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Weak Lensing from Strong Clustering

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ABSTRACT
We investigate the effect of weak gravitational lensing in the limit of small angular scales where projected galaxy clustering is strongly nonlinear. This is the regime likely to be probed by future weak lensing surveys. We use well-motivated hierarchical scaling arguments and the plane-parallel approximation to study multi-point statistical properties of the convergence field. These statistics can be used to compute the vertex amplitudes in tree models of hierarchical clustering; these can be compared with similar measurements from galaxy surveys, leading to a powerful probe of galaxy bias.

Key words: Cosmology: theory – gravitational lensing – large-scale structure of the Universe – Methods: analytical

1 INTRODUCTION

Weak gravitational lensing by large-scale density perturbations is likely to be one of the most useful tools for studying the statistical properties of the distribution of gravitating matter and the geometry of the background universe (Blandford et al. 1991; Miralda-Escudé 1991; Kaiser 1992). Studies of lensing in the weak limit usually revolve around the statistics of the *convergence* field κ, which is related to the line-of-sight fluctuations in gravitational potential and thus to the distribution of gravitating mass; for recent technical reviews see Kaiser (1996), Bernardeau (1999) and Mellier (1999).

Various methods, both analytical and numerical, have been applied to understand weak lensing. Bernardeau et al. (1997) have applied gravitational perturbation theory to compute statistical properties of κ. Numerical computations have mainly focussed on ray-tracing experiments using N-body simulations. Examples of this approach include: Jain & Seljak (1997); Jain, Seljak & White (1998, 1999a, 1999b); Premade, Martel & Matzner (1998); van Waerbeke, Bernardeau & Mellier (1998); Bartelmann et al. (1998); and Couchman, Barber & Thomas (1998). This method has the advantage that it can treat the small angular scales where perturbative calculations are no longer applicable. It is, however, difficult to probe a large parameter space of possible models using numerical experiments and it is consequently important to investigate analytic methods for studying the regime where non-linear gravitational fluctuations influence the lensing of light rays.

In this paper we use a hierarchical model, or *ansatz*, for the form of the n-point correlation functions of matter in the non-linear regime. Such models have a long history (Groth & Peebles 1977; Davis & Peebles 1977; Fry & Peebles 1978; Bernardeau & Schaeffer 1992; Szapudi & Szalay 1993) but have recently been shown to generate simple and accurate predictions for count-in-cell statistics (Munshi, Coles & Melott 1999a,b), the mass function of collapsed objects (Valageas & Schaeffer 1999) and the bias and higher-order correlations of over-dense cells (Bernardeau & Schaeffer 1992; Munshi et al. 1999a; Bernardeau & Schaeffer 1999; Coles et al. 1999). Numerical simulations have also demonstrated that the hierarchical ansatz is very accurate in predicting clustering properties (Munshi et al. 1999b). It is natural to use this model to study the statistical properties of the convergence field involved in weak lensing calculations.

2 MATTER CORRELATIONS AND CONVERGENCE STATISTICS

The statistics of κ are very similar to those of the projected density field. The formalism for handling higher-order correlation functions of projected galaxy clustering was developed by Tóth, Hollósi & Szalay (1989) and later applied by, e.g., Gaztañaga (1994) and Frieman & Gaztañaga (1994). The relevance and usefulness of these methods to studies of the convergence has been pointed out by Gaztañaga & Bernardeau (1998) and Hui (1999). The results we present here are new because they concern a different kind of statistical descriptor, the cumulant correlator. We begin this paper, therefore, by developing the basic relationship between cumulants (and cumulant correlators) of the convergence field κ with the underlying statistics of the matter distribution.

We adopt the following line element for background geometry:

$$d\tau^2 = -c^2dt^2 + a^2(t)[d\chi^2 + r^2(\chi)d^2\Omega], \tag{1}$$

where $a(t)$ is the expansion factor. The angular diameter distance $r(\chi) = K^{-1/2}\sin[K^{-1/2}\chi]$ for positive spatial curvature, $r(\chi) = (-K)^{-1/2}\sinh[(-K)^{-1/2}\chi]$ for negative curvature, and $r(\chi) = \chi$ for a flat universe. In terms of H_0 and Ω_0, $K = (\Omega_0 - 1)H_0^2$. In the following we consider a small patch of the sky so we can use the plane-parallel (or small-angle) approximation to replace spherical harmonics by Fourier modes. The convergence κ at a direction γ is given by a projection of the (3D) density contrast along the line of sight with a weight function $\omega(\chi)$:

$$\kappa(\gamma) = \int_0^{\chi_s} d\chi\omega(\chi)\delta(r(\chi),\gamma). \tag{2}$$

If all the sources are at the same redshift, one can write the weight function $\omega(\chi) = 3/4ac^{-2}H_0^2\Omega_m r(\chi)r(\chi_s - \chi)/r(\chi_s)$, where χ_s is the comoving radial distance to the source. (This approximation is not crucial and is easy to modify for a more accurate description.) Using a Fourier decomposition of δ we can write

$$\kappa(\gamma) = \int_0^{\chi_s} d\chi\omega(\chi)\int \frac{d^3\mathbf{k}}{(2\pi)^3}\exp(i\chi k_\parallel + r\theta k_\perp)\delta_k, \tag{3}$$

where we have used $k_\parallel$ and $k_\perp$ to denote the components of wave vector k parallel and perpendicular to the line of sight. In small angle approximation however one assumes that $k_\perp$ is much larger compared to $k_\parallel$, θ denotes the angle between the line of sight direction γ and the wave vector $\mathbf{k}$.

2.1 Cumulants

Using the definitions we have introduced above we can compute the projected two-point correlation function

$$\langle\kappa(\gamma_1)\kappa(\gamma_2)\rangle = \int_0^{\chi_s} d\chi_1 \frac{\omega^2(\chi_1)}{r^2(\chi_1)}\int \frac{d^2\mathbf{l}}{(2\pi)^2}\ \exp(i\theta l)\ \mathrm{P}\left(\frac{\mathbf{l}}{r(\chi)}\right) \tag{4}$$

(Peebles 1980; Kaiser 1992; Kaiser 1998) where we defined $\mathbf{l} = r(\chi)\mathbf{k}_\perp$, a scaled wave-vector projected on the surface of the sky. It is useful to consider the average of this smoothed over an angle θ_0, which is often used to reconstruct the matter power spectrum $P(k)$ (Jain, Selzak & White 1998). The convergence field smoothed by an angle θ_0 is represented by

$$\kappa_s(\gamma) = \int d^2\gamma W_{\theta_0}(\gamma)\int_0^{\chi} d\chi\omega(\chi)\delta(r(\chi),\gamma), \tag{5}$$

where W_{θ_0} is the "top hat" window function. We then get

$$\langle\kappa_s^2\rangle = \int_0^{\chi_s} d\chi\frac{\omega^2(\chi)}{r^2(\chi)}\int\frac{d^2\mathbf{l}}{(2\pi)^2}\ P\left(\frac{l}{r(\chi)}\right)W_2^2(l\theta_0), \tag{6}$$

where $W_2(x) = 2J_1(x)/x$ and $J_1(x)$ is a Bessel function; cf. Bernardeau (1995). Similar calculations for the three- and four-point correlation functions can be expressed in terms of integrals of the matter multi-spectrum B_p, defined by

$$\langle\delta(\mathbf{k}_1)\dots\delta(\mathbf{k}_p)\rangle = B_p(\mathbf{k}_1,\dots\mathbf{k}_p), \tag{7}$$

where the $\mathbf{k}_i$ are constrained to have $\sum\mathbf{k}_i = 0$. First define

$$I_N[r(\chi),\theta_0)] = \int\frac{d^2\mathbf{l_1}}{(2\pi)^2}W_2(l_1\theta_0)\dots\int\frac{d^2\mathbf{l_{N-1}}}{(2\pi)^2}W_2(l_{N-1}\theta_0)W_2(l_N\theta_0)B_N\left(\frac{l_1}{r(\chi)},\dots,\frac{l_N}{r(\chi)}\right)_{\sum\mathbf{l}_i=0}. \tag{8}$$

¿From this we obtain

$$\langle\kappa_s^3\rangle = \int_0^{\chi_s} d\chi_1\frac{\omega^3(\chi)}{r^6(\chi)}I_3[r(\chi),\theta_0] \tag{9}$$

and

$$\langle\kappa_s^4\rangle = \int_0^{\chi_s} d\chi_1\frac{\omega^4(\chi)}{r^8(\chi)}I_4[r(\chi),\theta_0] \tag{10}$$

For the derivation of the corresponding hierarchical parameters,

$$S_N = \frac{\langle\kappa_s^N\rangle}{\langle\kappa_s^{N-1}\rangle} \tag{11}$$

we need

$$\langle\kappa_s^N\rangle = \int_0^{\chi_s} d\chi\frac{\omega^N(\chi)}{r^{2(N-1)}(\chi)}I_N[r(\chi),\theta_0]. \tag{12}$$

In these results we have assumed that the smoothing angle θ_0 is very small.

2.2 Cumulant Correlators

Cumulant correlators were introduced by Szapudi & Szalay (1997) as a normalized two-point moments of the multi-point correlation functions. Later the concept of two-point cumulant correlators was generalized to multi-point cumulant correlators (Munshi, Melott & Coles 1999). The two-point cumulant correlators for galaxies have already been measured in projected surveys such as APM by Szapudi & Szalay (1997) and they have been shown to be an important diagnostic of hierarchical galaxy clustering models.

To calculate the two-point cumulant correlators we adopt the additional assumption that the separation between different angles are small. This is the regime in which we expect the hierarchical model presented in the next section to be most accurate. The general result is:

$$\langle \kappa_s^m(\gamma_1)\kappa_s^n(\gamma_2)\rangle = \int_0^{\chi_s} \frac{\omega^{n+m}(\chi)}{r^{2(n+m-1)}(\chi)}d\chi \int \frac{d^2\mathbf{l}_1}{(2\pi)^2}\dots\int \frac{d^2\mathbf{l}_{n+m-1}}{(2\pi)^2} W_2(l_1\theta_0)\dots W_2(l_{n+m}\theta_0)\exp[i(l_1+\dots+l_m)\theta] B_{m+n}\left(\frac{\mathbf{l}_1}{r(\chi)},\dots,\frac{\mathbf{l}_{m+n}}{r(\chi)}\right), \tag{13}$$

where again $\sum l_i = 0$, as it is in the following expressions. Relevant low-order examples of this result are:

$$\langle \kappa_s^2(\gamma_1)\kappa_s(\gamma_2)\rangle = \int_0^{\chi_s} \frac{\omega^3(\chi)}{r^4(\chi)}d\chi \int \frac{d^2\mathbf{l}_1}{(2\pi)^2}\int \frac{d^2\mathbf{l}_2}{(2\pi)^2} W_2(l_1\theta_0)W_2(l_2\theta_0)W_2(l_3\theta_0)\exp(il_2\theta_{12})B_3\left(\frac{\mathbf{l}_1}{r(\chi)},\frac{\mathbf{l}_2}{r(\chi)},\frac{\mathbf{l}_3}{r(\chi)}\right); \tag{14}$$

$$\langle \kappa_s^3(\gamma_1)\kappa_s(\gamma_2)\rangle = \int_0^{\chi_s} \frac{\omega^3(\chi)}{r^4(\chi)}d\chi \int \frac{d^2\mathbf{l}_1}{(2\pi)^2}\int \frac{d^2\mathbf{l}_2}{(2\pi)^2}\int \frac{d^2\mathbf{l}_3}{(2\pi)^2} W_2(l_1\theta_0)W_2(l_2\theta_0)W_2(l_3\theta_0)\exp(il_3\theta_{12}) B_4\left(\frac{\mathbf{l}_1}{r(\chi)},\frac{\mathbf{l}_2}{r(\chi)},\frac{\mathbf{l}_3}{r(\chi)},\frac{\mathbf{l}_4}{r(\chi)}\right); \tag{15}$$

$$\langle \kappa_s^2(\gamma_1)\kappa_s^2(\gamma_2)\rangle = \int_0^{\chi_s} \frac{\omega^3(\chi)}{r^4(\chi)}d\chi \int \frac{d^2\mathbf{l}_1}{(2\pi)^2}\int \frac{d^2\mathbf{l}_2}{(2\pi)^2}\int \frac{d^2\mathbf{l}_3}{(2\pi)^2} W(l_1\theta_0)W_2(l_2\theta_0)W_2(l_3\theta_0)\exp(i(l_1+l_2)\theta_{12}) B_4\left(\frac{\mathbf{l}_1}{r(\chi)},\frac{\mathbf{l}_2}{r(\chi)},\frac{\mathbf{l}_3}{r(\chi)},\frac{\mathbf{l}_4}{r(\chi)}\right). \tag{16}$$

The three-point cumulant correlators can be expressed in the following form

$$\langle \kappa_s^m(\gamma_1)\kappa_s^n(\gamma_2)\kappa_s^r(\gamma_3)\rangle = \int_0^{\chi_s} \frac{\omega^{n+m+r}(\chi)}{r^{2(n+m+r-1)}(\chi)}d\chi \int \frac{d^2\mathbf{l}_1}{(2\pi)^2}\dots\int \frac{d^2\mathbf{l}_{n+m+r-1}}{(2\pi)^2} W_2(l_1\theta_0)\dots W_2(l_{n+m+r}\theta_0) \exp[i(l_1+\dots+l_m)\theta_{12}+i(l_m+\dots+l_{m+n})\theta_{13}]B_{m+n+r}\left(\frac{\mathbf{l}_1}{r(\chi)},\dots,\frac{\mathbf{l}_{m+n+r}}{r(\chi)}\right). \tag{17}$$

3 TREE MODEL FOR CORRELATION FUNCTIONS

In deriving the expressions (9)-(17) we have not used any specific form for the matter correlation hierarchy. To go further we have to choose a particular form of hierarchical scaling. The most general "tree" model can be written as

$$\xi_N(\mathbf{r_1},\dots\mathbf{r_N}) = \sum_{\alpha,\mathrm{N-trees}} Q_{N,\alpha}\sum_{\mathrm{labellings}}\prod_{\mathrm{edges}}^{(N-1)}\xi(\mathbf{r_i},\mathbf{r_j}). \tag{18}$$

A similar hierarchy is developed in the quasi-linear regime in the limit of vanishing variance (Bernardeau 1992), but the hierarchical amplitudes $Q_{N,\alpha}$ are shape-dependent functions in such a case. In the highly nonlinear regime of interest here, there are some indications that these functions become independent of shape (e.g. Scocciamarro et al. 1998). In Fourier space, such a result means that the multi-spectra can be written

$$\begin{aligned}
B_2(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) &= Q[P(\mathbf{k_1})P(\mathbf{k_2})+P(\mathbf{k_2})P(\mathbf{k_3})+P(\mathbf{k_3})P(\mathbf{k_1})]\\
B_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) &= R_a[P(\mathbf{k_1})P(\mathbf{k_1}+\mathbf{k_2})P(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})+\text{cyc.perm.}]\\
&\quad +R_b[P(\mathbf{k_1})P(\mathbf{k_2})P(\mathbf{k_3})+\text{cyc.perm.}]\\
B_4(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4,\mathbf{k}_5) &= S_a[P(\mathbf{k_1})P(\mathbf{k_1}+\mathbf{k_2})P(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})P(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}+\mathbf{k_4})+\text{cyc.perm.}]\\
&\quad +S_b[P(\mathbf{k_1})P(\mathbf{k_2})P(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})P(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}+\mathbf{k_4})+\text{cyc.perm.}]\\
&\quad +S_c[P(\mathbf{k_1})P(\mathbf{k_2})P(\mathbf{k_3})P(\mathbf{k_4})+\text{cyc.perm.}]
\end{aligned} \tag{19}$$

(remembering that $\sum \mathbf{k_i} = 0$ in all these expressions.)

Particular hierarchical models differ in the way they fix the various amplitudes for the different topologies (Q, R_a, R_b, etc). For example, Bernardeau & Schaeffer (1992) considered the case where amplitudes can in general are factorizable (see

Munshi, Melott & Coles for a detailed description); see Szapudi & Szalay (1993) for an alternative prescription. We will avoid any specific prescription because our aim is to show that the statistics of κ can allow one to fix the hierarchical amplitudes empirically, as well as constraining the power spectrum and background geometry. We shall ignore smoothing corrections, for reasons discussed by Boschan, Szapudi & Szalay (1994). We first define

$$\begin{aligned}
\kappa_{\theta_0} &\equiv \int \frac{d^2\mathbf{l}}{(2\pi)^2} P\left(\frac{l}{r(\chi)}\right) W_2^2(l\theta_0), \\
\kappa_{\theta_{12}} &\equiv \int \frac{d^2\mathbf{l}}{(2\pi)^2} P\left(\frac{l}{r(\chi)}\right) W_2^2(l\theta_0)\exp(l\theta_{12}) \\
C_t[F(\chi)] &\equiv \int_0^{\chi_s} \frac{\omega^t(\chi)}{r^{2(t-1)}(\chi)} F(\chi) d\chi.
\end{aligned} \tag{20}$$

The expression for C_t can be computed numerically for specific models of background cosmology. $F(\chi)$ denotes the various products of κ_{θ_0} and $\kappa_{\theta_{12}}$ which appear in the following expressions containing χ dependence. The values of hierarchical amplitudes (which in general are insensitive to the background cosmology) can be computed from numerical simulations or with analytic methods such as hyper-extended perturbation theory (Scoccimarro et al. 1998; Scoccimarro & Frieman 1998). The one-point cumulants can then be expressed as

$$\begin{aligned}
\langle \kappa^3(\gamma) \rangle &= 3Q_3 C_3[\kappa_{\theta_0}^2] \\
\langle \kappa^4(\gamma) \rangle &= (12R_a + 4R_b) C_4[\kappa_{\theta_0}^3] \\
\langle \kappa^5(\gamma) \rangle &= (60S_a + 60S_b + 5S_c) C_5[\kappa_{\theta_0}^4]
\end{aligned} \tag{21}$$

The first of these expressions was derived by Hui (1999). He showed that this result agrees well with numerical ray-tracing experiments by Jain, Seljak & White (1998). The expressions for two-point cumulant correlators are

$$\begin{aligned}
\langle \kappa_s^2(\gamma_1)\kappa_s(\gamma_2) \rangle &= 2Q_3 C_3[\kappa_{\theta_0}\kappa_{\theta_{12}}] \\
\langle \kappa_s^3(\gamma_1)\kappa_s(\gamma_2) \rangle &= (3R_a + 6R_b) C_4[\kappa_{\theta_0}^2\kappa_{\theta_{12}}] \\
\langle \kappa_s^2(\gamma_1)\kappa_s^2(\gamma_2) \rangle &= 4R_b C_4[\kappa_{\theta_0}^2\kappa_{\theta_{12}}] \\
\langle \kappa_s^4(\gamma_1)\kappa_s(\gamma_2) \rangle &= (24S_a + 36S_b + 4S_c) C_5[\kappa_{\theta_0}^3\kappa_{\theta_{12}}] \\
\langle \kappa_s^3(\gamma_1)\kappa_s^2(\gamma_2) \rangle &= (12S_a + 6S_b) C_5[\kappa_{\theta_0}^3\kappa_{\theta_{12}}].
\end{aligned} \tag{22}$$

and the three-point cumulant correlators are

$$\begin{aligned}
\langle \kappa_s^2(\gamma_1)\kappa_s(\gamma_2)\kappa_s(\gamma_3) \rangle &= C_5[(4S_a\kappa_{\theta_{12}}\kappa_{\theta_{13}} + 4S_b\kappa_{\theta_{12}}\kappa_{\theta_{13}} + 4S_b\kappa_{\theta_{13}}\kappa_{\theta_{23}})\kappa_{\theta_0}^2] \\
\langle \kappa_s^3(\gamma_1)\kappa_s(\gamma_2)\kappa_s(\gamma_3) \rangle &= C_5[[(6S_a + 3S_b)(\kappa_{\theta_{12}}\kappa_{\theta_{23}} + \kappa_{\theta_{13}}\kappa_{\theta_{23}}) + (3S_c + 18S_b + 6S_a)\kappa_{\theta_{12}}\kappa_{\theta_{13}}]\kappa_{\theta_0}^2].
\end{aligned} \tag{23}$$

It is clear from these equations that at any order, measurements of the appropriate cumulant correlators provide enough information to determine the hierarchical coefficients of that order and below. Weak lensing statistics can therefore tell us directly about the form of hierarchical model appropriate for the underlying density field, just as it can with the distribution of galaxies (Munshi, Melott & Coles 1999).

4 DISCUSSION

Ongoing optical and radio experiments are intended to provide imaging data for studies of weak lensing. A large smoothing angle is required to probe the quasi-linear regime where most existing analytic calculations are valid. To be useful, such regions must be at least reach 10 degrees on a side. Existing CCD cameras have typical sizes $(0.25^\circ - 0.5^\circ)$. The initial lensing surveys will therefore provide information about matter fluctuations in the highly non-linear regime. This is the regime in which the model we have presented here is expected to be valid. Ongoing observational programmes in this regime allow the direct determination of the hierarchical scaling properties of cosmological density fluctuations. These can be combined with similar measurements of galaxy clustering to constrain models of the relationship between galaxies and mass. This is the key result of this paper.

Other studies of small-scale weak lensing have focussed on ray-tracing experiments through N-body simulations. It will definitely be very interesting to check our analytical results against such numerical simulations. This will also test different approximations we have used in deriving our analytical results. Results of this kind of analysis will be presented elsewhere (Munshi, Jain & White 1999a,b, in preparation).

Cumulant correlators of orders higher than those we have presented here can be written in a similar way to equations (21)-(23). We have not presented them here, however, because sampling variance plays an increasingly important role in these quantities at increasing order, and we have not yet developed a method for dealing with this in the context of weak lensing, although estimation of finite-volume corrections has been done for galaxies by several authors (Szapudi & Colombi 1997; Munshi et al. 1999; Munshi, Coles & Melott 1999b; Hui & Gaztanaga 1998). We also note that the hierarchical ansatz we have exploited furnishes a complete prescription of all the higher-order correlation functions of the matter distribution. This allows one to estimate the multi-point count probability distribution function for an arbitrary number of correlated volume elements

(Munshi, Coles & Melott 1999a,b; Bernardeau & Schaeffer 1999). These results can be used to compute the numbers of "hot" and "cold" spots in statistics of κ in the sky in much the same way as has been done for fluctuations in the cosmic microwave background (e.g. Coles & Barrow 1987). The development of our work in these directions will be presented elsewhere (Munshi & Coles, 1999, in preparion).

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