Equilibria for horizontal fine structures in quiescent prominences

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Abstract

We investigate the equilibrium of horizontal thread structures in quiescent prominences. We find that extended horizontal threads cannot lie along magnetic flux tubes unless the field is exactly horizontal. The reason for this is that the pressure scale height is very small. We have therefore constructed a class of exact magnetic equilibrium models for single horizontal threads which lie along the prominence axis. This model can be extended to describe approximate equilibria for multi–thread configurations. It is intended to use these models for future 2D radiative transfer calculations.

1 Introduction

Quiescent prominence can be characterised as thin vertical sheets of cool plasma which extend high into the solar corona. A more detailed analysis shows that these sheets are made up of many individual fine structures. There is both direct and indirect evidence for the existence of these fine structures. H_α filtergrams show elongated thin structures, generally called threads, as well as blob – like structures (Engvold, 1976). Zirker and Koutchmy (1990, 1991) analysed H_α spectra of prominences observed on the limb in detail and found convincing evidence that thread – like structures occur on spatial scales much smaller than the presently achieved resolution.

Indirect evidence for the existence of fine structures came from a recent study by Anzer and Heinzel (1998) in which they analysed the observations of Bommier et al (1994). They calculated models for magnetic equilibrium and for NLTE radiative transfer for all prominences described in the paper by Bommier et al. They found large differences of the physical parameters obtained with the two methods. From these result they concluded that the effects of prominence fine structure have to be included in any realistic model. In this paper we shall present some magnetic equilibrium models of such thread – like fine structures which could play an essential role in the description of quiescent prominences.

2 Vertical threads

High resolution observations show that fine structures can have all kinds of orientation. They can be inclined with respect to the vertical direction and they can also have finite angles relative to the prominence axis. But there seems to be some preference for vertical threads. For this reason Heinzel and Anzer (1999b) have developed equilibrium models for such vertical structures. Their models are based upon the one–dimensional Kippenhahn–Schlüter slab model. Thread–like equilibrium structures can be obtained by an appropriate variation of the physical parameters (column density, gas pressure, magnetic field etc.) along the prominence axis. Vertical variations are ignored and therefore these models are two–dimensional. The resulting 2D radiative transfer problem is also treated in the paper by Heinzel and Anzer (1999 b).

The 2D equilibrium models of Low (1982) are not very useful for our purposes. These models are derived under the assumption that all spatial functions are separable and therefore the set of possible equilibria is strongly restricted. One finds in particular that for sagging angles of the magnetic field smaller than 45° the temperature is highest where the density is highest and vice versa. And one also notes that in these models a vanishing magnetic sagging implies zero gas temperature. These are quite unrealistic properties and for these reasons we did not include these models into our investigation.

3 Horizontal threads

Such horizontal fine structures could in principle be aligned with the magnetic field vector like in coronal loops. But there is also the possibility of an arbitrary orientation with respect to the magnetic field.

3.1 Threads in direction of field lines

For this discussion we shall use the standard coordinate system for prominences: x–direction normal to prominence, y–direction along prominence, z–direction vertically upward. In this section we are interested in the structure of an individual magnetic flux tube which is filled with dense cool plasma. Anzer and Heinzel (1998) obtained for the width of a 1D prominence slab

$$D \approx 6H \frac{B_x}{B_{z1}} \tag{1}$$

where $H = kT/\mu g$ is the scale height,μ the mean mass per particle, B_x the value of the normal field component and B_{z1} the vertical field component at the prominence surface. If the field is sheared one obtains for the length measured along the field

$$L \approx 6H \frac{B_h}{B_{z1}} \tag{2}$$

where $B_h = \sqrt{B_x^2 + B_y^2}$ is taken. These equations were derived for a 1D slab but they also apply to individual fine structures. One can also derive a relation between the field line inclination, Ψ_1 and the values of the plasma beta (see Heinzel and Anzer, 1999a). With Ψ_1 = arctg (B_h/B_{z1}), β_c and β_0 the values of β in the center and at the surface, respectively, one has

$$tg\Psi_1 = \frac{1}{\sqrt{\beta_c - \beta_0}} \tag{3}$$

For typical values $\beta_c \gtrsim 0.1, \beta_0 = 0.01$ and H = 150 km one gets form Equations (2) and (3)

$$L \lesssim 3000 \ \ km \tag{4}$$

From this estimate one has to conclude that field aligned threads will be extremely short, typically $\lesssim 4$ arc sec. Therefore they will resemble blobs rather than elongated horizontal threads. (Even if we take $\beta = 0.03$ we get only L = 6000 km). From this we conclude that all extended threads will not be aligned with the magnetic field. They have to lie at a finite angle to the magnetic field.

3.2 Threads along the prominence axis

The simplest configuration of such elongated horizontal threads is therefore that of structures which lie along the prominence axis. In this section we shall construct equilibrium models for single horizontal threads which interact with a given background magnetic field. This background field has to be horizontal but it may have a shallow dip. In order to obtain simple equilibria we assume a uniform background field

$$\mathbf{B}_0 = B_{x0}\mathbf{e}_x + B_{y0}\mathbf{e}_y \tag{5}$$

where B_{x0} and B_{y0} are constant. In addition we assume that the threads have cylindrical symmetry. We take

$$r^2 = x^2 + (z - z_c)^2 \tag{6}$$

with z_c the height of the thread center. The thread extends to $r = r_1$. For $r < r_1$ we then have $\mathbf{B}_1 = B_\varphi(r)\mathbf{e}_\varphi, \rho = \rho(r), p = p(r)$ and $T = T_p$ is constant. Outside the thread we have a low β plasma with $p = p_0, \rho \approx 0, T = T_0$ and $\mathbf{B}_1 = B_\varphi(r_1)(r_1/r)\mathbf{e}_\varphi$, where p_0 and T_0 are the coronal values. The total magnetic field is then given by: $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1$

Magnetic equilibrium then requires

$$\frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B} + \rho \mathrm{g} - \nabla p = 0 \tag{7}$$

where all quantities have their standard meaning. Because of the special symmetry of our problem we can write Equation (7) in the following way:

$$\rho g = \frac{1}{4\pi} B_{x0} \frac{1}{r}\frac{d}{dr}(rB_\varphi) \tag{8}$$

and

$$\frac{dp}{dr} = -\frac{1}{4\pi} B_\varphi \frac{1}{r}\frac{d}{dr}(rB_\varphi) \tag{9}$$

Equation (8) describes the balance of those froces which are vertical everywhere and Equation (9) deals with the forces which are purely radial. Combining Equation (8) and (9) we arrive at

$$HB_{x0}\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(rB_\varphi)\right)+B_\varphi\frac{1}{r}\frac{d}{dr}(rB_\varphi)=0. \tag{10}$$

Here $p = gH\rho$ has been used. If we introduce the following dimensionless quantities

$$X=\frac{r}{H} \tag{11}$$

and

$$Y=\frac{rB_\varphi}{HB_{x0}} \tag{12}$$

we finally obtain the following equation:

$$XY''+(Y-1)Y'=0. \tag{13}$$

This is a non–linear ODE whose solutions can be found in text books (e.g. Kamke, p. 563, Equation (6.7.8)). It has the general solution.

$$Y(X)=\frac{(2+\gamma)X^\gamma+a(2-\gamma)}{X^\gamma+a} \tag{14}$$

Because of $B_\varphi \to 0$ for $r \to 0$ one also has the condition $Y \to 0$ for X $\to 0$. This can be only fulfilled if $\gamma = 2$ is taken. From Equation (14) we can then calculate the magnetic field:

$$B_\varphi(r)=4HB_{x0}\frac{r}{r^2+aH^2} \tag{15}$$

One also obtains

$$\rho(r)=\frac{2B_{x0}^2}{\pi g}aH^3\frac{1}{(r^2+aH^2)^2} \tag{16}$$

and

$$p(r)=\frac{2B_{x0}^2}{\pi}aH^4\frac{1}{(r^2+aH^2)^2} \tag{17}$$

The total mass of the thread per unit length in y-direction is given by:

$$M_{tot}=\frac{2B_{x0}^2H}{g}\frac{r_1^2}{r_1^2+aH^2} \tag{18}$$

If we now require that all field lines are connected directly to the photospheric boundary (i.e. no reconnection inside thread) then $B_\varphi < B_{x0}$ has to hold everywhere. This leads to the following condition:

$$a>4 \tag{19}$$

The condition that all field lines of the configuration are directly connected to the solar surface is not really necessary for our models. In principle reconnection could ocur in such configurations. Then $a < 4$ will be possible and the thread mass can be larger. The reconnected field lines are helices which extend to infinity in our simple models. In reality they are connected to the solar surface at the ends of the prominence.

The outer boundary is defined by the pressure balance $p(r_1) = p_0$, which gives:

$$r_1 = H\sqrt{4\sqrt{\frac{a}{\beta^*}} - a} \tag{20}$$

where we used the definition:

$$\beta^* = \frac{8\pi p_0}{B_{x0}^2} \tag{21}$$

Equation (20) gives an upper limit on a and therefore the parameter a is confined by

$$4 < a < \frac{16}{\beta^*} \tag{22}$$

The mass can now also be written in terms of β^*:

$$M_{tot} = \frac{2B_{x0}^2 H}{g}(1 - \frac{1}{4}\sqrt{a\beta^*}) \tag{23}$$

and it is confined by

$$M_{tot} < \frac{2B_{x0}^2 H}{g}(1 - \frac{1}{2}\sqrt{\beta^*}) \tag{24}$$

These equations define a one parametric set of thread equilibria where the governig parameter a has to lie in the range specified by the relations (22). M_{tot} decreases monotonically with increasing a. Field line plots for the cases a = 6 and a = 15 are shown in Figures 1 and 2. Here the lengths are in units of the scale heigth H and the center of the threads has been shifted to z = 0.

Our sequence of equilibria is in some sense the analogue of the sequence of 1D slab configurations of the Kippenhahn–Schlüter type. In this slab model all quantities depend only on the x–coordinate, where as in our new model they depend instead on the radial coordinate r. All physical quantities are axisymmetric with the exception of the magnetic field. The reason for this non–symmetry is the fact that the field represents a superposition of an azimuthal field and a constant horizontal field. An equivalent to our parameter a is the quantity B_{z1}, in the slab models. In our models a determines the total mass, whereas for slab models B_{z1} fixes the column mass.

4 Discussion

In the preceding section we have derived exact one–dimensional equilibrium solutions for horizontal threads. The models are characterised by the free parameter a and the mass of the thread ranges from zero to a maximum value of $2B_{x0}^2 H/g$ which is obtained for the limit of $\beta^* \to 0$.

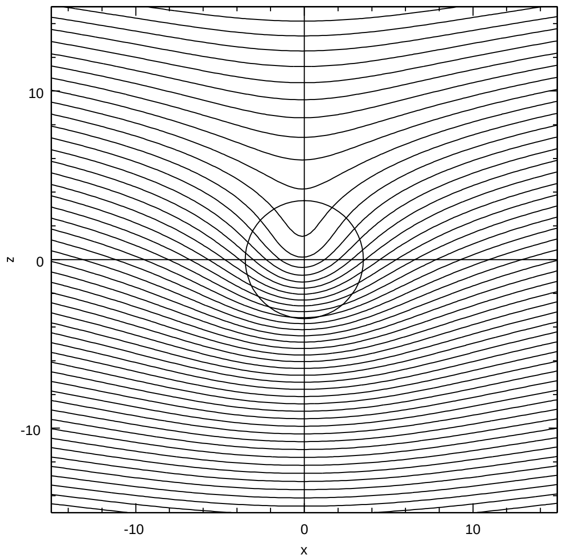

Figure 1: Plot of magnetic field lines for the case of *a* = 6 (heavy thread). Lengths are given in pressure scale heights. The circle $r = r_c \approx 3.5H$ represents the boundary inside which most of the thread mass is concentrated.

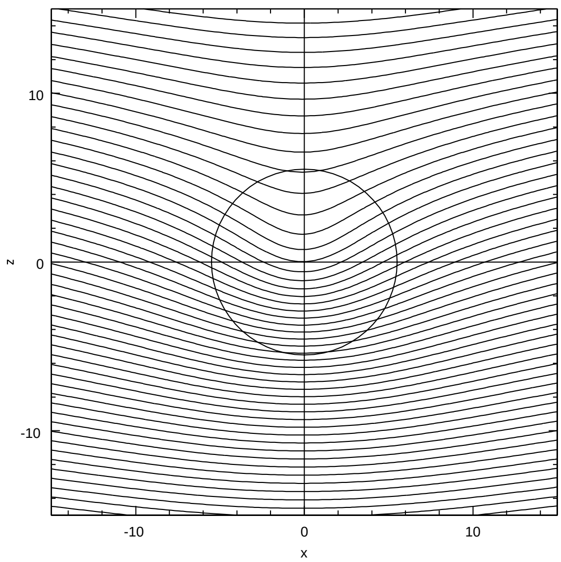

Figure 2: Same as Fig. 1, but for the case of *a* = 15 (light thread); $r_c \approx 5.5H$.

The radius r_1 is that distance at which the cool prominence configuration is matched to the hot corona. This radius of course will depend on the assumed value of the coronal pressure and therefore on β^*. As long as β^* is small and the parameter a not too large one has:

$$r_1 \approx \sqrt[4]{\frac{16a}{\beta^*}} H. \tag{25}$$

If we take typical prominence values of $B_{x0} = 5G$, H = 150 km and a coronal plasma $\beta^* = 10^{-2}$ we obtain a maximum thread mass of $M_{tot} = 3 \times 10^4$ g/cm and $r_1 = 1300$ km; for $\beta^* = 0.1$ one obtains $r_1 \approx 700km$. Since the outer parts of the cool threads have very low densities (in particular if β^* is very low) these regions will not be visible in the hydrogen lines. Therefore the observed thread radii can be much smaller than r_1. A more reasonable quantity to compare with observations would be a radius inside of which a sizeable fraction of the total thread mass is concentrated. If we take for example a compactness radius r_c inside of which 2/3 of the total mass is concentrated and also assume $\beta^* \ll 1$ we obtain $r_c = \sqrt{2a}H$. This new radius will be somewhere between 3*H* and 5*H* for a in the range 4 to 15. Such dimensions seem quite realistic for fine structure threads.

The model which we have developed in the previous section deals with the equilibrium of a single horizontal thread. But it can be extended easily to describe a multiple thread configuration with threads placed at different heights. As an ilustration we shall discuss the case of two threads. We assume that both threads can be modelled by the equilibrium equations (15) to (18). Then the background field at the position of each thread will be changed due to the presence of the other thread. The field at the center of the lower thread is increased by a factor $(1 + 4H\delta/(\delta^2 + a_2H^2))$ and that of the upper thread reduced by $(1 - 4H\delta/(\delta^2 + a_1H^2))$, with δ being the distance between the two threads, and a_1 and a_2 the thread mass parameters. Therefore the equilibrium masses will also have to be modified.

One also finds that the new background field is no longer uniform. The fieldstrength varies with height and the field is curved upward. This then means that our simple model is no longer in an exact equilibrium, but the resulting deviations will be small. If we take for example the radii of the two threads = 5*H* and $\delta = 20H$, then the field strength differnces are $\Delta B_0 \approx 0.1B_{x0}$ and $B_{z0} < 0.05B_{x0}$. Therefore the deviations from magnetohydrostatic equilibrium will be less than 10% everywhere in the threads and therefore this model seems quite acceptable.

In this paper we have shown that equilibrium solutions for horizontal threads along the prominence axis are theoretically possible. But the question to what extent such structures actually occur in quiescent prominences is still open. Limb observations show that most fine structures are vertical, but there are also a few horizontal ones. On the disc one finds both aligned and also inclined structures. We therefore conclude that at least some of the fine structures can be modelled by our equilibria.

The other question is how such elongated threads can actually form. One can imagine that somewhere along the prominence axis a local condensation occurs. Then the original field $\mathbf{B_0}$ will be distorted. In the lower parts of the condensation B_x will be increased, in the upper parts it will be decreased whereas B_y will remain unchanged.

Then the modified field lines lie no longer in planar surfaces. Therefore they will interact with neighbouring regions along the axis and this interaction can provide a seed mechanism fo the formation of elongated thread – like structures.

All our models, independent of the orientation of the threads, are based upon the assumption that quiescent prominences are in magnetic equilibrium and therefore the field has to be predominantly horizontal. This assumption has been challenged recently by Zirker et al. (1998). These authors have made high resolution disc observations of prominences. They found that sometimes the observed flows have fairly large vertical components. And there is evidence for counter streaming in neighbouring regions. From this they concluded that these flows have to be field–aligned and therefore the fields in these prominences cannot be horizontal. But this is not the only possible interpretation. One can also think of material suspended in locally horizontal fields which are anchored in the neighbouring photosphere. If the photospheric foot points of the field move around then the suspended material will be lifted upward or pulled downward, depending on the orientation of the photospheric flow. Now if neighbouring regions in the photosphere experience flows with opposite directions then this can lead to the observed counterstreaming. An additional advantage of our interpretation is the fact that it can naturally explain flow velocities which are orders of magnitude below the free fall velocieties in large prominences. From these arguments we conclude that at present there is no compelling evidence to abandon magnetic equilibrium modelling of quiescent prominences.

Acknowledgement

It is a pleasure to thank Petr Heinzel and Friedrich Meyer for clarifying remarks.

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