# No Pulsar Kicks from Deformed Neutrinospheres

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(August 10, 1998)

In a supernova core, magnetic fields cause a directional variation of the neutrino refractive index so that resonant flavor oscillations would lead to a deformation of the "neutrinosphere" for, say, $\tau$ neutrinos. The associated anisotropic neutrino emission was proposed as a possible origin of the observed pulsar proper motions. We argue that this effect was vastly overestimated because the variation of the temperature over the deformed neutrinosphere is not an adequate measure for the anisotropy of neutrino emission. The neutrino flux is generated inside the neutron star core and is transported through the atmosphere at a constant luminosity, forcing the temperature gradient in the atmosphere to adjust to the inflow of energy from below. Therefore, no emission anisotropy is caused by a deformation of the neutrinosphere to lowest order. An estimate of the higher-order corrections must take into account the modified atmospheric temperature profile in response to the deformation of the neutrinosphere and the corresponding feedback on the core. We go through this exercise in the framework of a simplified model which can be solved analytically.

PACS numbers: 97.60.Gb, 14.60.Pq, 98.70.Rz

## I. INTRODUCTION

After the supernova collapse of a massive star, neutrinos carry away about 99% of the gravitational binding energy $E_{\rm b}$ of the nascent neutron star, taking with them a huge amount of momentum which is of the order $10^{43}$ g cm s$^{-1}$ ($E_{\rm b}/3 \times 10^{53}$ erg). An anisotropy of the neutrino emission as small as 1% would suffice to account for a neutron star recoil of about 300 km s$^{-1}$ [1] and could thus explain the observed space velocities of most pulsars [2]. However, even such a small asymmetry is difficult to explain.

Pulsars tend to have strong magnetic fields, leading to the speculation that $B$-fields could be a natural agent to cause asymmetric neutrino emission. For some time it appeared as if for realistic field strengths the induced polarization of the nucleon spins, together with the parity-violating properties of the neutrino-nucleon cross-sections, was enough to explain the observed pulsar kicks [3]. Later it was recognized that this "cumulative parity violation effect" was in violation of fundamental symmetries required of the Boltzmann collision equation; a correct derivation leads to a much reduced anisotropy [4].

This observation, together with the impressive recent evidence for neutrino oscillations, leads one to take seriously another more indirect mechanism. The neutrino refractive index depends on the direction of the neutrino momentum relative to $\mathbf{B}$. For suitable conditions, resonant neutrino oscillations can occur between the neutrinospheres of electron neutrinos and, say, $\tau$ neutrinos, leading to a deformation of the effective $\nu_\tau$ sphere [5,6]. The $\tau$ neutrinos would thus be emitted from regions of varying effective temperatures, and thus, it was argued, would be emitted anisotropically. This idea was then taken up in several papers with modified neutrino oscillation scenarios [7–10].

Unfortunately, however, this elegant scenario and its variations also appear to be fundamentally flawed in at least two serious ways.

The first problem is caused by a common misunderstanding of the meaning of the "effective temperature" of the neutrino flux emerging from a supernova core. It is usually thought that the total energy carried away by neutrinos from a SN core is roughly equipartitioned between the flavors, yet the heavy-flavor neutrinos (we usually take $\nu_\tau$ as an example) have stiffer spectra, i.e. their *spectral* temperatures tend to be much larger than those of $\bar\nu_e$. Evidently, the neutrino luminosities are not given by the Stefan-Boltzmann law in a naive way—we will discuss this issue in some detail in Sec. II. For the moment it suffices to observe that in a situation of exact flavor equipartition, a spectral swap of two flavors by oscillations would not change the energy flux, except perhaps indirectly by a response of the thermal medium to the supposedly different spectra.

The flavor equipartition of the energy flux need not be exact, and for the sake of argument we may contemplate a situation where most of the energy is carried by $\nu_e$ and $\bar\nu_e$. If oscillations take place outside of the $\nu_\tau$ sphere, the oscillated $\nu_e$'s could escape from different depths according to the $B$-field deformed resonance sphere, and thus with different temperatures. Even then one will not achieve a large flux asymmetry because it is not justified to calculate the expected flux from the local gas temperature along the resonance surface.

All of the neutrino spectra formation and oscillation physics of the present problem take place in the "atmosphere" of the protoneutron star, the outer region where the density drops quickly from core values around nuclear density to effectively "zero". The neutrino fluxes, however, are determined in the core of the neutron star. The atmosphere has virtually no heat capacity relative to the core. Therefore, after a short time, typically of the order of a few hundred milliseconds at most, which is very short compared with the Kelvin-Helmholtz neutrino cooling time of the nascent neutron star, the neutrino luminosity is governed by the core emission and the surface-near layers have reached a state where the temperature gradient ensures that all energy streaming up from below is carried outwards with a luminosity that is independent of the radial position.

Therefore, the second serious problem of the oscillation kick scenario is that, to lowest order, a shift of the neutrinosphere will leave the neutrino luminosity unchanged.

A residual anisotropy effect obtains because the neutrino flux is not strictly fixed by the core alone; it depends on the temperature at the core-atmosphere interface. This temperature, in turn, depends on the atmosphere so that there is an indirect influence of the atmospheric structure on the neutrino fluxes. More precisely, the neutrino flux determines the temperature gradient in the atmosphere, and the atmosphere influences the temperature at the core-atmosphere interface. Without a self-consistent treatment of this sort there is no pulsar kick at all, and the kick that one does obtain is a higher-order effect.

In the following discussion we will elaborate our two arguments in more depth. In Sec. II we will explain the connection between the Stefan-Boltzmann law and the neutrino luminosities of a neutron star. We will stress the inadequacy of a simplistic application of the $R^2T^4$ scaling of the luminosity when $T$ is the spectral temperature. In Sec. III we will construct a simple self-consistent model in the so-called Eddington atmosphere approximation [11]. Our model leads to an estimate of the higher-order emission anisotropy from a changed neutrinosphere by direction-dependent resonant neutrino flavor conversions. Finally, Sec. IV is given over to a discussion and summary of our findings.

## II. NEUTRINO TRANSPORT IN NASCENT NEUTRON STARS

### A. Neutrino Fluxes and the Stefan-Boltzmann Law

We begin our more detailed discussion with a description of some crucial aspects of neutrino transport in nascent neutron stars. The picture thus developed will serve as background information for the analytical model of Sec. III. The most important insight to be presently explained is that the neutrino flux emerging from a supernova core is not trivially given by the Stefan-Boltzmann law; the spectral temperature does not fix the flux, in contrast with true blackbody radiation [12].

Lepton number is lost from the collapsed stellar core by the emission of electron neutrinos while energy is emitted in neutrinos and antineutrinos of all flavors. Electron neutrinos are produced efficiently via the $\beta$-process $e^- + p \to n + \nu_e$ during the first second after collapse because of the high electron chemical potential, so that the deleptonization, in particular of the surface-near layers, proceeds very fast. Most of the gravitational binding energy of the neutron star is radiated away after the collapsed stellar core has settled into the static, compact and hot protoneutron star when neutrinos and antineutrinos of all flavors take up approximately the same share of the total energy and are emitted with very similar luminosities from the thermal bath of the core.

The heat capacity and lepton number reservoir of the dense core are much larger than those of the less dense and much less massive atmosphere above. Roughly, core and atmosphere are discerned by the rather flat density gradient in the former, in contrast to the steep density decline in the latter. The density at the core-atmosphere interface is time-dependent and is typically between $10^{13}$ and $10^{14}$ g cm$^{-3}$. Its small heat capacity and short neutrino diffusion time imply that the atmosphere radiates away its binding energy in less than a few hundred milliseconds, to be compared with the neutrino diffusion timescale out of the core of a few seconds and the typical energy-loss timescale of several ten seconds.

Thus, after a brief initial relaxation phase, the neutrino luminosity of the nascent neutron star is governed by the energy loss from the core; it reaches its surface value already below the core-atmosphere interface. Throughout the atmosphere, the luminosity is independent of the radial position if gravitational redshift is ignored. The temperature and density profiles in the atmospheric layers adjust to the neutrino energy flux coming from inside to ensure its transport to the stellar surface under the constraint of hydrostatic equilibrium. This is equivalent to the situation in ordinary stars where the photon luminosity is produced in the nuclear burning zones while the stellar mantle and envelope adopt a structure in accordance with the transport of this energy to the photosphere. Evidently, the energy flux of neutrino or antineutrino species $\nu_i$ cannot be given simply by the local gas temperature according to $F_{\nu_i} \propto T^4$.

When thermal equilibrium between neutrinos and the stellar medium is assumed, the energy flux in the diffusion approximation can be expressed in terms of the atmospheric temperature gradient as

$$F_{\nu_i} = -D_{\nu_i}\,\frac{4\pi}{(hc)^3}\,4\mathcal{F}_3(0)\,(kT)^3\,\frac{\partial(kT)}{\partial r}\,. \qquad (1)$$

Here, $D_{\nu_i}$ is the diffusion coefficient, suitably averaged over the neutrino spectrum, while $h$, $c$ and $k$ are the

Planck constant, the speed of light, and Boltzmann's constant, respectively. Further,

$$\mathcal{F}_j(\eta_{\nu_i}) \equiv \int_0^\infty \mathrm{d}x\, \frac{x^j}{1+\exp(x-\eta_{\nu_i})}\,, \qquad (2)$$

where $\eta_{\nu_i}$ is the neutrino degeneracy parameter, i.e. the chemical potential divided by the temperature. In Eq. (1) it was assumed that the neutrino degeneracy parameter is zero, $\eta_{\nu_i} = 0$, which then gives $\mathcal{F}_3(0) \approx 3! = 6$. This is always true for $\mu$ and $\tau$ neutrinos and is also a good approximation for electron neutrinos and antineutrinos after the deleptonization of the atmosphere.

Another expression for the neutrino energy flux can be obtained by relating it to the neutrino energy density,

$$\varepsilon_{\nu_i} = \frac{4\pi}{(hc)^3}\,(kT)^4\,\mathcal{F}_3(0)\,, \qquad (3)$$

which yields

$$F_{\nu_i} = c\,\langle\mu\rangle_{E,\nu_i}\,\frac{4\pi}{(hc)^3}\,(kT)^4\,\mathcal{F}_3(0)\,. \qquad (4)$$

The factor $\langle\mu\rangle_{E,\nu_i}$ denotes the average cosine of the angle of neutrino propagation relative to the radial direction. It is calculated from the neutrino phase-space distribution function $f_{\nu_i}(r,t,\mu,\epsilon)$ according to

$$\langle\mu\rangle_{j,\nu_i} \equiv \frac{\int_{-1}^{+1}\mathrm{d}\mu\,\mu\int_0^\infty \mathrm{d}\epsilon\,\epsilon^{2+j} f_{\nu_i}(r,t,\mu,\epsilon)}{\int_{-1}^{+1}\mathrm{d}\mu\,\int_0^\infty \mathrm{d}\epsilon\,\epsilon^{2+j} f_{\nu_i}(r,t,\mu,\epsilon)}\,, \qquad (5)$$

where $r$ is the radial position, $t$ time, and $\epsilon$ the neutrino energy. Choosing $j=0$ gives us the "mean angle cosine" for the neutrino number flux, $\langle\mu\rangle_{N,\nu_i}$, while $j=1$ yields the corresponding quantity for the energy flux, $\langle\mu\rangle_{E,\nu_i}$. Comparing Eqs. (1) and (4) gives

$$\langle\mu\rangle_{E,\nu_i} = -\frac{4D_{\nu_i}}{cT}\,\frac{\partial T}{\partial r} = \frac{4}{3}\,\frac{\langle\lambda\rangle_{E,\nu_i}}{h_T}\,, \qquad (6)$$

where $h_T = (\partial \ln T/\partial r)^{-1}$ is the temperature scale height and $D_{\nu_i} = c\langle\lambda\rangle_{E,\nu_i}/3$ was used for the diffusion constant, with the mean free path $\lambda_{\nu_i}$ a suitable spectral average for the energy flux of neutrino species $\nu_i$.

From Eq. (4) together with Eq. (6) one verifies that $F_{\nu_i} \propto T^4$ is not the whole story. Instead, the flux factor $\langle\mu\rangle_{E,\nu_i}$ can be significantly different from the canonical value 1/4 which represents the Stefan-Boltzmann law. Put another way, the energy flux of "thermal" radiation is characterized by two parameters, the spectral temperature and the mean angle cosine which quantifies the deviation from an isotropic phase-space occupation.

Equation (6) reveals that $\langle\mu\rangle_{E,\nu_i}$ depends on the position in the atmosphere because $\langle\lambda\rangle_{E,\nu_i}$ becomes smaller for higher temperature and density. In the protoneutron star atmosphere the neutrino luminosity $L_\nu = 4\pi r^2 F_\nu$ (for an individual neutrino type or for the sum of neutrino and corresponding antineutrino) is fixed by the inflow from the core region. The flux factor $\langle\mu\rangle_{E,\nu_i}$, on the other hand, increases with radius (decreasing temperature) in accordance with Eq. (4).

### B. Neutrino Spheres

Another frequently misunderstood concept is that of a "neutrinosphere". We stress that actually for each type of neutrino two different kinds of neutrinospheres are defined, the "energy sphere" with radius $R_{E,\nu_i}$ and the "transport sphere" with radius $R_{\mathrm{t},\nu_i}$. The latter is what many authors mean with *the* neutrinosphere, i.e. the surface of "last scattering" at optical depth 2/3 which emits the neutrino flux. The energy sphere is where energy-exchanging reactions freeze out while energy-conserving collisions may still be important. Of course, the concept of well-defined neutrinospheres or -surfaces is always a simplification of the real situation because the neutrino-matter interactions are strongly energy dependent.

When $R_{L,\nu_i}$ is the radius at which the luminosity $L_{\nu_i}$ has reached its surface value, the three radii obey the relation $R_{L,\nu_i} < R_{E,\nu_i} < R_{\mathrm{t},\nu_i}$, because the luminosity is fixed already deep inside the nascent neutron star. Between $R_{L,\nu_i}$ and $R_{E,\nu_i}$, the luminosity is constant while the spectral distribution of the flux still evolves due to neutrino absorption and reemission as well as energy-exchanging collisions. Between $R_{L,\nu_i}$ and $R_{E,\nu_i}$ the number flux of a given neutrino flavor need not be conserved, whereas the lepton number flux (difference between neutrinos and antineutrinos of a given flavor) is conserved, even for the electron flavor, because after a few hundred milliseconds the atmosphere is in a relaxed state and does not gain or lose lepton number on short timescales.

Besides reactions in which energy is exchanged between neutrinos and the stellar medium, a significant fraction of the neutrino opacity of the stellar atmosphere is due to nearly iso-energetic neutrino-nucleon or neutrino-nucleus scatterings; for $\nu_\mu$ and $\nu_\tau$ this contribution in fact dominates. This has the consequence that even outside of $R_{E,\nu_i}$, where the flux spectrum is independent of radius, the neutrinos still undergo many scatterings and propagate outward by diffusion. Therefore, the local neutrino distribution function is nearly isotropic, implying $\langle\mu\rangle_{j,\nu_i} \ll 1$. Hence, it is between $R_{E,\nu_i}$ and $R_{\mathrm{t},\nu_i}$, the region of a "scattering atmosphere", where the naive Stefan-Boltzmann law is particularly poor at accounting for the neutrino luminosity [12].

The total opacity of electron (anti)neutrinos is dominated by $\beta$-processes so that the distinction between the energy and transport sphere is often not crucial—for this flavor the concept of *the* neutrinosphere is crudely justified. For the other flavors the distinction is crucial.

### C. Pulsar Kicks by Oscillations?

Our description of neutrino transport reveals that there is no simple relationship between the spectral temperature and luminosity of a given neutrino flavor [12]. In numerical simulations with nonequilibrium transport description one finds approximately equal luminosities of neutrinos and antineutrinos of all flavors, but vastly different spectral temperatures [13–15]. However, the most elaborate numerical models have so far not fully taken into account several important energy-exchange channels between heavy-flavor neutrinos and the nuclear medium such as nucleon recoils, (inverse) nucleon-nucleon bremsstrahlung, and collective as well as multiple-scattering effects [14,16]; therefore, the spectral temperatures between the flavors are likely far more similar than had been thought, and the difference between the energy and transport spheres may be less pronounced.

In the limit of exact equipartition, neutrino oscillations along an aspherical resonance surface could not produce any pulsar recoil. One caveat is that the spectral swap between the flavors modifies their interaction rate with the medium so that there could be a small indirect effect. Even this possibility is diminished if the spectra are more similar than had been thought previously.

Surely, it is not possible to calculate the pulsar recoil from a $\nu_\tau$ Stefan-Boltzmann flux, evaluated over the aspherical resonance surface with its varying temperature. In typical simulations one finds an equipartition of the total energy to within a few percent. Any possible flux anisotropy caused by neutrino oscillations is therefore far smaller than had been assumed in Refs. [5–10].

## III. EDDINGTON ATMOSPHERE MODEL FOR FLUX ANISOTROPY

### A. Description of the Model

In the preceding Section we have argued that in the limit of exact energy equipartition between the emitted neutrino flavors, an aspherical resonant oscillation surface could not cause a pulsar recoil, except perhaps by residual higher-order effects. But the equipartition need not be exact. This is especially true if the oscillations are into a sterile species $\nu_s$ which would not be emitted at all without oscillation effects [7]. Even in this case it is difficult to obtain a large recoil by oscillations because the magnitude of the anisotropy does not scale with the variation of the gas temperature along the deformed emission sphere.

An anisotropy of the neutrino emission can be established only in a much more indirect way. The aspherical escape surface of the neutrino flux quickly leads to a perturbation of the initial configuration by producing an aspherical density and temperature profile of the neutron star atmosphere. This has a feed-back effect on the temperature at the core boundary and thus modifies the neutrino flux. Therefore, one needs a self-consistent atmospheric model to estimate the neutrino flux anisotropy.

To this end we subdivide the star into the "core" and the "atmosphere" as indicated in Fig. 1. For simplicity we take the "luminosity sphere" $R_L$ as the interface; outside the neutrino luminosity is fixed. The core is characterized by a central temperature $T_c$, the temperature $T_L$ at the core-atmosphere interface, and a linear temperature gradient in between. Clearly, the neutrino luminosity is determined by the difference between $T_c$ and $T_L$. In order to estimate the flux anisotropy we need to calculate a self-consistent value for $T_L$ by matching a self-consistent atmospheric model to the interface.

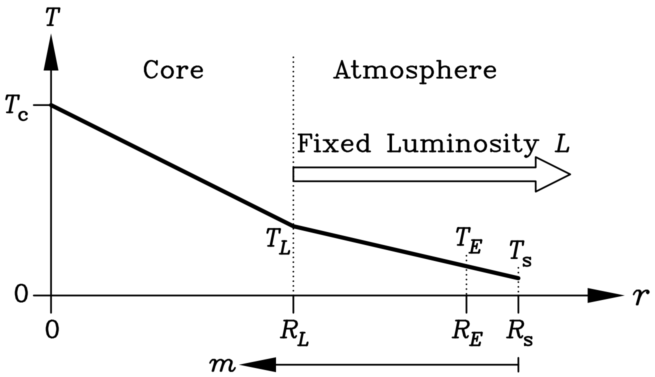


FIG. 1. Schematic model of a nascent neutron star during the Kelvin-Helmholtz cooling phase. The core is treated as one zone, the atmosphere is described self-consistently by the Eddington atmosphere model.

Our main task, therefore, is to construct a model for the atmosphere which is characterized by three parameters: its mass, the temperature $T_L$ at the bottom, and the neutrino flux $L$ which enters from below and depends on the central temperature $T_c$. The primary input quantity that varies as a function of direction is the atmospheric mass which is given by the neutrino resonance surface.

### B. Neutrino Eddington Atmosphere

We construct a self-consistent atmospheric model by virtue of the Eddington approximation; for neutrinos this was done in Ref. [11], a fundamental paper that we will closely follow. The Eddington atmosphere employs the assumption of plane-parallel geometry, i.e. the atmosphere is taken to be geometrically thin relative to the core size. Moreover, one uses the diffusion approximation, neutrinos and stellar medium are taken to be in thermal equilibrium, and the neutrino and antineutrino degeneracy parameters are taken to vanish everywhere, implying that there is no lepton number flux through the atmosphere. One uses neutrino and antineutrino opacities which are equal and vary with the square of the neutrino energy, $\Lambda_{\nu_i} = \Lambda_{\bar\nu_i} = \Lambda_0(\epsilon^2/\epsilon_0^2)$ with $\Lambda_0$ being a constant of dimension $\mathrm{cm}^2\mathrm{g}^{-1}$ and $\epsilon_0 = \mathrm{const.}$ This implies that the neutrino and antineutrino phase-space distribution functions are identical, $f_{\nu_i} = f_{\bar\nu_i}$, and can be written in terms of the Fermi-Dirac distribution $f^{\rm eq} \equiv [1 + \mathrm{e}^{\epsilon/(kT)}]^{-1}$ as

$$f_{\nu_i}(r,t,\mu,\epsilon) = f^{\rm eq} + \frac{\mu}{\Lambda}\,\frac{\partial f^{\rm eq}}{\partial m}\,. \tag{7}$$

Here, $\mu$ is the neutrino angle cosine and $m = \int_r^{R_{\rm s}} {\rm d}x\,\rho(x)$ the column mass density (g cm$^{-2}$) of the atmosphere measured from the surface inward (Fig. 1).

These assumptions are reasonably well fulfilled for electron neutrinos $\nu_e$ and antineutrinos $\bar\nu_e$ in the region of interest, i.e. between the $\nu_\mu$ and $\nu_\tau$ energy sphere $R_{E,\nu_x}$ and those of $\nu_e$ and $\bar\nu_e$, $R_{E,\nu_e} \approx R_{E,\bar\nu_e}$, which are very close to the neutron-star surface. On the other hand, the thermal coupling of muon and tau neutrinos to the stellar background ceases in the relevant regions so that we take the atmospheric structure to be determined by the electron (anti)neutrinos alone. Their combined luminosity $L_{\nu_e} + L_{\bar\nu_e} = 4\pi r^2 (F_{\nu_e} + F_{\bar\nu_e})$ is assumed to be constant and given by the inflow from the core. In our plane-parallel model this implies that both the area $4\pi r^2$ and the combined energy flux $F_E = F_{\nu_e} + F_{\bar\nu_e}$ are constant.

In this model one can derive an expression for the temperature as a function of the mass coordinate $m$ which is given by Eq. (27) of Ref. [11] as

$$(kT)^2(m) = \left(\frac{9(hc)^3}{2\pi^3\,c}\,\frac{\Lambda_0}{\epsilon_0^2}\,F_E\right) m + \left(\frac{30(hc)^3}{7\pi^5\,c}\,F_E\right)^{1/2}. \tag{8}$$

For $m = 0$ this equation yields the surface temperature, $(kT_{\rm s})^4 = F_E\,30(hc)^3/(7\pi^5 c)$, as a function of the energy flux $F_E$. Using Eq. (7) and the Rosseland mean opacity,

$$\Lambda_{\rm R} = \frac{7\pi^2}{5}\,\Lambda_0\,\left(\frac{kT}{\epsilon_0}\right)^2 \tag{9}$$

(Eq. 31 in Ref. [11]), one can write the energy flux $F_E$ in terms of the derivative of the neutrino and antineutrino energy density $\varepsilon \equiv \varepsilon_{\nu_e} + \varepsilon_{\bar\nu_e}$ as

$$F_E = \frac{c}{3\Lambda_{\rm R}}\,\frac{\partial\varepsilon}{\partial m} \tag{10}$$

(Eq. 28 in Ref. [11]).

Equations (9) and (10) can be used to relate the energy flux coming from the core to the temperature $T_L$ at the core-atmosphere interface. Taking Eq. (3) for the neutrino energy density with $2\mathcal{F}_3(0) = 7\pi^4/60$ [17] and $m_{\rm c}$ as the column mass density of the core, and evaluating Eq. (9) in the one-zone approximation for an average core temperature, $T^2 \equiv \frac{1}{2}(T_{\rm c}^2 + T_L^2)$ where $T_{\rm c}$ is the temperature at the center, one finds

$$F_E = \frac{2\pi^3}{9}\,\frac{c\,\epsilon_0^2}{\Lambda_0 (hc)^3\,m_{\rm c}}\,\left[(kT_{\rm c})^2 - (kT_L)^2\right]. \tag{11}$$

When one defines the column mass density of the atmosphere between radius $R_L$ and the surface at $R_{\rm s}$ as $m_L \equiv \int_{R_L}^{R_{\rm s}} {\rm d}r\,\rho(r)$ and plugs Eq. (11) into Eq. (8), one ends up with

$$(kT_L)^2 = \alpha\,\left[(kT_{\rm c})^2 - (kT_L)^2\right] + \sqrt{\beta\left[(kT_{\rm c})^2 - (kT_L)^2\right]} \tag{12}$$

where

$$\alpha \equiv \frac{m_L}{m_{\rm c}}\,,$$
$$\beta \equiv \frac{20}{21\pi^2}\,\frac{\epsilon_0^2}{\Lambda_0 m_{\rm c}} = \frac{4}{3}\,\frac{(kT_{\rm c})^2}{\Lambda_{\rm R}(T_{\rm c})\,m_{\rm c}}\,. \tag{13}$$

The second expression for $\beta$ was derived by using Eq. (9) with $T = T_{\rm c}$. The product $\Lambda_{\rm R}(T_{\rm c})\,m_{\rm c}$ is a measure of the optical depth of the core for $\nu_e$ and $\bar\nu_e$ with the Rosseland mean free path being computed for a neutrino spectrum with temperature $T_{\rm c}$. Equation (12) can be solved for $T_L^2$,

$$(kT_L)^2 = \frac{1}{2(1+\alpha)^2}\left(2\alpha(1+\alpha)(kT_{\rm c})^2 - \beta \pm \sqrt{\beta\left[\beta + 4(1+\alpha)(kT_{\rm c})^2\right]}\right). \tag{14}$$

Since $\alpha \ll 1$ and $\beta \ll (kT_{\rm c})^2$ (see below), this result can be approximated to first order in $\alpha$ by

$$kT_L \cong \sqrt{\alpha}\,kT_{\rm c}\,. \tag{15}$$

This and Eqs. (11) and (13) yield

$$F_E = \frac{7\pi^5\,c}{30(hc)^3}\,\beta\,\left[(kT_{\rm c})^2 - (kT_L)^2\right] \cong \frac{7\pi^5\,c}{30(hc)^3}\,\beta(1-\alpha)\,(kT_{\rm c})^2\,. \tag{16}$$

For conditions representative of the phase where the nascent neutron star loses most of its binding energy by neutrino emission, the central core temperature is around $kT_{\rm c} \approx 30$–$50\,{\rm MeV}$ and the temperature at the base of the atmosphere (near or somewhat inside the muon and tau neutrino energy sphere) is $kT_L \approx 10$–$15\,{\rm MeV}$. Equation (15) thus implies $\alpha \approx 1/10$ as a typical number. The optical depth of the core for $\nu_e$ and $\bar\nu_e$ is of the order $\Lambda_{\rm R}(T_{\rm c})m_{\rm c} \sim 10^5$, so that $\beta \sim 10^{-5}(kT_{\rm c})^2$. Using these numbers in Eq. (16) one gets a luminosity $L_E = 4\pi R_{\rm s}^2 F_E \sim 2\times 10^{51}\,(R_{\rm s}/10\,{\rm km})^2 (kT_{\rm c}/50\,{\rm MeV})^4\,{\rm erg\ s^{-1}}$ for $\nu_e$ plus $\bar\nu_e$. Assuming all neutrino and antineutrino flavors contribute equally, this corresponds to a total neutrino luminosity of $L_\nu \sim 6\times 10^{51}\,{\rm erg\ s^{-1}}$, in good agreement with detailed numerical models.

### C. Emission Anisotropy

In order to estimate the anisotropy of the neutrino emission we take the deformed effective neutrinosphere to be what corresponds to the atmospheric surface $R_{\rm s}$ in the previous section. The column mass density of the atmosphere, i.e. between the core boundary $R_L$ and $R_{\rm s}$, is taken as

$$
\begin{aligned}
m_L &= m_{L0} + \delta m_L \cos\phi \\
&= \frac{m_+ + m_-}{2} + \frac{m_+ - m_-}{2} \cos\phi
\end{aligned}
\tag{17}
$$

where $\cos\phi = (\mathbf{q}\cdot\mathbf{B})/q$ is the cosine of the angle of the neutrino momentum $\mathbf{q}$ relative to the direction of the magnetic field and $m_\pm$ are the atmospheric column densities that correspond to $\cos\phi = \pm 1$. With Eq. (17) inserted into Eq. (13) and Eq. (16) one obtains

$$
F_{\mathrm{E}}(\cos\phi) \propto \beta\,(kT_{\mathrm{c}})^2 \left(1 - \frac{m_{L0}}{m_{\mathrm{c}}} - \frac{\delta m_L}{m_{\mathrm{c}}}\cos\phi\right). \tag{18}
$$

The asymmetry in the third component of the neutrino momentum can now be estimated as

$$
\begin{aligned}
\frac{\Delta q}{q} &\approx \frac{1}{6}\,\frac{\int_{-1}^{+1} \mathrm{d}\cos\phi\,\cos\phi\, F_{\mathrm{E}}(\cos\phi)}{\int_{-1}^{+1} \mathrm{d}\cos\phi\, F_{\mathrm{E}}(\cos\phi)} \\
&\approx -\frac{1}{18}\,\frac{\delta m_L}{m_{\mathrm{c}}}\,.
\end{aligned}
\tag{19}
$$

This result is accurate to first order in $\alpha_0 \equiv m_{L0}/m_{\mathrm{c}} \ll 1$ and we have assumed, as in Refs. [5] and [6], that only one neutrino species is responsible for the anisotropy which carries off about 1/6 of the total energy.

The mass difference $\delta m_L$ is connected with the radial deformation $\delta r$ of the surface of resonance, which is defined by $r(\phi) = r_0 + \delta r\,\cos\phi$, through

$$
\delta m_L = \frac{1}{2}(m_+ - m_-) = \rho_0\,\delta r\,. \tag{20}
$$

Here, $\rho_0$ is the mean density at the surface of resonance. For the width $\delta r$ in dependence of the strength of the magnetic field $B$ one finds [5,6]

$$
\delta r = \frac{eB}{2}\left(\frac{3n_e}{\pi^4}\right)^{1/3}\left(\frac{\mathrm{d}n_e}{\mathrm{d}r}\right)^{-1} \approx \frac{3}{2}\,\frac{eB}{\psi_e^2}\,h_{n_e} \tag{21}
$$

where $h_{n_e} = |\partial \ln n_e/\partial r|_{r_0}^{-1}$ is the scale height for changes of the electron number density near $r_0$ and $\psi_e \approx hc(3n_e/8\pi)^{1/3}$ is the chemical potential of the electrons. With the definition

$$
\gamma \equiv \frac{3}{2}\,\frac{eB}{\psi_e^2} \approx 0.22\left(\frac{20\,\mathrm{MeV}}{\psi_e}\right)^2\left(\frac{B}{10^{16}\,\mathrm{G}}\right) \tag{22}
$$

and the density scale height $h_\rho = |\partial \ln\rho/\partial r|_{r_0}^{-1}$ near $r_0$, Eqs. (20) and (21) yield

$$
\delta m_L = \rho_0 h_\rho\,\gamma\,\frac{h_{n_e}}{h_\rho}\,. \tag{23}
$$

Finally, with Eq. (19) one ends up with

$$
\frac{\Delta q}{q} \approx -\frac{1}{18}\,\frac{\rho_0 h_\rho}{m_{\mathrm{c}}}\,\gamma\,\frac{h_{n_e}}{h_\rho}\,. \tag{24}
$$

Taking $\rho_0 h_\rho/m_{\mathrm{c}} < m_{L0}/m_{\mathrm{c}} \approx 1/10$ and $h_{n_e}/h_\rho \lesssim 1$ [6] leads to the numerical estimate

$$
\frac{\Delta q}{q} < -0.0012\left(\frac{20\,\mathrm{MeV}}{\psi_e}\right)^2\left(\frac{B}{10^{16}\,\mathrm{G}}\right). \tag{25}
$$

This result is at least 10 times smaller than the anisotropy derived in Ref. [6].

Therefore, the kick mechanism based on a deformation of the effective neutrinosphere requires more than an order of magnitude larger magnetic fields than estimated in Refs. [6,8,9] whose analysis already reduced the effect originally discussed by Kusenko and Segrè [5]. For a neutrino emission anisotropy of 1%, corresponding to a recoil velocity of the nascent neutron star of approximately $300\,\mathrm{km\,s^{-1}}$, one needs magnetic fields in excess of about $10^{17}$ G near the stellar surface.

The value of $\gamma$ in Eq. (22) is sensitive to the electron chemical potential. In Ref. [6] $\gamma$ was evaluated by using $Y_e = n_e/n_b \approx 0.1$ for the electron fraction ($n_b$ is the baryon number density) at a density $\rho \approx 10^{12}\,\mathrm{g\,cm^{-3}}$. A discussion of the uncertainties of this choice can be found in Appendix A where the structure of the protoneutron star atmosphere is self-consistently determined from a simple, analytical model. Typically, $\gamma$ decreases during the neutrino cooling of the nascent neutron star because the atmosphere becomes denser and more compact as the star cools and deleptonizes. This disfavors large $\delta r$ at intermediate and late times during the Kelvin-Helmholtz cooling when most of the gravitational binding energy of the neutron star is emitted in neutrinos. This makes large emission anisotropies even more unlikely.

## IV. SUMMARY AND DISCUSSION

We have argued that the neutrino-oscillation scenarios for neutron star kicks [5–10] suffer from two serious flaws; both problems are related to an incorrect picture of neutrino transport in the atmosphere of a protoneutron star.

First, when the neutrino luminosity is equipartitioned between the flavors, no significant recoil can be produced because only an indirect, higher-order effect remains which is associated with the spectral swap of two neutrino flavors. Spectral differences imply a change of the neutrino interaction with the stellar background and thus affect the neutrino transport from the core to the surface through a modified atmospheric temperature profile. While we cannot estimate the magnitude of the resulting small kick velocity, we are convinced that it is a very small effect. Moreover, current supernova models overestimate the spectral differences between $\nu_e$ and $\nu_\tau$ because neutrino interactions have not been taken into account which enhance the thermal coupling between $\nu_\tau$ and the stellar medium [15,16].

Second, when the luminosities are taken to be vastly different (as had effectively been assumed in previous papers), again no effect obtains in zeroth order because a shift of the location of the neutrinosphere does not affect the neutrino luminosity. The latter is governed by the core emission, not by local processes in the atmosphere.

However, the atmosphere adjusts to the oscillation-induced modification of its transport capabilities, causing a small, higher-order effect due to an altered temperature gradient in the core; in our treatment it was expressed as a change of the temperature at the core-atmosphere interface. In this sense the neutrino flux from the core fixes the atmospheric temperature profile and, conversely, the atmosphere determines the core emission. In contrast, in Refs. [5–10] it had been assumed that the unperturbed atmospheric temperature is a measure of the neutrino luminosity by virtue of the Stefan-Boltzmann law.

Both of our arguments imply a huge suppression of the pulsar recoils calculated in Refs. [5–10], but a realistic quantitative estimate of the residual effects is not possible with our simple analytic tools. A detailed numerical treatment would be extremely difficult, and the motivation for such an effort is minimal because most likely one would confirm what now looks like a non-effect. In any case, it is clear that the oscillation scenarios require much larger magnetic fields than had been contemplated in Refs. [5–10] and thus probably take one beyond what is astrophysically motivated.

We find it disappointing that both the cumulative parity violation and the neutrino oscillation scenario, which seemed to work with reasonable magnetic field strengths, do not survive a self-consistent discussion. Of course, it remains possible that huge magnetic fields with an asymmetric distribution cause sufficiently asymmetric neutrino opacities for a large neutrino rocket effect [18]. It is also possible that asymmetric neutrino emission has nothing to do with the pulsar kicks. Either way, it does not look as if the pulsar velocities can be attributed to neutrino oscillations within presently discussed scenarios.


## ACKNOWLEDGMENTS

This work was supported, in part, by the Deutsche Forschungsgemeinschaft under grant No. SFB-375.


## APPENDIX A: SIMPLE MODEL FOR PROTONEUTRON STAR ATMOSPHERE

A crucial parameter for estimating $\delta r$ in Eq. (21) is the electron number density $n_e$ in the protoneutron star atmosphere between the energy spheres of $\nu_e$ and heavy-flavor neutrinos $\nu_\mu$ and $\nu_\tau$. Making use of the fact that the degeneracy parameter of electron neutrinos approaches zero, $\eta_{\nu_e} = \eta_e + \eta_p - \eta_n \to 0$, as the protoneutron star atmosphere deleptonizes and the electron chemical potential $\psi_e = kT\eta_e$ decreases [15,19], one can easily estimate $n_e$ and the electron number fraction $Y_e = n_e/n_b = (n_{e^-} - n_{e^+})/n_b$ where $n_b = \rho/m_u$ is the number density of baryons and $m_u$ the atomic mass unit. With $Y_e = Y_p$, $Y_n = 1 - Y_p$, and $\eta_n - \eta_p \approx \ln(Y_n/Y_p)$ for Boltzmann gases of neutrons $n$ and protons $p$ with $m_n \approx m_p$, one finds as a very good approximation

$$\begin{aligned} Y_e &\approx \frac{8\pi m_u (kT)^3}{(hc)^3\rho}\,\frac{\eta_e}{3}\,\left(\pi^2+\eta_e^2\right) \\ &\approx 9.1\times10^{-3}\left(\frac{kT}{5\ \mathrm{MeV}}\right)^3\left(\frac{\rho}{10^{11}\ \mathrm{g\ cm^{-3}}}\right)^{-1} \\ &\quad\times\ln\left(\frac{1}{Y_e}-1\right)\left\{\pi^2+\left[\ln\left(\frac{1}{Y_e}-1\right)\right]^2\right\}. \end{aligned} \tag{A1}$$

This equation shows that $Y_e$ decreases with increasing density $\rho$ for a given temperature.

The temperature as a function of column mass density $m$ is given by Eq. (8) as $(kT)^2 = Am + (kT_\mathrm{s})^2$. In order to determine $\rho(m)$, we employ hydrostatic equilibrium which yields for the pressure $P = gm + P_\mathrm{s}$ as a function of $m$. Here $P_\mathrm{s}$ is the pressure at the surface and $g = GM_\mathrm{ns}/R_\mathrm{s}^2$ is the gravitational acceleration near the surface of the protoneutron star which is nearly constant in the thin, plane-parallel atmosphere. Since the layers between the energy spheres of electron neutrinos and muon and tau neutrinos are dense ($\rho \sim 10^{11}$–$10^{14}$ g cm$^{-3}$) and rather cool ($kT \sim$ 3–10 MeV), the pressure is dominated by baryons (see Fig. 8 of Ref. [20]) so that we can take $P \approx kT\rho/m_u$ to obtain

$$\rho(m) \approx \frac{(gm + P_\mathrm{s})m_u}{kT}\,. \tag{A2}$$

With Eqs. (8) and (A2) one derives

$$\frac{(kT)^3}{\rho} \approx \frac{\left[Am + (kT_\mathrm{s})^2\right]^2}{m_u gm + \rho_\mathrm{s} kT_\mathrm{s}}\,. \tag{A3}$$

From this relation and Eq. (A1) one can see that $Y_e$ first decreases only slightly, then increases with rising $m$, i.e. on the way inward into the atmosphere. Therefore, the minimum value of the electron fraction can be found very close to the electron neutrino energy sphere $R_{E,\nu_e} \approx R_\mathrm{s}$. This confirms $h_{n_e}/h_\rho \lesssim 1$ because $\rho$ as well as $Y_e$ have a negative gradient interior to $R_{E,\nu_e}$. Typical conditions near $R_{E,\nu_e}$ at early times during the protoneutron star evolution are [15,19] $kT_\mathrm{s} \approx 3$ MeV and $\rho_\mathrm{s} \approx 10^{11}$ g cm$^{-3}$ which yields $Y_e(R_\mathrm{s}) \approx 0.078$. At later times the temperature in the atmosphere drops due to cooling [15,19], and according to Eq. (A2) the density in the atmosphere must become higher and the density gradient steeper. Therefore, the electron neutrinosphere, which is assumed to be located at a certain value of the optical depth, moves to higher densities. Typical conditions then are $kT_\mathrm{s} \approx 5$ MeV and $\rho_\mathrm{s} \approx 10^{12}$ g cm$^{-3}$ for which one gets $Y_e(R_\mathrm{s}) \approx 0.050$. Even later, one has $kT_\mathrm{s} \approx 5$ MeV and $\rho_\mathrm{s} \approx 5\times10^{13}$ g cm$^{-3}$ which gives $Y_e(R_\mathrm{s}) \approx 0.004$.

Although $Y_e(R_\mathrm{s})$ decreases as the protoneutron star cooling goes on, the number density $n_e(R_\mathrm{s})$ nevertheless increases because of the rising density $\rho_\mathrm{s}$. In the listed examples, $n_e(R_\mathrm{s})$ changes from $4.7 \times 10^{33}$ cm$^{-3}$ through $3.0 \times 10^{34}$ cm$^{-3}$ to $1.2 \times 10^{35}$ cm$^{-3}$. From Eq. (21) we conclude that this disfavors large $\delta r$ at intermediate and late times during the Kelvin-Helmholtz neutrino cooling of the nascent neutron star.

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