orbital period at which $R_{\rm disc} = R_{\rm crit}$ (with $R_{\rm disc} > R_{\rm crit}$ for longer periods) is

$$P_{\rm crit} = 12{\rm d}\left(\frac{R_{\rm disc}/R_{1,R}}{0.7}\right)^{-3/2} m_1^{-0.675} m_2^{0.175} (b_1\,\dot{m}_{\rm E})^{3/4}, \tag{7}$$

where $R_{1,R}$ is the critical Roche radius of the primary. Thus, for practically all systems in question we also have $R_{\rm disc} > R_{\rm h}(\dot{M}_c)$.

As we shall argue below, during most of an outburst the mass flow rate through the outer parts of the active disc is highly super–Eddington in the sense that $-\dot{M}_{\rm d}(R_{\rm d}) \gg \dot{M}_{\rm E}$. Therefore, at least initially, in these systems $R_{\rm d} = R_{\rm h}(\dot{M}_{\rm E}) = $ const.

Now, if the viscosity at $R = R_{\rm d}$ does not depend on $\dot{M}_{\rm d}$, and $R_{\rm d}$ itself is constant, both the mass and the mass flow rate through the outer parts of the active disc decay exponentially on the viscous time scale

$$\tau_{\rm visc} = \frac{R_{\rm d}^2}{3\nu_{\rm visc}(R_{\rm d})} \tag{8}$$

at the outer radius of the active disc. If initially $-\dot{M}_{\rm d}(R_{\rm d}, t = 0) \gg \dot{M}_{\rm E}$, we have $R_{\rm d} = R_{\rm h}(\dot{M}_{\rm E}) = $ const. as long as $-\dot{M}_{\rm d} > \dot{M}_{\rm E}$ and the disc is essentially emptied during the super–Eddington phase which lasts for a time

$$t_{\rm outb} \simeq \tau_{\rm visc}(R_{\rm d}) \ln\left[\frac{-\dot{M}_{\rm d}(R_{\rm d}, t = 0)}{\dot{M}_{\rm E}}\right]. \tag{9}$$

This is the behaviour to be expected if the active part of the disc is dominated by irradiation, i.e. if $\nu_{\rm visc}$ is given by (5). If, on the other hand, viscous heating still dominates the vertical temperature structure, $\nu_{\rm visc}$ is given by (4). Using (4) in (8) we see that $-\dot{M}_{\rm d} \propto M_d^{10/7}$, i.e. that the mass flow rate and the mass in the active part of the disc do not decay exponentially but rather as

$$M_d(t) = M_d(0)\left[1 + \frac{3}{7}\frac{t}{\tau_0}\right]^{-3/7}, \tag{10}$$

where

$$\tau_0 = \frac{M_d(0)}{-\dot{M}_{\rm d}(0)} \tag{11}$$

is the time scale on which the disc mass decays initially. If, initially, $-\dot{M}_{\rm d} \gg \dot{M}_{\rm E}$, the disc is essentially emptied during the super–Eddington phase which lasts in this case for the time

$$t_{\rm outb} \simeq \frac{3}{7}(\tau_{\dot M_{\rm E}} - \tau_0), \tag{12}$$

where

$$\tau_{\dot M_{\rm E}} = \tau_0 \left[\frac{-\dot M_{\rm d}(0)}{\dot M_{\rm E}}\right]^{3/10} \tag{13}$$

is the time scale on which the disc mass decreases once $-\dot M_{\rm d}$ reaches $\dot M_{\rm E}$.

The duration of the quiescent phase is given by the time it takes to replenish the matter which has been lost (accreted/ejected) from the active part of the disc during the previous outburst. Because we have assumed that the active disc is fed at the rate $-\dot M_2$, irrespective of whether $R_{\rm disc}$ is larger or smaller than $R_{\rm d}$, the duration of quiescence is

$$t_{\rm q} \simeq \frac{M_d(0)}{-\dot M_2}. \tag{14}$$

The total length of an outburst cycle is then

$$t_{\rm cycle} = t_{\rm outb} + t_{\rm q} \tag{15}$$

and the duty cycle of the outbursts

$$d = \frac{t_{\rm outb}}{t_{\rm cycle}}. \tag{16}$$

To decide the question of how much the neutron star can accrete, the quantity we need is the accretion efficiency η. Because the mass accreted over one cycle is $\Delta M_{\rm accr} \simeq t_{\rm outb}\dot M_{\rm E}$, whereas the mass transferred over the same time is $\Delta M_{\rm transf} \simeq t_{\rm cycle}(-\dot M_2)$, we have

$$\eta \simeq \frac{t_{\rm outb}\cdot \dot M_{\rm E}}{t_{\rm cycle}(-\dot M_2)} \simeq d\frac{\dot M_{\rm E}}{(-\dot M_2)}. \tag{17}$$

Writing

$$x = \frac{-\dot M_{\rm d}(0)}{\dot M_{\rm E}} \tag{18}$$

we obtain

$$\eta \simeq \frac{\ln x}{x} \tag{19}$$

if the mass flow rate decays exponentially, i.e. if (5) is used, and

$$\eta \simeq \frac{7}{3}\frac{x^{0.3}-1}{x - x^{0.3}} \tag{20}$$

if (4), i.e. (10) holds. In both cases, it is immediately seen that η is small if $x \gg 1$, i.e. if during the initial phases of an outburst the mass flow rate is highly super–Eddington.

4. The mass flow rate through the outer disc

In order to get η, we need to know x, i.e. $-\dot{M}_{\rm d}(R_{\rm d}, t=0)$. Assuming again that $R_{\rm d} = R_{\rm h}(\dot{M}_{\rm E}) < R_{\rm disc}$, $-\dot{M}_{\rm d}(R_{\rm d}, t=0) > \dot{M}_{\rm E}$, and using (2), (5), (6) and (8) we find

$$\begin{aligned} -\dot{M}_{\rm d}(R_{\rm d}, t=0) = \;& 3.3\times 10^{-6}{\rm M}_\odot{\rm yr}^{-1}\alpha_{\rm h}{\alpha_c}^{-4/5}\mathcal{F} \\ & \left(\tfrac{\mu}{0.6}\right)^{-1}\left(\tfrac{T_{\rm H}}{6500K}\right)(b_1\,\dot{m}_{\rm E})^{1.3}{m_1}^{-0.37}. \end{aligned} \tag{21}$$

We note that $\dot{M}_{\rm d}(R_{\rm d}, t=0)$ as given in (21), and therefore also η, still depends on the orbital period. The dependence on P enters in a subtle way via the filling factor $\mathcal{F}$ which itself must be a function of $\dot{M}_2$ and hence via (1) of P. Unfortunately, this dependence is not explicitly known and can only be determined from full time–dependent calculations of the disc evolution.

Replacing ${\alpha_c}^{-4/5}\mathcal{F}$ in (21) by means of (6), noting that

$$M_d = -\dot{M}_2\cdot t_{\rm q}, \tag{22}$$

we get rid of the poorly known quantities α_c and $\mathcal{F}$ at the price of introducing $t_{\rm q}$. This yields

$$\begin{aligned} \dot{M}_{\rm d}(R_{\rm d}, t=0) = \;& 7.6\times 10^{-9}{\rm M}_\odot{\rm yr}^{-1}(\zeta_e-\zeta_R)^{-1}\left(\tfrac{\mu}{0.6}\right)^{-1}\left(\tfrac{T_H}{6500K}\right) \\ & (b_1\,\dot{m}_{\rm E})^{-1/4}m_1^{-1/2}m_2^{1.7425}p^{0.9281}t_{\rm q}({\rm yr}). \end{aligned} \tag{23}$$

Inserting typical values in (23), i.e $(\zeta_e-\zeta_R)\simeq 1, \alpha_{\rm h}\simeq 0.2, \dot{m}_{\rm E}=1.5, m_1 = 1.4, m_2 = 1$ we find $x\simeq 1.25t_{\rm q}({\rm yr})$ at $P=20^{\rm d}$ and $x\simeq 10.6t_{\rm q}({\rm yr})$ at $P=200^{\rm d}$. This shows that if the quiescent time of these discs is long, i.e. > a few decades, the accretion efficiency becomes very small. Unfortunately, we do not have any direct observational information about $t_{\rm q}$. However the analogy with the properties of the outbursts in black hole soft X–ray transients (BHSXTs) (see e.g. King & Ritter 1998) suggests that the quiescent times are likely to be very long, i.e. longer than several decades. In the Discussion below we show that the absence of any observed outbursts from the progenitors of pulsars in wide circular binaries supports this conclusion. We note in passing that $-\dot{M}_{\rm d}(R_{\rm d})$ must not be too large if this picture is to be self–consistent: the assumption that $\nu_{\rm visc}$ is dominated by external irradiation, i.e. that we may use (5), requires that

$$\begin{aligned} -\dot{M}_{\rm d}(R_{\rm h}(\dot{M}_{\rm E})) \;&< \tfrac{8\pi\sigma T_H^4(B_1\dot{M}_{\rm E})^{3/2}}{3GM_1} \\ &< 1.2\times 10^{-5}{\rm M}_\odot{\rm yr}^{-1}\left(\tfrac{T_H}{6500K}\right)^4(b_1\,\dot{m}_{\rm E})^{3/2}m_1^{-1}. \end{aligned} \tag{24}$$

If instead of (5) we use (4), i.e. a viscosity where the disc temperature is self–consistently determined by viscous dissipation, we obtain:

$$\begin{aligned} -\dot{M}_{\rm d}(R_{\rm d}, t=0) = \;& 2.0\times 10^{-9}{\rm M}_\odot{\rm yr}^{-1}(\zeta_e-\zeta_R)^{-10/7}\alpha_{\rm h}^{8/7} \\ & (b_1\,\dot{m}_{\rm E})^{-25/28}m_1^{-5/14}m_2^{2.4893}p^{1.3259}t_{\rm q}({\rm yr})^{10/7}. \end{aligned} \tag{25}$$

Inserting the same typical values as above in (25) we obtain $x \simeq 0.7 t_{\rm q}({\rm yr})^{10/7}$ at $P = 20^{\rm d}$ and $x \simeq 15 t_{\rm q}({\rm yr})^{10/7}$ at $P = 200^{\rm d}$. From this it is seen that if only $t_{\rm q} \gtrsim$few yr, disc accretion becomes even more inefficient if the viscosity is due to viscous dissipation rather than external irradiation. We also note that even if (24) is violated initially, the disc's evolution will eventually be dominated by external heating because $-\dot{M}_{\rm d}(R_{\rm h})$ decreases with time.

Finally, in the case of irradiation–dominated viscosity (5) the duty cycle becomes

$$\begin{aligned} d &\simeq \frac{\tau_{\rm visc}(R_{\rm h}(\dot{M}_{\rm E}))}{t_{\rm q}} \ln x \\ &\simeq 0.095 \alpha_{\rm h}^{-1} \left(\frac{\mu}{0.6}\right)^{-1} \left(\frac{T_H}{6500K}\right) (b_1\, \dot{m}_{\rm E})^{1/4} m_1^{1/2} t_{\rm q}^{-1}({\rm yr}) \ln x . \end{aligned} \tag{26}$$

Although at a first glance d does not seem to depend on P it nevertheless does: besides a weak dependence on P via $\ln x$ there is a significant one from the factor ${t_{\rm q}}^{-1} = -\dot{M}_2/M_{\rm d}$. On the one hand $\dot{M}_2$ depends explicitly on P (cf (1)), on the other hand a more subtle dependence which we have mentioned already earlier enters via the filling factor $\mathcal{F}$ in (6). Nevertheless, if typical parameters are used in (26), i.e. $\alpha_{\rm h} \simeq 0.2$, $\dot{m}_{\rm E} = 1.5$ and $m_1 = 1.4$, we have $d \simeq 0.12\ t_{\rm q}^{-1}({\rm yr}) \cdot \ln x$. Therefore, if $t_{\rm q} \simeq$ many decades, $d \ll 1$.

5. Discussion and Conclusions

Li & Wang (1998) have shown that the accretion efficiency of neutron stars which accrete from a giant is very small and, in fact, becomes too small for spinning up the neutron star to millisecond spin periods in systems with a final orbital period $P_{\rm f} \gtrsim 100^{\rm d}$ if the duty cycle $d \lesssim 10^{-2}$. Here we have worked out explicit expressions for d and the accretion efficiency η in terms of the binary parameters and the quiescent time $t_{\rm q}$ of the outburst cycle in the framework of an irradiated α–disc model. Our calculations show that both, d and η are small if $P > P_{\rm crit}$ and the mass flow rate at the onset of an outburst is highly super–Eddington. Whether or not d and η are really small enough to prevent neutron stars in systems with $P_{\rm f} \gtrsim 200^{\rm d}$ being spun up to ms–periods depends crucially on the duration of quiescence $t_{\rm q}$. If $t_{\rm q} \gtrsim$ many decades, as some observations of BHSXTs suggest, then both η and d are very small indeed. Unfortunately, determining $t_{\rm q}$ in the framework of our simple model is not possible. For this fully time–dependent calculations of the disc instability limit cycle would be needed.

However there is another way that we can get a lower limit on d. We know that pulsars in wide circular binaries must have descended from SXTs. Yet we do not know of a single transient with a period longer than $11.8^{\rm d}$ (GRO J1744–28), even though X–ray satellites would probably have detected an outburst from such a system anywhere in the Galaxy within the last ~ 30 yr. This must mean that outbursts are rather infrequent, i.e. $t_{\rm q}$ is long. We can make this more precise by defining the following quantities:

Let n be the frequency of X–ray outbursts from all systems in the Galaxy, $N_{\rm PSR,obs}$ the currently known number of the pulsar binaries in question, $N_{\rm PSR}$ the current number of the pulsar binaries in question in the Galaxy, $\tau_{\rm prog}$ the lifetime of a progenitor system in the phase of nuclear time scale mass transfer, and $\tau_{\rm PSR}$ the lifetime of the pulsar in the pulsar binaries in question. Here $\tau_{\rm prog}$ and $\tau_{\rm PSR}$ are suitable averages over the relevant population.

Now, the inventory of the binary pulsars in question is incomplete because of flux limitation, dispersion and beaming, whereas an X–ray outburst in one of the progenitor systems would probably be seen wherever in the Galaxy the outburst occurs. We correct for this incompletness by introducing a 'filling factor' $f < 1$. In a stationary situation we then must have

$$t_{\rm q} = \frac{N_{\rm PSR}}{n} \frac{\tau_{\rm prog}}{\tau_{\rm PSR}} = \frac{N_{\rm PSR,obs}}{n\ f} \frac{\tau_{\rm prog}}{\tau_{\rm PSR}}. \qquad (27)$$

$\tau_{\rm PSR}$ must be of the order of the spin-down time scale $P/2\dot{P} \approx 10^8$ yr, $\tau_{\rm prog}$, on the other hand, is of the order of the nuclear time scale t_∞ defined in Ritter (1999, Eq. 14) which, for the systems in question, i.e. those which end their evolution with a long orbital period $P_{\rm orb} \gtrsim 200^{\rm d}$, is of the order 10^8 yr. With $N_{\rm PSR,obs} = 2$ (i.e. PSR B0820+02 and PSR J1803-2712), $f \approx 0.2$ and from the fact that we have not seen a single outburst in the past 30 yr this yields the lower limit $t_{\rm q} > (60\ {\rm yr})/f \gtrsim 300$ yr. We would also like to emphasize that we have adopted a very conservative value for f. The real value is likely to be smaller still and thus the estimate for $t_{\rm q}$ even larger because already the canonical beaming factor for pulsars is of order 0.2.

Thus it is very likely that the lack of ms–pulsars in binaries with white dwarf companions and $P_{\rm orb} \gtrsim 200^{\rm d}$ is entirely a consequence of dwarf nova–like disc instabilities in long–period binaries.

Finally, we note that spinning up a NS in a long–period binary faces yet another obstacle: during the long quiescent phases the NS acts as a propeller (Illarionov & Sunyaev 1975) and thereby is spun *down*, making the spin-up to ms spin periods even more difficult.

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