Effects of a torsion field on Big Bang nucleosynthesis

M. Brüggen*

Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str.1, 85740 Garching, Germany; and

Churchill College, Storey's Way, Cambridge CB3 0DS, United Kingdom

(June 28, 1999)

Abstract

In this paper it is investigated whether torsion, which arises naturally in most theories of quantum gravity, has observable implications for the Big Bang nucleosynthesis. Torsion can lead to spin flips amongst neutrinos thus turning them into sterile neutrinos. In the early Universe they can alter the helium abundance which is tightly constrained by observations. Here I calculate to what extent torsion of the string theory type leads to a disagreement with the Big Bang nucleosynthesis predictions.

keywords: Big Bang nucleosynthesis – theories of gravitation – nonstandard neutrino physics

04.50.+h,14.60.St,98.80.Cq,98.80.Es

Typeset using REVTEX

*email: marcus@mpa-garching.mpg.de, phone: +49 89 3299 3245

I. INTRODUCTION

In this paper I investigate the spin flip of neutrinos due to a torsion field. Torsion fields arise naturally in most quantum theories of gravitation. In these theories the spin of a particle is related to the torsion in the same manner as the mass is related to the curvature. Motivated by the work of Yang and Mills [1], Kibble [2] and Sciama [3] developed a gauge theory of gravity that contains torsion as a necessary ingredient. More recently, torsion fields have appeared in classical string theory [4], supergravity theories [5,6] and twistor theories of gravity [7]. Torsion is defined as the antisymmetric part of the affine connection and introduces an additional coupling term in the Dirac equation which can cause spin flips. Hammond [8] derived a theory of gravity based on the assumption that torsion can be derived from an antisymmetric potential. It was found that this leads to a coupling which is of the same type as that obtained from string theory.

To avoid confusion I should stress that spin flips can also be caused by the interaction between spin and curvature. This is a separate mechanism related to the frame-dragging effect in standard General Relativity. The flipping of the neutrino's helicity in the vicinity of rotating proto-neutron stars has been treated, e.g., in Ref. [9].

Capozziello *et al.* [10] have described the spin flip in a semiclassical formalism by introducing a torsion term into the Hamiltonian. They speculate that spin flips due to torsion could become significant in the early Universe with profound implications. In this paper I calculate the spin flip probability due to torsion in the early Universe and investigate its consequences on Big Bang nucleosynthesis (BBN). The effect of torsion on the structure formation in cosmology has been calculated in Ref. [11].

II. SPIN FLIP DUE TO TORSION

The spin-flip cross section in a torsion field of the string theory type has been derived in Ref. [12], and I will only briefly sketch the derivation here. Details can be found *ibid.* Torsion is defined as the antisymmetric part of the affine connection, i.e. $\Gamma^{\sigma}_{[\mu\nu]} = S^{\sigma}_{\mu\nu}$, where $\Gamma^{\sigma}_{\mu\nu}$ is the Christoffel symbol.

To derive the corresponding coupling in the Dirac equation one can rewrite the connection in terms of the tetrads e^{a}_{μ}, where the Latin indices refer to the locally inertial frame and Greek indices to a generic non-inertial frame. The nonholonomic index a labels the tetrad, while the holonomic index μ label the components of a given tetrad. In nonholonomic coordinates the connection can be expressed as

$$\Gamma_{abc} = -\Omega_{abc} + \Omega_{bca} - \Omega_{cab} + S_{abc}, \tag{1}$$

where $\Omega^{c}_{\alpha\beta} = e^{c}_{[\alpha,\beta]}$ and $\Omega^{c}_{ab} = e^{\alpha}_{a} e^{\beta}_{b} e^{c}_{\sigma} \Omega^{\sigma}_{\alpha\beta}$.

Assuming that torsion dominates gravitation, one can neglect the Ω_{abc} (curvature) terms in Eq. (1), and the Dirac equation can be written in the form

$$\gamma^{\mu}\psi_{,\mu} + \frac{imc}{\hbar}\psi = \tfrac{1}{4} S_{\mu\nu\sigma}\gamma^{\mu}\gamma^{\nu}\gamma^{\sigma}\psi, \tag{2}$$

where c denotes the speed of light and m mass. Subsequently, the scattering cross section in the high energy limit ($E \gg m$) can be calculated by solving Eq. (2) in the Born approximation. Assuming a minimally coupled Lagrangian, the cross section for a neutrino with mass m and spin S to undergo a spin flip by scattering from a fixed particle is given by [12]:

$$\sigma \simeq 8.28\pi \left(\frac{9GSm}{4\hbar c^2}\right)^2, \tag{3}$$

where G is the gravitational constant.

III. BIG BANG NUCLEOSYNTHESIS

The striking agreement between the predictions of primordial nucleosynthesis with the observed light element abundances is regarded as a big success of Big Bang nucleosynthesis [13]. Thus the predictions of BBN have become a stringent test for theories of cosmology and particle physics.

The observed ^{4}He abundance of $Y = 0.235 \pm 0.01$ [13] constrains the number of neutrino species (active and sterile) to $N_\nu < 3.6$. When additional neutrino species are introduced in the early Universe, the Universe would expand faster due to the increased energy density, which in turn would lead to a higher neutron to proton ration and hence to a higher helium yield. The effect of neutrino mixing between a sterile and an active neutrino on BBN has been studied by various authors [14–17]. For instance, the observed helium abundance has led Shi *et al.* [17] to exclude large angle sterile neutrino mixing as an MSW (Mikheyev-Smirnov-Wolfenstein) solution to the solar neutrino problem.

An additional neutrino must interact weakly enough with the Z^0 in order not to violate the constraints from the Z^0-decay experiment at LEP, which requires the total number of active neutrino species to be 2.993 ± 0.011 [18]. Moreover, it has to interact weakly enough not to be counted as a full species during BBN, so that any additional neutrino species must be sterile. If the sterile neutrinos were brought into chemical equilibrium before the active neutrinos freeze out, they would increase the helium yield and bring it in disagreement with the observed abundance. A sterile neutrino, ν_{s}, may be produced by helicity flip of active neutrinos.

The ν_{s} will roughly achieve chemical equilibrium if their production rate Γ_{s} is larger than the Hubble constant, H, before the active neutrinos decouple. Hence one can approximately constrain Γ_{s} by requiring that $\Gamma_{\mathrm{s}} < H$, i.e.

$$\sigma c \int_{t_{\rm QH}}^{t_{\rm dec}} n(t)\, dt\, \Gamma_\nu \leq H, \tag{4}$$

where $n(t)$ is the number density of scattering centres; $t_{\rm dec}$ is the time when neutrinos decouple from the ambient matter, which is at around 3 MeV for electron neutrinos, and $t_{\rm QH}$ is the time of the quark-hadron transition at around 100 MeV.

The production rate of the active neutrinos is given by

$$\Gamma_\nu \simeq \frac{2}{\hbar} G_{\rm F}^2 T^5, \tag{5}$$

where $G_{\rm F}$ is Fermi's constant. Between $t_{\rm dec}$ and $t_{\rm QH}$ the Universe is almost exclusively composed of photons, electrons, positrons and neutrinos. Nucleons are rarer than the other particle species by a factor of about 10^{-10}. In the temperature region we are concerned with we can assume that the photons and the e^+e^- pairs, which interact via the electromagnetic interaction, are in chemical equilibrium. Since the e^+e^- pairs are far more abundant than protons, one can neglect the chemical potential of the electrons (and positrons). Hence, the number density of electrons and positrons in the early Universe is given by [19]:

$$n_e \simeq \frac{3\zeta(3)}{\pi^2} \left(\frac{kT}{\hbar c}\right)^3, \tag{6}$$

where ζ is the Riemann Zeta-function, k Boltzmann's constant and T temperature. Substituting Eqs. (3), (5) and (6) into Eq. (4), one finds that for a neutrino mass of 10 eV and a Hubble constant of 50 km s^{-1} Mpc^{-1}:

$$\sigma c \int_{t_{\rm QH}}^{t_{\rm dec}} n_e(t)\, dt\, \Gamma_\nu / H \approx 10^{-45}, \tag{7}$$

i.e. much less than 1, which is the value where BBN would be affected.

IV. SUMMARY

As shown by Eq. (7), the effects of spin flips of neutrinos by torsion of the string theory type are completely insignificant for the nucleosynthesis predictions. Although the estimate presented in Eq. (7) is somewhat crude, it is so small that more sophisticated calculations are unlikely to yield a significant result.

One generalization of the above estimate would be to allow for nonminimal coupling as suggested by renormalization arguments and some string theories [20]. In the case of non-minimal coupling the coupling term in the Dirac equation assumes the form $\frac{C}{4} S_{\mu\nu\sigma}\gamma^{\mu}\gamma^{\nu}\gamma^{\sigma}\psi$, where C plays the role of an undetermined coupling constant. However, bounds derived from electron-electron interactions constrain C to $< 1.6 \times 10^{14}$ [21], which is still too small to raise the expression in Eq. (7) to a significant figure.

Consequently, theories of gravity that contain torsion of the string theory type have no noticeable effect on BBN and, conversely, helium abundance observations cannot serve to place more stringent constraints on the coupling constant. Thus one can conclude that quantum gravity theories with a coupling of the kind considered here are not ruled out as alternative theories of gravity by observations of the helium abundance in the Universe.

REFERENCES

[1] C.N. Yang, R.L. Mills, Phys. Rev. 96, 191 (1954).

[2] T. Kibble, J. Math. Phys. 2, 212 (1961).

[3] D. Sciama, Rev. Mod. Phys. 36, 463 (1964).

[4] M. Kalb, P. Ramond, Phys. Rev. D 6, 2271 (1974).

[5] E. Cremmer, B. Julia and J. Scherk, Phys. Lett. B76, 409 (1978).

[6] G. Papadopoulos, P.K. Townsend, Nucl. Phys. B444, 245 (1995).

[7] P.S. Howe, G. Papadopoulos, Phys. Lett. B379, 80 (1996).

[8] R.T. Hammond, Gen. Rel. Grav. 26, 247 (1994).

[9] M. Brüggen, Phys. Rev. D 55, 5876 (1997).

[10] S. Capozziello, G. Iovane, G. Lambiase and C. Stornaiolo, astro-ph/9904199 (1999).

[11] S. Capozziello, C. Stornaiolo, Il Nuovo Cimento 113B, 879 (1998).

[12] R.T. Hammond, Class. Quant. Grav. 13, 1691 (1691).

[13] E.W. Kolb, M.S. Turner *The Early Universe*, Reading (1990).

[14] M. Khlopov, S. Petcov, Phys. Lett. B99, 117 (1981).

[15] R. Barbieri, A. Dolgov, Phys. Lett. B237, 440 (1990).

[16] J.M. Cline, Phys. Rev. Lett. 68, 3137 (1992).

[17] X. Shi, D.N. Schramm and B.D. Fields, Phys.Rev. D 48, 2563 (1993).

[18] The LEP Electroweak Working Group, and the SLD Heavy Flavor Group, CERN/PPE/97-154 (1997).

[19] P.J.E. Peebles, *Principles of Physical Cosmology*, Princeton (1993).

[20] I.L. Buchbinder, S.D. Odintsov and I.L. Shapiro, Phys. Lett. B162, 92 (1985).

[21] R.T. Hammond, Phys. Rev. D 52, 6918 (1995).