# Colour transformations for isochrones in the $VI$-plane

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**Abstract.** The increase in high-quality $VI$-photometry for globular clusters requires the transformation of theoretical isochrones to the $(V-I)$ colour. We present a combination of a theoretical transformation for main-sequence stars combined with an empirical one for giants. This combination allows isochrone fits, which agree well with observations at all metallicities under consideration ($-2 \lesssim$ [Fe/H] $\lesssim -0.7$) for cluster parameters derived from $BV$-photometry. Vice versa, cluster ages and reddenings derived from $VI$-data agree with those from $BV$-photometry. In contrast to the presented transformation, many other published transformations predict inconsistent colours as we demonstrate by mutual comparison. This implies that the quality of isochrone fits is not necessarily equivalent to that of the stellar models or derived cluster parameters. However, our isochrones are now well-tested and reliable for $BV$ and $I$ photometric data.

**Key words:** Stars: Population II – low-mass – late-type – Hertzsprung-Russell-Diagram – Globular clusters : general

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## 1. Introduction

The last step in comparing theoretical isochrones to globular cluster colour-magnitude diagrams (CMD) is the transformation of luminosity and effective temperature into brightness and colour. While there are a number of both theoretical and empirical sets of transformations available, this last step is, nonetheless, difficult and critical. Known shortcomings in the theoretical transformations, which are obtained from theoretical atmosphere calculations are, e.g. incomplete line lists for the opacity computation which affect the derived spectra and broad band colours, treatment of the atmospheric convection which also affects the colour determination (Gratton et al. 1996), incomplete coverage of the $\log g$–$\log T_{\rm eff}$ and composition parameter space, while empirical relations are naturally based on a limited number of stars. The transformations of Kurucz and coworkers (Kurucz 1992; Castelli et al. 1997) have well-known problems with the colours of red giants (Gratton et al. 1996; Salaris et al. 1997; Grundahl et al. 1998), but are nevertheless widely used. It is generally believed that the bolometric corrections, which are more important for most methods of cluster age determinations, are reliable, at least in their differential properties, and indeed results based on different transformations are in reasonable agreement (Salaris et al. 1997).

While there is also the persistent concern about the unsolved problem of convection in the envelopes of cool stars, it appears that with the present input physics for stellar evolutionary models there is in fact not too much of freedom to vary the mixing-length parameter, if one wants to match both the solar radius and the effective temperature of metal-poor giants (Salaris & Weiss 1998). The existence of a rather constant value for this parameter is also supported by comparing stellar models with the structure of convective envelopes computed by means of 2-dimensional hydrodynamical models (Ludwig et al. 1997; Freytag 1998).

The confidence in ages derived from theoretical isochrones is higher if the isochrone is able to reproduce the complete CMD from the lower main sequence (MS) to the tip of the red giant branch (RGB) to high accuracy, even if in practice only single points along the isochrone may be used for determining the cluster age (e.g. the turn-off and horizontal branch). Reliable colours become crucial for methods that use colour differences, e.g. that between turn-off (TO) and RGB, for (absolute) age determinations, for determining distance and age from the MS-fitting technique, or for deriving correct integrated colours from theoretical isochrones, which constitutes an necessary step for studying the GC stellar populations in unresolved galaxies.

In our previous papers on this subject (Salaris et al. 1997; Salaris & Weiss 1997, "SW97"; Salaris & Weiss 1998, "SW98") we have taken great care to find transformations from the theoretical Hertzsprung-Russell-Diagram (HRD) to the CMD in $V-(B-V)$, which give a satisfying fit for *all* metallicities ranging now from $[\mathrm{Fe/H}] = -2.3$ to $-0.6$. We found that a combination of the transformations of Buser & Kurucz (1978) and Buser & Kurucz (1992) with an appropriate shift in colour as to make the transition smooth satisfied our needs. The importance of reliable, RGB colours was demonstrated later by our finding (SW98) that disk clusters (47 Tuc, M 71, NGC 6352) probably have almost solar helium content, which was based on the inconsistency between relative ages with respect to clusters of similar metallicity as derived from the colour difference between TO and RGB and from the brightness difference between TO and zero age horizontal branch (ZAHB).

Work on globular cluster dating during the last few years has received new attention. Due to improved models and methods and lately due to HIPPARCOS-based distances to clusters, much lower ages are obtained than before, such that, at this time, no serious conflict between cluster ages and the age of the universe exists (SW98; Chaboyer 1998). In order to extend our work to a larger sample of galactic halo clusters and in particular to those of the bulge (both needed for questions about the formation of the Galaxy), it is necessary to work in the $V-(V-I)$–CMD[1], because there is now an increasing number

[1] If not stated differently, the $I$-band refers to the Cousins system

of high-quality observations of clusters in $V-(V-I)$. However, up to now, well-tested isochrones for this colour have not been available.

In this paper, we demonstrate that presently no single transformation from $\log L$–$\log T_{\rm eff}$ to $M_V-(V-I)$ exists that appears to be completely correct or reliable. The requirements we must impose are the following: (i) the transformation should give RGB colours consistent with the determined age; (ii) an isochrone that fits a cluster well in $(B-V)$, must do so in $(V-I)$, too; (ii) there should be agreement between theoretical and empirical transformations at least for those cases where the latter ones are well-defined. In Sect. 2 we will present a number of existing theoretical and empirical transformations and apply them to a few theoretical isochrones. We will show that considerable differences in the resulting colours exist and that there is no preferred transformation to be selected. However, at the same time the reliability of the bolometric correction in $V$ will be reconfirmed. In Sect. 3 we present our pragmatic approach to the problem of $(V-I)$-transformations by introducing an effective temperature–colour transformation based on different published transformations and show tests verifying its usefulness. By no means this is what one really wants for isochrone fittings, but at present it appears to be the only approach promising convincing results. Finally, a discussion and summary closes the paper (Sect. 4).

## 2. A comparison of different published colour-transformations

### *2.1. Transformation sources*

For the purpose of this investigation we have used a number of published transformations, which we briefly describe in the following:

–Buser & Kurucz (1978; 1992, "BK92"): This is the source for synthetic colours we have been using in our previous papers, showing that an appropriate combination (henceforth called "BK") yields satisfactory CMD-fits for all metallicities in the $V-(B-V)$ plane. The transformations are based on theoretical atmospheres and are available for all metallicities and both for dwarfs and giants. $(V-I)$ colours are available only for the cooler stars on the lower MS and on the RGB (BK92).

–ATLAS9: This is a set of spectra and colours obtained from a model atmosphere grid computed with the ATLAS9 code (Kurucz 1992), covering the range of O-K stars. The first set of ATLAS9 colours was discussed in Kurucz (1992). The convective models of this set were later revised ("K95"; see Bessell et al. 1998); the influence of the treatment of convection in the ATLAS9 model atmospheres on some colour indices has been discussed by Castelli et al. (1997). The latest grid of ATLAS9 colours, computed from "no-overshoot" models is presented in Bessell et al. (1998) for the solar metallicity. In this paper we will use this latest set of colours (hereinafter "BCP98") extended to lower metallicities by Castelli (1997, private communication).

–Lejeune et al. (1997, 1998; "LCB"): Synthetic spectra and colours based on various sets of theoretical atmospheres, among them K95 (covering almost completely the range of gravities and $T_{\rm eff}$ spanned by our isochrones). The theoretical spectra for the solar metallicity have been corrected to match empirical colour-$T_{\rm eff}$ relations; since comprehensive empirical calibration data have only been available for the full temperature sequences of solar-abundance giant and dwarf stars, the spectra (and derived colours) of lower metallicity models have been consequently corrected in such a way as to preserve the monochromatic flux ratios predicted by the models.

–Green (1988, "Yale"): An empirical UBVRI recalibration of VandenBerg & Bell (1985) and Kurucz (1979) synthetic colours and bolometric corrections.

Note that all theoretical work up to now is for solar metal ratios only, i.e. no enhancement of the $\alpha$-elements has been considered. Work along this line is in progress (Castelli 1998, in preparation).

–Alonso et al. (1996, "AAM96"): Empirical determination of the $T_{\rm eff}$-colours relation for main sequence stars as a function of metallicity, using a large sample of dwarfs and subdwarfs. Since AAM96 use the Johnson–$I$-band, we have transformed Johnson–$(V-I)$ into Johnson–Cousin–$(V-I)$ colours according to Fernie (1983) throughout this paper.

–Montegriffo et al. (1998, "M98"): An empirical determination of bolometric correction-colour, colour-colour and $(V-K)-T_{\rm eff}$ relations obtained from photometric observations of 6500 GC giants covering metallicities from [Fe/H] $= -2.2$ to 0.0. The results are averaged over two mean metallicity ranges below ("poor") and above ("rich") [Fe/H] $= -1.0$.

–von Braun et al. (1998, "BCM98"): Empirical colour–$(V-K)$ relations for giants in GC of low and intermediate metallicity as well as one open cluster. These relations, together with an empirical $T_{\rm eff} - (V-K)$ relation taken from the literature, provide direct $T_{\rm eff}$-colour transformations for GC giants, taking into account their dependence on metallicity.

In the empirical work, $\alpha$-element enhancement is considered only as far as the observed stars show it. There is no systematic differentiation between solar-type and $\alpha$-enhanced populations.

### *2.2. Isochrones in the $V-(B-V)$–CMD and the bolometric correction*

The theoretical isochrones used throughout this paper were computed for metallicities ranging from [Fe/H] $= -2.3$ up to [Fe/H] $= -0.6$, and are based on the same stellar evolutionary tracks as in our previous papers (e.g. SW98), except for the fact that meanwhile we have extended the calculations to the tip of the RGB, i.e. to the onset of the core helium flash for all masses considered. All isochrones used here are for $\alpha$-element enhanced mixtures. [2]

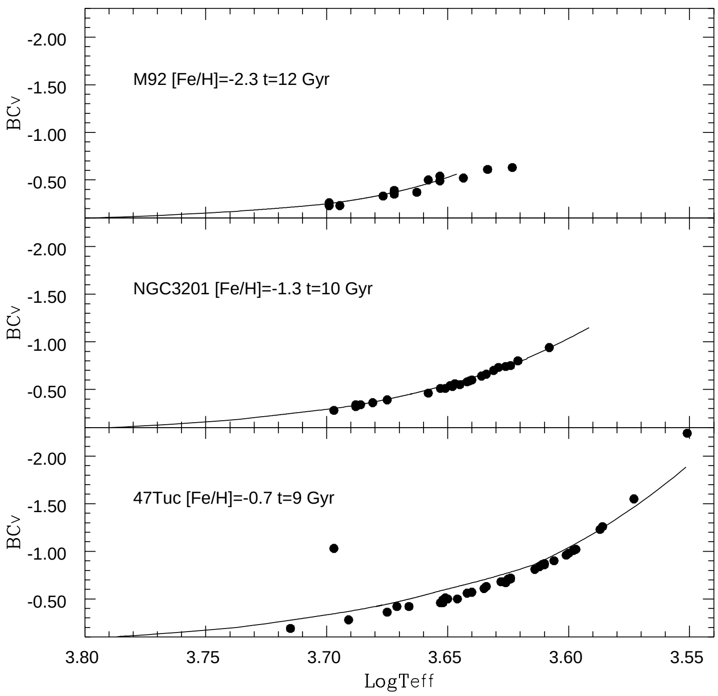


**Fig. 1.** Comparison of $BC_V$ from BK with empirical data by Frogel et al. (1981) (see text). The outlier in panel c is in the original data

[2] The isochrones are available on request from the authors.

As previously mentioned, our source for $(B-V)$ colours and bolometric corrections ($BC_V$) is BK. In the previous papers (e.g. SW98) we have shown that by using these colours we were able to obtain a good fit to all clusters sequences covered by our isochrones, to derive GC reddenings in good agreement with the results from Zinn (1985), and to reproduce the HIPPARCOS subdwarf colours given by Gratton et al. (1997). As for the $BC_V$ scale, the zero point was calibrated to the empirical scale by Alonso et al. (1995, 1996) for metal-poor MS stars; it reproduces the empirical $BC_V$-values for the whole range of metallicities and MS temperatures covered by the isochrones within $\pm 0.02$ mag.

With the isochrones extended now up to the RGB-tip, we can also compare our $BC_V$ scale for RGB stars with the empirical results by Frogel et al. (1981). In Fig. 1 (panels a-c) we show the comparison between the empirical $BC_V$ values derived for three template clusters (M92, NGC3201, and 47Tuc, spanning almost the whole metallicity range of galactic GC) and the $BC_V$ scale from our theoretical isochrones. After correcting for the slightly different (by 0.04 mag) zero points (i.e. to identical solar bolometric correction), we find very good agreement between the two sets of $BC_V$. Also the $BC_V$ scale derived from the ATLAS9, Yale or LCB transformations agree very well with the BK one. In particular, once the $BC_V$ zero point is calibrated as we did for BK (see above and SW98), the different scales agree within 0.05 mag all along the isochrones and on the ZAHB; moreover, the brightness differences between TO and ZAHB for fixed age and metallicity agree again within 0.05 mag. This confirms once more the absolute ages derived in SW97 and SW98.

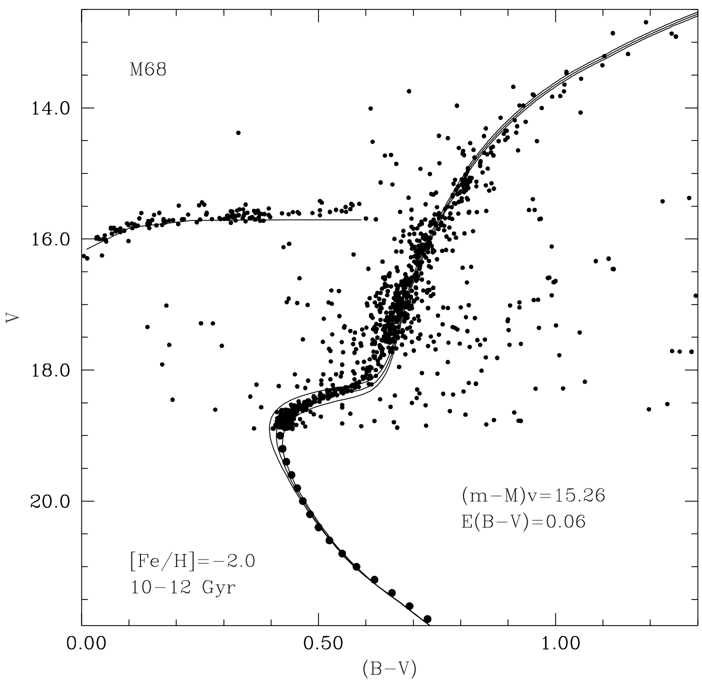


**Fig. 2.** The $V-(B-V)$–CMD of M68 (data from Walker 1994) and isochrones of 10–12 Gyr. For stars fainter than 19th mag only the ridge line is shown; for stars brighter than that only the data points

As a final demonstration of the usefulness of our BK-transformations, we show in Figs. 2 and 3 the fits to M68 and 47 Tuc in the $V$-$(B-V)$ plane with our extended isochrones (in SW97 and SW98 only fits up to the HB were displayed). The data for

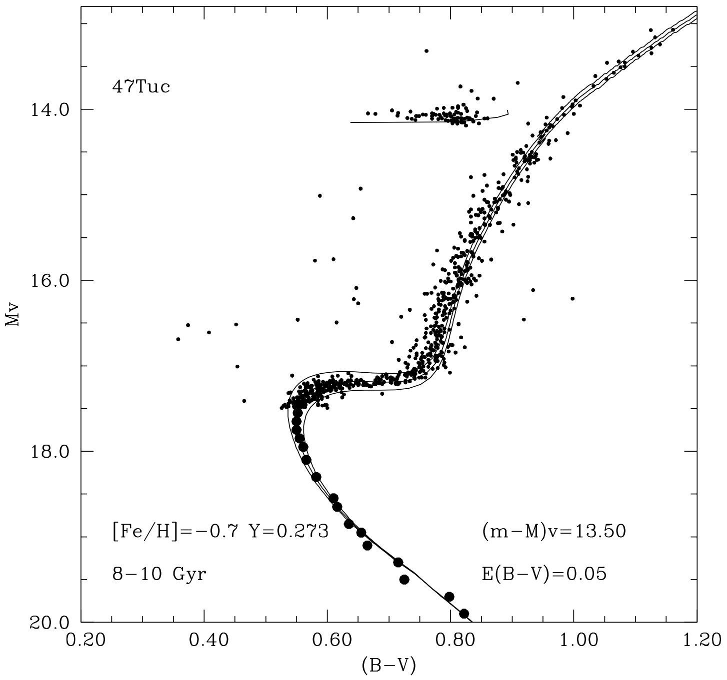


**Fig. 3.** The $V$-$(B-V)$-CMD of 47 Tuc (data from Kaluzny et al. 1998) with isochrones of 8–10 Gyr. For MS stars only the ridge line is shown

M68 (Walker 1994) are the same as in our previous papers, while we used the latest photometry by Kaluzny et al. (1998) for 47 Tuc. We briefly recall that the ages of the clusters are derived from the brightness difference between the ZAHB (defined as the lower envelope of the observed HB stellar distribution; see SW97 for more details) and TO; this permits to derive the cluster age independently of the knowledge of the cluster distance and reddening (due to the horizontal nature of the HB), but of course the theoretical ZAHB models provide a distance. Therefore, the distance modulus is derived by comparing the theoretical ZAHB level to the observed one, while the reddening is obtained from fitting the unevolved theoretical MS to the cluster one. For 47 Tuc we recover with the new photometric data exactly the same age, reddening and distance modulus as in SW98, where we were using the older composite photometry by Hesser et al. (1987). It is evident from the pictures that the extended isochrones reproduce well the observed RGB of both clusters.

### *2.3. Isochrones in the $V-(V-I)$ CMD*

For transforming our isochrones to the $V-(V-I)$ plane we have tested all transformations mentioned above. The BK transformations to $(V-I)$ colours do not extend to temperatures higher than 6000 K such that neither ZAHB nor TO models are covered for all metallicities and ages. Moreover, it is not guaranteed automatically that if this set of models reproduces well the observations in one colour band, the same is true for any other colour.

To show the comparison between the different $T_{\rm eff}-(V-I)$ relations, we have used a representative isochrone ([Fe/H] $= -1.6/10$ Gyr), transformed to the $M_V$–$(V-I)$ plane in Fig. 4 for the MS and in Fig. 5 for the RGB. In the case of the BCM98 empirical transformations, we have used the empirical $T_{\rm eff}-(V-K)$ relation they give (Ridgway

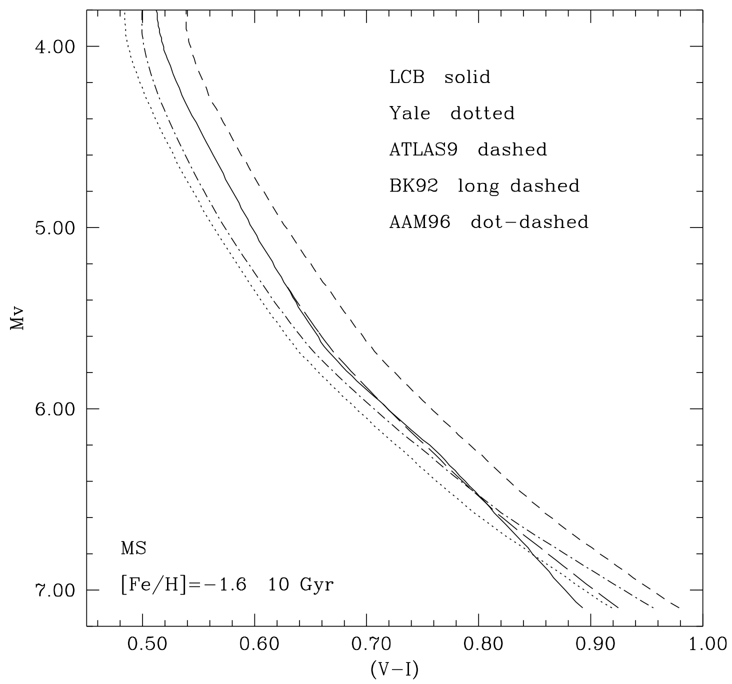


**Fig. 4.** Result of various colour-transformations (indicated in the figure) applied to a theoretical isochrone of [Fe/H] $= -1.6$ and 10 Gyr; shown is the main-sequence part. The various labels are explained in Sect. 2.1

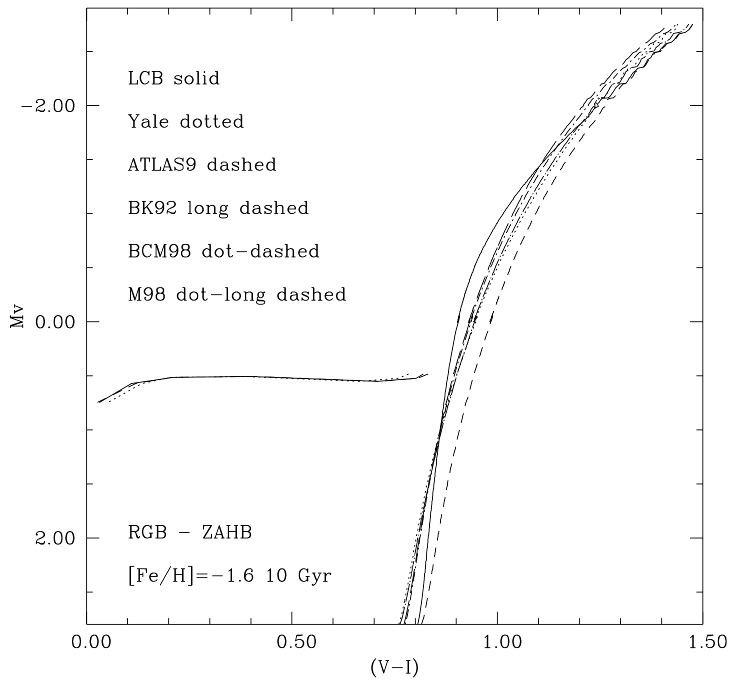


**Fig. 5.** As Fig. 4, but for the red giant branch and ZAHB

et al. 1980) together with their empirically determined $(V-I)-(V-K)$ relations. As they discuss in that paper, the Ridgway et al. (1980) relation is obtained for solar metallicity stars, but there are good indications that it is not affected by metallicity. As for M98, the authors derive an empirical $T_{\rm eff}$-(V-K) relation for metal-rich stars; following the arguments in BCM98, it appears to be safe to use the same relation for the most metal-poor stars, too. In conjunction with the empirical $(V-I)-(V-K)$ relation we obtain again a direct $T_{\rm eff}-(V-I)$ transformation which is displayed in Fig. 5.

It immediately is evident from these illustrations that colour differences between 0.05 and 0.10 mag exist all along the isochrone and that there are no two transformations yielding the same result for all evolutionary stages. In particular, the empirical relation by Alonso et al. (1996) does not support any theoretical transformation. While on the MS the isochrones run more or less parallel to each other (except for LCB), they differ in their slopes along the RGB as well. However, the two empirical relations tested on the RGB (M98 and BCM98) agree with each other rather well (see Sect. 3.2). Part of the mismatch between the different transformations might be attributed to differences in the composition (of model atmospheres or observed stellar samples), to different or inappropriate assumptions in theory (e.g. convection theory or LTE-conditions) and to the internal spread in the empirical relations shown here as fitcurves only. The result of this exercise is pretty much the same for other metallicities.

Since the width of the upper MS and the RGB in high-quality photometry is at most as large as 0.05 mag, it is evident that whatever transformation is chosen as it was published, isochrones can deviate from the CMD structures by at least the width of the branches in colour. *It is therefore an illusion to assume that a perfect isochrone fitting is a strong indication for excellent theoretical models!*

Recently, Bessell et al. (1998) have independently compared a number of theoretical colour transformations with empirical relations for MS stars between spectral types O and M. Since they were interested in aspects of population synthesis, they concluded that there is in general an excellent agreement between theoretical and empirical $T_{\rm eff}$–colour, bolometric correction–colour, and colour–colour relations. However, inspecting their figures (e.g. Fig. 6, where the $T_{\rm eff}$–$(V-I)$ relation for the lower MS is shown) one realizes that the deviations between the different sources are of same order as in our case. The same spread is also visible in the empirical $T_{\rm eff}$–$(V-I)$ relation (Fig. 2 as corrected in their Erratum). Only due to the enormous scale of the colour axis the agreement appears to be excellent. Globally, over all temperatures, the agreement is indeed very good, but not sufficient for such narrow structures as the RGB in galactic GC.

### *2.4. Colour-colour relations*

As demonstrated, there is no preferred transformation from the theoretical into the observational HRD available. Therefore, we also investigated a simpler relation, the transformation between $(B-V)$ and $(V-I)$. The idea is that since we have isochrones available in $(B-V)$ (obtained using BK), which have proven to yield satisfactory fits and reliable relative ages (Pulone et al. 1998) for many galactic GC, isochrones for $(V-I)$ could be obtained by just adding the corresponding colour-colour relation. Testing the existing relations, we found only for the bluer stars on the MS a sufficiently good agreement between BCP98, BK92 and the empirical relation of Alonso et al. (1996). On the lower MS and the RGB the different sources disagree again by up to 0.1 mag. The Yale transformation deviate most significantly from all other sources. We have to conclude that there is no convincing way to decide which colour–colour relation – either empirical or theoretical – to use and that a mismatch of observed colours by 0.05 mag or more is within the general uncertainty.

## 3. A composite $T_{\rm eff}$–$(V-I)$ transformation

After the apparently unclear situation as to which relation one should use to transform theoretical isochrones to the $V$–$(V-I)$ plane, we are now presenting our solution to this problem, the justification of which is mainly based on the fact that it successfully fulfills the following requirements we are imposing: (i) for a given cluster, the fits in $(B-V)$ and $(V-I)$ should be of the same quality; (ii) the reddening obtained from both colours should be consistent; (iii) derived quantities as age and distance should not depend on the colour used.

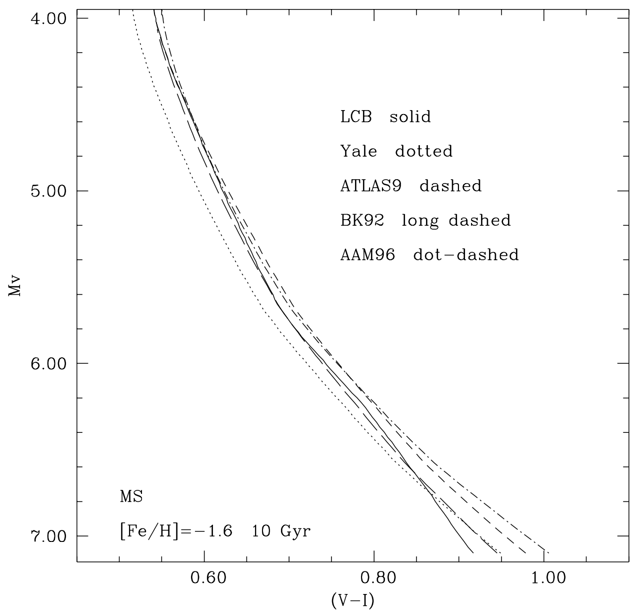


**Fig. 6.** As Fig. 4, but now after adding a constant to the various relations such that they reproduce $(V-I)_{\odot}$

### *3.1. The main sequence*

In Fig. 4 some of the different lines run almost parallel to each other, such that one suspects that a correction in colour zero-points could yield agreement. According to Bessell et al. (1998) $(V-I)$ of the Sun is 0.726, a value which is reproduced within 0.01 mag by BCP98, while BK92, AAM96, LCB and Yale yield colours too blue by, respectively, $\approx$0.02, 0.05, 0.03 and 0.03 mag. By applying corresponding shifts one obtains, for example, Figs. 6 and 7 for two metallicities. After this simple correction, BCP98 (a theoretical relation; labeled as ATLAS9 in the figures) and AAM96 (an empirical one) agree rather well with each other at all metallicities, while all the other colours differ from these two by different amounts at different metallicities. This latter fact implies that the derivative $\delta(V-I)/\delta[Fe/H]$ at a fixed value of the MS brightness, a fundamental quantity needed for applying the MS–fitting method in the $V$-$(V-I)$ plane, does depend on the particular colour transformation used. Based on this comparison, we decided to apply the BCP98 transformations for MS stars of all metallicities. Our choice is substantiated by Pinsonneault et al. (1998), who, for multicolor main-sequence fitting of open clusters, found that in the case of the Hyades consistent distances in both colours can be obtained by using the AAM96 colour-metallicity sensitivity, instead of that of the Yale colours.

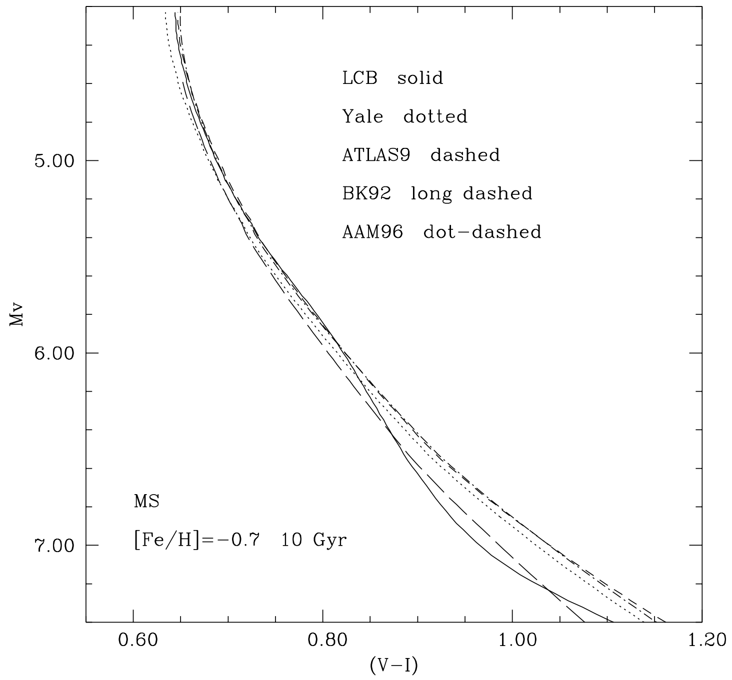


**Fig. 7.** As Fig. 6, but for [Fe/H] = −0.7

### *3.2. The Red Giant Branch*

While several theoretical transformations disagree with empirical ones on the RGB (see Fig. 5), the empirical relations by Montegriffo et al. (1998) and von Braun et al. (1998) confirm each other. In Fig. 8 we show the direct comparison between the two empirical colour–colour relations for $(V-K)-(V-I)$. Since the definition of the $K$-band differs between both sources, we have transformed $K$-magnitudes to a common scale according to Bessel & Brett (1988). In the left panel, the comparison is done for the metal-poor ([Fe/H] $< -1$) subsample of Montegriffo et al. (1998) using three different representative metallicities for the von Braun et al. (1998) relations. The agreement is rather good except for the reddest colours. Only for the highest metallicities, the RGB will extend to $(V-I) \gtrsim 1.3$ such that we can claim agreement for the larger part of all RGBs. In the right panel, the same comparison is shown for the metal-rich subsample ([Fe/H] $> -1$). Here, disagreement is found for the lower part of the RGB ($(V-I) \lesssim 1.1$). We suspect that this might be due to the inclusion of HB stars by Montegriffo et al. (1998). In the $T_{\rm eff}-(V-I)$ relation of Montegriffo et al. (1998) a similar behaviour is noticeable for the same $(V-I)$-range

For the giants we therefore choose the analytical fits to observational data by von Braun et al. (1998), who in their paper used the $T_{\rm eff}$–$(V-K)$ relation by Ridgway et al. (1980) and the empirical $(V-I)$–$(V-K)$ relation. These we combined to obtain the adopted $T_{\rm eff}$–$(V-I)$ transformation. In Fig. 9 we show how this relation (BCM98-Ridgway) compares to the one using the more recent di Benedetto (1993; “DB93”) $T_{\rm eff}$–$(V-K)$ relation instead and to the empirical one by Montegriffo et al. (1998). The agreement is, except for the brightest RGB part, better than 0.02 mag. The BCM98-Ridgway relation we then used for the giants of all cluster metallicities. Between the MS and the RGB relation we smoothly switch from one to the other relation, which overlap over a sufficiently wide range. There is no need for any additional correction or shift. For lowest metallicities they almost agree (at the level of $\approx 0.01$ mag).

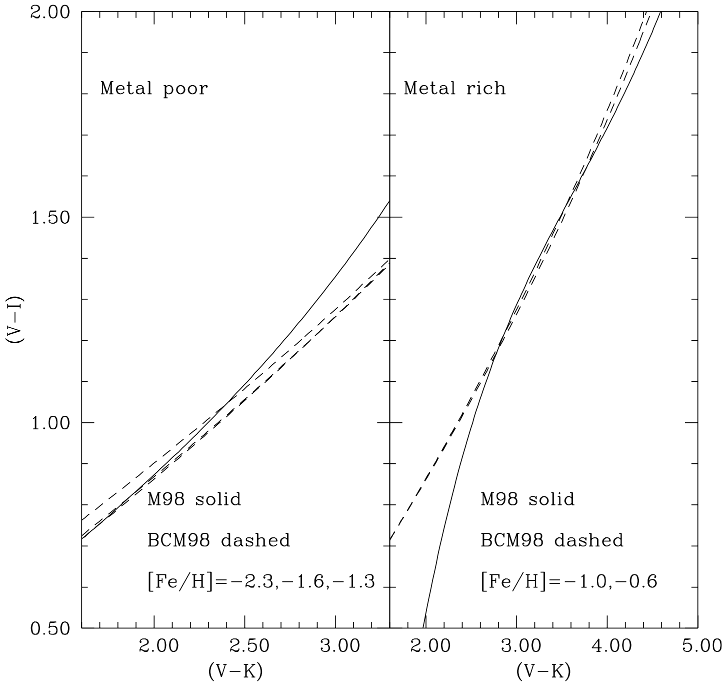


**Fig. 8.** Comparison of the $(V-K)$–$(V-I)$ transformation of Montegriffo et al. (1998) and von Braun et al. (1998); left panel: metal-poor, right: metal-rich subsample of Montegriffo et al. (1998)

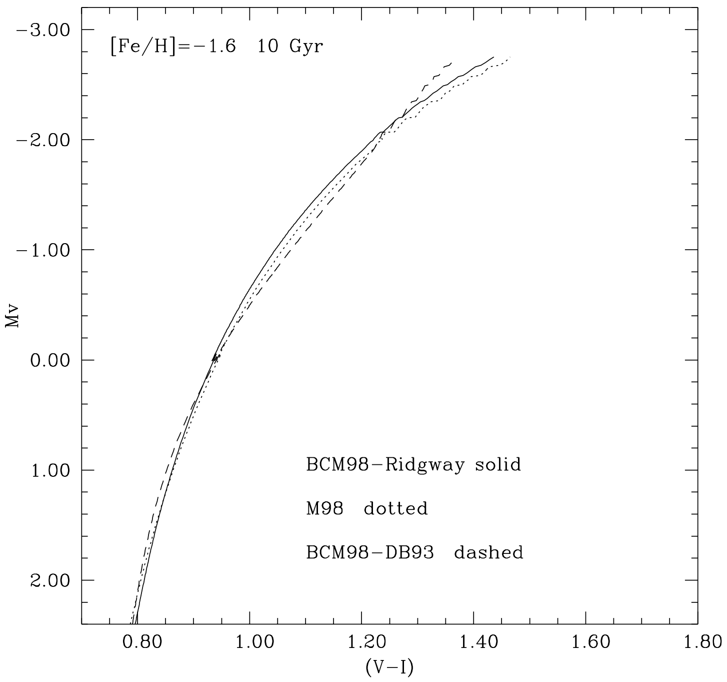


**Fig. 9.** Comparison of empirical $T_{\rm eff}-(V-I)$ relations on the RGB (**see text**)

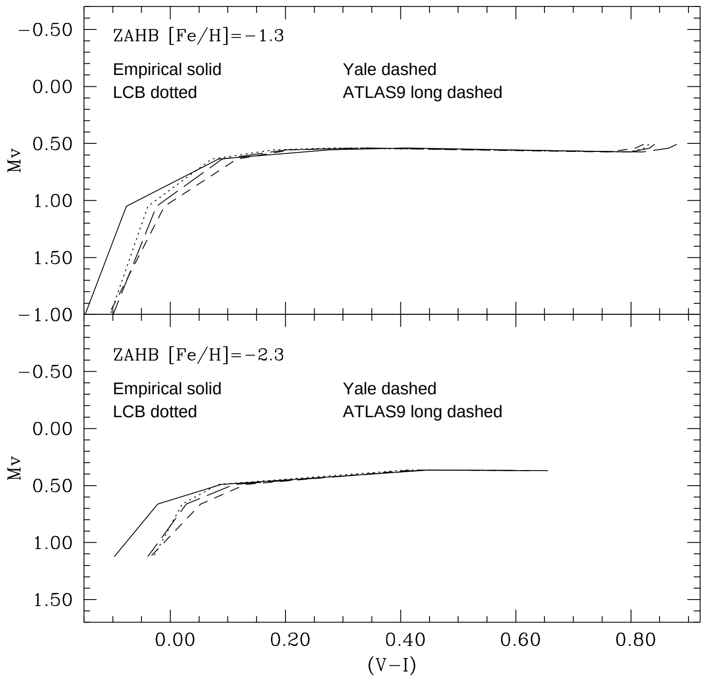


**Fig. 10.** The ZAHB at two metallicities in $(V-I)$: shown is our own relation ("empirical"; see text) and three different transformations. Except for the most extreme colours the agreement is good

### *3.3. The Horizontal Branch*

For HB stars we have derived an empirical $(B-V)-(V-I)$ relation using multicolor photometries of clusters spanning the relevant range of metallicities. We used HB data for M68 (Walker 1994), M3 (Ferraro et al. 1997), M5 (Sandquist et al. 1996), and 47Tuc (Kaluzny et al. 1998), and we applied this relation to our ZAHB models in the $(B-V)$ plane. In Fig. 10 the comparison between the ZAHB transformed by using this empirical colour-colour relation and different theoretical transformations is shown for two different metallicities. Evidently, the theoretical relations agree quite well with each other in predicting the turn-down of the ZAHB in the blue. But there are differences not visible due to the horizontal nature of the HB. The predicted colour along the HB differs in some cases by 0.05–0.10 mag, just as for the RGB. Also, the ATLAS9 transformation yields consistently redder stars at the cool edge (similar to the known effect on the RGB) with increasing metallicity. The empirical relation deviates at the blue end, possibly because of the effect that evolved stars show up in the relation. Since observed HBs usually have a large scatter at the blue, less luminous end, this part is not putting any strong constraint on the ZAHB level, such that the difference between the transformations as demonstrated in Fig. 10), is not really significant. Execept for the mentioned problem of too red colours from the ATLAS9 transformation, we consider all relations as being effectively equivalent. Since we have used empirical data from those clusters we will discuss below, we will use the empirical relation throughout the remainder of this paper.

*3.4. Globular cluster test cases*

To test our $T_{\rm eff} - (V-I)$-relation we use four GC of different metallicity for which we have determined the age already in our previous papers and for which $BVI$ data exist. We transform the corresponding isochrones into $(V-I)$ by our new relation and determine the reddening $E(V-I)$. For the transformation of reddenings we take the extinction law by Cardelli et al. (1989), which gives $E(V-I) = 1.3\,E(B-V)$.

As an example for metal-poor clusters we use M68 ([Fe/H] $= -1.99$; Carretta & Gratton 1997); the $(V-I)$ data are from Walker (1994). The $(B-V)$-CMD has been shown in Fig. 2 and the corresponding $(V-I)$-CMD is displayed in Fig. 11 (for sake of clarity the ridge-line only is displayed for the MS). The age determined in Salaris & Weiss (1998) from the $V$ brightness difference between TO and ZAHB is $11.4 \pm 1.0$ Gyr, in good agreement with the 11 Gyr isochrone displayed Fig. 11. The MS ridge-line was determined by us from the original data by taking the mode of the colour-distribution within brightness bins. The transformed isochrones appear to become too blue in the brightest RGB parts, while the corresponding part in $(B-V)$ is fitting very well. The reddening (obtained, as described in Salaris & Weiss 1998) is 0.06 mag in $(B-V)$ and 0.08 mag in $(V-I)$; their ratio is as predicted by Cardelli et al. (1989).

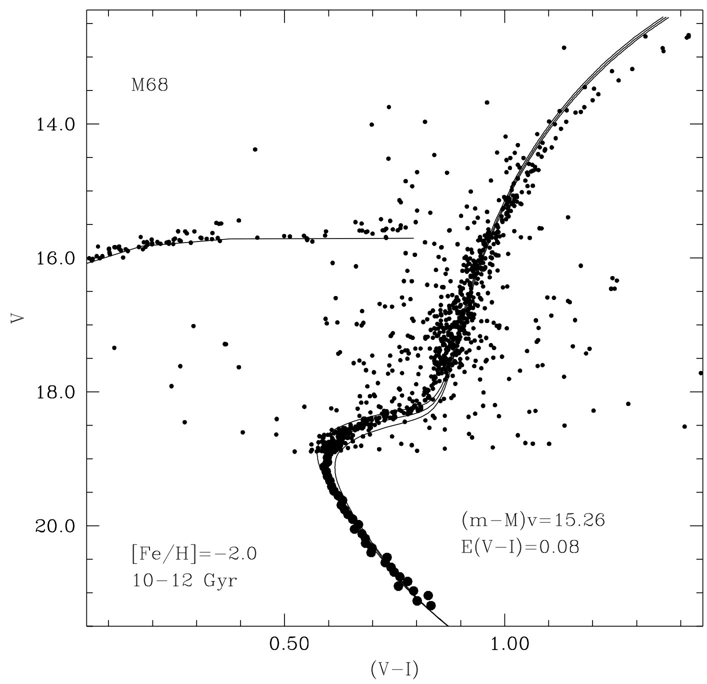


**Fig. 11.** The $V-(V-I)$ isochrones for M68. Data are from Walker (1994) and isochrone parameters as in Salaris & Weiss (1998). For simplicity, only data points brighter than $V = 18.5$ are shown. Large filled circles mark the ridge-line

As an example of an intermediate metal-poor cluster, Fig. 12 shows the comparison between the ridge-line from Johnson & Bolte (1998) and the 10 Gyr-isochrone for M3 ([Fe/H] $= -1.34 \pm 0.06$; Carretta & Gratton 1997), for which SW98 determined differentially the age from $(B-V)$-data (Ferraro et al. 1997) to be $10.1 \pm 1.1$ Gyr. The agreement is excellent except for a small deviation in the upper part of the RGB. The derived $E(V-I)$-reddening is 0.03 mag, in perfect agreement with $E(B-V) = 0.02$ (SW98; see also Buonanno et al. 1994). The effect of using the Yale or ATLAS9 $T_{\rm eff} - (V-I)$

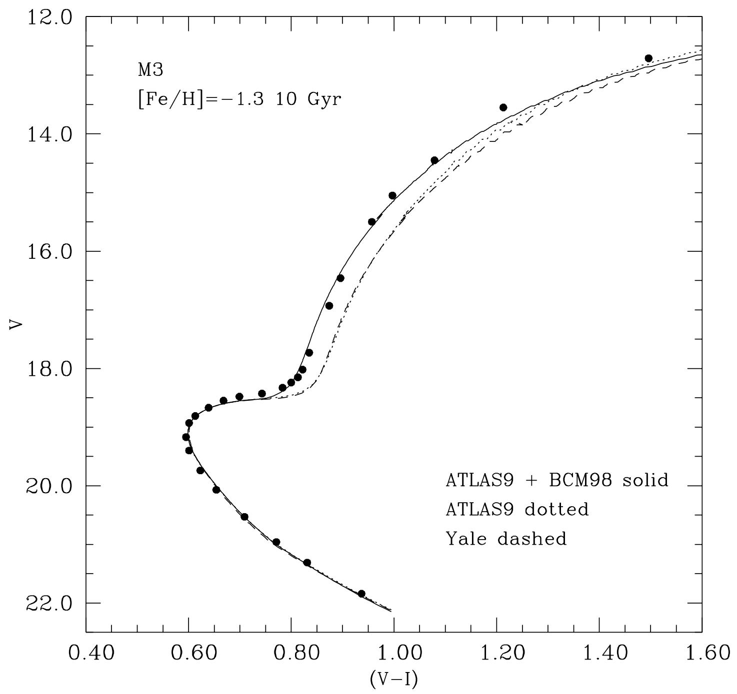


**Fig. 12.** The $V$–$(V-I)$ 10-Gyr isochrone and the ridge line for M3. Data are from Johnson & Bolte (1998). Also displayed is the same isochrone transformed by using the ATLAS9 and Yale colours, always matching the observed main sequence by changing the adopted reddening

relation is shown, too. It is evident that, when the isochrone main sequence is forced to match the observed one (the derived reddening from the Yale colours then is higher by $\approx$0.05 mag!), the Yale and ATLAS9 RGB colours differ substantially from the cluster data (by $\approx$0.05 mag) up to half of the RGB extension; the agreement improves in the upper part of the RGB. This also implies that the RGB slope as derived from ATLAS9 or Yale colours is very different from the observed one. This example emphasizes the need for and justification of the relation we are using.

From the third of our metallicity groups (Salaris & Weiss 1997) we have selected M5; the cluster absolute age as derived from the $V-(B-V)$ diagram (Sandquist et al. 1996) is $9.9 \pm 0.7$ Gyr (SW98). Very recently Stetson et al. (1998) have published a new $V-(V-I)$ diagram of M5, and found some discrepancy with respect to the scale and zero points of the Sandquist et al. (1996) $V$ and $I$ magnitudes. However, their estimated TO-luminosity agrees with the one given by Sandquist et al. (1996) within 0.01 mag, and the same is true for the $V$ brightness of the HB and RGB stars in common (with $V$ ranging between 14.5 and 16.5 mag). Therefore, the absolute age as determined from the $V$ difference between ZAHB and TO remains basically unchanged, and the same is true for the cluster distance modulus determined from the fit of theoretical ZAHB sequences to the observed one (in the following we will therefore use the observed ZAHB-$V$ brightness as determined by SW98 employing the more populated HB in the diagram by Sandquist et al. 1996). The Stetson et al. (1998) $V-(V-I)$ diagram (they provide the ridge-line) is shown in Fig. 13. The two panels contain the comparison with two of our isochrones, whose metallicities bracket the cluster metallicity $[\mathrm{Fe/H}] = -1.11 \pm 0.11$, as determined by Carretta & Gratton (1997); the distance modulus is fixed by the observed ZAHB brightness. The quality of the fit is good in both cases; there is only a difference in the upper part of the RGB, where the more metal-rich isochrones are too red (as expected).

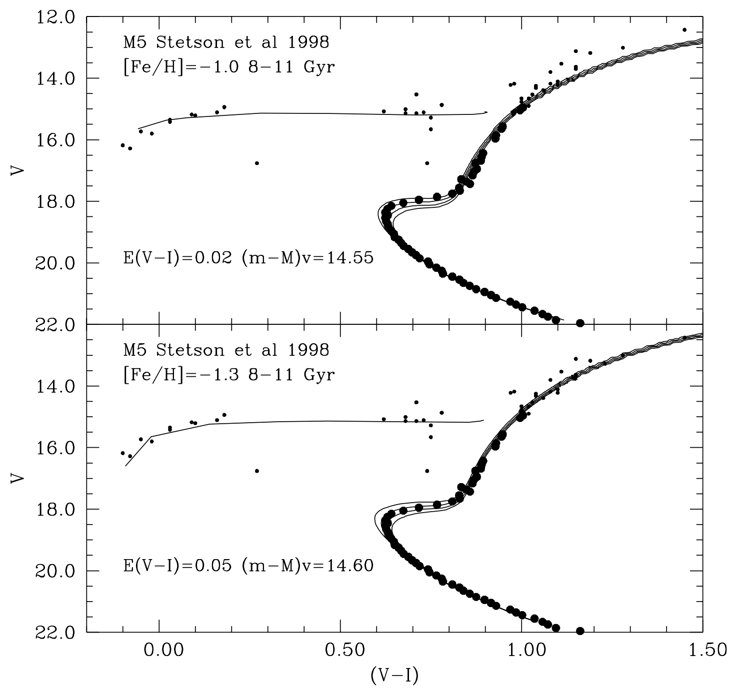


**Fig. 13.** The $V$–$(V-I)$ ridge-line for M5 (from Stetson et al. 1998) and isochrones of 8, 9 and 10 Gyr; upper panel: [Fe/H] = −1.30; lower: [Fe/H] = −1.3

By interpolating for the value of the cluster metallicity we get $E(V-I) = 0.03$, in good agreement with $E(B-V) = 0.02$ as determined by SW98 (due to a misprint in Table 2 of SW98, the apparent cluster distance modulus was quoted $(m - M_V) = 14.53$, while the correct value is $(m - M_V) = 14.57$).

Finally, we compare the data by Kaluzny et al. (1998) for 47 Tuc ([Fe/H] = −0.70; Carretta & Gratton 1997) with our isochrones. Since in SW98 we used the data by Hesser et al. (1987) for determining the age from the $(B-V)$ data, we showed the same fit for the new data in Fig. 3 (the ridge-line shown for the cluster MS is derived as previously described). The theoretical isochrones are identical to those in SW98 (including the higher helium content of $Y = 0.273$) except for the fact that they are now extended to the tip of the RGB. The newly derived age, distance and reddening are the same as in SW98. In Fig. 14 the corresponding $(V-I)$-diagram is shown. The reddening determined in this colour is 0.08, which, when transformed to $E(B-V)$ is 0.06, differing from the expected value of 0.05 by only 0.01 mag. Note that while in $(V-I)$ the isochrone appears to be a bit too blue for the lower RGB, it is too red by the same amount in $(B-V)$. The fit is of the same quality in both colours and slightly better than that shown in SW98 for the Hesser et al. (1987) data.

Inspecting the HB of 47 Tuc in $(V-I)$, one has the impression that is inclined towards the blue. This effect is not so evident in $(B-V)$ (Fig. 3). We think it results from the fact that not all stars have both $(B-V)$ and $(V-I)$ colours and that in $(V-I)$ a number of redder HB stars are missing, thus making the inclination, which we ascribe to evolution on the HB, more apparent. In any case, the inclination could only be a brightness effect, independent of colour.

To conclude this section, we have demonstrated that the $T_{\rm eff}$–$(V-I)$ transformation we have constructed on the basis of two existing transformations, which show reasonable agreement with (other) empirical relations, results in transformed isochrones which fit clusters of all metallicities in $(V-I)$ equally well as in $(B-V)$ and yield consistent reddening.

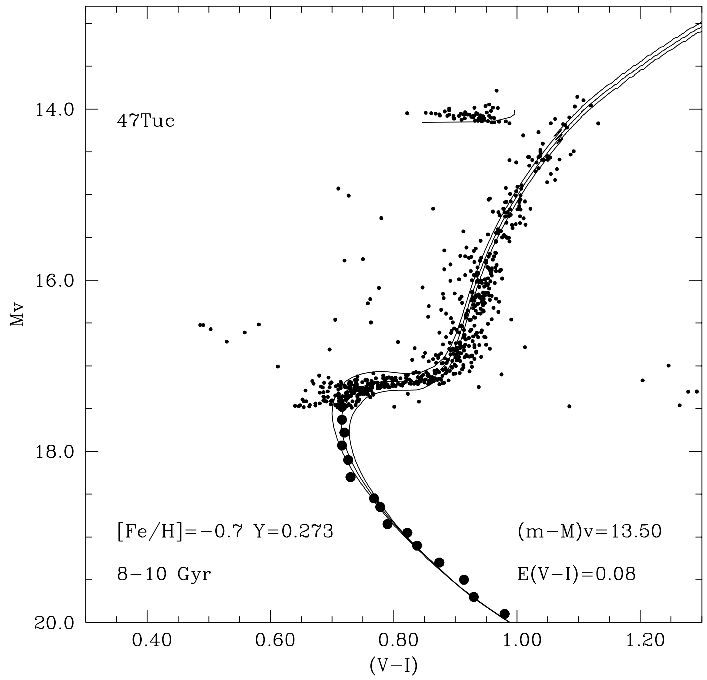


**Fig. 14.** The $V - (V - I)$-CMD of 47 Tuc (data from Kaluzny et al. 1998) with the same isochrones as in Fig. 3

## 4. Discussion

In this paper we tried to approach the problem of fitting theoretical isochrones to globular cluster data in $(V - I)$. This is important since more and more $I$-band photometry is becoming available and for some objects (e.g. bulge clusters) will be the only data of quality possible. Absolute ages of clusters determined by the turn-off brightness or the $\Delta V$-method (see Salaris & Weiss 1998) depend almost exclusively on the bolometric correction $BC_V$. Theoretical values agree, when properly calibrated, with empirical values and also between different transformation sources.

On the other hand, transformations of effective temperatures to colours differ from source to source by a rather constant level of 0.05–0.10 mag. From the comparisons we have shown, it is evident that (i) there is no straightforward way to decide which transformation is the correct or best one and (ii) that the accuracy with which an isochrone fits a colour-magnitude-diagram does not indicate the quality of the underlying stellar evolution calculations, if the mismatch is of the order of 0.05–0.1 mag. While in Sect. 2 we displayed $(V - I)$–fits, the situation is the same in $(B - V)$ and corresponding examples and conclusions can be found in the literature (e.g. Grundahl et al. 1998; D'Antona et al. 1997).

The problem of inaccurate colours is not just a cosmetic one, but can influence age determinations which make use of colour differences (e.g. that between turn-off and red giant branch), distances derived by means of the main-sequence–fitting technique, or integrated colours of unresolved stellar populations. Finally, *if* a set of transformation is available which is known to reproduce colours accurately, any deviation of an isochrone from the observed CMD will indicate a real inconsistency in the isochrone. As an example we repeat that a colour mismatch of the RGB indicated to us a higher helium content for 47 Tuc (Salaris & Weiss 1998). Thus, reliable colour transformations are a source for obtaining additional knowledge about cluster stars. We therefore attempted to find a preliminary solution to this problem, such that we can continue our work about cluster dating with $(V-I)$ data, expecting improved theoretical transformations in the meantime.

We found that for MS stars the Bessell et al. (1998) transformation is the only one reproducing $(V-I)$ for the Sun to an accuracy of 0.01 mag. When adjusting the zero-point of the empirical relation by Alonso et al. (1996) such that it, too, gives the correct solar $(V-I)$ colour, Bessell et al. (1998) and Alonso et al. (1996) agree very well for the whole GC metallicity range and we decided, on this ground, to use Bessell et al. (1998) for the MS $T_{\rm eff}$–$(V-I)$ transformation.

On the red giant branch, no theoretical transformation was found to be in agreement with empirical data, but two recent empirical sources (Montegriffo et al. 1998; von Braun et al. 1998) confirm each other quite well. Since von Braun et al. (1998) provide metallicity-dependent relations, we use this one for the red giant branch. Both parts (MS and RGB) can be connected smoothly along the subgiant branch.

While the arguments just given have only led the way to our rather pragmatic combination of two $T_{\rm eff}$–$(V-I)$ transformations, the justification for the combined and final relation comes from the tests we have performed. For each of the four metallicity ranges defined in Salaris & Weiss (1997), which reach from the most metal-poor halo to the half-solar disk clusters, we selected one cluster with $BVI$-photometry (three of them had already been used for absolute age determinations in Salaris & Weiss 1998). We then showed successfully that (i) the quality of the isochrone fit is comparable in both colours and (ii) the reddenings determined independently from these two fits fulfill the reddening law by Cardelli et al. (1989) either exactly or within 0.01 mag.

We therefore conclude that the combination of the (theoretical) transformation of Bessell et al. (1998) for the main-sequence and the (empirical) one by von Braun et al. (1998) for giants is of sufficient accuracy as to allow isochrone fits to globular cluster colour-magnitude-diagrams in $V$–$(V-I)$.

In a forthcoming paper we will determine cluster ages based on $VI$-photometry, both for clusters we already have investigated in our previous papers and for clusters with $VI$-data only. Although $(V-I)$ is less sensitive to metallicity than $(B-V)$, we found that $\triangle(V-I)$ – the colour difference between turn-off and red giant branch, which we need for relative age determinations – at fixed age is actually *more* metal-sensitive. Our approach of grouping clusters into four metallicity bins will therefore not be accurate enough. Either a finer metallicity grouping is needed (implying a higher number of clusters for absolute age determination), or a purely relative age approach is needed as in Pulone et al. (1998).

Independent of this, we are still waiting for improved theoretical colour transformations, because for more metal-rich clusters (e.g. bulge clusters) the von Braun et al. (1998) results cannot be applied. The study of such clusters therefore actually requires to some extent an extrapolation of our adopted colour transformations.

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