A Limit on Primordial Small-Scale Magnetic Fields from CMB Distortions

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Spatially varying primordial magnetic fields may be efficiently dissipated prior to the epoch of recombination due to the large viscosity of the baryon-photon fluid. We show that this dissipation may result in observable chemical potential μ and Compton y distortions in the cosmic microwave background radiation (CMB) spectrum. Current upper limits on μ and y from FIRAS constrain magnetic fields to have strength $B_0 < 3 \times 10^{-8}$Gauss (scaled to the present) between comoving coherence length ≈ 400 pc and ≈ 0.6 Mpc. This represents the strongest upper limit on small-scale primordial magnetic fields to date.

I. INTRODUCTION

There is a plethora of scenarios for the generation of magnetic fields in the early universe [1,2,4,3]. Such scenarios rely on some mechanism for separation of charges, which, in combination with differential rotational flows, may result in electric currents and seed magnetic fields. It is often argued that these seed magnetic fields may be significantly amplified after their formation by magneto hydrodynamic effects in a turbulent primordial plasma. Although the details of such amplification remain to be understood, a natural outcome of turbulent amplification is expected to be an equipartition between magnetic field energy and peculiar flow energy, possibly amounting to magnetic field energy densities as large as a significant fraction of the total ambient radiation energy density. Since these scenarios operate by causal mechanism, coherence lengths of the resulting magnetic fields are usually limited to the horizon scale at the epoch of magnetic field generation. The coherence length of magnetic fields may be much larger for generation mechanism before, or during, an inflationary epoch, or when magnetic field energy density is transferred to larger scales by inverse cascades [5] subsequent to the epoch magnetogenesis.

At present not much is known about the existence or absence of magnetic fields in the early universe. Limits have been placed on the contribution of magnetic field energy density to the total energy density during the epoch of cosmological weak freeze-out ($T_W \approx 1$MeV) [6]. These limits are obtained from an increased ^{4}He production during Big Bang nucleosynthesis (BBN) beyond the observationally allowed abundances and give approximately $B_0 \lesssim 3 \times 10^{-6}$Gauss (here, and henceforth, we quote magnetic field strengths at the values they would have at the present epoch, i.e. after cosmological expansion). Nevertheless, even when magnetic field energy density is in approximate equipartition with radiation energy density well before BBN, dissipation of these fields by neutrino-induced viscosity is likely to significanlty diminish magnetic energy before BBN [7], rendering the above limit inpractical in most cases.

On larger scales, limits can be set by the cosmic microwave background (CMB) and by Faraday rotation measures of light from high redshift quasars. A homogeneous cosmic magnetic field on the scale of the present horizon is limited by its effect on the CMB due to an induced anisotropic cosmic expansion to fall below $B_0 \lesssim 3 \times 10^{-9}$Gauss [8]. For smaller scale coherence fields, Faraday rotation measures limit a present day field to be $B_{50\rm Mpc} \lesssim 6 \times 10^{-9}$ Gauss on 50 Mpc scales, $B_{Mpc} \lesssim 10^{-8}$ Gauss on Mpc scales [9]. Recently, a spectrum-dependent limit on primordial magnetic fields, typically resulting in $B_0 \lesssim 10^{-7}$ Gauss on ~ 0.1 Mpc scales for steep power spectra, has been proposed due to a possible generation of gravitational waves and concomitant CMB distortions [10].

Future high-precision CMB satellite observations should be able to detect (or limit) large-scale magnetic fields. Observational proposals include the detection of rotational (vector) perturbations below [11] and above [12] the Silk damping scale, a depolarization of the cosmic microwave background by Faraday rotation [13], as well as, distortions of the acoustic peaks due to a changed (magneto-) sonic propagation velocity at the epoch of recombination [14]. These effects may be observable for magnetic field strengths above $B_0 \gtrsim 3 \times 10^{-9} - 10^{-7}$Gauss given anticipated sensitivities of the sattelite missions, and under the assumption that magnetic field coherence lengths are not much smaller than the comoving Silk damping scale (~ 10Mpc).

In this paper, we impose a limit on pre-recombination era magnetic fields on small scales. In the following we elucidate the physical origin of the limit and apply the resulting constraint on a stochastic magnetic field characterized by power spectral index, minimum coherence length, and normalization amplitude.

II. CMB DISTORTIONS FROM MAGNETIC FIELD DISSIPATION

We have recently shown that spatially varying, stochastic, magnetic fields undergo significant dissipation before recombination beyond that of magnetic diffusion [7]. Imagine initial conditions of a randomly magnetized baryon-photon fluid without peculilar flows and at uniform density. The magnetic field accelerates the fluid by magnetic stresses which results in the conversion of magnetic field energy into kinetic energy of the fluid. For all length scales λ with magnetic field relaxation time $\tau_r \sim \lambda/v_A$, where v_A is the Alfvén velocity, shorter than the Hubble time, $\tau_r < \tau_H$, the above initial conditions result in a physical state of the fluid which may be envisioned as a superposition of Alfvén waves and slow magnetosonic waves with approximate equipartition between magnetic and kinetic energy When the fluid is non-ideal, as is the case before recombination due to the long photon mean free path, shear viscosity η will affect a dissipation of kinetic energy into heat. The dissipation of energy drawn from the intial magnetic field energy resumes until the magnetic field reaches a force-free state, i.e., does not exert any magnetic stresses on the fluid. For most models of magnetic field generation in th early universe, if an initially random magnetic field relaxes to a final force-free field configuration (e.g. straight field lines), the final state will only have a negligible fraction of the initial magnetic field energy. Most models of primordial magnetogenesis give much lower amplitude fields for large scales versus small scales.

The calculation of dissipation times for Alfvén, slow- and fast- magnetosonic perturbations on a uniform magnetic field background as a function of mode wavelength λ has been performed in great detail in [7]. Similar to the damping of baryon-photon sound waves by photon- shear viscosity and heat conductivity, i.e. Silk damping, damping of magneto-hydrodynamic perturbations may occur in the photon diffusion limit, when the photon mean free path $l_\gamma \ll \lambda$. Dissipation then occurs for modes smaller than the approximate photon diffusion length, $d_\gamma \sim (l_\gamma t)^{1/2}$, where t is cosmic time and with the speed of light, $c = 1$, in natural units. It should be noted that dissipation scales have been computed in [7] in the linear limit, whereas generic stochastic magnetic fields are typically in the non-linear limit. It seems, however, difficult to avoid dissipation by non-linear effects unless an increase of the magnetic field coherence length is more rapid than the increase of the dissipation scale. Note also that a subset of the full non-linear MHD equations exihibits the same damping as linear modes [16].

In general, damping of Alfvén and slow magnetosonic waves is significantly different from damping of sound and fast magnetosonic waves for Alfvén velocities below the speed of light, $v_A \ll 1$, corresponding to magnetic field strength $B_0 \ll 3\times 10^{-6}$Gauss. When the magnetic field relaxation time, $\tau_r \sim \lambda/v_A$, is longer than the dissipation time scale (by photon diffusion), $\tau_d \sim \lambda^2/l_\gamma$, a magnetic field configuration becomes overdamped. In this limit it takes modes longer than τ_r to accelerate the fluid since the fluid is strongly "dragged". This occurs for some modes still in the photon diffusion limit, i.e. $l_\gamma \ll \lambda$. Such modes may only be damped later when the photons are free-streaming on the scale, $l_\gamma \gg \lambda$, with an effective damping scale much shorter than the photon diffusion length, i.e. on scale $\sim v_A d_\gamma$.

In this letter, we are mostly concerned with damping of magnetic fields in the photon diffusion limit, since this leads to the strongest constraints on small scales. In [7] the dissipation times were computed in the WKB approximation, i.e. with mode amplitudes evolving as $\tilde{b}_k(t) = \tilde{b}_k(t_0)\exp(i\int_{t_0}^{t}\omega \mathrm{d}t)$, where ω is mode frequency and k is comoving wavevector. The expression $\tilde{b}$ denotes a "comoving" magnetic field strength, scaled to the present epoch, with $\tilde{b} = ba^2$, where b is the redshift dependent magnetic field and a is the scale factor ($a = 1$ at the present epoch). Dissipation of magnetic fields manifests itself through an imanginary part in the mode frequency. For Alfvén and slow magnetosonic waves in the photon diffusion limit mode freqeuncies are identical to lowest order in b

$$\omega_{\rm osc}^{\rm SM,A} = v_A \cos\theta\left(\frac{k}{a}\right) + \frac{3}{2}i\frac{\eta'}{(1+R)}\left(\frac{k}{a}\right)^2 . \qquad (1)$$

In this expression $3(\rho_\gamma + p_\gamma)\eta' = \eta$, where η is shear viscosity, ρ_γ and p_γ are photon energy density and pressure, respectively, and $R = 3\rho_b/(4\rho_\gamma) \approx 0$ at redshifts $z \gg 10^4$, with ρ_b baryon density. Photon shear viscosity is given by $\eta = (4/15)\rho_\gamma l_\gamma$ with the photon mean free path $l_\gamma = 1/(n_e\sigma_{Th})$ determined by Thomson scattering on electrons.

One may Fourier transform a stochastic magnetic field, $\tilde{b}(x) = (V^{1/2}/(2\pi k_N)^{3/2})\int \mathrm{d}^3k\,\tilde{b}_k\exp(ikx)$, where V and k_N are a normalization volume and wavevector, respectively. The total energy density of the field ρ_B^0 at the present epoch, and in the absence of dissipation, is then given by spatial average, $\langle ...\rangle$, which yields $\rho_B^0 = \langle\tilde{b}^2\rangle/(8\pi) = 1/(8\pi k_N^3)\int \mathrm{d}^3k\,|\tilde{b}_k|^2$. We are interested in statistically isotropic magnetic field configurations which may be described by power spectra, $|\tilde{b}_k|^2 = B_0^2(k/k_N)^n(n+3)/4\pi$ for $k < k_N$ and zero otherwise, normalized such that $\langle\tilde{b}^2\rangle = B_0^2$.

It is long known that injection of energy into the CMB in a nonthermal form between redhifts $10^6 - 10^7$ and recombination, $z \sim 10^3$, may yield observable distortions in the CMB blackbody [17]. Given current upper limits on chemical potential μ and Compton y distortions of the CMB spectrum, one finds that no more than $\sim 6 \times 10^{-5}$ of the CMB energy density may have been injected in this redshift regime in nonthermal form [18]. Equilibration of photons to a blackbody distribution has to proceed via photon-electron scattering, i.e. Compton scattering and double-Compton scattering, slow processes due to the smallness of the baryon-to-photon ratio. Here, we are mainly concerned with blackbody distortions resulting from diffusion (or mixing) of photons from different regions with different peculiar velocities. This process initially yields a spectrum which may be described by a superposition of blackbodies of different temperature, i.e. a Compton y distortions [19]. When the mixing occurs at high redshift $z \gg 4 \times 10^4$ the optical depth of photons towards Compton scattering is still large, redistribution of photons in frequency space is possible, significantly diminishing the amplitude of the final y-distortion. Nevertheless, for redshifts $z \lesssim 2 \times 10^6$ the final distribution will not contain the same photon number as a blackbody at the same temperature T, due to the inefficiency of photon number creating processes at large frequencies, of which double-Compton scattering is the most rapid one. The resultant distribution will be of the Bose-Einstein type at large frequencies with non-vanishing chemical potential μ.

It has been recently noted that the presence of pre-recombination magnetic fields affects the emission of gyroradiation by electrons into the CMB [20]. The authors erroneously argue that this emission leads to large chemical potential distortions, even for very small magnetic fields, due to the efficiency of creating "extra" photon number at the fundamental, and small, frequency of gyration, $h\nu_g/T \approx 10^{-6}(B_0/10^{-8}\mathrm{Gauss})(z/10^6)$. Nevertheless, the authors did not consider the inverse processes, i.e. absorption of gyroradiation and inverse bremstrahlung, which even for very low redshifts are more rapid than the Hubble expansion due to the low frequency they occur at [17]. In spite of the emission of gyroradiation, the spectrum at low frequency remains Plankian.

The evolution of chemical potential distortions at large frequencies may be well approximated by [21]

$$\frac{d\mu}{dt} = -\frac{\mu}{t_{DC}(z)} + 1.4\,\frac{Q_B}{\rho_\gamma}\,, \tag{2}$$

where $Q_B = d\rho_B/dt$ is the instantaneous energy dissipation rate of the magnetic field. In this expression $t_{DC} = 2.06 \times 10^{33}\mathrm{s}\,(1-Y_p/2)^{-1}(\Omega_b h^2)^{-1} z^{-9/2} \equiv t_{DC}^0 z^{-9/2}$ is a characteristic time scale for double-Compton scattering, with Y_p the primordial helium mass fraction, Ω_b the fractional contribution of baryons to the critical density, and h the Hubble constant in units of 100 km s^{-1} Mpc^{-1}. The solution to Eq. 2 at low redshifts is given by

$$\mu(z) = 1.4\int_0^{t(z\ll z_\mu)} \mathrm{d}t\,\frac{Q_B}{\rho_\gamma}\exp - \left(\frac{z}{z_\mu}\right)^{5/2}\,, \tag{3}$$

where $z_\mu = (4\tilde{t}_0/5t_{DC}^0)^{-2/5}$ denotes the characteristic redshift for "freeze-out" from double-Compton scattering. The quantity $\tilde{t}_0 = 2.39 \times 10^{19}$s determines the time-redshift relationship via $t = \tilde{t}_0/z^2$ for cosmic times well before matter-radiation equality. The characteristic redshift equals $z_\mu = 2.49 \times 10^6$ for $\Omega_b h^2 = 0.0125$, and $Y_p = 0.24$.

The energy dissipation rate may be computed as the sum of the contribution from all different magnetic field Fourier modes

$$Q_B = \frac{1}{8\pi k_N^3}\int \mathrm{d}^3k\,\frac{d|\tilde{b}_k|^2}{dt} = \frac{1}{8\pi k_N^3}\int \mathrm{d}^3k\,|\tilde{b}_k|^2\;(2\mathrm{Im}\omega)\exp\left(-2\int \mathrm{Im}\omega dt\right). \tag{4}$$

Note that by employing $\tilde{b}$ the decrease in magnetic field energy density due to the expansion of the universe, which is not of dissipative character, does not contribute to Eq. 4. Similarly, the real part of the frequency Eq. 1 does not change the total energy density in magnetic fields and fluid flow. When $|\tilde{b}_k|$ and Im ω are inserted in Eq. 4 one obtains

$$\frac{Q_B}{\rho_\gamma} = \frac{1}{\rho_\gamma^0}\frac{\tilde{B}^2}{8\pi}\frac{(n+3)}{k_N^{n+3}}\int_0^{k_N}\mathrm{d}k\frac{k^{n+4}}{n_e^0\sigma_{Th}}\frac{1}{5(1+z)}\exp\left(-\frac{2}{15}\frac{k^2\tilde{t}_0}{n_e^0\sigma_{Th}}\frac{1}{z^3}\right), \tag{5}$$

It remains to insert Eq. 5 into Eq. 3 and perform the double-integral. An analytic solution may be obtained for $k_N \gg k_D^0 z_\mu^{3/2}$ with $k_D^0 = (15 n_e^0\sigma_{Th}/\tilde{t}_0)^{1/2}$, with the physical interpretation that the cutoff in the spectrum k_N should be at much larger wavevectors than the characteristic wavevector damped by photon viscosity at redshift z_μ (see below). The solution is then simply

$$\mu = K\frac{B_0^2}{8\pi\rho_\gamma^0}\left(\frac{k_D^0}{k_N}z_\mu^{3/2}\right)^{(n+3)}\,, \tag{6}$$

with $K = 1.4\,\Gamma(n/2+5/2)\,\Gamma(3n/5+9/5)\,2^{-(n+5)/2}\,(6/5)\,(n+3)$ a numerical constant of order unity. For example, $K = 2.09, 1.10, 0.77, 0.78$ for magnetic field spectral indices $n = 1, 0, -1, -2$, respectively. The scale $k_D^0 z_\mu^{3/2}$ has a simple interpretation. It is the scale which at redshift z_μ is damped by one e-fold. For the above values of $\Omega_b h^2$ and Y_p the corresponding comoving wavelength is $\lambda_D = (2\pi)/(k_D^0 z_\mu^{3/2}) = 395$pc. Note that the ratio of magnetic field energy density and CMB energy density is given by $B_0^2/(8\pi\rho_\gamma^0) = 10^{-4}(B_0/3.24\times 10^{-8}\mathrm{Gauss})^2$.

The limit may be put into context by comparing the prediction of Eq. 6 with the present upper limit from COBE/FIRAS, $|\mu| < 9\times 10^{-5}$ at (95% confidence level) [18] for a given realization of the magnetic field spectrum. For example, when the spectral cutoff is taken at $k_N = \pi/r_H^{\mathrm{QCD}}$ with $r_H^{\mathrm{QCD}} \approx 0.9\mathrm{pc}(T_{\mathrm{QCD}}/100\mathrm{MeV})^{-1}$ the approximate comoving QCD-horizon, as well as under the assumption that the magnetic field energy density is in approximate equipartition with the radiation energy density (i.e. $B_0 = 3.24\times 10^{-6}$ Gauss), one derives $n \gtrsim -1.35$. Note that there are no constraints on such magnetic field spectra from BBN, since a significant fraction of the initial magnetic field energy density is already dissipated by neutrino viscosity before neutrino decoupling. (The magnetic field strength dependent damping scale before neutrino decoupling is about ~ 50pc for $B_0 \approx 3\times 10^{-6}$ Gauss, but only ~ 0.05pc for $B_0 \approx 3\times 10^{-8}$ Gauss.) For comparison, a scenario which produces uncorrelated magnetic dipole fields in different QCD horizon domains, has a super-horizon power spectrum tail with $n = 0$ [1], and is unconstrained. Nevertheless, magnetic field generation mechanism with resultant spectral indices of $n = -1$ [4] and $n = -2$ [3] have also been proposed. Furthermore, it is not unconceivable that processes, such as inverse cascades in a turbulent primordial fluid, may significantly increase an initially small coherence length of magnetic fields [5].

The derived limit may be restated in the following form: Primordial magnetic fields of strength $\gtrsim 3\times 10^{-8}$Gauss, and comoving coherence length ≈ 400pc may have not existed, since they would yield chemical potential distortions $\mu \gtrsim 10^{-4}$. This is due to a nonthermal energy injection $\Delta\rho_B/\rho_\gamma \gtrsim 10^{-4}$ induced by magnetic energy dissipation at redshift $z \approx 2\times 10^6$. One may derive a similar limit for magnetic field dissipation at redshifts $z \lesssim 4\times 10^4$ which would yield Compton y distortions in the CMB. For a magnetic field strength of $\approx 3\times 10^{-8}$Gauss there is magnetic field damping in the photon diffusion limit above redshift $z \gtrsim 2\times 10^4$ with maximum damping scale ≈ 0.3Mpc [7]. Some additional damping of even slightly larger scales ≈ 0.6Mpc may occur shortly before recombination in the radiation free-streaming limit. Since a magnetic field strength of $B_0 \approx 3\times 10^{-8}$Gauss corresponds to a fractional CMB energy density of $\approx 10^{-4}$, magnetic fields on such coherence scales would produce y distortions above the observational upper limit.

III. CONCLUSIONS

In this paper, we have shown that spatially varying magnetic fields extant prior to the epoch of recombination, may lead to observable distortions in the CMB blackbody. Distortions may result from the injection of non-thermal energy into the CMB due to the dissipation of magnetic field induced peculiar flows in the baryon-photon fluid. Our main result is, that magnetic fields of strength $\gtrsim 3\times 10^{-8}$Gauss, and comoving coherence length ≈ 400pc, may have not been present at redshifts $z \approx 2\times 10^6$, since they would yield chemical potential distortions $\mu \gtrsim 10^{-4}$. Similarly, magnetic fields of $\gtrsim 3\times 10^{-8}$Gauss on scales ~ 0.6 Mpc are also disallowed since they would induce y distortions shortly before recombination. Limits on the spectral index of a stochastic magnetic field distribution, depending on the total energy density in the field and its minimum coherence length, are also given. This limit may be regarded as complimentary to a limit on a homogeneous primordial magnetic field, i.e. $B_0 \lesssim 3\times 10^{-9}$Gauss data. The limit may also be compared to present day upper limits on magnetic fields from observations of Faraday rotation of light from distant quasars, which is significantly weaker, $B_0 \lesssim 7\times 10^{-7}$ Gauss, on a scale of ~ 400 pc [22].

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