Primordial Black Hole Formation during First-Order Phase Transitions

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Primordial black holes (PBHs) may form in the early universe when pre-existing adiabatic density fluctuations enter into the cosmological horizon and recollapse. It has been suggested that PBH formation may be facilitated when fluctuations enter into the horizon during a strongly first-order phase transition which proceeds in approximate equilibrium. We employ general-relativistic hydrodynamics numerical simulations in order to follow the collapse of density fluctuations during first-order phase transitions. We find that during late stages of the collapse fluctuations separate into two regimes, an inner part existing exclusively in the high-energy density phase with energy density $\epsilon_{\rm h}$, surrounded by an outer part which exists exclusively in the low-energy density phase with energy density $\epsilon_{\rm h}-L$, where L is the latent heat of the transition. We confirm that the fluctuation density threshold $\delta\epsilon/\epsilon$ required for the formation of PBHs during first-order transitions decreases with increasing L and falls below that for PBH formation during ordinary radiation dominated epochs. Our results imply that, in case PBHs form at all in the early universe, their mass spectrum is most likely dominated by the approximate horizon masses during epochs when the universe undergoes phase transitions.

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I. INTRODUCTION

It is long known that only moderate deviations from homogeneity in the early universe may lead to abundant production of PBHs from radiation [1]. Slightly non-linear fluctuations with excess density contrast $\delta\epsilon/\epsilon \approx 1$, if horizon-size, are already very close to their own Schwarzschild radius. Therefore, already moderate collapse of such fluctuations may lead to the formation of a black hole. Under such conditions primordial angular momentum of fluctuations does generally not halt the collapse. The ultimate fate of an initially super-horizon density fluctuation, upon horizon crossing, is mainly determined by a competition between dispersing pressure forces and the fluctuation's self-gravity. For an ordinary radiation dominated equation of state (i.e. $p=\epsilon/3$, where p is pressure) there is approximate equality between the Jeans-, $M_{\rm J}^{\rm RD}$, and horizon-, $M_{\rm h}$, masses. For fluctuation overdensities exceeding a critical threshold at horizon crossing $(\delta\epsilon/\epsilon)_{\rm hc}\geq\delta_{\rm c}^{\rm RD}\approx 0.7$ [2] gravity dominates and the formation of a PBH with mass $M_{\rm pbh}\sim M_{\rm h}$ results. Fluctuations with $(\delta\epsilon/\epsilon)_{\rm hc}<\delta_{\rm c}^{\rm RD}$ disperse by pressure forces.

Since PBH formation from pre-existing adiabatic density fluctuations is a fine competition between self-gravity and pressure forces any decrease of the pressure response $(\partial p/\partial\epsilon)_{\rm S}=v_{\rm s}^2$ of the radiation fluid, or equivalently, decrease of the Jeans mass $M_{\rm J}\approx(4\pi/3)\epsilon\,(v_{\rm s}^2/4\pi G\epsilon)^{3/2}$, may yield a reduction of the threshold density contrast $\delta_{\rm c}$ for PBH formation. Here $v_{\rm s}$ is the adiabatic speed of sound, with $v_{\rm s}=1/\sqrt{3}$ during ordinary radiation dominated epochs. A decrease in the pressure response of the radiation fluid is, in fact, anticipated to occur during cosmological first-order phase transitions. In essence, during a first-order transition between a high-energy density phase with energy density, $\epsilon_{\rm h}$, and a low-energy density phase with energy density $\epsilon_{\rm l}=\epsilon_{\rm h}-L$ both phases may coexist in pressure equilibrium, $p_{\rm h}=p_{\rm l}$ at a coexistence temperature $T_{\rm c}$. In a state where a fluid element is permeated by both phases, compression leads to an increase of energy density, since some low-energy density phase is converted into high-energy density phase. Nevertheless, there is no increase in pressure such that $v_{\rm s}^{\rm eff}\approx 0$ [3,4] for a fluid element substantially larger than the mean separation between high-energy- and low-energy density phases. (see [3] for a more detailed discussion). These considerations are, of course, only valid under the assumption of approximate maintenance of thermodynamic equilibrium, in particular, negligible super-cooling and -heating. Less dramatic reductions of $v_{\rm s}^{\rm eff}$ may also occur during higher-order phase transitions or particle annihilation periods in the early universe.

A reduction of the PBH formation threshold for fluctuations which enter the cosmological horizon during first-order phase transitions (i.e. $\delta_{\rm c}^{\rm FPT}<\delta_{\rm c}^{\rm RD}$) may have cosmological implications even if it is only modest. Conversion of cosmic radiation into PBHs at early epochs must be an extremely inefficient process if the contribution of PBH mass density to the present closure density, $\Omega_{\rm pbh}$, is not too exceed unity. For Gaussian fluctuations this implies that PBH formation results only from those overdense fluctuations well within the exponentially declining tail of the density distribution function. PBH number density is dominated for fluctuations with $(\delta\epsilon/\epsilon)_{\rm hc}$ in a very narrow range between δ_c and $\delta_c + \sigma^2/\delta_c$, where σ is the variance of the Gaussian distribution. Typically, $\sigma^2/\delta_c \lesssim 10^{-2}$ for $\Omega_{\rm pbh} \lesssim 1$. For approximately scale-invariant Harrison-Zel'dovich spectra of the primordial density fluctuations, such as resulting from a multitude of proposed inflationary scenarios, fluctuations re-enter into the horizon with equal amplitudes on all mass scales. In this case, the slightest reduction of $\delta_c^{\rm FPT}$ as compared to $\delta_c^{\rm RD}$ may result in the formation of PBHs on essentially only the horizon scale during the first-order phase transition, yielding a highly peaked PBH mass function. We note here that there are other proposed scenarios for PBH formation during first-order phase transitions [6]. These usually involve production of seed fluctuations during the transition which collapse to PBHs.

The above considerations have led one of us to propose PBHs formed during the QCD color deconfinement transition at temperature $T_c \approx 100$MeV and with typical masses $M_{\rm pbh} \sim M_{\rm h}^{\rm QCD} \approx 2M_\odot(T/100{\rm MeV})^{-2}$ as a candidate for halo dark matter [3,5]. Compact halo dark matter with approximate masses in the range $0.1M_\odot \lesssim M \lesssim 1M_\odot$ may have been detected by the MACHO [7] and EROS [8] collaborations. They monitored the light curves of millions of stars within the Large- and Small- Magellanic Clouds (LMC and SMC), searching for gravitational microlensing of LMC (SMC) stars by foreground compact objects. The simplest interpretation of their findings is that a significant fraction of halo dark matter is within compact objects of unknown nature. A dynamically significant population of halo red- or white- dwarfs is subject to stringent constraints from either direct observations [9], or considerations of chemical evolution and contribution of MACHO baryon density to the critical density [10]. Nevertheless, recent observations of microlensing of a star in the SMC by a binary lens [11] may favor a cosmologically less interesting interpretation of the MACHO/EROS observations. It has been shown that the binary lens responsible for the microlensing event 98-SMC-1 most likely resides within the SMC itself, and not within the galactic halo. It may well be that lenses responsible for other microlensing events are not within the galactic halo but represent so far unknown stellar populations within the LMC and SMC. Clearly, further observations are needed to reveal the nature and location of the lens population. Note that a population of halo PBHs could be revealed by detection of gravitational waves emitted during PBH-PBH mergers [12].

In this paper we employ a general-relativistic hydrodynamics code to follow the evolution of density fluctuations which enter the cosmological horizon during a first-order phase transition. In Section 2. we briefly summarize the adopted numerical technique which has been discussed in detail by [13] and [2] and introduce the equation of state describing the phase transition. Results and discussion of the PBH formation process are presented in Section 3.

II. THE PHASE TRANSITION AND NUMERICAL SIMULATION TECHNIQUE

The algorithm employed to follow the evolution of spherically symmetric density fluctuations in an expanding Friedman-Robertson-Walker (FRW) universe is the same as the one adopted and described in detail in [13,2]. The method, based on an algorithm developed by Baumgarte *et al.* [13] and modified for use in a FRW background, is particularly well suited not only to simulate the formation of a black hole, but also to follow the late-time evolution of PBHs by virtue of the chosen Hernandez-Misner coordinates [14]. For details concerning the algorithm, zoning, size of time steps and computational domain, etc. the reader is referred to [2]. Initial conditions for the metric perturbations are chosen to be pure energy density fluctuations with Mexican-hat profile

$$\epsilon(R) = \epsilon_0 \left[1 + A\left(1 - \frac{R^2}{R_{\rm h}^2}\right)\exp\left(-\frac{3R^2}{2R_{\rm h}^2}\right)\right] \ , \tag{1}$$

in unperturbed uniform Hubble expansion (essentially synchronous gauge with uniform Hubble expansion condition). Here, R is circumferential radius (i.e. R appears in the angular piece of the metric $R^2 d\Omega^2$, where $d\Omega$ is the solid angle element), which in the absence of perturbations, corresponds to what is commonly referred to as proper distance in cosmology, $r_{\rm p} = a r_{\rm c}$, where a is the scale factor of the universe and $r_{\rm c}$ is the comoving cosmic distance. The distance $R_{\rm h}$ is chosen to be the cosmological horizon distance, $R_{\rm h}(t_0)$, at the beginning of the simulation t_0, i.e.

$$R_{\rm h}(t_0) = a(t_0)\int_0^{t_0} \frac{dt}{a(t)} \ , \tag{2}$$

The increase of horizon distance with cosmic time in the presence of a phase transition is computed numerically. The amplitude A is adjusted such that fluctuation amplitudes are characterized by a certain average overdensity at horizon crossing

$$\left(\frac{\delta\epsilon}{\epsilon}\right)_{\rm hc} \equiv (V_{\rm h}\epsilon_0)^{-1} 4\pi \int_0^{R_{\rm h}(t_0)} (\epsilon(R) - \epsilon_0) R^2 dR - 1 \ , \tag{3}$$

where $\epsilon_0 = \epsilon_0(t_0)$ is the unperturbed FRW energy density at t_0 and $V_{\rm h} = (4\pi/3)R_h^3(t_0)$ is the initial horizon volume.

The properties of the equilibrium first-order phase transition are fully described by a choice for the equation of states of the low- and high-energy density phases, respectively. We adopt a phenomenological bag model for the high-energy density phase, where the energy density is given by the contributions of a quasi-free, extremely relativistic gas with statistical weight g_h plus a temperature-independent self-interaction correction, the bag constant B. We approximate the low-energy density phase as a non-interacting, extremely relativistic gas with statistical weight g_l. Thermodynamic quantities of the individual phases are [15]

$$\epsilon_l(T) = g_l\frac{\pi^2}{30}T^4 \quad ; \quad \epsilon_h(T) = g_h\frac{\pi^2}{30}T^4 + B \ ; \tag{4}$$

$$p_l(T) = \frac{1}{3}g_l\frac{\pi^2}{30}T^4 \quad ; \quad p_h(T) = \frac{1}{3}g_h\frac{\pi^2}{30}T^4 - B \ ; \tag{5}$$

$$s_l(T) = \frac{4}{3}g_l\frac{\pi^2}{30}T^3 \quad ; \quad s_h(T) = \frac{4}{3}g_h\frac{\pi^2}{30}T^3 \ , \tag{6}$$

where s denotes entropy density. Requiring the existence of a first-order phase transition at temperature T_c, i.e. $p_l(T_c) = p_h(T_c)$, we find $\epsilon_h(T_c) - \epsilon_l(T_c) = 4B$, which together with the definition of latent heat, $L = T_c(\partial/\partial T)(p_h - p_l) = T_c(s_h - s_l)$, implies $L = \epsilon_h(T_c) - \epsilon_l(T_c)$. We define the useful quantity

$$\eta = \frac{L}{\epsilon_l(T_c)} = \frac{\epsilon_h(T_c) - \epsilon_l(T_c)}{\epsilon_l(T_c)} \ , \tag{7}$$

describing the strength of the phase transition. Since we assume the transition to proceed in close-to-equilibrium (i.e. negligible super -cooling and -heating) we may use conservation of entropy to relate the ratio of scale factors at the beginning of the transition, a_1, and the end of the transition, a_2, to η

$$\frac{a_2}{a_1} = \left(\frac{g_h}{g_l}\right)^{1/3} = \left(1 + \frac{3}{4}\eta\right)^{1/3} \ . \tag{8}$$

During ordinary radiation dominated epochs the dynamics of fluctuations is self-similar for fluctuations on small length scales collapsing at early times and fluctuations on large length scales collapsing at late times, only dependent on the shape and density contrast of the fluctuation. Similarly, fluctuation dynamics in the presence of a phase transition should be independent of the temperature, energy density, and horizon mass at which the transition occurs. It is dependent, however, on the strength of the transition, η, as well as on the exact time at which the fluctuation crosses into the horizon, in particular, if shortly before onset, during, or shortly after completion of the transition. This "time" may be parameterized by the ratio of cosmic average energy density at fluctuation horizon crossing and some typical energy density at the transition, $\tau_{hc} \equiv \epsilon_0(t_0)/\epsilon_h(T_c)$. The threshold for PBH production is thus a function of τ_{hc} and η only

$$\delta_{\rm c}^{\rm FPT} = \left(\frac{\delta\epsilon}{\epsilon}\right)_{\rm c,hc}(\tau_{\rm hc},\eta) \ . \tag{9}$$

In contrast, the resulting PBH masses for collapsing fluctuations are approximately determined by the horizon mass, $M_{\rm h} \approx (4\pi/3)\epsilon_{\rm h}R_{\rm h}^3(\epsilon_{\rm h},\eta)$, at which the transition occurs, and are only weakly dependent on η and $\tau_{\rm hc}$. For a more accurate determination of the PBH mass spectrum it is necessary to convolve the distribution function for the pre-existing density fluctuations, $P(\delta)d\delta$, with a scaling relation for the resulting PBH masses, $M_{\rm pbh} = kM_{\rm h}(\delta - \delta_{\rm c}^{\rm FPT})^{\gamma}$ as shown in [16]. Here k is a dimensionless constant. This mass spectrum may be somewhat dependent on η and $\tau_{\rm hc}$.

In order to follow the hydrodynamic evolution of fluid elements at T_c, on length scales much larger than the mean separation between high- and low- energy density phase, it is necessary to specify an effective equation of state describing the mixture of phases in thermodynamic equilibrium. Note that the assumption of a near-to-equilibrium, first-order phase transition, implies negligible super -cooling and -heating and a typical mean separation between phases much shorter than the cosmological horizon. These assumptions are met if nucleation of new phase is efficient. Since a fluctuation scale length is of the order of the cosmological horizon the details of the distribution of phases on small scales should have vanishing influence on the all-over fluctuation dynamics. The fluid is in phase mixture, for energy densities between $\epsilon_{\rm h}(T_c)$, where the volume fraction occupied by high-energy density phase, $f_{\rm h} = 1$, and, $\epsilon_{\rm l}(T_c)$, where $f_{\rm h} = 0$. The effective equation of state may thus be written

$$p(\epsilon) = \frac{1}{3}(\epsilon - L) \quad ; \quad v_s^2 = \frac{1}{3} \quad ; \quad \epsilon \geq \epsilon_h(T_c) \; ; \tag{10}$$

$$p(\epsilon) = \frac{1}{3}\epsilon_l(T_c) \quad ; \quad v_s^2 = 0 \quad ; \quad \epsilon_l(T_c) < \epsilon < \epsilon_h(T_c) \; ; \tag{11}$$

$$p(\epsilon) = \frac{1}{3}\epsilon \quad ; \quad v_s^2 = \frac{1}{3} \quad ; \quad \epsilon \leq \epsilon_l(T_c) \; , \tag{12}$$

where $\epsilon_l(T_c)$ and $\epsilon_h(T_c)$ are constants. Note again that the pressure response, v_s, of mixed phase is exactly zero in thermodynamic equilibrium, whereas pressure itself is substantial.

III. RESULTS AND DISCUSSION

We followed the evolution of density fluctuations upon horizon crossing during a cosmological phase transition with scaled latent heat $\eta = 2$ (in particular, we chose $\epsilon_h(T_c) = 1$ and $\epsilon_l(T_c) = 0.5$). Figures 1-3 show results for a fluctuation with overdensity $(\delta\epsilon/\epsilon)_{hc} = 0.535$ entering the cosmological horizon at time $\tau_{hc} = 1$, i.e. when the surrounding universe is at average energy density $\epsilon_h(T_c)$ at the onset of the phase transition. In Figure 1 we show the evolution of the radial energy density profile of the fluctuation from the initial horizon crossing time t_0 to $20.1t_0$. We choose constant proper time slicing, i.e. an individual curve in Figure 1 represents the energy density of all fluid elements evaluated at identical proper time. Energy density is shown as a function of scaled circumferential radius $R_{sc} = (R/R_h(t_0))(a_0/a)$ such that $R_{sc} = constant$ for a fluid element in simple FRW expansion. The horizon at t_0 is located at $R_{sc} = 1$. The two horizontal dotted lines in Figure 1 indicate the regime of energy densities in which fluid elements exist within mixed phases.

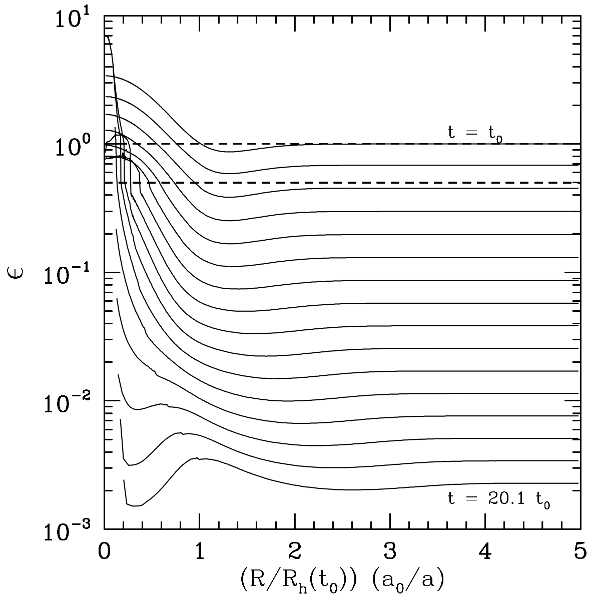

FIG. 1. Energy density, ϵ, as a function of scaled circumferential radius, $R_{sc} = (R/R_h(t_0))(a_0/a)$, for a fluctuation with initial density contrast, $(\delta\epsilon/\epsilon)_{hc} = 0.535$, at horizon crossing. The initial horizon at t_0 is located at $R_{sc} = 1$. From top to bottom, solid lines show the fluctuation at 1., 1.22, 1.49, 1.82, 2.23, 2.72, 3.32, 4.06, 4.95, 6.05, 7.39, 9.03, 11.0, 13.5, 16.4, and 20.1 times the initial time t_0. Constant proper time slicing was used. The horizontal dashed lines indicate the energy densities at onset and completion of the phase transition. The formation of a PBH with $M_{pbh} \approx 0.34M_h(t_0)$ results.

The formation of a PBH with final mass $M_{pbh} \approx 0.34M_h(t_0)$ results from the evolution of the fluctuation shown in Figure 1-3. Here we define the horizon mass at the initial time $M_h(t_0) = \epsilon_0 V_h(t_0)$. The fluctuation's selfgravity exceeds pressure forces such that the fluctuation separates from Hubble flow and recollapses to high-energy densities at the center until an event horizon forms. Subsequent accretion of material on the young PBH continues until the immense pressure gradients close to the event horizon launch an outgoing pressure wave which significantly dilutes the PBH environment. Accretion thereafter is negligible. The existence of a phase transition facilitates the PBH formation process as is evident from Figure 2. Figure 2 is a zoom into the core of the fluctuation shown in Figure 1. For comparison, this figure also shows the evolution of a fluctuation with the same initial conditions, but entering the cosmological horizon during an ordinary radiation dominated epoch, by the dotted line. The strong pressure gradients experienced by the fluctuation entering the horizon during an epoch with equation of state $p = \epsilon/3$ prevent the formation of a PBH.

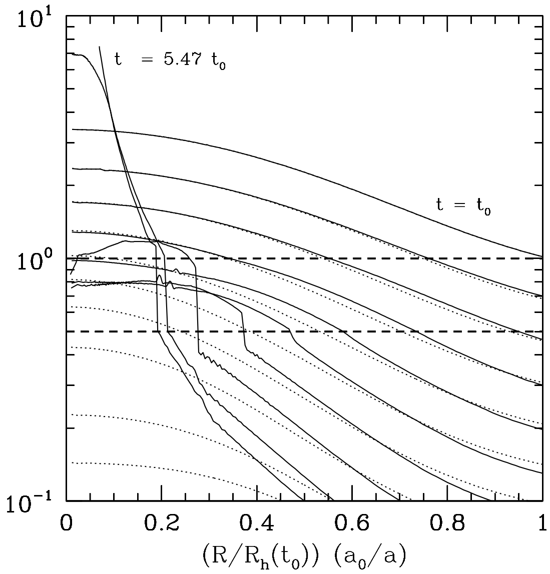

FIG. 2. A zoom into the central region of Figure 1. From top to bottom, solid lines show the fluctuation at 1.0, 1.22, 1.49, 1.82, 2.22, 2.72, 3.32, 4.06, 4.95, and 5.47 times the initial time t_0. The horizontal dashed lines indicate the energy densities at onset and completion of the phase transition. The dotted lines shows, for comparison, the evolution of a fluctuation with the same initial fluctuation parameters, but entering the cosmological horizon during an epoch with equation of state $p = \epsilon/3$.

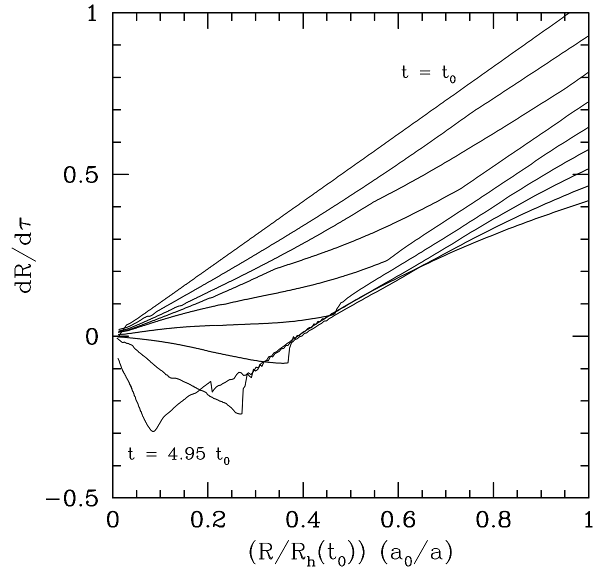

FIG. 3. Coordinate velocity, $\partial R/\partial\tau$, with τ proper time of fluid elements, as a function of scaled circumferential radius, $R_{\rm sc} = (R/R_{\rm h}(t_0))(a_0/a)$. The solid lines show, from top to bottom, the coordinate velocities at times 1.0, 1.22, 1.49, 1.82, 2.22, 2.72, 3.32, 4.06, and 4.95 times t_0 for the same fluctuation and equation of state as shown in Figures 1 and 2 by the solid lines.

The evolution of the fluctuation in the presence of a phase transition proceeds as follows. The initial phase between times t_0 and $\sim 2t_0$ is characterized by a core in high-energy density phase surrounded by an envelope in mixed phase. Cosmological expansion results in the decrease of energy densities in core and envelope. Nevertheless, the fluctuation's overdensity decelerates the material with respect to FRW expansion. This is evident from Figure 3, which shows the coordinate velocity $\partial R/\partial\tau$ for the fluctuation shown in Figures 1 and 2, with τ proper time. In unperturbed Hubble flow, coordinate velocities would be straight lines with decreasing slope for increasing cosmic time. The deceleration of the expanding fluid is stronger for those fluid elements existing in mixed phase due to the absence of pressure gradients. Material in the core remains in high-energy density phase until $t \approx 2.2$. Its deceleration is first stronger than FRW due to selfgravity until at $t \approx 2t_0$, the increasing pressure gradient begins to counteract the collapse. The pressure gradient would give rise to the launching of a pressure wave and the dispersion of the fluctuation if core and envelope wouldn't join at $t \approx 2.2t_0$ and exist in mixed phase. In the subsequent phase between times $\sim 2.2t_0$ and $\sim 4t_0$, a distinctively separated core develops. It is distinct through discontinuities in energy density and coordinate velocity. The core first undergoes homologous, but sub-Hubble, expansion which turns into homologous contraction at $t \approx 3.3t_0$. The core energy density is almost uniform and increases with time. Initially the core mass-energy still reduces somewhat. Shells on the boundary of the core are expelled when they experience strong pressure gradients as their energy density falls below $\epsilon_{\rm l}$. In the final phase of collapse of the core region between $\approx 4t_0$ and $\approx 5t_0$ the fluid exists exclusively in high-energy density phase surrounded by fluid in low-energy density phase. The collapse proceeds no further homologously, rather, inner parts of the core contract faster than outer parts. At $t \approx 5t_0$ an event horizon forms. The resulting young PBH rapidly increases its mass up to $M_{\rm pbh} \approx 0.06 M_{\rm h}(t_0)$ at $t \approx 5.5t_0$. Subsequent slow long-term accretion onto the PBH increases the PBH mass further to $M_{\rm pbh} \approx 0.34 M_{\rm h}(t_0)$.

The dynamics of PBH formation during a first-order phase transition is different from the one experienced during ordinary radiation dominated epochs. In the latter there is no obvious segregation between collapsing fluctuation and expanding universe, in particular, there are no discontinuities. Further, the collapse of fluctuations during ordinary radiation dominated epochs does not proceed in a homologous fashion. Figures 2 and 3 reveal the existence of instabilities in the evolutionary calculations of the PBH formation process. It may be observed that material within mixed phase develops a small-scale perturbation of growing amplitude. This is not surprising, since with $v_{\rm s} = 0$ in mixed phase the fluid is Jeans-unstable to collapse on all scales. The perturbation extends over many zones and its amplitude and size are unchanged with an increase of numerical viscosity and/or number of zones. We note that there may be other physical instabilities for $v_{\rm s} = 0$ which we, however, do not observe due to our choice of initial conditions. It has been argued that an instability of non-gravitational origin exists for sound waves which experience a sudden drop in the speed of sound [4,17]. Finally, we also observe a numerical instability of the adopted algorithm which occurs when the energy density of fluid shells falls slightly below $\epsilon_{\rm l}(T_{\rm c})$. Such shells experience immense acceleration when leaving the regime of mixed phase with large energy density gradients. The amplitude of the perturbations may be regulated by our choice of numerical viscosity, nevertheless, the final black hole mass is virtually unaffected by an increase/decrease of numerical viscosity.

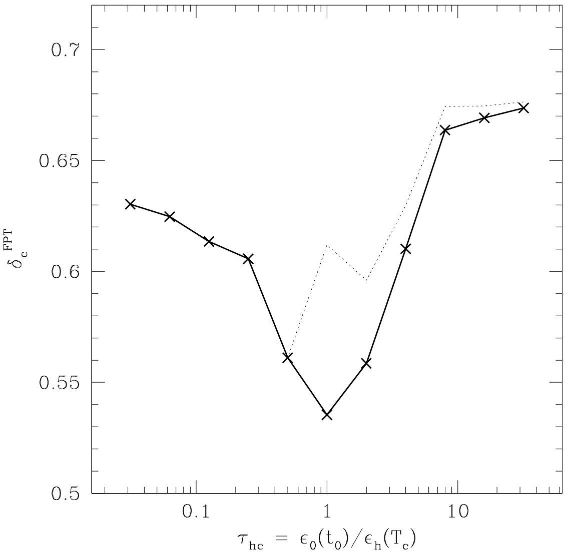

FIG. 4. Energy overdensity threshold for PBH formation, $\delta_{\rm c}^{\rm FPT}$ (solid line), for fluctuations entering the cosmological horizon during, or close to a first-order phase transition, as a function of horizon crossing time, $\tau_{\rm hc} = \epsilon_0(t_0)/\epsilon_{\rm h}(T_{\rm c})$. The energy densities at the onset and completion of the transition are chosen $\epsilon_{\rm h}(T_{\rm c}) = 1$ and $\epsilon_{\rm l}(T_{\rm c}) = 0.5$, respectively. The crosses are points determined from numerical simulation. The solid line is an interpolation between crosses. The dotted line shows $\delta_{\rm c}^{\rm FPT}/(1+w)$, with w the cosmic average p/ϵ at horizon crossing of the fluctuation. (See text for further explanations).

The main result of our study is displayed in Figure 4. This figure shows the threshold for PBH formation, $\delta_{\rm c}^{\rm FPT}$, for a phase transition with scaled latent heat, $\eta = 2$, as a function of the horizon crossing time of the fluctuation, $\tau_{\rm hc}$, by the solid lines. Crosses represent the lowest $(\delta\epsilon/\epsilon)_{\rm hc}$ for which PBH formation results. The relative accuracy in $\delta_{\rm c}^{\rm FPT}$ is estimated to be on the 1% level. It is evident that over a range of horizon crossing times the energy overdensity required for PBH formation is below that for PBH formation during ordinary radiation dominated epochs. It should be clear from the introductory remarks that the slightest bias in favor of forming PBH during a phase transition may imply that essentially all PBH are formed on the scale associated with minimum $\delta_{\rm c}^{\rm FPT}$. However, assuming a Harrison-Zel'dovich, exactly scale-invariant spectrum for the underlying density perturbations, the amplitudes of energy density perturbations in uniform Hubble constant gauge upon horizon crossing are not *exactly* constant during epochs with varying equation of state. Rather, modes with varying length scale cross into the horizon with equal $\xi = (1+w)^{-1}(\delta\epsilon/\epsilon)_{\rm hc}$ [18]. Here $w = p/\epsilon$, such that for w smaller than 1/3, as in Eq. (10) - Eq. (12) for $\epsilon > \epsilon_{\rm l}(T_{\rm c})$, the horizon crossing amplitude $(\delta\epsilon/\epsilon)_{\rm hc}$ of perturbations is reduced. This partially decreases the bias to form PBH during phase transitions. In Figure 4 the dotted line shows a $\tilde{\delta}_{\rm c}^{\rm FPT}$ which corrects for this effect. Remarkably, the combined effects of reducing the threshold for PBH formation by the dynamics of fluctuations during a first-order transition and the decrease of the horizon crossing amplitude of energy density fluctuations due to the changed equation of state, result in a double dip structure.

We have also computed test runs for fluctuations entering the horizon during a phase transition with $\eta = 10$. Our results indicate that with increased strength of the phase transition $\delta_{\rm c}^{\rm FPT}$ is further decreased. With the discovery of critical phenomena in general relativity it has been recognized that the resulting black hole masses in nearly critically collapsing space-times obey scaling relations [19], such as $M_{\rm pbh} = K(\delta - \delta_{\rm c})^{\gamma}$. For a given matter model, γ is independent of initial conditions, while K depends on the specific choice of the control parameter δ. Specifically, a collapsing radiation fluid has $\gamma \approx 0.36$ [20]. In our case, δ may be associated with $(\delta\epsilon/\epsilon)_{\rm hc}$ and K with $kM_{\rm h}$. We have explicitly verified that $\gamma \approx 0.36$ also holds for the cosmological PBH formation process during ordinary radiation dominated epochs [2]. We have attempted to derive a scaling exponent γ for the PBH formation process during epochs with equation of state given by Eq. (10) - Eq. (12). Our numerical simulations do not clearly indicate a simple scaling relation as above. Preferred values of γ were rather large (i.e. $\gamma > 2$), even though somewhat dependent on the range of black hole masses to which we fit the scaling relation. In contrast to our work in [2], convincing verification of a scaling relation and a value for the exponent γ would require the use of adaptive mesh techniques. This would allow to simulate the formation of PBH with masses far smaller than $0.1M_{\rm h}$. We note that convolving a Gaussian density perturbation probability function with the preferred scaling relation of our numerical simulations would predict average masses $M_{\rm pbh}$ far below $0.1M_{\rm h}$. Clearly, resolution of this issue would be desirable.

IV. CONCLUSIONS

We have performed hydrodynamic general-relativistic simulations of the PBH formation process when initially super-horizon energy density fluctuations enter the cosmological horizon during a first-order phase transition. We have verified that the significantly diminished pressure forces for fluid undergoing the phase transition facilitate the PBH formation process. We have shown that the threshold of the amplitude of energy density fluctuations above which PBH formation results is smaller for fluctuations crossing into the horizon during a first-order phase transition than for fluctuations crossing into the horizon when the equation of state is ordinary (i.e. $p = \epsilon/3$). PBH formation from pre-existing adiabatic fluctuations is more probable during first-order transitions, even when the reduction of fluctuation amplitudes of Harrison-Zel'dovich type which cross into the horizon for fluid pressures smaller than $\epsilon/3$, is taken into account. Our simulations could not clarify if simple scaling relationships with scaling exponent $\gamma \approx 0.36$ for the resulting PBH masses, as found in many studies of critical phenomena in general relativity, also apply to the PBH formation process during cosmological first-order phase transitions. Our simulations favor much larger scaling exponents, possibly yielding small typical PBH masses (in units of the horizon mass). Further study with adaptive mesh techniques is required to resolve this issue.

Our results may have cosmological implications for the PBH mass spectrum. It was so far believed that PBH formation from pre-existing adiabatic perturbations in the early universe is equally likely on all scales, only dependent on the statistics of the pre-existing density perturbations. The dynamics of fluctuations during cosmological phase transitions may easily introduce a strong enough bias to form PBH only on the approximate horizon mass scales during epochs with phase transitions. This may yield a highly peaked mass function for PBHs. Here the term cosmological phase transition may be loosely interpreted, since a reduction of pressure forces is not only anticipated during first-order phase transitions, but may also occur during higher-order transitions, cross-overs, and particle annihilation periods. A prime candidate for such a transition is QCD color-confinement due to the large number of degrees of freedom participating in the transition. Whether, or not, PBHs have been produced in the early universe in cosmologically interesting numbers remains an open question.

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