The sign of temperature inhomogeneities deduced from time-distance helioseismology

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Abstract.
Inhomogeneities in wave propagation conditions near and below the solar surface have been detected by means of time-distance helioseismology. Here we calculate the effect of temperature inhomogeneities on the travel times of sound waves. A temperature increase, e.g. in active regions, not only increases the sound speed but also lengthens the path along which the wave travels because the expansion of the heated layers shifts the upper turning of the waves upward. Using a ray tracing approximation we find that in many cases the net effect of a temperature enhancement is an *increase* of the travel times. We argue that the reduced travel times that are observed are caused by a combination of magnetic fields in the active region and *reduced* subsurface temperatures. Such a reduction may be related to the increased radiative energy loss from small magnetic flux tubes.

Keywords: convection, waves, helioseismology

1. Introduction

In recent years it has become possible to measure the travel times of acoustic waves travelling through the outer layers of the Sun via 'time-distance helioseismology'. These travel times are used to infer information about the sub-surface structure of the Sun and have revealed inhomogeneities in the wave propagation conditions. (Duvall et al. 1993, Duvall 1995, D'Silva & Duvall 1995, Duvall 1997, Duvall et al. 1998). These inhomogeneities indicate shorter sound-wave travel times and have been associated with active regions.

Slight shifts in global p-mode frequencies in the course of a solar cycle have also been detected (Woodard & Libbrecht 1993, Dziembowski et al. 2000). As in the case of the time-distance measurements, they have been found to be closely related to magnetic activity (Dziembowski et al. 2000), and indicate shorter travel times in the outermost layers of the convection zone. A common interpretation for both effects is that the sound speed in active regions is somewhat higher because of an increase in temperature. However, it was shown by Goldreich et al. (1991) that, though intuitively appealing, such a temperature increase is unlikely to be the cause of the mode frequency shifts. Contrary to expectation, a higher temperature causes *longer* travel times, because a temperature increase causes a slight expansion of the envelope. The path length increase caused by this expansion dominates over the increase in sound speed since the expansion is proportional to the temperature T, while the sound speed increases only as $T^{1/2}$. Instead of a temperature increase, Goldreich et al. propose that most of the effect is due to the photospheric magnetic field. A magnetic field increases the stiffness of the gas as experienced by pressure waves.

The expansion argument would also affect the interpretation of travel-time anomalies discovered by time-distance helioseismology. The purpose of this paper is to verify to what extent the expansion of the envelope associated with a temperature rise increases the wave travel times. In Sec. 2 we show that in most cases a temperature enhancement does in fact lead to *longer* travel times. To explain shorter travel times one might then invoke magnetic fields. Alternatively, the subsurface temperatures could be slightly *lower* in the active regions, the opposite of what is concluded in most analyses of time-distance measurements. These possibilities are discussed briefly in Sec. 3.

2. Model

One would like to investigate the effects of temperature inhomogeneities without making too many assumptions about their origin. On the other hand, for actual wave propagation calculations a well defined equilibrium model is needed. A simple model is a *geostrophic* one, in which the pressure balance within the inhomogeneity is attained through Coriolis forces associated with the solar rotation. This balance is probably valid on large scales (of the order of an active region). On smaller scales, this is probably not a good model; see Sec.2.3 for a detailed discussion.

For definiteness in discussing the model, we regard a region of increased temperatures. Since the effects are nearly linear in δT, the results apply equally with opposite sign to regions of lower than average sub-surface temperatures.

2.1. Equilibrium model

In a plane-parallel envelope in hydrostatic equilibrium under constant gravitational acceleration g, the pressure, p, satisfies

$$\frac{dp}{dz} = g\rho, \tag{1}$$

where z is the depth beneath some reference layer. Assuming a polytropic relation between p and density, ρ, i.e.

$$p = K\rho^{1+1/n}, \tag{2}$$

one finds that the pressure varies with depth as

$$p = p_0 \left(\frac{z}{z_0}\right)^{n+1} \tag{3}$$

and similarly

$$\rho = \rho_0 \left(\frac{z}{z_0}\right)^{n}, \tag{4}$$

where z_0 denotes the depth of some reference layer from where the polytropic layer extends downward. Above z_0 one may, for example, want to match the polytrope onto an isothermal atmosphere. Here we only consider rays that lie completely within the polytrope.

The sound speed is thus

$$c^2 = \frac{\gamma g}{n+1} z, \tag{5}$$

where γ is the ratio of the specific heats. The acoustic cut-off frequency is approximately given by

$$\omega_{\rm c} = \frac{c}{2H} = \left(\frac{\gamma g}{n+1}\right)^{1/2} \frac{n}{2} z^{-1/2}, \tag{6}$$

where H is the density scale height defined as

$$H = \left(\frac{\mathrm{d}\ln\rho}{\mathrm{d}z}\right)^{-1} = z/n. \tag{7}$$

This yields a depth of the upper turning point of

$$z_{\rm t} = \left(\frac{n}{2\omega}\right)^2 \frac{\gamma g}{n+1}. \tag{8}$$

Assuming the gas to be ideal and the ionization to be constant, the temperature is given by

$$T = \frac{\mu p}{R\rho} = \frac{\mu g}{R(n+1)} z, \tag{9}$$

where μ is the mean molecular mass and R the gas constant. Thus, the temperature profile is determined by the polytropic index n.

The entropy is given by

$$S \sim \ln p/\rho^{\gamma} = (1 + 1/n - \gamma) \ln \rho + \text{cst.}, \tag{10}$$

where an ideal gas equation of state has been assumed. Since the stratification of the convection zone is close to adiabatic, the values for the polytropic index and γ are related by

$$\gamma \approx 1 + 1/n. \tag{11}$$

2.2. Time-distance calculations

We now investigate the travel times of those waves that enter a column of hotter material, are then reflected near the surface and subsequently leave the hotter region again. We treat this problem in two dimensions; we ignore the effect of the advection of the waves, assuming that to equal parts the waves travel in and out of the inhomogeneity and that therefore the net effect of advection vanishes to first order. The flows may cause second-order effects ($\sim v^2$, and independent of the direction of the flow) on the travel times. This is discussed in section 2.3.

In the spirit of the JWKB approximation we will regard the sound waves as locally plane (see Gough 1993). In this approximation, which is commonly made in local helioseismology, the waves are assumed to follow rays that obey the laws of geometrical acoustics. For simplicity, we will regard the medium as plane-parallel.

We let the ray traverse a column in which the temperature profile is raised to $T_1(z)$. This will be achieved using two very simple models. In model 1 the temperature in the hotter column is raised by lowering the polytropic index to $n_1 < n_0$ (subscript 0 shall denote the corresponding values of the 'normal' Sun). The temperature difference in this model is thus proportional to the depth z, and its depth is assumed infinite.

Model 2 mimics some proposed models for temperature enhancements associated with active regions (Kuhn and Stein 1996). In these models the source is assumed to be an entropy increase located at some depth D, the effect of which extends to the surface. This can be incorporated in a model with a higher, but still depth-independent entropy S. In a polytropic model, this corresponds to an increased value for the polytropic constant K. If this model is in lateral pressure balance at the source depth D, vertical equilibrium implies a vertical shift, such that the top of the polytrope is at some depth $z_1 < 0$. If K_1 is the polytropic constant of this model, one finds that

$$\left(\frac{K_1}{K_0}\right)^{-\frac{n}{n+1}} = 1 + \frac{z_1}{D}. \tag{12}$$

For small entropy changes $\delta K = K_1 - K_0$ we have

$$\frac{z_1}{D} = -\frac{n}{n+1}\frac{\delta K}{K_0}. \tag{13}$$

The vertical shift is thus proportional to the depth of the source. In the example shown in Fig. 2 we have chosen a depth of $D = 21$ Mm and $\frac{\delta K}{K_0}$ was chosen to be $6 \cdot 10^{-3}$ which yields a vertical shift upward of $z_1 \approx -120$ km.

In the hotter column the upper turning point is raised over the one in the adjacent colder medium. In case 1, $n_1 < n_0$, which decreases the depth z_t of the upper turning point (see Eq.(8)) and in case 2 the upper turning point is shifted upward due to expansion.

The ray equations follow from the dispersion relation, which can be written as

$$k^2 = k_\mathrm{v}^2 + k_\mathrm{h}^2 = \frac{\omega^2 - \omega_\mathrm{c}^2}{c^2}, \tag{14}$$

where k_v and k_h are the vertical and horizontal components of the wavevector $\mathbf{k}$.

The advantage of using a polytropic approximation (aside from the fact that the approximation is quite good for the upper layers of the Sun) is that the ray equations can be written down analytically. If x is the horizontal coordinate in the plane of the wave and assuming that $k_x^2 + k_z^2 = \omega^2/c^2$ (i.e. ignoring ω_c), one can write

$$\begin{aligned} x &= \int \frac{dx}{dz}dz = \int \frac{k_x}{k_z}dz = \int \frac{k_x}{\sqrt{\omega^2/c^2 - k_x^2}}dz = \int \frac{dz}{\sqrt{a/z - 1}} \\ &= a\left[\sin^{-1}(z/a)^{1/2} - (z/a)^{1/2}(1 - z/a)^{1/2}\right], \end{aligned} \tag{15}$$

where a is the depth of the lower turning point given by $a = \omega^2/c_0^2 k_x^2$. The time taken for this traverse is given by the integral along the ray

$$\begin{aligned}\tau &= \int \frac{k}{\omega} ds = c_0^{-1} \int \; z^{-1/2}(1-z/a)^{-1/2} \; dz \\ &= c_0^{-1} a^{1/2} \sin^{-1}(z/a)^{1/2} \; . \end{aligned} \tag{16}$$

When the cut-off frequency is included in the ray equations, the expressions become a bit more complicated, but the integrals can still be solved analytically.

At the vertical interface between the two regions of different temperatures, the rays are refracted. We are using the approximations of geometrical acoustics and calculate the angle of refraction by Snell's law. Then we compare the travel times in the homogeneous Sun with the corresponding times in the scenario depicted in Fig. 1. In all cases, we consider rays which bounce (i.e. have their upper turning point) inside the inhomogeneity (see Fig. 1). In Fig. 2 we have plotted the relative travel time difference $\delta\tau/\tau = (\tau_{\rm inhom} - \tau_{\rm hom})/\tau_{\rm hom}$ (between rays in a homogeneous and inhomogeneous Sun) for rays of different interskip distances (and depths) for a fixed width of the hotter region in models 1 and 2. In Fig. 3 $\delta\tau/\tau$ is shown for a ray of fixed depth but as a function of the width of the hotter region.

Fig. 2 shows that the travel-time can be positive, i.e. that the waves can take longer in the presence of the hot column. This implies that the hot column retards the waves, because the effect of the raised upper turning point outweighs the increased sound speed in the hotter column. However, $\delta\tau/\tau$ decreases with increasing depth of the ray, since the waves spend more time in the hotter (and therefore faster) region whereas the effect of the raised upper turning point depends very little on the depth of the rays (since the rays are almost vertical near the upper turning point).

In Fig. 3 one can note that the travel-time is positive as long as the width of the hot region is small compared to the length of the ray. But $\delta\tau/\tau$ decreases with increasing w because the sound speed is higher in the hotter region, and this eventually dominates the effect of the lengthening of the ray through the raised upper turning point: the travel-time difference becomes negative.

2.3. Horizontal equilibrium of a temperature inhomogeneity

So far we have evaded the question about what restores the horizontal pressure equilibrium. The difference in temperature between the inhomogeneity and its surroundings causes a difference in gas pressure that is subject to adjustments on the short hydrodynamic time scale. Consider a temperature enhancement in a patch (an active region, say) at colatitude θ, extending below the surface as a column of width L (Fig. 4). In geostrophic balance, the pressure excess δp is balanced by the Coriolis force acting on a flow $\mathbf{v}$ around the column, i.e.

$$(\nabla \delta p)_{\mathrm{h}} = 2(\rho \mathbf{v} \times \Omega)_{\mathrm{h}}, \tag{17}$$

where the subscript h indicates the horizontal components.

The flow is concentrated at the boundary of the column, where the pressure gradients are greatest. Inside the column, the flow vanishes and the excess pressure δp is just given by hydrostatic equilibrium,

$$p_r \delta p = -g \delta \rho. \tag{18}$$

The column is assumed to extend down to some depth D where $\delta p = 0$. Below this depth, the temperature excess vanishes. The flow speed is then maximal at the surface, and vanishes at depth D.

In the preceding paragraph, we have proposed a geostrophic balance for the pressure changes. For relatively small inhomogeneities the effects of rotation are small, and a geostrophic balance is not realistic. Instead one could consider upwellings and downdrafts. If there is a source of heat at some depth below the surface, a circulation is set up, with upwelling above the heat source. This flow is driven by the higher gas pressure inside the rising column, and its velocity $\mathbf{v}$ is such that the dynamic pressure of the flow, $\rho \mathbf{v} \cdot \nabla \mathbf{v} \approx \rho v^2 / r_c$, balances the pressure difference (r_c is the gradient length scale of the velocity).

The flow advects the p-modes (to first order in v) but also has a second-order effect on their propagation speed. The second-order effect is proportional to, and of the same order of magnitude as the temperature increase and likely to give a positive contribution to the sound speed. This is because flows with vorticity speed up on compression. In the previous section we neglected the contribution to the sound speed which is provided by the compressibility of the flows. Its effect is to increase the sound speed, but the extent of the effect is difficult to model.

For a qualitative estimate, we assume the effect of the flow on the waves to be *local* and *isotropic*. In this case, the effect on wave propagation would be a mere increase of the propagation speed. Thus, we repeat the calculations presented above with an increased sound velocity inside the hotter region:

$$\delta c/c = \alpha \frac{1}{2} \delta T/T, \tag{19}$$

where α is a factor of order unity. In the absence of flows, we would have $\alpha = 1$. Fig. 5 shows the same case as depicted in Fig. 2 only with $\alpha = 1.01$. As expected, the relative time delay is smaller than in the absence of flows and even for a slight relative increase of the sound speed by 1 % the time delay quickly becomes negative. Therefore, one will have to know the properties of the turbulent medium very accurately before one can definitly predict its effect on the travel times.

In the above, the flows were presumed to have no effect on the position of the upper turning point. In reality, a 'turbulent' pressure increase due to small scale flows expands the region vertically. This would raise the upper turning point levels, and increase the travel times. In the case of granulation flows, this effect is the main contribution to the p-mode frequency anomalies associated with the outer envelope (Rosenthal, et al. 1999). By leaving it out, we are probably underestimating the travel time increase (decrease) in hotter (cooler) regions.

3. Summary and discussion

Time-distance helioseismology indicates the presence of subsurface inhomogeneities. In order to map these inhomogeneities quantitatively, a propagation model is needed. In most analyses it is assumed that shorter travel times correspond to higher propagation speeds. While this is correct if the inhomogeneities are due to a vertical magnetic field, the most obvious possibility, i.e. a change in temperature, requires careful treatment. This is because a temperature change has two side effects in addition to a change in propagation speed. Since hot gas is less dense, the resulting positive buoyancy causes the envelope to expand vertically in regions of higher temperature. Secondly, the resulting horizontal imbalance also sets up a circulation flow. Both have effects on acoustic wave propagation.

We have analysed here the effects of a temperature increase in a model in which horizontal imbalance is compensated geostrophically by a circulation (i.e. by Coriolis forces acting on a horizontal flow, as in high- and low-pressure systems in the Earth's atmosphere). We find that if temperatures are increased, but vertical expansion is ignored, the changes in travel time as seen in time-distance measurements can be of either sign, depending on the wavenumber and the inter-skip distance. This is because the increased propagation speed is offset in part by the fact that the upper turning point of the waves is higher in a hotter model.

This effect is enhanced if the vertical expansion of the perturbed model due to vertical pressure balance is taken into account. We have calculated this in a model in which the entropy has been increased locally (corresponding to an increase in the polytropic constant K). The results show that the travel times are *increased* by a temperature enhancement, as long as the horizontal extent of the inhomogeneity is not too large. We found it difficult to obtain time delays for inhomogeneities that have widths of more than about 10 Mm without making unrealistic assumptions. If one takes into account the effect of turbulence onto the sound speed inside the inhomogeneity, one finds that the sign of the time delay can change. This implies that predictions become sensitive to parametrizations of poorly known turbulent flows and therefore less robust (see Sec. 2.3).

The effects of temperature enhancements have been considered before by Kuhn and Stein (1996), whose conclusions differ from ours. In a numerical convection simulation these authors calculate the effect of a temperature increase applied at a depth of 2.6 Mm on a part of the lower boundary. This increase causes both a circulation and a vertical expansion of the model above the hotter boundary region. The temperature change as a function of depth in their figure 2 mimics the *vertical gradient* of the unperturbed model. The vertical shift implied by the figure is about 10 km at the photosphere. This can be compared with a value of roughly 12 km expected from approximate vertical hydrostatic balance for an entropy change as applied by the authors. The authors then perform a ray tracing calculation similar to ours and find *reduced* travel times.

The cause of the disagreement can be traced to the treatment by Kuhn and Stein of the layers below 2.6 Mm, i.e. at depths not covered by the numerical simulation. In their ray tracing calculations, they assume higher temperatures, where $\delta T/T$ declines linearly from 0.006 at 2.6 Mm to 0.003 at a depth of 50 Mm. The vertical expansion of the model, however, is based only on the 2.6 Mm layer included in the simulation. The expansion is approximately proportional to the depth over which the increased temperatures extend (see Eqn. 12). On the one hand, the convection simulation by Kuhn and Stein demonstrates the vertical expansion effect, but on the other hand their ray tracing calculation underestimates its effect on travel times by a large factor.

Kuhn and Stein do not specify the cause of their assumed temperature enhancement at a depth of 50 Mm but they suggest a source in those layers where the magnetic field of the solar cycle is produced, near the base of the convection zone ($z \sim 200$ Mm) (see also Kuhn, Libbrecht and Dicke 1988). If this were the case, for a given temperature increase at the surface, the vertical expansion effect would be four times larger than for an assumed depth of 50 Mm. The travel time increase by the vertical expansion would then certainly dominate over the reduction due to the higher sound speed for all time-distance measurements published so far (which reach depths of the order of 20 Mm).

3.1. Lower temperatures in active regions?

The shorter sound travel times in active regions found by time-distance seismology are consistent with the increase of mode frequencies with magnetic activity found by Woodard et al. (1993) and Dziembowski et al. (2000). As noted by Goldreich et al. (1991), the increased mode frequencies are not consistent with increased temperatures, for the same reason as in our time-distance calculations. Instead, Goldreich et al. suggest that the magnetic field of active regions causes the increase in propagation speeds. For the vertical magnetic fields seen near the surface there would be no associated vertical expansion of the envelope. While this explanation is consistent with the data available then, it is no longer compatible with recent data. This is because a magnetic change in propagation speed is confined to a thin layer near the surface (unless very large magnetic fields are assumed at a depth of 10–20 Mm). The mode frequency changes measured by Woodard et al. and very accurately by Dziembowski et al. (2000) (with MDI) show that while most of the effect is concentrated near the surface, there are also significant changes at depths of 10 Mm. This would require field strengths of the order 20 kG covering large fractions of the surface.

Instead of getting involved in a discussion about the difficulties that such large unobserved magnetic fields would cause, we suggest here a more radical and simple solution. One could infer that the subsurface temperatures in active regions, in spite of the increased emission at the surface, are in fact *reduced*.

If the effect of the small scale magnetic fields were an increase in convective efficiency, or some other effect that increases the radiation losses at the surface, then the increased cooling would cause the intergranular downdrafts to be cooler than average (since by this assumption the granules would have lost more heat). These lower temperatures would be carried down with the downdrafts, and cause horizontal average temperatures to be reduced below active regions. The depth dependence of the effect would depend on the details of the rate of spreading of the downflows by entrainment.

An effect that would cause just such an increased cooling associated with active region fields was proposed by Spruit (1977). There it was shown that the 'dimples' in the photosphere caused by the reduced opacity in the magnetic elements allow more radiation to escape. The magnetic elements are effectively small leaks through which more heat escapes than from the normal photosphere. This increased radiation at the same time implies a larger average cooling rate in active regions. The downflows inferred from time-distance helioseismology (Duvall et al. 1998) are consistent with this interpretation, but are hard to understand in models with increased sub-surface temperatures.

References

S. D'Silva, T.L. Duvall, Jr.: 1995, ApJ, 438, 454.

T.L. Duvall Jr., S.M. Jefferies, J.W. Harvey, M.A. Pomerantz: 1993, Nature, 362, 430.

T.L. Duvall Jr. 1995: GONG '94: Helio- and Astero-Seismology from the Earth and Space, ed. Roger K. Ulrich, Edward J. Rhodes, Jr., Werner Däppen, San Francisco, p.465.

T.L. Duvall, et al.: 1997, Sol. Phys., 170, 63.

T.L. Duvall Jr., A.G. Kosovichev, P.H. Scherrer: 1998, Sounding solar and stellar interiors, Proceedings of the 181st symposium of the International Astronomical Union, edited by Janine Provost, Francois-Xavier Schmider, Dordrecht, p. 83.

W.A. Dziembowski, P.R. Goode, A.G. Kosovichev, J. Schou: 2000, preprint.

P. Goldreich, N. Murray, G. Willette: 1991, ApJ, 370, 752.

D.O. Gough: 1993, in *Astrophysical Fluid Dynamics*, edited by J.-P. Zahn & J. Zinn-Justin, North-Holland, Amsterdam, p.399-560.

J.R. Kuhn and R.F. Stein: 1996, ApJ, 463, L117.

J.R. Kuhn, K.G. Libbrecht, R.H. Dicke: 1988, Science, 242, 908.

C.S. Rosenthal, J. Christensen-Dalsgaard, Å. Nordlund, R.F. Stein, R. Trampedach: 1999, A&A, 351, 689.

H.C. Spruit: 1977, Sol. Phys., 55, 3.

M.F. Woodard, K.G. Libbrecht: 1993, ApJ, 402, L77.

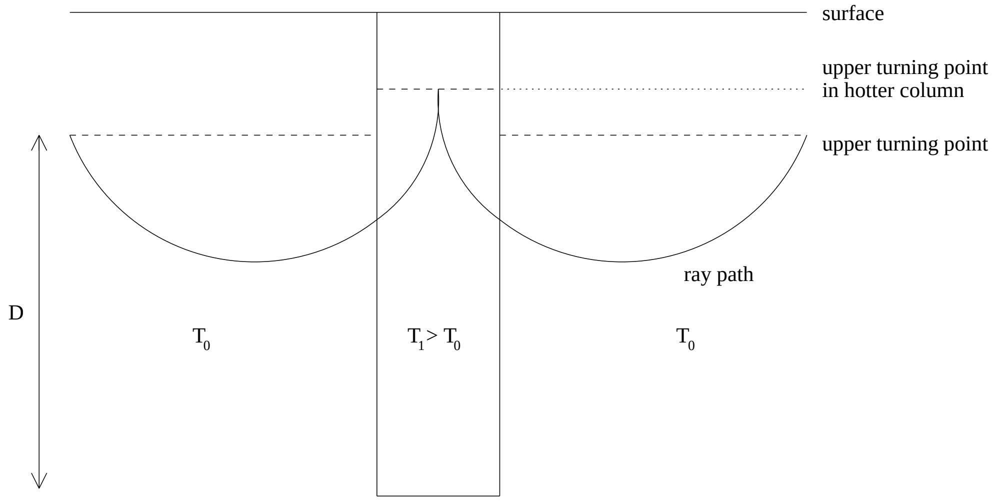

Figure 1. Sketch of a ray which traverses and is reflected in a hot region.

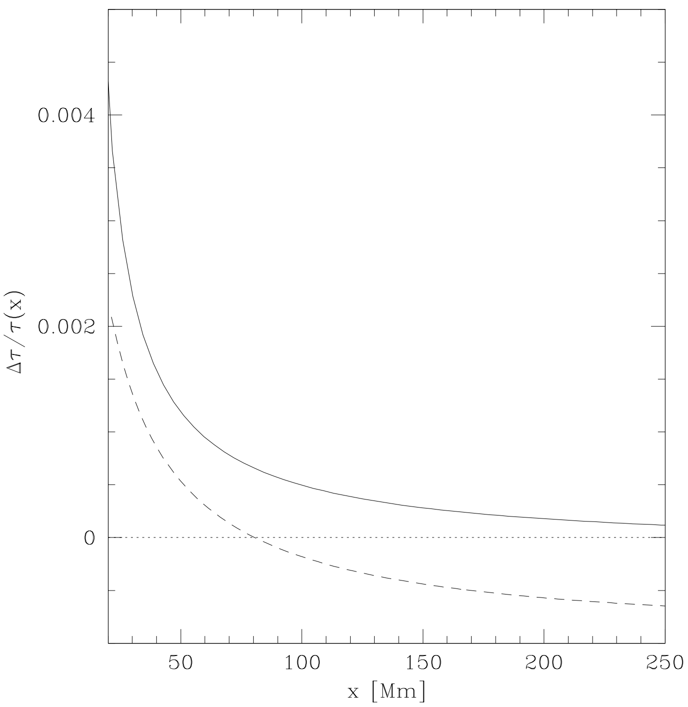

Figure 2. Relative travel-time differences between the homogeneous and an inhomogeneous Sun as a function of interskip distance. The dashed line shows the result from our model 1 with polytropic indices of $n_0 = 3$ and $n = 2.7$ inside the inhomogeneity which has got a presumed width of $w = 3.5$ Mm. The solid line shows the results from our model 2, in which the entropy has been increased down to a width of $D = 20$ Mm. ($\delta K/K_0 = 6 \cdot 10^{-3}$, $w = 7$ Mm)

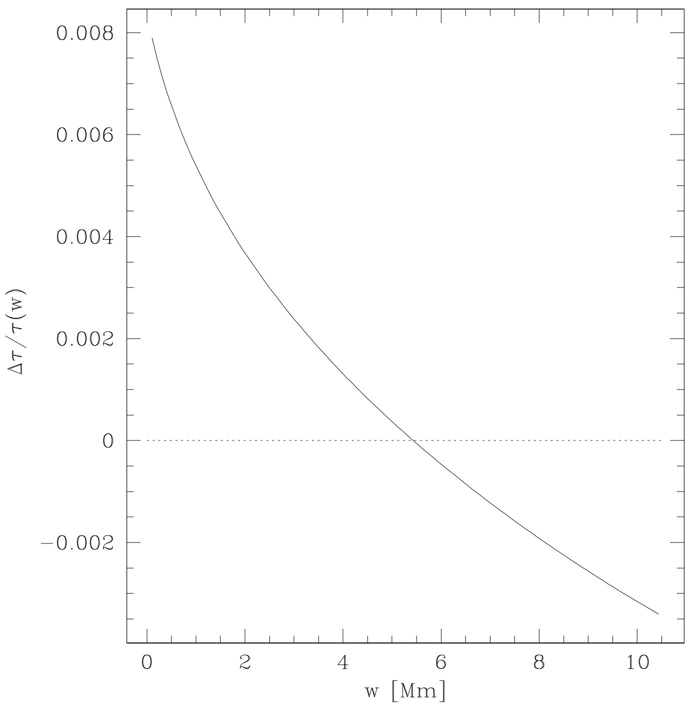

Figure 3. Relative travel-time difference between the homogeneous and an inhomogeneous Sun for our model 1 as a function of the width of the hot column for a ray with an interskip distance of 7 Mm.

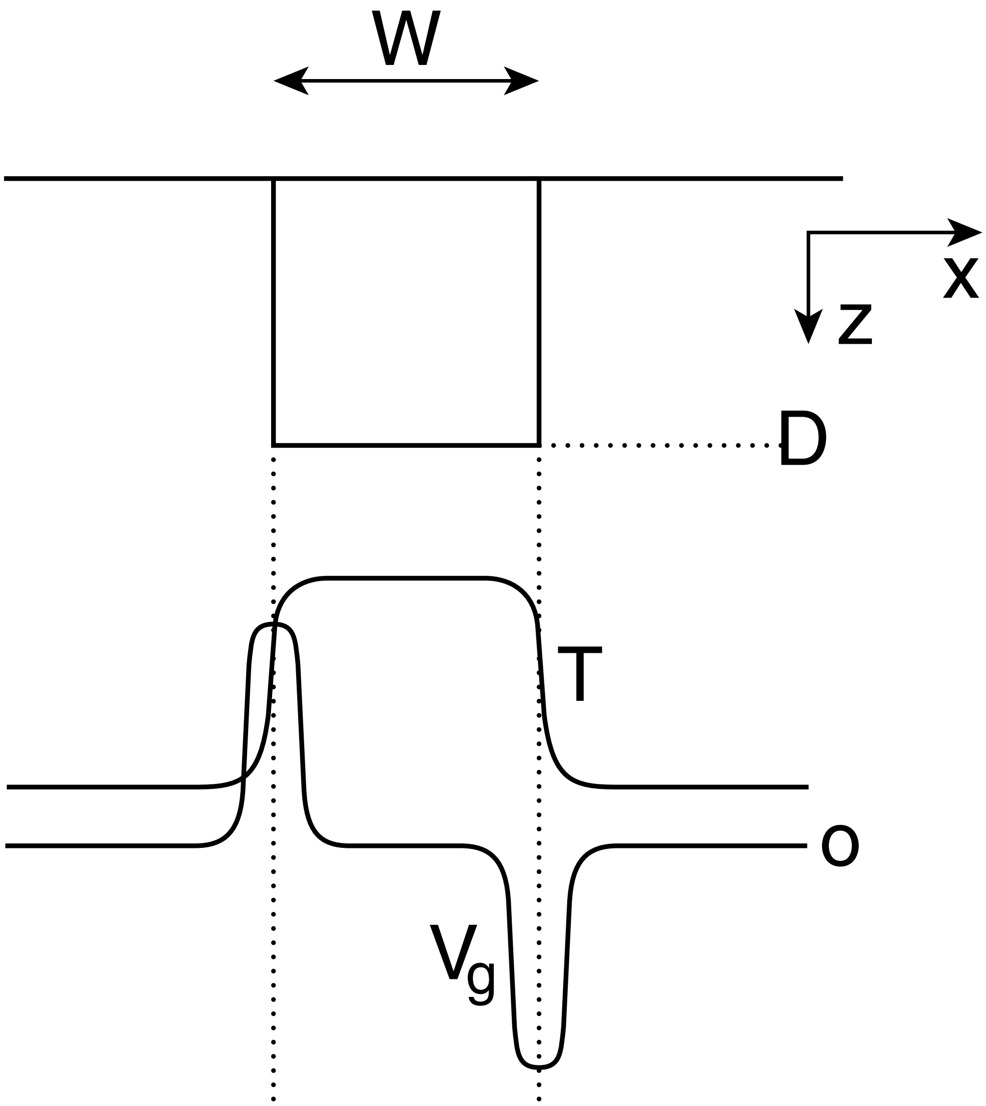

Figure 4. Schematic picture showing a model of geostrophic equilibrium of a column of enhanced temperature. The solid line indicates the azimuthal flow speed and the dashed line the temperature distribution.

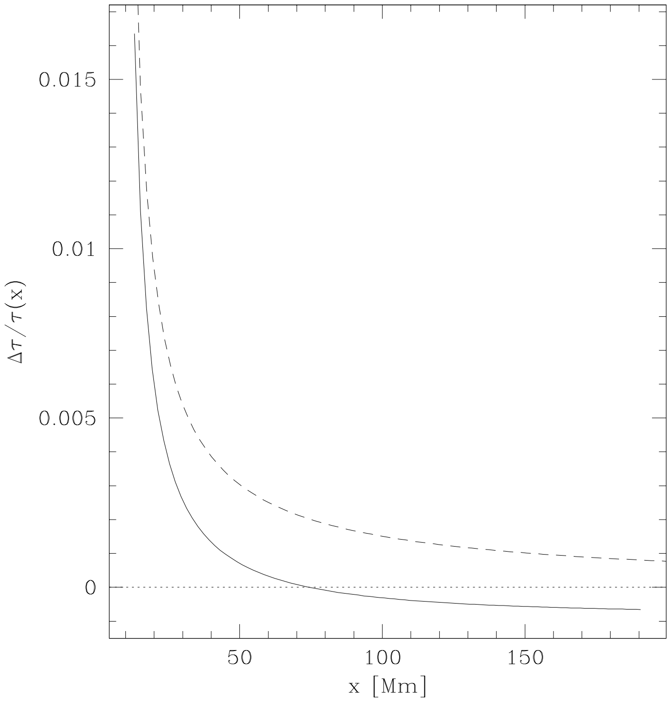

Figure 5. Relative travel-time difference for model 2 ($\delta K/K_0 = 6 \cdot 10^{-3}$, $w = 7$ Mm, $D = 40$ Mm) but with $\alpha = 1.01$ (see text). The dashed line shows the result with $\alpha = 1.00$.