Cosmic Microwave Background Radiation in the Direction of a Moving Cluster of Galaxies with Hot Gas: Relativistic Corrections

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ABSTRACT

It has been recently realized (Rephaeli 1995) that the relativistic corrections to the spectral distortions of the cosmic microwave background (CMB) measured in the directions of galaxy clusters containing hot gas are significant and should be detectable with the forthcoming experiments. In the present paper we calculate the correction terms that are proportional to $(V/c)^n$, with $n = 2$–4, and $V_{\rm r}/c \times kT_e/m_e c^2$ to the standard formulae describing the spectral distortions caused by the bulk motion (kinematic effect) and thermal motion (thermal effect) of free electrons for the case of a cluster having a peculiar velocity V ($V_{\rm r}$ being its radial component). The results of our analytical calculations are confirmed by direct Monte-Carlo simulations (Sazonov & Sunyaev 1998).

Subject headings: cosmic microwave background — cosmology: theory — galaxies: clusters: general — radiation mechanisms: thermal

1. Introduction

Thomson scattering by hot electrons in the intergalactic gas bound in clusters of galaxies modifies the spectrum of the cosmic microwave background (CMB) radiation (Sunyaev & Zel'dovich 1972). Zel'dovich & Sunyaev (1969), basing on the Kompaneets equation (1957), derived a simple formula describing the spectral form of the resulting distortion, which is proportional to the parameter $y = (kT_e/m_e c^2)\tau$, where τ is the Thomson

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optical depth along the line of sight. The effect has now been observed from a number of galaxy clusters (Birkinshaw 1998).

Recently, interest to this effect has been reactivated in view of the perspectives of accurate measurement of the CMB distortions in a number of experiments, both ground-based and on balloons, by the MAP spacecraft, and especially by the Planck Surveyor mission scheduled to be flown in the middle of the next decade. These activities were motivated by the fact that the gas temperature is so high in the clusters of galaxies (ranging between ~ 3 and ~ 17 keV, Tucker et al. 1998) that the scattering electrons have thermal velocities of the order of $0.1 - 0.3\ c$, so one has to include into consideration the relativistic corrections to the differential scattering cross-section and the electron velocity distribution to obtain an accurate result. Rephaeli (1995), basing on extensive previous work (Wright 1979; Fabbri 1981; Taylor & Wright 1989; Loeb et al. 1991) has demonstrated by means of numerical calculations the importance of the relativistic corrections for the future experiments. Stebbins (1997), Itoh et al. (1998), and Challinor & Lasenby (1998) used a Fokker-Planck approximation of the relativistic photon kinetic equation to obtain corrections, written as series in powers of kT_e/m_ec^2, to the standard nonrelativistic solution. These results have proved to be in excellent agreement with those of Rephaeli, thus demonstrating the applicability of the analytical solutions obtained using the diffusion approximation to the problem at hand, despite the small optical depths of galaxy clusters ($\tau \sim 0.01$). This consistency is interesting and not pre-determined, as 1) only a very small ($\sim \tau$) part of the incident photons experience scatterings in the gas of the cluster, and 2) these scattered photons acquire a significant change in their energy.

A gas cloud moving rapidly with respect to the CMB along the observer's line of sight must significantly modify the spectrum of the CMB in addition to the thermal effect. The change in the brightness temperature caused by this "kinematic" effect is to first order simply proportional to the radial component of the cluster velocity $\sim (V_{\rm r}/c)\tau$ (Sunyaev & Zel'dovich 1980). The effect should be detectable in the future, and will enable measurement of cluster peculiar velocities, with significant implications for studies of the large-scale structure of the universe. It is obvious that corrections similar to those found for the thermal effect must exist and should be taken into account for the kinematic effect as well. In this paper, we calculate the next-order changes in the spectrum of the CMB related to the cluster peculiar motion by solving the photon kinetic equation. We have obtained simple formulae that give the correction terms of the orders of $(V/c)^n$ (n up to 4) and $V_{\rm r}/c \times kT_e/m_ec^2$. Our method is similar to that used by Psaltis & Lamb (1997) who considered the more general problem of comptonization in a moving media. The solution these authors have obtained, although applicable to many astrophysical situations, does not contain the $O(V_{\rm r}/c \times kT_e/m_ec^2)$ term, because this term is third-order in electron velocity, whereas their solution is accurate only to second order in it. We also confirm the existence of the term of order $(kT_e/m_ec^2)^2$ found earlier using techniques different from ours (Rephaeli 1995; Stebbins 1997; Itoh et al. 1998; and Challinor & Lasenby 1998). Earlier, we found all the relativistic corrections mentioned above numerically using Monte-Carlo simulations (Sazonov & Sunyaev 1998).

2. Scattering of the CMB by a directed beam of electrons – relativistic description of the kinematic effect

From the point of view of the observer each electron in the intercluster gas scatters CMB photons independently. We can therefore first consider the problem of scattering of the CMB by a directed beam of monoenergetic electrons having a density $N_{\rm e}$ and moving at velocity $\mathbf{v}$. Once we have found a solution to this simplified problem, we will be able to consider the more general problem of scattering of the CMB by a cloud of thermal electrons having a peculiar motion, which corresponds to the real situation of a galaxy cluster, by simply averaging the result obtained for the directed beam over a drifting Maxwellian distribution of electron velocities. We will use two coordinate frames. Quantities with subscript 0 will refer to the system where the electrons are at rest, while quantities without subscript will refer to the frame that is fixed to the CMB, hereafter referred to as the laboratory frame. It is easy to show that our results hold for a cluster at any redshift (see a discussion in Sazonov & Sunyaev 1998). In the laboratory frame, the initial occupation number in the photon phase space is Planckian with a temperature $T_{\rm CMB}$: $n = 1/(e^x - 1)$, where $x = h\nu/kT_{\rm CMB}$. The corresponding spectral intensity is

$$I_\nu = \frac{2(kT_{\rm CMB})^3}{(hc)^2}\frac{x^3}{e^x - 1}. \tag{1}$$

The occupation number is invariant with respect to the Lorentz transformations of the frequency and direction of motion of a photon:

$$\nu_0 = \frac{\nu}{\gamma(1+\beta\mu_0)} = \nu\gamma(1-\beta\mu) \text{ and } \mu_0 = \frac{\mu-\beta}{1-\beta\mu}, \tag{2}$$

where $\beta = v/c$, $\gamma = (1-\beta^2)^{-1/2}$, $\mu = \cos\theta$, and θ is the angle between $\mathbf{v}$ and the photon velocity (Landau & Lifshitz 1975). Therefore, in the electron rest frame the occupation number depends on the photon incident direction:

$$n_0 = \frac{1}{e^{x_0\gamma(1+\beta\mu_0)} - 1}. \tag{3}$$

Ignoring induced scattering and the change in the photon frequency in each scattering due to the recoil effect, we can write down the photon kinetic equation in the electron rest frame as follows:

$$\frac{d}{dt_0} n_0(\mu_0, \nu_0) = cN_{\rm e0} \int \frac{d\sigma}{d\Omega_0'} [n_0(\mu_0', \nu_0) - n_0(\mu_0, \nu_0)]\, d\Omega_0'. \tag{4}$$

Integrating the Thomson differential cross-section over the azimuthal angle (Chandrasekhar 1950) one derives

$$\frac{d}{dt_0} n_0(\mu_0, \nu_0) = \frac{3cN_{\rm e0}\sigma_{\rm T}}{16} \int_{-1}^{1} (3 + 3\mu_0^2\mu_0'^2 - \mu_0^2 - \mu_0'^2)[n_0(\mu_0', \nu_0) - n_0(\mu_0, \nu_0)]\, d\mu_0'. \tag{5}$$

We now evaluate the collision integral in equation (5) by expanding n_0 up to forth order in β. The integral is then easily taken, yielding

$$\begin{aligned}
\frac{1}{n_0(\mu_0, \nu_0)} \frac{dn_0(\mu_0, \nu_0)}{dt_0} = cN_{\rm e0}\sigma_{\rm T} \Bigg\{ & \frac{x_0 e^{x_0}}{e^{x_0} - 1} \left[\mu_0\beta + \frac{3(-1+3\mu_0^2)}{20} x_0\beta^2 + \frac{\mu_0}{2}\beta^3 + \frac{\mu_0}{2} x_0\beta^3 \right. \\
& \left. + \frac{\mu_0^3}{6} x_0^2\beta^3 + \frac{3(-1+3\mu_0^2)}{20} x_0\beta^4 + \frac{3(-1+3\mu_0^2)}{40} x_0^2\beta^4 + \frac{-6 - 3\mu_0^2 + 35\mu_0^4}{840} x_0^3\beta^4 \right] \\
& + \left(\frac{x_0 e^{x_0}}{e^{x_0} - 1}\right)^2 \left[\frac{3+\mu_0^2}{10}\beta^2 - \frac{\mu_0}{2}\beta^3 - \frac{\mu_0(3+\mu_0^2)}{20} x_0\beta^3 + \frac{3+\mu_0^2}{10}\beta^4 + \frac{3-\mu_0^2}{8} x_0\beta^4 \right. \\
& \left. + \frac{4 - \mu_0^2 - \mu_0^4}{40} x_0^2\beta^4 \right] + \left(\frac{x_0 e^{x_0}}{e^{x_0} - 1}\right)^3 \left[\frac{\mu_0(3+\mu_0^2)}{10}\beta^3 - \frac{3+\mu_0^2}{10}\beta^4 \right. \\
& \left. + \frac{-36 + 3\mu_0^2 + 7\mu_0^4}{140} x_0\beta^4 \right] + \left(\frac{x_0 e^{x_0}}{e^{x_0} - 1}\right)^4 \left[\frac{3(2+\mu_0^2)}{35}\beta^4 \right] \Bigg\}.
\end{aligned} \tag{6}$$

Our next step is to calculate the corresponding scattering rate as measured in the laboratory frame. This can be done by making use of the Lorentz-invariance property of the photon occupation number. It is easily shown (see e.g. Peebles 1971) that

$$\frac{dn(\mu, \nu)}{dt} = \frac{1}{\gamma(1+\beta\mu_0)} \frac{dn_0(\mu_0, \nu_0)}{dt_0}. \tag{7}$$

Using relations (2), (7) and substituting $N_{e0} = N_e/\gamma$ (due to the Lorentz-transformation of volume) for the electron density, we derive from equation (6)

$$\frac{1}{n(\mu,\nu)}\frac{dn(\mu,\nu)}{dt} = cN_{\rm e}\sigma_{\rm T}\frac{xe^x}{e^x-1}\left\{\beta\mu+\beta^2\left[-1-\mu^2+\frac{3+11\mu^2}{20}F\right]+\beta^3\mu\left[2-\frac{31+11\mu^2}{20}F\right.\right.$$
$$\left.+\frac{9+13\mu^2}{120}(2F^2+G^2)\right]+\beta^4\left[-1-\mu^2+\frac{17+53\mu^2}{20}F-\frac{9+66\mu^2+13\mu^4}{120}(2F^2+G^2)\right.$$
$$\left.\left.+\frac{3+33\mu^2+28\mu^4}{420}F(F^2+2G^2)\right]\right\}, \quad (8)$$

where $F = x\coth(x/2)$, and $G = x/\sinh(x/2)$. Formula (8) presents a description of the *kinematic effect for any moving gas cloud accurate to fourth order in* $\beta = V/c$, where V is the cloud peculiar velocity.

Compton scattering retains the total number of photons. We have verified that all the β terms in equation (8) indeed vanish after the integration over photon direction and frequency: $d/dt\int n\nu^2\, d\nu\, d\mu = 0$.

Another known integral property of the process of Compton scattering is the energy exchange rate between an electron and an isotropical radiation field. The radiation energy density $\epsilon_{\rm r}$ (see e.g. Pozdnyakov et al. 1983) should increase with time as

$$\frac{d\epsilon_{\rm r}}{dt} = \frac{4}{3}cN_e\sigma_{\rm T}\epsilon_{\rm r}(\gamma^2-1) \quad (9)$$

Taking an integral $d/dt\int n\,\nu^3\, d\nu\, d\mu$ we find in our case

$$\frac{d\epsilon_{\rm r}}{dt} = \frac{4}{3}cN_e\sigma_{\rm T}\epsilon_{\rm r}(\beta^2+\beta^4), \quad (10)$$

which is identical to dependence (9) to forth order in β.

3. Scattering of the CMB by the hot gas in a moving cluster

Consider now a cluster of galaxies moving with a peculiar velocity V at an angle θ ($\mu = \cos\theta$) relative to the vector drawn from the cluster to the observer. The cluster contains hot gas, and the distribution of the electron velocities in the cluster rest frame is assumed to be relativistic Maxwellian with a temperature T_e: $dN_e \sim \exp\left[-E_0(\mathbf{p}_0)/kT_e^2\right] d\mathbf{p}_0$, where $\mathbf{p}_0$ is the electron momentum, and E_0 the electron energy. The corresponding distribution in the laboratory frame is obtained via the Lorentz-transformation of $\mathbf{p}$:

$$p_x = \gamma(p_{x0} + \frac{V}{c^2}E_0);\; p_y = p_{y0};\; p_z = p_{z0}, \tag{11}$$

where $\gamma = (1 - V^2/c^2)^{-1/2}$, and axis X is drawn along the direction of the cluster peculiar motion (Landau & Lifshitz 1975). We can average equation (8) over the resulting electron velocity distribution expanded in powers of V/c and kT_e/m_ec^2 to get

$$\begin{aligned}\frac{\delta n(\nu)}{n(\nu)} = \tau\frac{xe^x}{e^x-1}\left\{\frac{V}{c}\mu + \frac{kT_e}{m_ec^2}(-4+F) + \left(\frac{V}{c}\right)^2\left[-1-\mu^2+\frac{3+11\mu^2}{20}F\right]\right.\\ +\frac{V}{c}\frac{kT_e}{m_ec^2}\mu\left[10-\frac{47}{5}F+\frac{7}{10}(2F^2+G^2)\right]\\ \left.+\left(\frac{kT_e}{m_ec^2}\right)^2\left[-10+\frac{47}{2}F-\frac{42}{5}F^2+\frac{7}{10}F^3+\frac{7}{5}G^2(-3+F)\right]\right\}.\end{aligned} \tag{12}$$

Here we have replaced an integral over time $\int N_e\sigma_{\rm T}c\,dt$ by an integral taken along the line of sight $\tau = \int N_e(r)\sigma_{\rm T}\,dr$, where $\tau \ll 1$. A Monte-Carlo computation that takes into account only single-scattering events proves the correctness of this transition (Sazonov & Sunyaev 1998).

The subsequent terms in equation (12) physically correspond to increasing orders in β in equation (8), which is, in the current problem, a sum of the electron thermal and peculiar velocities. The first term (of order $V_{\rm r}/c$) (hereafter $V_{\rm r} = V\mu$) describes the *kinematic effect*. The second term, which is proportional to the squared thermal velocity, describes the *thermal effect*. The $O[(V/c)^2]$ term is the main relativistic correction to the kinematic effect for a cloud of cold electrons. The "*interference*" term that is proportional to $V_{\rm r}/c \times kT_e/m_ec^2$ draws from the term of order β^3 in equation (8). We first ascertained its existence by means of Monte-Carlo simulations (Sazonov & Sunyaev 1998). This term constitutes the leading relativistic correction to the kinematic effect and is the main subject of the present paper. Finally, the term of order $(kT_e/m_ec^2)^2$ (forth-order in β) is the *leading relativistic correction to the thermal effect* found earlier using a Fokker-Planck approximation (Stebbins 1997; Itoh et al. 1998; Challinor & Lasenby 1998). We note that the term of order $(V/c)^2 \times kT_e/m_ec^2$, whose contribution would be negligible, does not appear in equation (12) (although it is of fourth order in β), because in expanding the Maxwellian velocity distribution above we restricted ourselves to linear terms in V/c and kT_e/m_ec^2. The terms $O[(V/c)^3]$ and $O[(V/c)^4]$ are not given either in formula (12) for the same reason, but they are already present in expression (8).

We should specially emphasize that the spectral effect caused by the "interference" term in equation (12) can be either positive, or negative depending on the direction of the cluster motion ($\mu > 0$ or $\mu < 0$). This behavior is similar to that of the main kinematic effect.

4. Properties of the CMB Spectral Distortion

The distortion of the spectral intensity of the CMB is related to the corresponding change in the photon occupation number by the equation

$$\delta I_\nu = I_\nu \frac{\delta n(\nu)}{n(\nu)}. \tag{13}$$

In Fig. 1 we plot this distortion at $kT_e/m_e c^2 = 0.02$, $V/c = 0.01$ (the large value for the peculiar velocity was chosen for illustration purposes), for two opposite directions of the cluster motion, $\mu = 1$ and $\mu = -1$. One can see that the contribution from the newly found term $O(V_\mathrm{r}/c \times kT_e/m_e c^2)$ to the total effect is significant. Note that this contribution reaches its local maximum at $x = 3.34$, i.e. near the frequency $x_\mathrm{c} \sim 3.83$ at which the thermal effect vanishes.

In our previous paper (Sazonov & Sunyaev 1998) we have performed Monte-Carlo simulations to determine the spectral changes in the CMB for various sets of parameters. As evident from Fig. 1, which uses the result of that paper, the good accuracy of the analytical formula (12) is confirmed by the numerical calculations (except very far into the Wien part of the spectrum, where the next-order corrections, $\sim V_\mathrm{r}/c \times (kT_e/m_e c^2)^n$, $n = 2, 3, ...$, become important).

Using equation (13) we have obtained a simple approximation formula describing the position of the crossover frequency (X_0), i.e. the frequency at which the distortion of the incident microwave spectrum is zero:

$$X_0 = 3.830 \left[1 - 0.30 \frac{V_r}{c} \frac{m_e c^2}{kT_e} + 1.1 \frac{kT_e}{m_e c^2} - 1.5 \frac{V_r}{c}\right]. \tag{14}$$

The correction term of order $kT_e/m_e c^2$ in this expression was known before our work (Rephaeli 1995; Itoh et al. 1998; Challinor & Lasenby 1998). The $O(V_\mathrm{r}/c \times m_e c^2/kT_e)$ term is due to the kinematic effect. The term of order V_r/c contains the contribution from the "interference" effect. One should note, that the form of the expansion used in relation (14), which is different from that in equation (12), is a natural mathematical result of the balancing of the small changes in the total effect due to the frequency gradient of the main thermal effect on one side, and the contributions from the other effects on the other.

We have obtained similar approximate formulae for the positions (X_{min} and X_{max}) and values ($(\delta I_\nu)_{min}$ and $(\delta I_\nu)_{max}$) of the minimum and maximum of the spectrum of the CMB distortion:

$$X_{min} = 2.266\left[1 - 0.20\frac{V_r}{c}\frac{m_ec^2}{kT_e} - 0.2\frac{kT_e}{m_ec^2} - 0.04\left(\frac{V_r}{c}\frac{m_ec^2}{kT_e}\right)^2 + 0.0\frac{V_r}{c}\right], \qquad (15)$$

$$(\delta I_\nu)_{min} = -2.059\,\frac{2(kT_R)^3}{(hc)^2}\frac{kT_e}{m_ec^2}\tau\left[1 - 0.83\frac{V_r}{c}\frac{m_ec^2}{kT_e} - 3.8\frac{kT_e}{m_ec^2} + 0.09\left(\frac{V_r}{c}\frac{m_ec^2}{kT_e}\right)^2 + 3.1\frac{V_r}{c}\right], \qquad (16)$$

$$X_{max} = 6.511\left[1 - 0.09\frac{V_r}{c}\frac{m_ec^2}{kT_e} + 2.6\frac{kT_e}{m_ec^2} - 0.01\left(\frac{V_r}{c}\frac{m_ec^2}{kT_e}\right)^2 + 0.4\frac{V_r}{c}\right], \qquad (17)$$

$$(\delta I_\nu)_{max} = 3.390\,\frac{2(kT_R)^3}{(hc)^2}\frac{kT_e}{m_ec^2}\tau\left[1 + 0.40\frac{V_r}{c}\frac{m_ec^2}{kT_e} - 7.3\frac{kT_e}{m_ec^2} + 0.05\left(\frac{V_r}{c}\frac{m_ec^2}{kT_e}\right)^2 + 0.8\frac{V_r}{c}\right]. \qquad (18)$$

Note that the term of order $(V_{\rm r}/c \times m_ec^2/kT_e)^2$ in these formulae is due to the presence of a small frequency gradient in the kinematic effect. This term is negligible (thus not appearing) in relation (14).

Finally, one can calculate the excess in the CMB energy flux in the direction of the cluster, $\delta I = \int \delta I_\nu\, d\nu$:

$$\delta I = \tau I\left[4\frac{V_{\rm r}}{c} + 4\frac{kT_e}{m_ec^2} + (7\mu^2 - 1)\left(\frac{V}{c}\right)^2 + 20\frac{V_{\rm r}}{c}\frac{kT_e}{m_ec^2} + 10\left(\frac{kT_e}{m_ec^2}\right)^2\right], \qquad (19)$$

where $I = \sigma_R T^4_{\rm CMB}/\pi$, and σ_R is the Stefan-Boltzmann constant.

One can see again that the relativistic correction of order $V_{\rm r}/c \times kT_e/m_ec^2$ is important. Equation (19) allows one to calculate the rate of energy exchange between the hot gas and the CMB. We immediately see that the $O(kT_e/m_ec^2)$ and $O[(kT_e/m_ec^2)^2]$ terms are the two leading terms in the relativistic formula that gives the energy transfer rate averaged over a Maxwellian distribution of electron velocities: $d\epsilon_{\rm r}/dt = 4/3\, cN_e\sigma_{\rm T}\epsilon_{\rm r}\langle\gamma^2 - 1\rangle$. Integration of the term of order $(V/c)^2$ in formula (19) over the observing angle (μ) leads again to relation (10) for the energy transfer rate due to the bulk motion of the electrons, as one should have expected.

We have also obtained approximate formulae that give separately the flux from the "negative" source, i.e. integrated over the frequency range $(0, X_0)$ and that from the positive source (integrated from X_0 to ∞), taking into account dependence (14):

$$\delta I_{-} = \tau I \left[1.5 \frac{V_{\mathrm{r}}}{c} - 1.35 \frac{kT_e}{m_e c^2} - 0.4 \left(\frac{V_{\mathrm{r}}}{c} \right)^2 \frac{m_e c^2}{kT_e} - 2.4 \frac{V_{\mathrm{r}}}{c} \frac{kT_e}{m_e c^2} + 3.3 \left(\frac{kT_e}{m_e c^2} \right)^2 \right], \tag{20}$$

$$\delta I_{+} = \tau I \left[2.5 \frac{V_{\mathrm{r}}}{c} + 5.35 \frac{kT_e}{m_e c^2} + 0.4 \left(\frac{V_{\mathrm{r}}}{c} \right)^2 \frac{m_e c^2}{kT_e} + 22.4 \frac{V_{\mathrm{r}}}{c} \frac{kT_e}{m_e c^2} + 6.7 \left(\frac{kT_e}{m_e c^2} \right)^2 \right]. \tag{21}$$

Terms of order $(V/c)^2$ have been neglected in these formulae.

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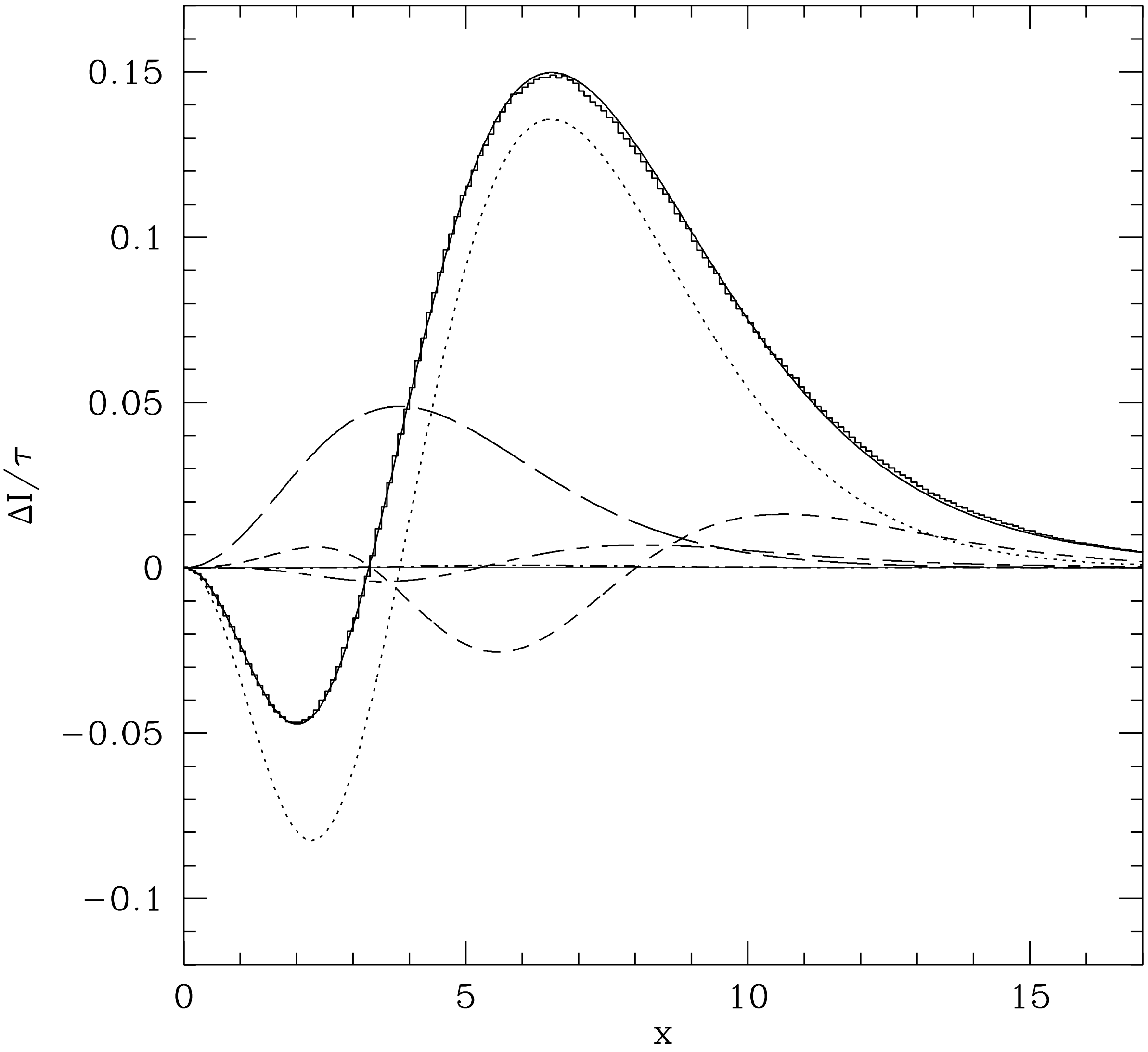

Fig. 1.— (a) An example of the CMB spectral distortion (in units of $2(kT_e)^3/(hc)^2$) due to the Sunyaev-Zel'dovich effect for the following set of cluster parameters: $kT_e/m_ec^2 = 0.02$ ($T_e = 10.2$ keV), $V/c = 0.01$ ($V = 3000$ km/s), and $\mu = 1$ (the cluster moves towards us). The solid line shows the cumulative effect, which was calculated by summing the analytical expressions for terms of different orders in kT_e/m_ec^2 and V/c as given in Itoh et al. (1998) and the present paper. The contributions from the following components are shown: $O(kT_e/m_ec^2)$ (dotted curve), $O((kT_e/m_ec)^2)$ (short-dashed curve), $O(V/c)$ (long-dashed curve), $O((V/c)^2)$ (dash-dotted curve), and $O(V/c \times kT_e/m_ec^2)$ (long-short dashed curve). For comparison, the result of Monte-Carlo simulations is shown as a histogram.

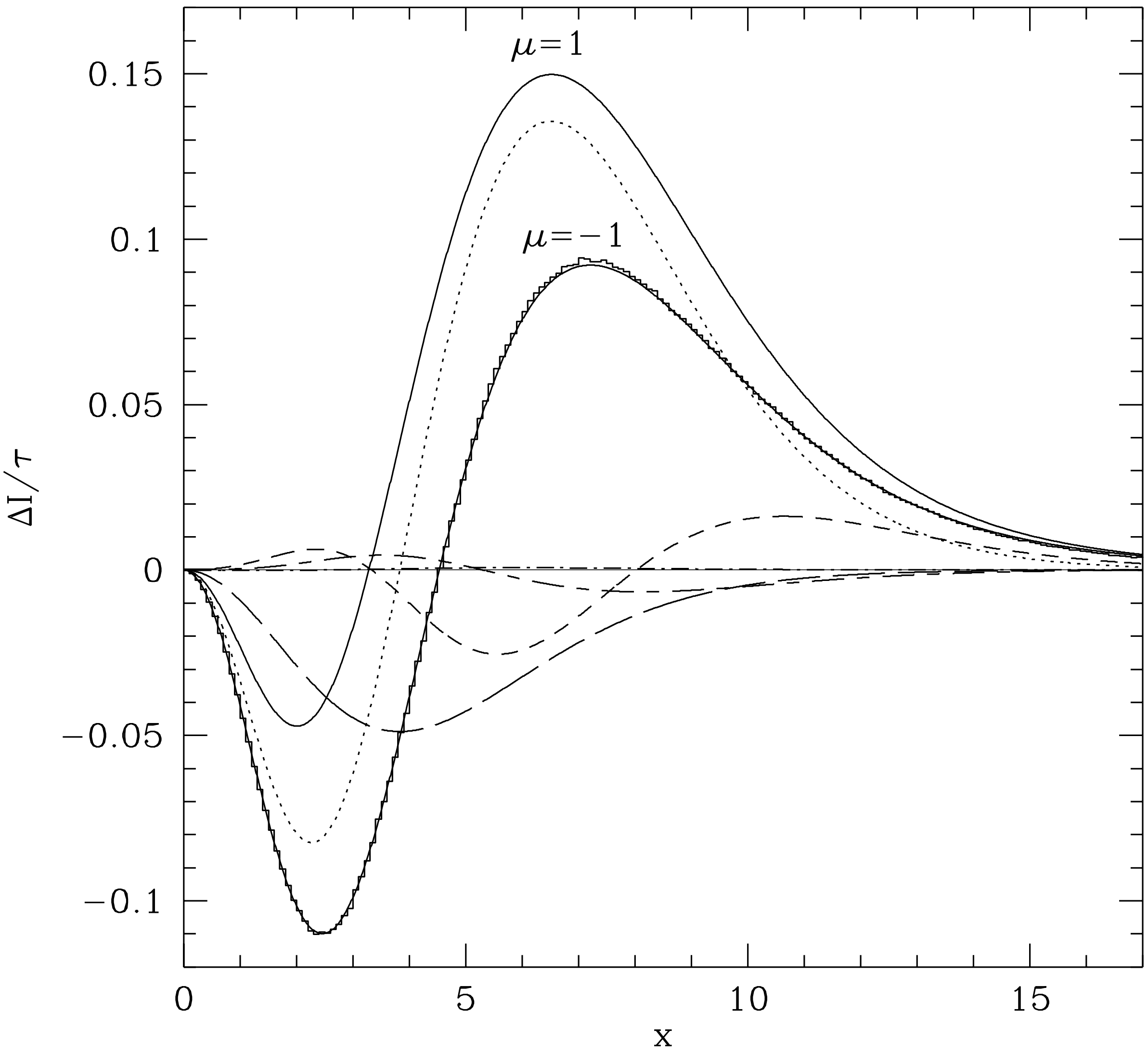

Fig. 1.— (b) Same as (a), but the cluster moves outwards from the observer ($\mu = -1$). For comparison, the result shown in (a) is repeated.