Results from a Gyrotropic Two Ion Fluid Model for the Comet Interaction with the Solar-Wind

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Abstract. A two-and-a-half-dimensional two ion fluid model (with gyrotropic pressure tensor) for the interaction of cometary pickup plasma with the solar wind has been developed. Whereas previous models have, in general, assumed a single fluid plasma, this generalized model follows the evolution of two fluids - solar wind protons and cometary ions. The two fluids are coupled by the magnetic field and by coupling terms which are the velocity moments of the time relaxation model of Bhatnagar-Gross-Krook. Further more, a gyrotropic pressure tensor replaces the isotropic pressure assumed in other models. Isotropization terms couple the pressures in the directions perpendicular and parallel to the magnetic field. Among the results received, the stand-off distance of the shock increased significantly when the isotropization rate was reduced. The large cometary plasma pressure forces at the shock accelerated the cometary plasma along the shock. As a result, the cometary plasma drifted ahead of the solar wind along the shock flanks parallel to the magnetic field. Close to the nucleus of the comet, in the far tail lobes, and in the far inner tail, the cometary plasma lagged behind the solar wind.

Introduction

The physics of the pickup ions is a long standing problem in space physics. Spacecraft encounters with comets provided us with measurements in the presence of an extended and relatively intense source of pickup ions. The understanding of the evolution of the pickup ions' velocity distribution has been further improved by kinetic models. The knowledge gained elucidated the nature of the local interaction but the effect on the global phenomena has not yet been elaborated. That motivated us to develop a detailed fluid model for the global interaction of cometary plasma with the solar wind [Gan-Baruch, 1996; Gan-Baruch et al., 1997].

The atmosphere of a comet is created by evaporation of neutral molecules from the surface of its nucleus due to solar heating. The evaporating molecules expand away from the nucleus with their thermal velocity of about 1 km/s. They are ionized with a typical time scale of $\approx 10^6$ seconds. A new ion is injected into the solar wind with the velocity $\mathbf{V}_{inj} \equiv \mathbf{W}_n - \mathbf{V}_{sw}$ where $\mathbf{W}_n$ is the neutrals velocity, and $\mathbf{V}_{sw}$ is the solar wind velocity. The motion of the ionized particle across the interplanetary magnetic field carried by the solar wind generates an electric field which forces the particle to gyrate around the magnetic field lines.

In figure 1 the pickup process appears on a solar wind magnetic field aligned (SWB) frame. In this frame, the z axis is parallel to the magnetic field $\mathbf{B}$, the y axis is parallel to the electric field $\mathbf{E} = \mathbf{V}_{inj} \times \mathbf{B}$ at the moment of the particle's injection, and the x axis is in the direction of the cross product of the fields. The velocity of the solar wind in this frame is zero. Perpendicular to the magnetic field the ion assumes the solar wind velocity, but parallel to the field it retains its initial velocity. This causes a drift of the cometary ions relative to the solar wind plasma. The gyration of the new ion around the magnetic field creates a ring distribution in velocity space with a radius $V_{inj} \sin\theta$. Since the neutral velocity is very small in comparison to the solar wind velocity, θ can be considered as the angle between the solar wind direction and the magnetic field. The angle θ is also the pitch angle of the new ions because its tangent equals the ratio of the perpendicular to the parallel velocity. The ring distribution created is unstable to the ion cyclotron instability. Alfvén waves will be excited and will interact with the new ions.

Galeev et al. [1991] applied quasilinear theory to examine the ion cyclotron instability in the pickup process. They supposed that the ion distribution evolves to an asymptotic state $f = f_0 + f_1(\theta)$ where f_0 is the isotropic part and f_1 is the anisotropic part of the distribution function, which represents the injected ions at angle θ. Although $f_0 \gg f_1$ there is sufficient anisotropy to maintain wave growth as new particles are added at the ring position. As a result, the ions are scattered in pitch angle and form to a first approximation a shell distribution with radius V_{inj}. It can be seen in figure 1 that the bulk velocity of a uniformly filled ring is $\left(0, 0, V_{inj\parallel}\right)$, and of a simple shell is $(0, 0, 0)$, which is also the solar wind velocity in the SWB frame. In other words the ring distribution of new cometary plasma lags behind the solar wind. The drift velocity of the cometary plasma, relative to

the solar wind is along the magnetic field, and equals the parallel component of the injection velocity in the SWB frame. The simple shell is only a first approximation while the actual distribution created is a bispherical shell. The Alfvén waves which are generated by the instabilities move mostly upstream along the field but also a small fraction moves downstream. The conservation of energy of waves and particles requires that the particles' velocities will appear in the velocity space as two spherical shells which are centered along the field at $\pm V_A$. Since pickup of new cometary particles proceeds, the ring distribution is recreated. The asymptotic distribution function is, therefore, a bispherical shell combined with a ring at the injection angle. This non spherically symmetric asymptotic distribution results in anisotropic pressure. The pitch angle scattering, and shell formation is not the last step of the evolution of the particles' distribution. Velocity diffusion as well as other energization mechanisms would cause broadening of the shell.

A. Coates [1991] analysed Giotto measurements of the velocity distribution of the water group ion distribution. His results confirm that while approaching the bow-shock the pickup ions distribution evolves from a ring to a shell distribution which then broadens in energy. The three phases of the pickup process, i.e. the formation of a ring distribution, the scattering to a bispherical shell and its approximate "thermalization" were obtained also by kinetic and hybrid simulations [Wu et al., 1986; Miller et al., 1991; Gary et al., 1991]. Miller et al. [1991], showed that for injection angles larger than 40° the velocity distributions are bispherical because of the propagation of waves along and against the beam direction while in smaller angles, the distribution is monospherical.

The models yield the local evolution of the plasma in the aforementioned pickup process. Because of computer time and memory limitations, it is still not possible to develop a global model of the interaction by means of kinetic or hybrid simulations. This motivated us to develop a global fluid model which is a step beyond the Magneto-Hydro-Dynamics (MHD) for being able to incorporate more of the macroscopic effects that originate from the kinetic processes, that is the pressure anisotropies and the two fluid interaction of cometary plasma with the solar wind. The generalizations introduced alter the large scale phenomena significantly.

The Generalized Formulation of the Interaction

The time scales for the evolution of the particles' distribution function are the main factor to be considered when examining which of the kinetic effects will relax within the fluid time scales and which will result in macroscopic effects that will have to be taken into account in the fluid simulations. The fluid time scales are of the order of the characteristic distance divided by the characteristic propagation speeds. In the case of a comet, taking L_{mhd} to be $10^5 - 10^6$km, and $V_{mhd} = V_{Alfven}$ with $B = 5\gamma$ and $\rho = 5\mathrm{m_p}$ cm^{-3} yields $\tau_{mhd} \approx 10^3 - 10^4$s. The kinetic time scales are the photoionization time τ_i, the pickup time τ_p which is the time needed for

the creation of a ring distribution in the phase space by the cyclotron motion, the pitch angle scattering time τ_s for creation of a shell, and the energy diffusion time τ_d for a filled shell creation. By means of hybrid simulations, Gaffey et al. [1988] obtained time scales for the evolution of the distribution function of the pickup ions to a ring, to a shell, and finally to a filled shell for H_2O^+ in the vicinity of Halley's comet. Far upstream τ_p was 150 sec, τ_s was 260 sec, and τ_d was 1400 sec. Near the bow-shock τ_p was again 150 sec, τ_s was 153 sec, and τ_d was 142 sec. Coates et al. [1989] derived from the measurements of Giotto at Halley an upper limit of 10,250 seconds for the pitch angle scattering in the far upstream region, and energy scattering time ranging from 10^4 seconds downstream to 10^5 seconds far upstream. Their results indicate slower evolution than obtained by theory. The time scales above, lead to the conclusion that in the presence of a continuous source of pickup ions, the isotropization and the thermalization of the pickup plasma would be of the same order as the MHD time scale. The MHD approximation is, therefore, inaccurate. The pickup definitely destroys the isotropy of the pressure and might lead to non-vanishing higher moments of the distribution function. Isotropic pressure is not sufficient to describe the comet - solar wind interaction. Furthermore, the relatively slow scattering of the ring distribution to a shell distribution which moves with the solar wind velocity, and the slow energy diffusion time, imply that the one fluid approximation of MHD should be replaced by two coupled fluids, each with its own velocity field. The fast pickup time which is also the ion cyclotron time implies that in the direction perpendicular to the magnetic field there is an immediate assimilation of the velocities of the two fluids. The generalized formulation of the interaction presented here incorporates the two effects of two interacting fluids, and anisotropic pressure. Raeder [1990] incorporated the two effects in his model which was solved by a hybrid numerical method called Particle In Cell (PIC). Wegmann [1997] incorporated the anisotropic pressure in his one fluid model.

The first to derive transport equations in a dilute plasma without interparticle collisions for a distribution function of higher order than the Maxwell distribution function were Chew, Goldberger and Low (CGL) [1956]. In order to truncate the hierarchy of the transport equations, CGL assumed that all moments higher than the thermal pressure tensor vanish. The neglect of the heat flux vector is controversial for solar system plasmas. Kulsrud [1983] pointed out that one of the consequences of the neglect of the heat flux is the inconsistency of the condition for the excitation of the mirror instability [Rosenbluth, 1956; Chandrasekhar, 1961; Sagdeev, 1966]. The condition derived from the guiding center formalism required much lower pressure anisotropy with perpendicular pressure larger than the parallel pressure as a sufficient potential for the instability growth than the condition derived from the double adiabatic formalism. The Firehose instability [Rosenbluth, 1956; Parker, 1958] requires that the parallel pressure be larger than the perpendicular pressure and the criterion for its excitation obtained from both formalisms is identical, [Kulsrud, 1983] since parallel heat flow plays no role. The

extent of the above inconsistency is lowered in this model by the use of properly adjusted isotropization terms which limit the anisotropies.
In the present phase of the model development, the velocity moments of the Bhatnagar-Gross-Krook (BGK) approximation [Bhatnagar et al., 1954] for the collision term

$$\left(\frac{\partial f}{\partial t}\right)_{coll.} = -\frac{1}{\tau(v)}\left(f - f_M\right)$$

where f_M is the Maxwell distribution function, and $\tau(v)$ is the relaxation time of the distribution function to the equilibrium distribution function, are used as pressure isotropization terms and as coupling terms of the two fluids in the direction parallel to the magnetic field. The BGK model is only a first approximation for the collision terms. Furthermore, the dependence of $\tau(v)$ on the particles' velocity cannot be included in fluid simulations. In the present state of the code development, the relaxation times taken are constant, and equal for all velocity moments. However, one must keep in mind that the velocity moments of the collision term yield different relaxation times for its different moments [Livi and Marsch, 1986], and that the wave turbulence which isotropizes the fluids, and couples them, depends on time and distance from the comet. Information from measurements, and kinetic, or hybrid simulations can yield estimates for the time scales. The phenomenological terms, in their present form, cannot describe the isotropization process and fluids coupling very accurately.
From the discussion above we can draw two conclusions. First, the pickup process leads to anisotropic pressure. The pressure is a tensor quantity not necessarily gyrotropic. Still it is a step forward to assume in a pickup environment, a gyrotropic instead of an isotropic pressure. Hence in our model the double adiabatic energy equations of Chew-Goldberger-Low modified by isotropization terms replace the adiabatic energy equation. Second, the interaction of the cometary plasma with the solar wind plasma is a two fluid phenomena. The two fluids adjust their velocities in the direction perpendicular to the magnetic field, however, in the parallel direction their coupling is slow. The third fluid, the electron component, is assumed cold in this model and therefore degenerate. It has no dynamic effect. The equations which satisfy the requirements of this model are presented in the next section.

The Model Equations

The one fluid approach of MHD sums over all ion species, and electrons in order to get the total plasma variables. In this model the solar wind plasma, and the cometary plasma, are treated separately as two coupled fluids. It is not possible to consider each of these fluids in the one fluid approach, and sum over its ions and electrons for the total fluid variables, because the electron population cannot be partitioned and related to each of the ion populations. Hence, the equations derived are the ion equations for the solar wind plasma, and the ion equations for

the cometary plasma.

Derivation of the model equations

The different components of the plasma will be indicated by the following indices:

1. cometary ions (one species) - ci.
2. solar wind ions (one species) - si.
3. electrons - e.

The electrons which cannot be distinguished according to their origin, are indicated by one index.
The macroscopic variables of the plasma are:

The species mass density:

$$\rho_{si} = n_{si} m_{si}$$

$$\rho_{ci} = n_{ci} m_{ci}$$

$$\rho_e = n_e m_e$$

where m_{si} and m_{ci} are the average molecular masses of the solar wind ions, and of the cometary ions respectively, and m_e is the electron mass.
The species velocity:

$$\mathbf{U_{si}}, \quad \mathbf{U_{ci}}, \quad \mathbf{U_e}$$

The total current density:

$$\mathbf{J} = n_{si} e \mathbf{U_{si}} + n_{ci} e \mathbf{U_{ci}} - n_e e \mathbf{U_e}$$

The species pressure dyads:

$$\hat{\mathbf{P}}_{\mathbf{si}}, \quad \hat{\mathbf{P}}_{\mathbf{ci}}, \quad \hat{\mathbf{P}}_{\mathbf{e}}$$

where the gyrotropic non viscous pressure dyad is

$$\hat{\mathbf{P}} = P_\perp \hat{\mathbf{I}} + (P_\parallel - P_\perp) \mathbf{bb}$$

Here $P_\parallel$ is the parallel pressure and $P_\perp$ is the perpendicular pressure with respect to the magnetic field. Also $\hat{\mathbf{I}}$ is the unit dyad, and $\mathbf{bb}$ is the dyad formed from the unit vector $\mathbf{b} = \frac{\mathbf{B}}{|B|}$ where $\mathbf{B}$ is the magnetic field vector.
To derive the model equations without source terms, the transport equations of mass, momentum, and the double adiabatic energy equations of CGL are applied to the solar wind ions with q=si, and to the cometary ions with q=ci (equ. (1)-(4)). For the electrons, which are assumed a cold component, a momentum equation is applied (equ. (5)). In addition, Faraday's magnetic induction law (equ. (6)) after neglect of the displacement current, and Ampère's law (equ. (7)) for the current density are used. Source terms and coupling terms for the transport equations are

presented in as well as isotropization terms which modify the double adiabatic energy equations.

$$\frac{\partial \rho_q}{\partial t} + \nabla \cdot \rho_q \mathbf{U}_q = 0 \tag{1}$$

$$\rho_q \left(\frac{\partial \mathbf{U}_q}{\partial t} + \mathbf{U}_q \cdot \nabla \mathbf{U}_q \right) - e n_q \left(\mathbf{E} + \mathbf{U}_q \times \mathbf{B} \right) + \nabla \cdot \hat{\mathbf{P}}_q = 0 \tag{2}$$

$$\frac{\partial \left(P_{\perp q} \right)}{\partial t} + \mathbf{U}_q \cdot \nabla (P_{\perp q}) + 2 P_{\perp q} \nabla \cdot \mathbf{U}_q - \left(P_{\perp q} \mathbf{bb} \cdot \nabla \right) \cdot \mathbf{U}_q = 0 \tag{3}$$

$$\frac{\partial \left(\frac{P_{\| q}}{2} \right)}{\partial t} + \mathbf{U}_q \cdot \nabla \left(\frac{P_{\| q}}{2} \right) + \left(\frac{P_{\| q}}{2} \right) \nabla \cdot \mathbf{U}_q + \left(P_{\| q} \mathbf{bb} \cdot \nabla \right) \cdot \mathbf{U}_q = 0 \tag{4}$$

$$\rho_e \left(\frac{\partial \mathbf{U_e}}{\partial t} + \mathbf{U_e} \cdot \nabla \mathbf{U_e} \right) + e n_e \left(\mathbf{E} + \mathbf{U_e} \times \mathbf{B} \right) + \nabla \cdot \hat{\mathbf{P}}_\mathbf{e} = 0 \tag{5}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \tag{6}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \tag{7}$$

The assumptions made for the electron momentum equation (5) are:
The electron inertia vanishes,

$$\rho_e \frac{d \mathbf{U_e}}{dt} = \rho_e \left(\frac{\partial \mathbf{U_e}}{\partial t} + \mathbf{U_e} \cdot \nabla \mathbf{U_e} \right) = 0$$

A second assumption

$$\nabla \cdot \hat{\mathbf{P}}_\mathbf{e} = 0$$

which is more crude, considers the electrons as a cold negative background which does not influence the dynamics but is required for charge neutrality. The relation obtained for the electric field under the two assumptions is:

$$\mathbf{E} = -\mathbf{U_e} \times \mathbf{B} \tag{8}$$

The total current density, defined above, can be substituted into equation (7), together with the quasi neutrality condition for the total plasma

$$n_e = n_{si} + n_{ci}$$

From the relation obtained

$$\nabla \times \mathbf{B} = \mu_0 \ e \left(n_{si} \mathbf{U_{si}} + n_{ci} \mathbf{U_{ci}} - (n_{si} + n_{ci}) \mathbf{U_e} \right)$$

the electron velocity

$$\mathbf{U_e} = \frac{n_{si} \mathbf{U_{si}} + n_{ci} \mathbf{U_{ci}}}{n_{si} + n_{ci}} - \frac{\nabla \times \mathbf{B}}{\mu_0 e (n_{si} + n_{ci})} \tag{9}$$

is extracted. It is then substituted into equation (8) for the electric field, which is substituted into the ion momentum equations (2). The resulting equations are

$$\rho_{si}\left(\frac{\partial \mathbf{U_{si}}}{\partial t}+\mathbf{U_{si}}\cdot\nabla\mathbf{U_{si}}\right)-e\frac{n_{si}n_{ci}}{n_{si}+n_{ci}}\left(\mathbf{U}_{si}-\mathbf{U}_{ci}\right)\times\mathbf{B}$$
$$-\frac{n_{si}}{\mu_0(n_{si}+n_{ci})}(\nabla\times\mathbf{B})\times\mathbf{B}+\nabla\cdot\hat{\mathbf{P}}_{\mathbf{si}}=0 \tag{10}$$

$$\rho_{ci}\left(\frac{\partial \mathbf{U_{ci}}}{\partial t}+\mathbf{U_{ci}}\cdot\nabla\mathbf{U_{ci}}\right)-e\frac{n_{si}n_{ci}}{n_{si}+n_{ci}}\left(\mathbf{U}_{ci}-\mathbf{U}_{si}\right)\times\mathbf{B}$$
$$-\frac{n_{ci}}{\mu_0(n_{si}+n_{ci})}(\nabla\times\mathbf{B})\times\mathbf{B}+\nabla\cdot\hat{\mathbf{P}}_{\mathbf{ci}}=0 \tag{11}$$

In the second term in the momentum equations there is a cross product of a velocity difference of solar wind and cometary ions with the magnetic field. Since this difference is cross multiplied with the magnetic field, its component parallel to the field does not contribute. The velocities' difference perpendicular to the magnetic field gives a small contribution within one cyclotron period which is short relative to the time scales possible in fluid simulations. This consideration justifies the neglect of the second term in the momentum equations.
The electric field in Faraday induction law, (6), is also replaced by relation (8) with the electron velocity extracted from the total current. The equation derived is known as the Magnetic flux equation.

$$\frac{\partial\mathbf{B}}{\partial t}=\nabla\times\left(\frac{n_{si}\mathbf{U_{si}}+n_{ci}\mathbf{U_{ci}}}{n_{si}+n_{ci}}\times\mathbf{B}-\frac{1}{(n_{si}+n_{ci})e}\mathbf{J}\times\mathbf{B}\right) \tag{12}$$

The second term on the right hand side of equation (12) is the Hall term which relative to the other terms can be neglected for the long time scales considered.
The model equations ((13)-(21)) with source, coupling, and isotropization terms (presented in detail in) are:

$$\frac{\partial\rho_{si}}{\partial t}+\nabla\cdot\rho_{si}\mathbf{U_{si}}=S_{si1} \tag{13}$$

$$\frac{\partial\rho_{si}\mathbf{U_{si}}}{\partial t}+\nabla\cdot(\rho_{si}\mathbf{U_{si}}\mathbf{U_{si}})-\frac{n_{si}}{\mu_0(n_{si}+n_{ci})}(\nabla\times\mathbf{B})\times\mathbf{B}+\nabla\cdot\hat{\mathbf{P}}_{\mathbf{si}}=\mathbf{S_{si2}} \tag{14}$$

$$\frac{\partial\left(P_{\perp si}\right)}{\partial t}+\mathbf{U_{si}}\cdot\nabla(P_{\perp si})+2P_{\perp si}\nabla\cdot\mathbf{U_{si}}-(P_{\perp si}\mathbf{bb}\cdot\nabla)\cdot\mathbf{U_{si}}=S^{\perp}{}_{si3} \tag{15}$$

$$\frac{\partial\left(P_{\parallel si}\right)}{\partial t}+\mathbf{U_{si}}\cdot\nabla\left(P_{\parallel si}\right)+\left(P_{\parallel si}\right)\nabla\cdot\mathbf{U_{si}}+\left(2P_{\parallel si}\mathbf{bb}\cdot\nabla\right)\cdot\mathbf{U_{si}}=S^{\parallel}{}_{si3} \tag{16}$$

$$\frac{\partial\rho_{ci}}{\partial t}+\nabla\cdot\rho_{ci}\mathbf{U_{ci}}=S_{ci1} \tag{17}$$

$$\frac{\partial\rho_{ci}\mathbf{U_{ci}}}{\partial t}+\nabla\cdot(\rho_{ci}\mathbf{U_{ci}}\mathbf{U_{ci}})-\frac{n_{ci}}{\mu_0(n_{si}+n_{ci})}(\nabla\times\mathbf{B})\times\mathbf{B}+\nabla\cdot\hat{\mathbf{P}}_{\mathbf{ci}}=\mathbf{S_{ci2}} \tag{18}$$

$$\frac{\partial\left(P_{\perp ci}\right)}{\partial t}+\mathbf{U_{ci}}\cdot\nabla(P_{\perp ci})+2P_{\perp ci}\nabla\cdot\mathbf{U_{ci}}-(P_{\perp ci}\mathbf{bb}\cdot\nabla)\cdot\mathbf{U_{ci}}={S^{\perp}}_{ci3} \tag{19}$$

$$\frac{\partial\left(P_{\parallel ci}\right)}{\partial t}+\mathbf{U_{ci}}\cdot\nabla\left(P_{\parallel ci}\right)+\left(P_{\parallel ci}\right)\nabla\cdot\mathbf{U_{ci}}+\left(2P_{\parallel ci}\mathbf{bb}\cdot\nabla\right)\cdot\mathbf{U_{ci}}={S^{\parallel}}_{ci3} \tag{20}$$

$$\frac{\partial\mathbf{B}}{\partial t}=\nabla\times\left(\frac{n_{si}\mathbf{U_{si}}+n_{ci}\mathbf{U_{ci}}}{n_{si}+n_{ci}}\times\mathbf{B}\right) \tag{21}$$

Source terms, Coupling terms, and Isotropization terms

a. *Source terms*

The comet is an extended source of neutral particles which are ionized and then picked up by the solar wind. The ionization mechanisms are charge exchange with the solar wind, photoionization by solar radiation, and electron impact ionization. In this model, the chemical processes will not be treated, and only photoionization will be considered.

The number density production rate at a distance R from the nucleus under the assumption of spherical symmetry, and neutral velocity W_n, lifetime against ionization τ_i, and total production rate of cometary neutrals, $G(s^{-1})$, is

$$A=\frac{G}{\tau_i 4\pi R^2 W_n}\ \exp\left(-\frac{R}{\tau_i W_n}\right) \tag{22}$$

This model describes the R^{-2} dependence of the density in spherical symmetry modified by exponential exhaustion of the neutrals because of ionization. In this two fluid model, ionization of cometary neutrals gives the source for the transport equations of the cometary ions.

b. *Perpendicular velocity assimilation*

The acceleration of a new ion during the pickup process can be well described by kinetic models, in which the cyclotron motion can maintain the drift velocity perpendicular to the magnetic field equal for both types of ions. In a two fluid model, the assimilation of the perpendicular velocities of the two fluids can be maintained by the following averaging procedure:

(a) The bulk velocity of the total plasma is calculated,

$$<\mathbf{U}>=\frac{\rho_{si}\mathbf{U_{si}}+\rho_{ci}\mathbf{U_{ci}}}{\rho_{si}+\rho_{ci}}$$

(b) The perpendicular component of the bulk velocity $<\mathbf{U}>$ replaces the perpendicular component of the velocity of each of the fluids,

$$<\mathbf{U}>_{\parallel}=(<\mathbf{U}>\cdot\mathbf{B})\frac{\mathbf{B}}{B^2}$$

$$< \mathbf{U} >_{\perp} = < \mathbf{U} > - < \mathbf{U} >_{\parallel}$$
$$\mathbf{U_{si}}^{new} = \mathbf{U}_{\parallel si} + < \mathbf{U} >_{\perp}$$
$$\mathbf{U_{ci}}^{new} = \mathbf{U}_{\parallel ci} + < \mathbf{U} >_{\perp}$$

(c) The change in the kinetic energy because of the momentum averaging in the perpendicular direction transforms to perpendicular thermal energy density.

$$P_{\perp si}^{new} = P_{\perp si} + \frac{1}{2}\rho_{si}(\mathbf{U}_{\perp si} - < \mathbf{U} >_{\perp})^2$$

$$P_{\perp ci}^{new} = P_{\perp ci} + \frac{1}{2}\rho_{ci}(\mathbf{U}_{\perp ci} - < \mathbf{U} >_{\perp})^2$$

In the numerical calculation this procedure is applied in the end of each time step. $\mathbf{U}_q$ and $P_{\perp q}$ are replaced by ${\mathbf{U}_q}^{new}$ and $P_{\perp q}^{new}$ such that the constraint $\mathbf{U}_{\perp si} = \mathbf{U}_{\perp ci}$ is enforced and momentum and energy are conserved.

c. *Parallel velocity coupling*
The parallel velocity components of the two fluids are coupled by relaxation-time model collision terms. This phenomenological coupling has its origin in the kinetic process that scatters the bimodal distribution of cometary plasma and solar-wind to a unimodal distribution.

d. *Isotropization terms*
The parametrized isotropization terms are also based on a relaxation-time model. They should describe the kinetic process of the pitch angle scattering. The extent to which the anisotropy grows is limited by the fitted isotropization rates.

Below, the full source terms, coupling terms, and isotropization terms of this model are presented. The relaxation rates which were discussed in are designated $\nu_{\parallel}$ for the parallel velocity relaxation rate, and $\nu_{isot.}$ for the isotropization rate.

$$S_{si1} = 0$$
$$\mathbf{S_{si2}} = -\nu_{\parallel}\rho_{si}(\mathbf{U_{si}} - < \mathbf{U} >)_{\parallel}$$
$$S^{\perp}{}_{si3} = \frac{1}{3}\nu_{isot.}(P_{\parallel si} - P_{\perp si})$$
$$S^{\parallel}{}_{si3} = 2\nu_{\parallel}\rho_{si}(\mathbf{U_{si}} - < \mathbf{U} >)_{\parallel}^2 + \frac{2}{3}\nu_{isot.}(P_{\perp si} - P_{\parallel si})$$
$$S_{ci1} = Am_{ci}$$
$$\mathbf{S_{ci2}} = Am_{ci}\mathbf{W_n} - \nu_{\parallel}\rho_{ci}(\mathbf{U_{ci}} - < \mathbf{U} >)_{\parallel}$$
$$S^{\perp}{}_{ci3} = \frac{1}{2}Am_{ci}\left((\mathbf{W_n} - \mathbf{U_{ci}})^2 - (\mathbf{W_n} - \mathbf{U_{ci}})_{\parallel}^2\right) + \frac{1}{3}\nu_{isot.}(P_{\parallel ci} - P_{\perp ci})$$
$$S^{\parallel}{}_{ci3} = Am_{ci}(\mathbf{W_n} - \mathbf{U_{ci}})_{\parallel}^2 + 2\nu_{\parallel}\rho_{ci}(\mathbf{U_{ci}} - < \mathbf{U} >)_{\parallel}^2 + \frac{2}{3}\nu_{isot.}(P_{\perp ci} - P_{\parallel ci})$$

Simulation Results

The numerical method used to solve the model equations [Gan-Baruch et al., 1997] is a Godunov type upwind difference scheme [Godunov, 1959; Wegmann, 1997]. It uses an approximate Riemann solver which is based on the linearized equations. Presently the model is two-and-a-half-dimensional, namely, three dimensional vectors with dependence on two spatial dimensions. The spatial derivatives perpendicular to the plane of symmetry of the comet which is defined by the interplanetary magnetic field vector and the solar wind velocity vector are assumed to be zero. The calculations are carried out on an equidistant cartesian grid. On the subsolar boundary, solar wind conditions are applied. The other boundaries form open boundaries. They are placed so far from the nucleus such that there is no gradient of the flux across the three downstream boundaries. The initial conditions, comet parameters, and grid and time parameters used in the calculations are presented in table 1. The calculations start from an undisturbed solar wind which contains, for technical reasons, a homogeneous background of cometary plasma with a negligible mass density.

Two cases were calculated. In the first case, the model was reduced to one fluid with isotropic pressure by taking high velocity coupling rates and high isotropization rates. The isotropization rate taken was $\nu_{isot.} = 10^{-2} s^{-1}$, and the parallel velocity coupling generated immediate assimilation. In the second case, the full generalized model with two fluids and gyrotropic pressures was applied. The isotropization rate was reduced to $\nu_{isot.} = 10^{-3} s^{-1}$ for both of the fluids, and the parallel velocity coupling rate taken was $\nu_{\|} = 10^{-3} s^{-1}$.

Figure 2 shows the Mach number along the sun-comet line obtained in the first (dashed line) and in the second (dashed dotted line) calculations. The Mach number in these calculations is the flow velocity over the maximal wave velocity along the symmetry axis of the comet. The fastest wave is the wave which reduces in the special case of Magneto-Hydro-Dynamics to the fast magnetosonic wave. The bow shock stand off distance can be estimated at $Mach = 1$ where the reversal of the supersonic flow to subsonic flow occurs. For the isotropic case, the shock distance was 85,000 km while for the gyrotropic case the shock distance was 320,000 km. The error of the numerical result cannot be less than the grid distance. Due to the low resolution of the present calculation grid, the error bars correspond to $\pm 0.75 \cdot 10^5$. Hence, shock distances are not determined accurately. Nevertheless, the relations between the estimated shock distances reflects the significance of the inclusion of pressure gyrotropy. The shock distances for isotropic and gyrotropic plasmas obtained from theoretical arguments obey the same relations. Biermann et al. [1967] showed that the solar wind, loaded by the heavy cometary ions, can remain supersonic only as long as the mean molecular weight does not exceed $\frac{\gamma^2}{\gamma^2-1}$. In this relation $\gamma = \frac{C_p}{C_v} = \frac{N+2}{N}$ is the ratio of the specific heats in constant pressure C_p and in constant volume C_v which can also be written in terms of N, the number of degrees of freedom of the particle. When the mean molecular weight exceeds

this value, a shock must be formed. Biermann et al. [1967] derived an approximate lower limit for the shock distance

$$R_B \geq \frac{G m_{ci} \sigma (\gamma^2 - 1)}{4\pi W_n \rho_{s0} U_{s0}} \tag{23}$$

where ρ_{s0}, and U_{s0} are the initial density and velocity of the solar wind. Equation 23 was used to obtain the approximate lower limits of the shock distances for the plasma parameters taken in the two calculations (table 1). For an isotropic plasma with three degrees of freedom, or $\gamma = 5/3$, the distance obtained was 140,000 km while for a gyrotropic plasma with two degrees of freedom perpendicular to the magnetic field, or $\gamma = 2$, the distance was 240,000 km. The numerical estimates for the shock distances are, therefore, consistent with the theoretical estimates. When the pressure is gyrotropic the shock is created further out compared with isotropic pressure. This result can be interpreted in the following way. For $\gamma = 2$ the magnetosonic velocity is higher than for $\gamma = 5/3$ due to the lower compressibility of the plasma. Hence, it reaches the solar wind velocity earlier such that the flow must become subsonic at larger distances.

The rest of the results obtained in the second calculation are presented below. Figure 3 presents the contour lines of the logarithm of the solar wind density. The density increases from its initial value of $5\mathrm{M_p\,cm^{-3}}$, where M_p is the proton mass, to a maximum value of $15.5\frac{M_p}{cm^3}$ which appears downstream the shock ahead of the nucleus. Downstream the nucleus the density decreases due to the divergence of the solar wind flow.

The logarithm of the density contours of the cometary plasma in figure 4 appear as a long narrow tail. The cometary plasma is generated mainly in the inner coma and advected down the tail. The maximum of the density is $31.6\frac{M_p}{cm^3}$ and it appears downstream the nucleus because of the fast advection.

Figures 5, and 6 show the velocity field of the solar wind and of the cometary plasma respectively. The flow is decelerated and deflected in the bowshock. Close to the comet nucleus the streamlines are deflected around it. Behind the nucleus the flow is again accelerated due to the pressure gradient force.

Figure 7 shows that the magnetic field drapes the comet, and that its strength increases with a maximum in front of the nucleus where the magnetic pile-up is expected. Theory [Schmidt and Wegmann, 1982; Flammer, 1991] shows that the drag of the outflowing cometary neutrals is necessary to create a ionopause, or magnetic cavity at the observed place. Pure cometary plasma pressure will stand off the solar wind but further inward. In this model, the neutrals' dynamics is not considered, and the flow feels only the effect of pickup ions. A hard obstacle is not fixed at the ionopause location since in a two dimensional calculation, the magnetic field would pile up indefinitely. In any case, the magnetic cavity is much smaller than the grid size hence it cannot be resolved here at all. The magnetic field is expected to reduce below the interplanetary magnetic field values in the far tail in order to conserve magnetic flux. This reduction is not obtained because of the

relatively small domain of calculation.
The solar wind parallel and perpendicular pressure obtained along the sun-comet line is presented in figure 8. Comparison of figure 8 with figure 9 for the cometary plasma pressure shows that the cometary pressure is about an order of magnitude higher than the solar wind pressure. The pickup which is a source for the cometary pressure only, is the cause for this ratio.
Figure 10 is a contour plot of the pressure anisotropy $A = P_\perp/P_\parallel$ of the solar wind. The anisotropy of the solar wind does not exceed A=2. The maximum appears behind the shock. Since the shock is perpendicular, i.e., shock front normal is oriented at right angle to the interplanetary magnetic field, the compression at the shock acts on the perpendicular degrees of freedom, hence, it results in a jump in $P_\perp$. The anisotropy reduces towards the shock flanks where the shock compression tends to become more and more parallel. The minimal anisotropy is 0.96 and it appears upstream of the shock flanks. Downstream the shock the perpendicular pressure still increases, since the stagnating flow is compressed perpendicular to the field. Expansion behind the nucleus reduces the anisotropy. Figure 11 is a contour plot of the pressure anisotropy of the cometary plasma. Far upstream the new pickup plasma takes a large fraction of the total cometary plasma. Since the particles are picked up with $\theta = 90°$, the far upstream cometary anisotropy reaches a high value of A=5. Isotropization reduces the pressure anisotropy as the flow approaches the shock. Compression at the shock creates a peak of A=3 after which the anisotropy drops again. The minimal anisotropy is 0.8 and it appears upstream the shock flanks. Figure 12 presents the anisotropy of the solar wind A_{si}, and of the cometary plasma A_{ci} along the sun comet line. While the upstream behavior of the two fluids is different, downstream both tend to have A=1. The isotropization time applied, 10^3 seconds, becomes small compared with the convection time, hence, the low anisotropy.
In figure 13 one can see the relative drift velocity vectors $\mathbf{U_d}$ of the two fluids, where $\mathbf{U_d} = \mathbf{U_{ci}} - \mathbf{U_{si}}$. The vectors are parallel to the magnetic field. Along the shock flanks as well as above and below the symmetry axis in the inner tail, the solar wind lags behind the cometary plasma. However, close to the nucleus, in the far inner tail along the symmetry axis, and in the far tail lobes, the cometary plasma lags behind the solar wind.
In figure 14, the contour lines of the drift velocity $\mathbf{U_d}$ are presented. The maximal drifts are 79 km/s in front of the flanks of the shock, and 4.7 km/s in the flanks of the inner tail. The minimal drifts are -9.7 km/s which occur $3 \cdot 10^5$ km away from either side of the nucleus, -13 km/s in the tail lobes $1.5 \cdot 10^6$ km away from the axis, and -1. km/s in the far inner tail.
The acceleration of the pickup ions during the pickup process was described in section . The pickup ions get in the solar wind frame an initial velocity $V_{inj\parallel}$ and therefore lag behind the solar wind. In case of perpendicular pickup, the lag is zero. The relative drift velocity $\mathbf{U_d}$ showed that a lag of the cometary plasma was not obtained in the inner tail and along the shock flanks. On the contrary, the solar

wind lagged behind the cometary fluid. An acceleration mechanism of the cometary fluid overtook its lag. The acceleration mechanism of the pickup ions along the magnetic field lines can be explained by comparison of the parallel component of the momentum equations of the two fluid. At the shock region, the source terms and the interaction terms , $\mathbf{S}_q$, in the momentum equations,

$$\rho_q \left(\frac{\partial \mathbf{U}_q}{\partial t} + \mathbf{U}_q \cdot \nabla \mathbf{U}_q \right) - \frac{\acute{n}}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} + \nabla \cdot \hat{\mathbf{P}}_q = \mathbf{S}_q$$

where $\acute{n} = n_q / (n_{si} + n_{ci})$, are smaller by an order of magnitude than the rest of the terms. Neglecting $\mathbf{S}_q$, and taking the parallel component of the momentum equations yields

$$\frac{dU_{q\parallel}}{dt} = -\frac{1}{\rho_q} \mathbf{b} \cdot \nabla \cdot \hat{\mathbf{P}}_q$$

$$\frac{dU_{q\parallel}}{dt} = -\frac{1}{\rho_q} \left(\left(P_{q\parallel} - P_{q\perp} \right) \nabla \cdot \mathbf{b} + \mathbf{b} \cdot \nabla P_{q\parallel} \right)$$

The ratio of the mass densities of the cometary plasma to the solar wind ρ_{ci}/ρ_{si} is presented in figure 15. At the shock region, this ratio is lower than 0.5. From figures 16-19 for the perpendicular and the parallel pressure of the solar wind and the cometary plasma, one can obtain the parallel component of the pressure forces. In this calculation it was found to be an order of magnitude larger for the cometary plasma. Hence, the pressure force accelerates the cometary plasma along the shock parallel to the magnetic field lines more rapidly than in the solar wind, and is therefore responsible for the outward relative drift $\mathbf{U_d}$ at the shock flanks. The same acceleration mechanism causes the positive drift in the inner tail above and below the symmetry axis. However, due to the low pressure forces of the cometary plasma and its high mass density in the inner tail, the acceleration in this region is small, and the relative drift obtained is low. In regions where the cometary plasma lags behind the solar wind, the acceleration works to reduce the lag. It is expected that an isotropization rate smaller than 10^{-3} would generate via isotropization a lower parallel component of the pressure force. In this case, the lag of the cometary ions would be more pronounced.

Summary

A global model for the comet interaction with the solar wind has been developed. The long time scales that cometary pickup plasma needs in order to isotropize and assimilate with the ambient plasma are beyond the time scales for advection of an MHD fluid. In this environment, pressure anisotropies may evolve. Although the plasma is not necessarily gyrotropic, it is still a step forward to introduce a gyrotropic pressure tensor instead of the isotropic pressure used in MHD models. Hence, the double adiabatic energy equations of Chew-Goldberger-Low [Chew et al., 1956], modified by isotropization terms, replace the adiabatic

energy equation. In addition, the long time needed for the assimilation of the pickup plasma with the ambient plasma indicates that the interaction should be formulated as a two fluid interaction, i.e. solar wind ions, and cometry ions, instead of the one fluid MHD approach. The two ion fluids interact on longer time scales by coupling terms based on the velocity moments of the BGK relaxation time model [Bhatnagar et al., 1954]. Presently, the electron component in our model, is assumed cold and has no dynamic effect. First results, calculated on a two dimensional equidistant rectangular grid, are presented. The results show that the generalizations introduced alter the phenomena obtained in MHD models. The stand-off distance of the shock increased significantly when the isotropization rate was reduced to $10^{-3}s^{-1}$. For that rate, the solar wind pressure anisotropies did not exceed $P_{\perp}/P_{\parallel} = 2$, the maximal anisotropy which appeared upstream of the nucleus. The large cometary plasma pressure forces at the shock accelerated the cometary plasma along the shock to create a relative drift of the solar wind behind the cometary plasma parallel to the magnetic field. For a field aligned drift relaxation rate of $10^{-3}s^{-1}$, the maximal relative drift between the two fluids which appeared at the shock flanks was 79 Km/s. Acceleration of cometary plasma was obtained to a low extent also in the inner tail. Close to the nucleus of the comet, in the far tail lobes, and in the far inner tail, the cometary plasma lagged behind the solar wind.

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