MAGNETIC DIPS IN PROMINENCES

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Abstract

Magnetic dips are generally assumed to be basic equilibrium configurations in quiescent solar prominences. Here we discuss two types of the magnetic dips which were considered in the literature: (1) dips resulting from a force-free magnetic equilibrium in the corona, and (2) magnetic dips which are formed in situations where the Lorentz force balances the weight of the prominence plasma. An important parameter which decides between these two cases is the plasma β. For $\beta \ll 1$, the effect of the prominence material on the equilibrium structure is quite negligible and the case (1) holds. If, however, β is larger, say between 0.1 and 1 or even higher, magnetic dips of the second kind are formed and they can be characterized by the angle ψ_1 between the vertical and the direction of the field lines at the surface of the prominence structure. A simple and illustratory formula is derived to relate this angle to the plasma β at the prominence center, namely $\beta_c \simeq \cot^2 \psi_1$. $\beta_c = 1$ thus corresponds to $\psi_1 = 45°$. Finally, we discuss the range of values of both β_c and ψ_1 as deduced from various observations and conclude that the dips of the second kind are important for the prominence equilibria. We also suggest a new method for determination of the field-line inclination.

1 Introduction

The most widely accepted view of modelling solar prominences is that they are supported by magnetic fields (Kippenhahn and Schlüter, 1957; Kuperus and Raadu, 1974; Tandberg-Hanssen, 1995). For an alternative interpretation see Jensen (1986). In order to have a vertical Lorentz force which can balance the downward directed gravity of the dense prominence material one needs a horizontal magnetic field in the center of the prominence. Such fields then could be curved either upward or downward; the latter case is usually referred to as a configuration with 'magnetic dips'. Theoretical prominence models require dips for several reasons:

1. the initial coronal field has to be curved upward to make the resulting equilibrium stable (Kippenhahn and Schlüter, 1957; Anzer, 1969);
2. dips are also required for the siphon mechanism of Pikel'ner (1971) and the injection mechanism of Poland and Mariska (1986); and
3. the weight of the prominence material will in general also lead to a dip.

There are two different types of mechanism which can produce such magnetic dips:

1. effects resulting from the global field configuration in the corona; and
2. the deformation of the original field by strong gravitational forces, in the presence of a dense prominence plasma.

The coronal plasma has under most circumstances a very low plasma β defined as $\beta = 8\pi p/B^2$ with p and B being the gas pressure and the magnetic field strength, respectively. Therefore any dips of the first kind have to be produced by force-free fields, or as a special case of this by a potential field. Such configurations have been studied by Priest, Hood and Anzer (1989) for the case of a giant twisted flux rope, Aulanier and Démoulin (1998) for linear force-free fields (i.e. taking $\alpha = \text{const}$) and Anzer (1990) for a quadrupolar potential field configuration.

The supporting magnetic field configuration could be either of normal (N) polarity or inverse (I) polarity; for the definition of N and I-polarity see Priest (1989). But since the dips are a local property of the field configuration, our investigation will apply to both N and I-polarity prominences. In the following sections we shall demonstrate the properties of these dips and their relation to the values of β.

2 Coronal Dips

As discussed in the Introduction, the formation of a prominence requires the existence of dips in the coronal magnetic field which exists prior to the prominence formation. In the case of the quiet corona one obtains as typical values for the pressures

$$p = 10^{-3} - 10^{-2}\,\mathrm{dyn/cm^2}\,\mathrm{and}\, p_{mag} = 0.1 - 1\,\mathrm{dyn/cm^2},$$

where p_{mag} denotes the magnetic field pressure. This implies that the relation $\beta = p/p_{mag} \ll 1$ holds everywhere in the corona and the coronal magnetic field has to be force-free. In this case one can put estimates on the possible size of the magnetic dips. From the condition that the vertical component of the Lorentz force vanishes one obtains

$$\frac{\partial}{\partial z}(\frac{B^2}{8\pi}) - \frac{1}{4\pi}(\mathbf{B}\cdot\nabla)B_z = 0. \tag{1}$$

If we denote by h the height of the prominence and by D its total width we can get the following estimate from Equation (1):

$$\frac{B^2}{8\pi h} \simeq \frac{1}{4\pi} B \frac{2B_{z1}}{D}, \tag{2}$$

where B_{z1} denotes the value of B_z at the prominence-corona interface. This gives

$$B_{z1} \simeq \frac{D}{4h} B. \tag{3}$$

If we take h=40000 km and D=5000 km as typical values for prominences we obtain

$$B_{z1} \simeq 0.03\,B. \tag{4}$$

This result might be modified by the following two effects:

1. the field strength at the bottom of the prominence does not vanish, therefore $\partial(B^2/8\pi)/\partial z < B^2/8\pi h$ will hold in reality;

2. D is the prominence width, but the derivative is along field lines which in general show some shear. This means that D in Equation (3) should be replaced by the quantity $D\sqrt{(B_x^2 + B_y^2)}/B_x$.

Since these two corrections go in opposite directions, Equation (3) should represent a reasonable order-of-magnitude estimate.

This estimate shows that the dips which are produced in coronal force-free fields are extremely shallow and the vertical field at the surface of the prominences resulting from coronal effects alone will be practically undetectable. It should be made clear that our estimates of the field line dips refer to the relatively narrow region within the prominence. If one considers the entire coronal region then the dips can be much larger. This is in particular true for the twisted flux rope model of Priest, Hood and Anzer (1989). However, this is of no relevance for the actual field inside the prominence. One also sees from Equation (3) that if one replaces the height h by a vertical length scale Δz which is comparable to D then locally a deeper dip can be procuced. But such a tiny vertical region, if it would exist, can not have any effect on the global model of solar prominences.

3 Dips Produced by the Prominence Plasma

The weight of the prominence itself can also produce dips of the magnetic field. If we use the one dimensional slab model as discussed by Anzer and Heinzel (1998) and sketched in Figure 1, we get the vertical equilibrium condition given by

$$Mg = \frac{B_x B_{z1}}{2\pi,} \quad (5)$$

where M is the column mass, g the gravitational acceleration, B_x the field component normal to the prominence and B_{z1} the vertical field strength at the prominence surface. The investigations of Bommier et al. (1994) (hereafter referred to as BLLS) showed that in many of their prominences B_{z1} is comparable to the total horizontal field which we denote by $B_h = \sqrt{(B_x^2 + B_y^2)}$. This then implies that the column mass in the prominence has to be sufficently large to produce these deep dips. One also finds that the inclination angles of the magnetic-field lines in these prominences are at least one order of magnitude larger than the coronal values which we have discussed in the previous section. This means that the coronal dips will play almost no role for the magnetic equilibrium condition for these prominences.

We shall now derive a relation between the field line inclination (see Figure 1) and the value of the plasma β inside the prominence. In our one dimensional isothermal slab models one has

$$\beta(x) = \frac{8\pi p(x)}{B_h^2 + B_z^2(x)}. \tag{6}$$

In a one dimensional geometry B_x =const follows from $\operatorname{div} \mathbf{B} = 0$ and B_y = const is required for magnetic equilibrium in the y-direction. Therefore, B_h will be also constant everywhere. Force balance in the x-direction gives a condition on the total pressure

$$\frac{d}{dx}[p(x) + \frac{B_h^2 + B_z^2(x)}{8\pi}] = 0. \tag{7}$$

This leads to the relation

$$p(x) = p_0 + \frac{B_{z1}^2 - B_z^2(x)}{8\pi}, \tag{8}$$

where p_0 is the gas pressure at the surface of the prominence which is equal to the coronal gas pressure. For the value of the plasma β at the surface we then obtain

$$\beta_0 = \frac{8\pi p_0}{B_h^2 + B_{z1}^2}. \tag{9}$$

If we define the *local* value of ψ by $\cos\psi(x) = B_z(x)/B(x)$ with $B(x) = \sqrt{[B_h^2 + B_z^2(x)]}$ then one can derive the following basic relation between the local plasma β and the angle ψ

$$\beta(x) = (\beta_0 + \cos^2\psi_1)\frac{B^2(x_1)}{B^2(x)} - \cos^2\psi(x), \tag{10}$$

where x_1 is the surface coordinate. This will lead to a central value of β

$$\beta_c = \frac{\beta_0}{\sin^2\psi_1} + \cot^2\psi_1 = \beta_{0c} + \cot^2\psi_1, \tag{11}$$

where $\beta_{0c} = 8\pi p_0/B_h^2$ is the component of β_c due to the external coronal pressure. Since usually $\beta_{0c} \ll \beta_c$ holds, Equation (11) can be simplified to give

$$\beta_c \simeq \cot^2\psi_1. \tag{12}$$

The dependence of the dip angle ψ_1 on β_c is shown in Figure 2, where we plot the relation

$$\psi_1 = \arctan \sqrt{\frac{1}{\beta_c - \beta_{0c}}} \tag{13}$$

for β_{0c}=0.0, 0.001, 0.01 and 0.1. Note that $\beta_c > \beta_{0c}$. Several important conclusions can be immediately drawn from this plot:
For a given value of ψ_1 (e.g. taken from BLLS - see Discussion), we get immediately a lower limit for β_c neglecting the coronal pressure. $\psi_1 = 45°$ is a common value given by BLLS and this corresponds to $\beta_c = 1$.
On the other hand, for a given β_c and neglecting p_0, we get a lower limit for ψ_1. For non-zero p_0, ψ_1 will be larger and dips shallower.
When $p \to p_0$, the vertical component of the field vanishes and the dip disappears - however, the dips of the first kind may be still present.
For $p_{mag} = 1$ (this corresponds to $B_h = 5$ G and is also consistent with a mean value derived from BLLS), one can replace β_c in Figure 2 simply by the gas pressure p. Then Figure 2 shows how the dips will vary with changing the gas pressure.

Equation (11) was derived from a one dimensional slab model. But a similar relation can be obtained if the prominence consists of individual vertical threads (details of such thread models will be presented in a separate paper). In such models the column mass, the field vector and the prominence width all will vary along the prominence axis. But in this class of models, the total pressure $p_{tot} = p + B^2/8\pi$ has to be constant and the magnetic field is 'laminar', i.e. the field lines lie in parallel planes which requires that B_y/B_x is constant everywhere. In this case Equation (10) will hold along each individual field line; but now the quantities ψ_1, x_1, and also $\mathbf{B}$ can all be functions of the y-coordinate. From these results one can conclude that the relations (10) to (12) which were originally derived for the special case of a one dimensional slab can also be applied to much more general configurations. Therefore, we expect that the results which we have presented here are valid under quite general circumstances.

4 Discussion

Our Equation (1) shows that the coronal dips are due to an increase of field strength with height. But it would be misleading to take the observations by Leroy, Bommier and Sahal-Bréchot (1983) as an evidence for the existence of such coronal dips. They found that the field strength in quiescent prominences increases systematically with height. It is important to realise that they measure the total magnetic field of the prominence which has to be considered as the superposition of the coronal background field and the field due to currents inside the prominence. As has been shown these prominence currents will modify the initial coronal field considerably. Therefore the increase of the total field strength with height does not prove the existence of coronal dips.

We shall now discuss in detail the measurements of Bommier et al. (1994) (all relevant quantities are taken from tables given by Anzer and Heinzel (1998)).

In Figure 3 we show again the dependence of ψ_1 on the plasma beta β_c, neglecting p_0 for a simplicity (Equation (12)). The values of ψ_1 obtained by BLLS for a set of 13 quiescent prominences are $45°, 60°$ and $75°$. The corresponding values of β_c follow from Figure 3 - see the three crosses. We thus see that BLLS polarization measurements lead to rather high values of the plasma beta, ranging from $\beta_c \simeq 0.07$ to 1. For non-zero p_0, these will be even larger (see Figure 1). Therefore, the assumption of $\beta \ll 1$ can be ruled out for most, if not all, prominences of this sample. Since the determination of B_z and ψ_1 as obtained from polarization analysis is very complicated, the values given here might be associated with large uncertainties (Leroy - private communication). But even taking this aspect into account, one can conclude that finite values of β have to be considered when one tries to model these prominences.

One can also obtain values for the mean gas pressure $\overline{p}$ in these prominences by a NLTE line analysis (see Anzer and Heinzel, 1998). Then the central gas pressure is $p_c = 3/2\,\overline{p}_{NLTE}$ and we obtain

$$\beta_{NLTE} = \frac{12\pi\overline{p}_{NLTE}}{B^2}, \tag{14}$$

where now we interpret the value for B given by BLLS as our B_h (in a sense this is correct since BLLS assume that their B is constant everywhere in the prominence). The values for β_{NLTE} are in general smaller than those represented by the corresponding crosses in Figure 3, they range from 0.004 to 0.4. This again reflects the discrepancies between MHD and NLTE solutions found by Anzer and Heinzel (1998), namely that the gas pressure derived from MHD models is up to a factor of 10 larger than that obtained from NLTE spectral diagnostics. Moreover, these β_{NLTE} are *not* consistent with the values of ψ_1 given by BLLS. Using Equation (12), we have computed new angles ψ_1 for all values of β_{NLTE} and these are displayed in Figure 3 as diamonds. We see that in this case the dips are shallower due to lower β_{NLTE}. Note, however, that the NLTE gas pressure was derived by Anzer and Heinzel (1998) consistently with the electron densities of BLLS, which are rather low as compared to some other diagnostics.

Therefore, using Equation (12) to evaluate ψ_1, if from some other diagnostics the value of β_c has been determined, represents an interesting new method. It can give *directly* a condition on the size of the magnetic dip which is required in order to keep the prominence in equilibrium. To specify β_c, we must determine the gas pressure at the center of the prominence structure and the horizontal magnetic field. The former can be done by analyzing the pressure-sensitive line ratios like CaII 8542 Å/ Hβ (Milkey and Heasley, 1978; Gouttebroze, Vial and Heinzel, 1997). The magnetic-field intensity at the dip center can be derived from polarization measurements, but only one line is sufficient and the resulting B is model-independent (the field is horizontal). In this way one can avoid the very complex and still model-dependent determination of ψ as applied by BLLS, with the result which will be probably less uncertain. As a 'prototype' determination of ψ_1 in such a way, see the diamonds in Figure 3 and the relevant discussion above.

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Figure Captions

Figure 1
A sketch of the magnetic field lines with a dip supporting the prominence sheet. At the surface, the inclination of the field lines (vector **B**) with respect to the z-axis is represented by the angle ψ_1.

Figure 2
Dependence of the inclination angle ψ_1 on the plasma beta β_c at the prominence center. Four values of the external coronal pressure are considered: $p_0 = 0.0$ dyn cm^{-2} (solid line), 0.001 (dash), 0.01 (dot-dash) and 0.1 (threedot-dash).

Figure 3
Values of ψ_1 from BLLS and their corresponding β_c (crosses). Diamonds represent β_{NLTE} and the corresponding values of ψ_1. Assuming $B_h = 5$ G, one can simply replace β_c by the gas pressure p in this figure.