On an overshooting approach to the solar Li problem

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Abstract. To solve the solar Li-problem and to reduce the deviation between the sound speed of standard solar models and that infered from helioseismology, which is a persistent feature immediately below the convective envelope in most solar models, some additional mixing effect can be postulated. We investigate, how a recently proposed parametric prescription of overshooting, which is based on two-dimensional hydrodynamical calculations, affects both the lithium content and the seismic properties of the models. While we conclude that for a certain choice of the free parameter the observed lithium depletion can be reproduced, we also show that this parameter is neither constant within the Sun, nor during its evolution. Further, all models with overshooting agree less with the seismic model than the standard model.

Key words: Convection – Sun: abundances – Sun: evolution – Sun: interior – Stars: interiors

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1. Introduction

The contemporary standard solar model (SSM) appears to be a highly accurate representation of the true solar structure, as is infered from the inversion of solar p-mode frequencies. This is demonstrated, for example, in Bahcall et al. (1998), whose model has a rms deviation from the seismic model of Basu et al. (1996) of only 0.001. This gain in accuracy, as compared to models calculated only a few years ago (e.g. Turck-Chièze et al. 1993) is due to the inclusion of new opacities and equations of state and the consideration of microscopic particle diffusion (see Bahcall et al. 1995 for details of the SSM).

Notwithstanding the success of the SSM – which in particular has helped to identify the reason for the solar neutrino problem to be found in particle physics – there are still significant discrepancies (significant because of the high precision of the p-mode frequency measurements) and the Li-problem. With regard to the first point, all SSM have the largest deviation from the seismic model just below the convective envelope's boundary, which itself is very well reproduced by the models (at $r/R_\odot = 0.7135 \pm 0.0005$; Basu 1998). The relative deviation in sound speed is as large as 0.003 and therefore at least comparable to the error in the seismic model. This situation is illustrated in Fig. 1, which shows the comparison between our own SSM ("GARSOM4"; Schlattl et al. 1999) and the seismic model of Basu et al. (1996), along with the error range of the seismic model as given by Degl'Innocenti et al. (1997). We note that the model by Christensen-Dalsgaard et al. (1996), which better reproduces the seismic model in the critical region below the convective zone, was calculated using the OPAL92 opacities (Rogers & Iglesias 1992), while ours and that by Bahcall et al. (1998) employ the OPAL96 opacities (Iglesias & Rogers 1996). The latter two SSM use the most recent compilation of nuclear reaction rates (Adelberger et al. 1998), too. The effect that these improvements in the physical input lead to a slight deterioration in the comparison has been discussed already by Brun et al. (1998a) and Turck-Chièze et al. (1998). Nevertheless, this also demonstrates that the good agreeement with the seismic model is not just a fortunate coincidence.

The solar lithium problem manifests itself in the fact that the ^{7}Li abundance in the solar atmosphere (and therefore the solar convective envelope) is only about 1/140 of the meteoritic value of $\log^7\mathrm{Li} = 3.31 \pm 0.04$[1] (Anders & Grevesse 1989), while the SSM predicts a depletion of only a factor 2–3, if the evolution is started on the zero-age main sequence (ZAMS; for a discussion of this subject, see Chaboyer 1998). The situation changes, if pre-main sequence (PMS) evolution is taken into account. Here, a significant depletion can be obtained, depending on the treatment of convection and the particular set of opacities used, because these factors determine to which temperature the convective envelope extends. D'Antona & Mazzitelli (1994) find a PMS depletion of a factor of 10, but with an updated version of their models also complete destruction of ^{7}Li during the PMS. They discuss these new results, their sensitivity on physical and stellar parameters (notably metallicity) and possible solutions to this *inverse* lithium problem of complete destruction in great detail (Ventura et al. 1998). Our own standard solar model, calculated with the PMS included leads to a lithium abundance of 0.107 of the initial one when arriving on the ZAMS, and a further reduction to 0.092 thereafter. Since the lithium abundance in open clusters appears to decline with age (Charbonnel et al. 1994; Chaboyer 1998), with young clusters such as the Hyades at ≈ 0.8 Gyr showing a depletion of only a factor of two, this amount of PMS-depletion might potentially be in contradiction with observed lithium abundances in young open clusters, if one ignores all differences between the cluster compositions and assumes that the derived cluster age–lithium relation is the same as that for the Sun (but see Jones et al. 1999). In any case, our SSM including PMS-evolution and diffusion does not reproduce the present solar lithium abundance.

Since the solution to the Li-problem lies in an additional mixing from the bottom of the convective layer to those hot enough to burn ^{7}Li at $T \gtrsim 2.5 \cdot 10^6$ K and this is exactly the region of highest deviation of the SSM from the seismic model, it is reasonable to try

[1] We are using the standard logarithmic notation, where $\log \mathrm{N(H)} \equiv 12$

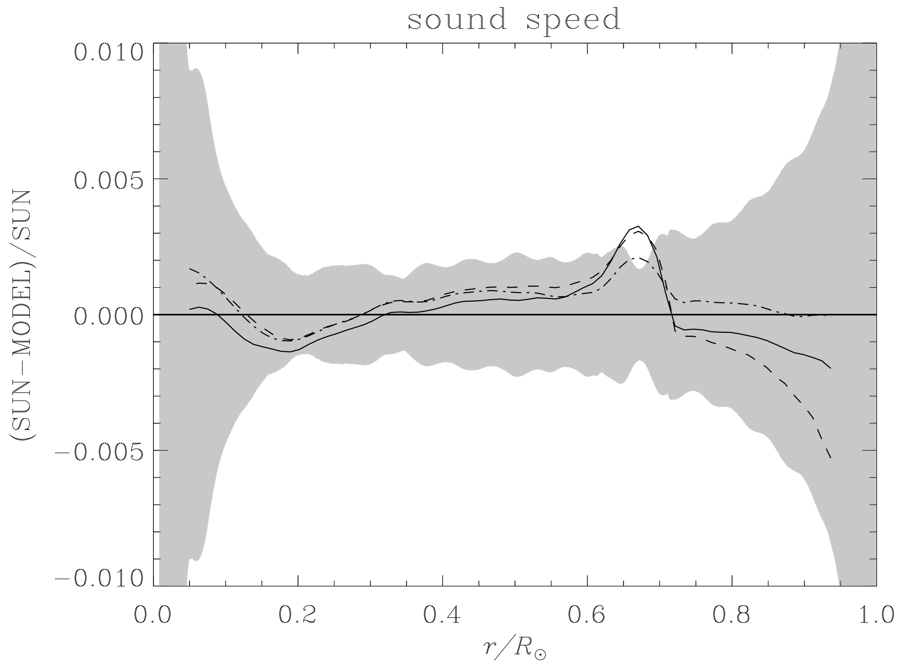

Fig. 1. Comparison of modern solar models with the seismic model by Basu et al. (1996): shown is the relative (seismic-solar model) difference of sound speed for the model of Bahcall et al. (1998) (dashed line), Christensen-Dalsgaard et al. (1996) (dot-dash) and our best model GARSOM4 (solid). The grey area indicates a conservative error range of seismic models according to Degl'Innocenti et al. (1997)

to solve both problems at the same time. Richard et al. (1996) have, rather successfully, done so by introducing additional diffusive mixing due to differential rotation in their solar model. This solution, of course, contains free parameters, and an approximative treatment of the poorly known effects of rotation on mixing inside a star.

Recently, a different attempt to solve the Li-problem has been reconsidered with a new approach. Motivated by the results of 2d-hydrodynamical simulations of the (thin) convective envelopes of A-stars, Blöcker et al. (1998) introduced a parametrized treatment of overshooting from the convective envelope to reach ^{7}Li-burning temperatures. However, their model did not comply with the definition of the SSM. In particular, it did not contain particle diffusion. In addition, the influence of the additional mixing process on the solar structure was not discussed and a comparison with a seismic model or with measured p-mode frequencies was lacking. Stimulated by a preliminary result by Richard and Charbonnel (Richard 1999 and Charbonnel, private communication) that the agreement with the seismic model worsens for the overshooting case, we decided to perform a study about the effect of the new overshooting approach, which complies with the standard ways of comparisons for any model of the Sun. In the next section, we will briefly introduce the diffusion approach and its implementation in our solar model code, and summarize the results of Blöcker et al. (1998). In Sect. 3 we will present our own results including the comparison with the seismic model (Basu et al. 1996). A short summary will close the paper.

2. Overshooting as a diffusive process

In a seminal paper Freytag et al. (1996) have investigated the role of overshoot from the lower boundary of outer stellar convective layers. Based on two-dimensional hydrodynamical models they showed convincingly how the dynamics of turbulent convection is governed by the fast narrow downdrafts, which give rise to overshooting beyond the formal convective boundary. The simulations showed that the assumption of a finite overshooting distance does not represent the situation appropriately. Rather, the velocity field of the downward motions extends beyond the region with significant convective flux and declines exponentially. The scale length is correlated with the pressure scale height, but, and this is an important point, depends on the particular stellar parameters. Although the velocities decline rapidly, they are still of order of typical diffusion speeds, such that convective motions can counteract the sedimentation process. The temperature structure below the formal convective boundary remains dominated by radiative diffusion, as is necessary for SSM, which already without overshooting reproduce the depth of the solar convective envelope extremely well. This is the important difference to other overshooting approaches, in which the temperature gradient below the formally stable boundary is changed as well.

The complete convective envelope and the adjacent overshooting region fit into the simulation box only for hot stars (A-type and DA white dwarfs). From this, Freytag et al. (1996) derived a relation for the diffusion coefficient appropriate for the overshooting:

$$D(z) = D_0 \exp \frac{-2z}{H_v} \tag{1}$$

Here, z is the radial distance from the formal lower boundary of the convective zone, H_v the velocity scale height, and D_0 is a "typical" diffusion coefficient for convection at $z = 0$, e.g. resulting from the convection speed derived in the mixing-length picture. Freytag et al. (1996) give examples for H_v: in terms of the pressure scale height H_P it varies from $H_v = (0.25 \pm 0.05)H_P$ for an A-type star to $(1.0 \pm 0.1)H_P$ for the white dwarfs. Formulated this way, overshooting can easily be incorporated into SSM-codes, which already have implemented particle diffusion and treat convection as a fast diffusive process, too. The varying proportionality factor is then expressed a free parameter, $H_v = f\,H_P$.

Since the solar convective envelope could not be simulated completely, Freytag et al. (1996) could argue only qualitatively about the effect on the photospheric ^{7}Li and ^{9}Be abundance. They concluded that $H_v \approx 7$ Mm ("with appreciable uncertainty") could be a reasonable choice to destroy ^{7}Li during the solar main-sequence evolution. This value for H_v corresponds to $f \approx 0.14$.

Blöcker et al. (1998) have calculated solar models using the qualitative results of Freytag et al. (1996) to investigate the effect of such an overshooting approach on the solar ^{7}Li. Although the bottom-line of this work is that for $f = 0.07$ – applied from the ZAMS on – the observed depletion of ^{7}Li is obtained, the authors mention some potential problems: (i) for the PMS, f had to be adjusted (no number is quoted) to obtain the depletion by -0.3 dex observed in young open clusters; (ii) the MS-value for f is larger than that derived by Herwig et al. (1997) for overshooting in the same description from convective cores and deep convective envelopes ($f \approx 0.02$); (iii) no other mixing/diffusion process had been taken into account. As such, Blöcker et al. (1998) concluded correctly that their study is a demonstration of the potential effect of the overshooting prescription on ^{7}Li, deserving more detailed follow-up investigation.

3. Calculations and results

We have added eq. (1) into the diffusion part of our SSM program, which already takes into consideration particle diffusion (coefficients calculated according to Thoul et al. 1994) of hydrogen, helium and 7 heavier elements, among them ^{7}Li. Turbulent convection is treated as a diffusive process as well, with the diffusion velocity calculated within our convection theory approach.

The solar models we calculate correspond to those described in Schlattl et al. (1997), with some modifications: we now use the nuclear reaction cross sections recommended by Adelberger et al. (1998) and treat nuclear burning and diffusion simultaneously as one system of equations. In addition, the T–P–stratification of the stellar envelopes throughout the whole evolution is taken from 2d-hydro-models provided by H.-G. Ludwig (private communication). These are compatible with those of Freytag et al. (1996). They are extending down to an optical depth of $\tau = 1000$ and are continued by convective layers with convection treated according to Canuto & Mazzitelli (1991; 1992), where the usual mixing-length parameter α is used. All calculations are full solar model calculations, implying the inclusion of the PMS phase and a complete calibration of the free parameters. Fig. 1 shows that our SSM is fully competitive with the best ones published so far.

case	envelope	core	$^7\mathrm{Li}(t_\odot)/^7\mathrm{Li}_\mathrm{i}$	Y_i	α
1	0.070	0.000	0.005	0.270	0.960
1a	0.070	0.000	0.0	0.271	0.955
2	0.016	0.000	0.004	0.273	0.967
3	0.070	0.070	0.0	0.283	0.789
4	0.006	0.006	0.056	0.274	0.971
5	0.005	0.005	0.063	0.274	0.971
G4	0.000	0.000	0.092	0.274	0.972

Table 1. Parameter f (eq. (1)) for overshooting from envelope and core, final ^{7}Li abundance and calibrated values of initial helium content Y_i and mixing length parameter α for the SSMO calculations; for comparison, the standard case (G4=GARSOM4) is listed as well

We have performed calculations of standard solar models including overshooting (SSMO) for the cases listed in Tab. 1. For overshooting from the convective envelope (downwards) and the core (upwards) we have applied different values of f. The final solar models were investigated for their agreement with the seismic model of Basu et al. (1996). Helium and metal diffusion is always present. Except for case 1a, all calculations were started on the PMS.

We first applied $f = 0.07$ to reproduce the result by Blöcker et al. (1998), with the difference that no overshooting during the PMS evolution was included. This is because Blöcker et al. (1998) did not specify the value for f they used but only quoted the change in lithium abundance (-0.3 dex) until the ZAMS. Fig. 3 demonstrates that we obtain a final depletion of 200 in our SSMO. The seismic properties, however, change only partially as intended (Fig. 2, short-dashed line). Although the deviation just beneath the convective boundary is removed, a new region of similar deviation but larger extent exists below, such that the total deviation has grown somewhat. The core remains almost unaffected, although the neutrino flux increases slightly (Tab. 2) due to a temperature increase.

The assumption that the overshooting parameter might be the same also for the PMS evolution results, for $f = 0.07$, in a complete destruction of lithium (case 1a). Therefore, a smaller (but still constant) value of $f = 0.016$ was chosen for case 2 (this value is the one Herwig et al. 1997 employed for envelope overshooting in asymptotic giant branch stars). In this case, almost the same final ^{7}Li abundance is reached already on arrival on the ZAMS, with only minor further depletion thereafter due to sedimentation. This constancy during the main-sequence evolution is in contradiction to the observed

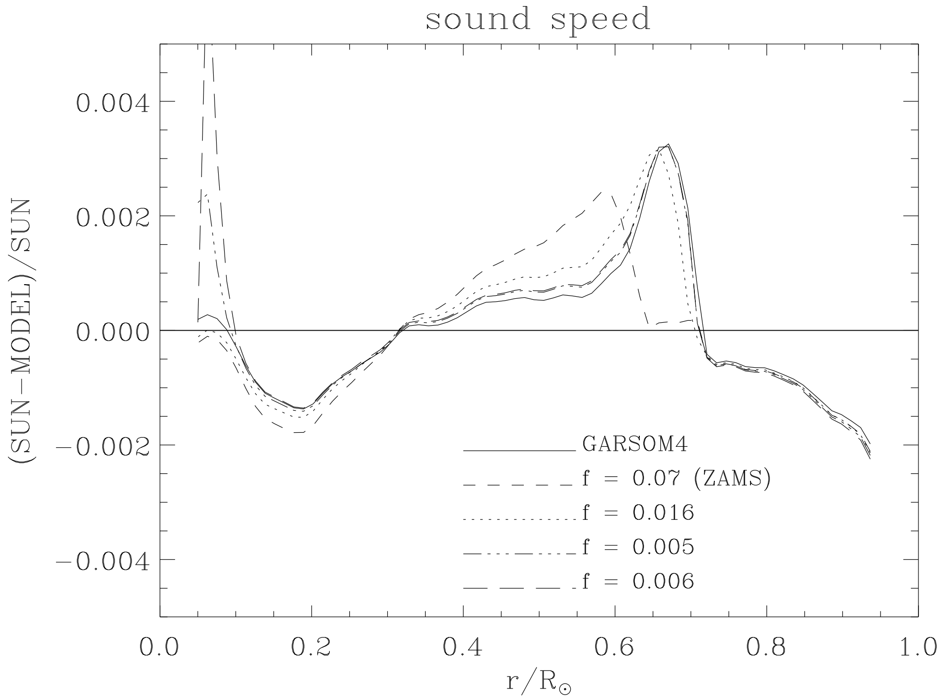

Fig. 2. Comparison of solar models with overshooting (see text) with the seismic model by Basu et al. (1996). Our standard model (GARSOM4, solid line) is shown for comparison

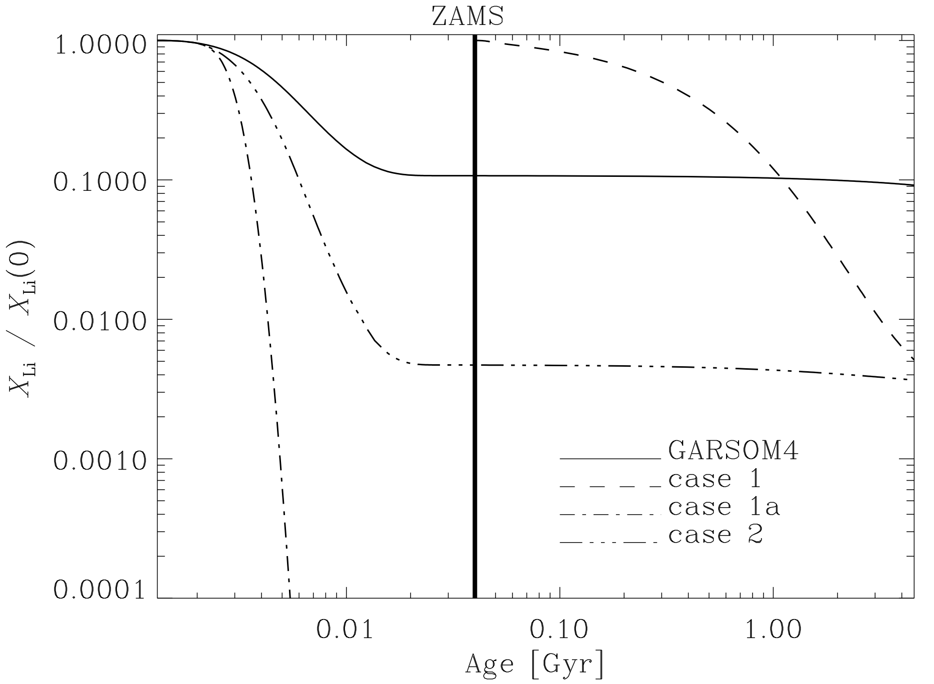

Fig. 3. ^{7}Li-abundance as a function of time for the different values of the (envelope) overshooting parameter

correlation between the lithium abundance in open cluster stars (of solar mass) and time, which indicates a gradual depletion during the main-sequence phase (Chaboyer 1998; Jones et al. 1997; Jones et al. 1999). The sound speed profile and the neutrino rates lie in between the standard case and case 1, with hardly any noticable change (Fig. 2, dotted line, and Tab. 2). Brun et al. (1998b) found a rather similar effect on the sound speed profile, when introducing turbulent diffusion in the same region, while Richard et al. (1996) noticed a small improvement for their implementation of rotation-induced mixing.

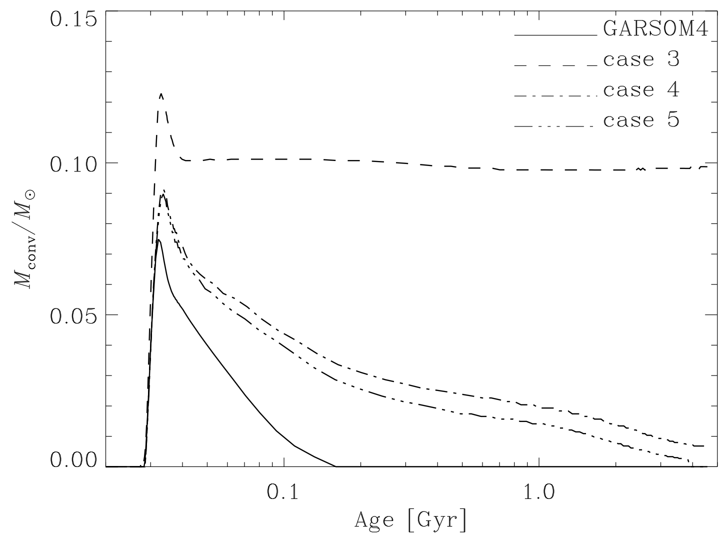

Fig. 4. Evolution of the convective core for different assumptions about the overshooting parameters

case	Ga [SNU]	Cl [SNU]	SK [$10^6 {\rm cm}^{-2}{\rm s}^{-1}$]
1	126.5	7.18	4.76
1a	126.2	7.12	4.72
2	127.8	7.42	4.94
3	115.4	6.02	4.12
4	129.9	8.08	5.48
5	128.7	7.66	5.13
G4	128.7	7.60	5.08

Table 2. Predicted event rates of the Gallex ("Ga"), Homestake ("Cl") and Super-Kamiokande ("SK") neutrino experiments for the SSMO models and the standard case (G4=GARSOM4)

Next, we tested the effect of overshooting on the small convective core, which exists for a few 10 million years during the PMS evolution. Assuming $f = 0.07$ for the core (case 3) leads to a completely non-standard solar evolution (Fig. 4) with a persistent convective core of $\approx 0.1 M_\odot$ and strongly reduced neutrino rates (Tab. 2). We do not show the comparison with the seismic model, because even in cases 4 and 5 ($f = 0.006$ and 0.005), where the convective core vanishes shortly after respectively before the solar age (Fig. 4), the SSMO deviate strongly from the seismic models (Fig. 2, long-dashed and dash-dotted lines). While the effect on the core structure is well known (see, e.g., Richard & Vauclair 1997), this experiment demonstrates that for overshooting from the core one can allow only for values of f much smaller than those required for lithium depletion, which in all these three cases amounts to either the same as in case 1 (for $f = 0.07$) or to that of the standard case GARSOM4 (for $f \leq 0.006$).

4. Conclusions

While it would be desirable to solve the solar lithium problem and to reduce the remaining solar model deficits (Fig. 1) simultaneously, it appears that the overshooting approach by Blöcker et al. (1998) is not able to do so. The arguments for this conclusion are that the overshooting parameter f (eq. (1)) would have to be adjusted to different values both for the different phases of evolution (PMS and main-sequence) and for envelope and core overshooting. Since the seismic model leaves almost no room for overshooting from the transient convective core, which exists for some time during the PMS evolution, $f < 0.005$ is indicated. Second, overshooting from the convective envelope seems to be suppressed during the PMS evolution as well, because otherwise lithium would be destroyed too much (or completely) before the Sun has reached its ZAMS position. Recall that even the SSM model (without overshooting) is depleting lithium more efficiently than observed in the young open clusters (α Per or Pleiades). This requirement restricts f to be smaller than 0.016 in that phase, which is the value used by Herwig et al. (1997) to obtain a main-sequence width compatible with the observations for intermediate mass stars, and which resulted in significant third dredge-up in AGB stars. (Note that Herwig et al. 1997 keep the value of f constant during the evolution and for core and envelope overshooting.) However, for this value there is no further lithium depletion during the main-sequence evolution, which is in contradiction to the anti-correlation between lithium abundance and age for open clusters (such as Praesepe, NGC 752 and M67). Finally, the case presented by Blöcker et al. (1998) of $f = 0.07$ during the main-sequence evolution (only envelope) indeed solves the solar lithium problem. However, the resulting solar model shows an even stronger disagreement with the solar sound speed, such that the combined solution of both mentioned problems cannot be achieved with this approach.

To conclude, we consider the obvious need for a "variable" parameter in the overshooting approach and the failure to improve the solar model below the convective region as being discouraging. Rather, the turbulent diffusion approach (Chaboyer et al. 1995; Richard et al. 1996) seems to be more promising and should be investigated further. In this case, the solar lithium abundance can be reproduced and the sound speed profile improves slightly with respect to the seismic model. Whether our results allow negative conclusions for other situations, where the present overshooting approach has been used (main-sequence, AGB), is beyond the scope of the present paper. Actually, the non-constancy of f is an argument *not* to draw conclusions beyond the solar case. It would be highly useful to have 2d-hydrodynamical calculations available, which accomodate the whole convective region under consideration. This would also clarify, whether the overshooting description, which was derived from A-type stellar convective envelopes, is applicable at all to the solar case, where convection is much more effective than in the former stars.

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