The Physical Origin of Galaxy Scaling Relations

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Abstract. A standard paradigm is now available for the recent evolution ($z < 10$) of structure on galactic and larger scales. Most of the matter is assumed to be dark and dissipationless and to cluster hierarchically from gaussian initial conditions. Gas moves under the gravitational influence of this dark matter, settling dissipatively at the centres of dark halos to form galaxies. The evolution of the dark matter component has been studied extensively by N-body simulations. The abundances, density profiles, shape and angular momentum distributions and the formation histories of the dark halo population can all be predicted reliably for any hierarchical cosmogony. The systematic variation of these properties with halo mass can produce scaling relations in the galaxy population. Simple hypotheses for how galaxies condense within dark halos lead to characteristic luminosities, sizes, and spins which are close to those of real spiral and elliptical galaxies. Furthermore, correlations similar to the Tully-Fisher, Faber-Jackson and luminosity-metallicity relations arise quite naturally. A quantitative explanation of the fundamental plane of elliptical galaxies appears within reach.

1 Introduction

The current popularity of the Cold Dark Matter (CDM) model and its variants stems from a variety of sources. Most of the mass in the Universe appears to be in a dark collisionless form which is concentrated towards galaxies but extends far beyond their visible boundaries. If this matter has interacted only gravitationally since early times it is possible to reconcile the very small observed amplitude of fluctuations in the Microwave Background with the existence of massive nonlinear structures in the present Universe. Furthermore, the large-scale distribution of galaxies looks quite similar to the patterns which result from the gravitational amplification of gaussian density fluctuations. This is a simple and natural initial condition in CDM-like models, and in such models the galaxies are indeed expected to trace the dark matter distribution on large scales.

The idea that hierarchical clustering under gravity has given rise to galactic and larger structures predates the CDM model (e.g. Peebles 1980), and almost two decades of simulation using N-body methods have provided a reasonably complete understanding of the structure produced by this process. If galaxy formation results from the dissipative collapse of gas within the potential wells provided by dark halos, then it is the internal structure of such halos and their formation history which must regulate the global properties of galaxies. In recent years there has been substantial progress in understanding the predictions of hierarchical clustering models for these and other aspects of halo structure. In the next section I summarise the aspects of this work most relevant for a discussion of galaxy scaling relations.

The structure and evolution of dark halos may determine the mass and angular momentum of the material available for galaxy formation, as well as the rate of interactions between galaxies. The global properties of galaxies must in addition depend on how gas cools to form dense clouds, on how star-formation proceeds in such clouds, and on how this star formation affects the surrounding material through the injection of heavy elements and energy. These processes interact in a highly nonlinear way and involve a very wide range of scales; there is little hope of simulating them realistically. If, however, they are parametrised by an appropriate set of efficiencies, to be assigned physically reasonable values by comparison with available observational or simulation data, then it is possible to make simple "semi-analytic" models which can predict a very wide range of properties of the galaxy population for any specific cosmogony, for example any of the popular CDM variants (e.g. luminosity functions, colours, sizes, morphologies, gas contents, and the dependence of all of these on environment and redshift). In sections 3 and 4 I discuss how such models can be used to predict galaxy scaling relations, and I emphasise, in particular, the inferences which can be drawn from the observed tightness of some of these relations.

2 Structure and Evolution of Dark Halos

N-body simulations have provided a good understanding of the structure and evolution of the dark halos which form through hierarchical clustering of dissipationless matter. For example, the angular momentum of dark halos, best characterised by the dimensionless spin parameter,

$$\lambda \equiv J|E|^{1/2}/GM^{5/2}, \tag{1}$$

where J, E and M are the total angular momentum, energy and mass of the halo, is found to have a broad distribution with a median near $\lambda \sim 0.04$. This distribution appears "universal" in the sense that it has no strong dependence on the mass of the halo or on the parameters of the particular cosmogony in which clustering occurs (e.g. initial power spectrum, $P(k)$, cosmic density, Ω, cosmological constant, Λ,...; see Barnes & Efstathiou 1987, Frenk et al 1988, Warren et al 1992, Cole & Lacey 1996). For a cold rotationally-supported self-gravitating disk one finds $\lambda \sim 0.4$. Hence $\lambda \sim 0.04$ implies a system in which rotation velocities are an order of magnitude smaller than needed for centrifugal support. Any gas component in such a system must shrink in radius by a similar factor if it is to make a centrifugally supported disk (see below). Within individual dark halos rotational streaming usually varies quite weakly with radius, but there are large variations from halo to halo.

The axial ratios of dark halos have also been extensively studied and also appear to show a broad distribution which depends at most weakly on mass or cosmogony (Frenk et al 1988; Warren et al 1992; Cole & Lacey 1996). Nearly spherical halos are quite uncommon. There is a slight preference for near-prolate over near-oblate shapes, and major-to-minor axis ratios in excess of two are common. It is interesting to ask whether such a distribution is consistent with the fact that deviations from axisymmetry in observed disks are typically quite small (e.g. Rix & Zaritsky 1995). I will not pursue this question further here.

A third regularity in the structure of dark halos has emerged only recently. Navarro et al (1996, 1997; hereafter NFW) studied halo density profiles in a wide variety of hierarchical cosmogonies. Their work is distinguished from earlier studies in that they simulated the evolution of each halo separately. This allowed them to set the resolution limits in mass and in linear scale to be constant fractions of the characteristic mass and radius of each halo, even though these characteristic values ranged over several orders of magnitude. NFW found the remarkable result that the spherically averaged density profiles of halos of *all* masses in *all* the cosmogonies they considered could be adequately represented by a suitable scaling of the same analytic form:

$$\rho(r)/\rho_{crit} = \delta_c r_s^3 / r(r+r_s)^2. \tag{2}$$

In this formula r_s sets the "core" radius of the halo and δ_c is its characteristic density in units of the critical density, ρ_{crit}. Thus the inner regions have a density cusp with $\rho \propto 1/r$ while at larger radii the profile steepens towards $\rho \propto 1/r^3$. The bounding radius of a virialised halo is conventionally defined as the radius r_{200} within which the mean density is 200 times the critical value; the "total" halo mass is then the mass within this radius M_t. Defining the concentration of a halo to be $c \equiv r_{200}/r_s$, one immediately finds c to be determined implicitly from

$$\delta_c = \frac{200}{3} \frac{c^3}{[\ln(1+c) - c/(1+c)]}, \tag{3}$$

and, of course, we have

$$M_t = \frac{4\pi}{3}\, 200 \rho_{crit}\, (c r_s)^3. \tag{4}$$

NFW found that for any particular hierarchical cosmogony the two parameters δ_c and r_s are strongly correlated with each other and so with M_t. This correlation is always in the sense that lower mass halos have higher characteristic densities and so greater concentrations. It turns out that this correlation can be understood as a reflection of the fact that smaller mass halos form earlier, and indeed, for a suitable definition of the formation epoch of a halo z_f, NFW showed that all the halos in all their cosmogonies obey the simple relation

$$\delta_c \approx 5 \times 10^3 \Omega_0 (1+z_f)^3. \tag{5}$$

To a good approximation it seems that equilibrium dark halos in all hierarchical cosmogonies have similar density profiles and furthermore that the characteristic density of a halo is just proportional to the density of the universe at the time it formed. It is hard to imagine a simpler situation.

Of course, the properties of the galaxies within a dark halo depend not only on its current structure but also on the details of its formation history. There has been substantial progress over the last five years in understanding how individual nonlinear objects are built up by hierarchical clustering. This is primarily a result of the discovery that extensions of the original argument of Press & Schechter (1974) can provide a remarkably detailed and accurate description of the statistics of merging and accretion in N-body simulations of hierarchical clustering (Bond et al 1991; Bower 1991; Lacey & Cole 1993,1994). Indeed, these formulae provide a basis for Monte Carlo realisations of the full merging tree which describes how any particular object, for example a rich cluster, is built up by successive merging of smaller systems (Kauffmann & White 1993).

Armed with such a tree one can attempt to model all the additional processes which determine the galaxy population within a dark halo (gas cooling, star formation, energy injection from young stars, chemical enrichment, stellar population evolution, galaxy (as opposed to halo) merging, etc.). A major success of recent galaxy formation studies has been the demonstration that even very simple physical models for these processes lead to explanations not only for the luminosities, colours, morphologies, metallicities and abundances of galaxies, and for scaling relations between these properties, but also for the fact that $10^{15}\mathrm{M}_{\odot}$ halos typically contain many bright early-type galaxies while $10^{12}\mathrm{M}_{\odot}$ halos typically contain a single central spiral and a few satellites (Kauffmann et al 1993; Baugh et al 1996a). Furthermore, since such models automatically specify the full history of the galaxy population, they can be compared directly with observational indicators of galaxy evolution, for example with counts and redshift distributions of faint galaxies (Cole et al 1994; Kauffmann et al 1994; Heyl et al 1995; Baugh et al 1996b) or with the properties of damped Lyα systems in QSO spectra (Kauffmann 1996a). The results so far are encouraging, and it seems that a reasonably complete, if schematic, picture of galaxy formation is now in place. In the next two sections I discuss the scaling relations this picture predicts for disk and elliptical galaxies.

3 Scaling Relations for Disks

The defining characteristic of galaxy disks is that they are made of stars which are almost all on near-circular orbits confined close to the disk plane. Since it appears impossible to create a thin centrifugally supported disk without very substantial dissipation, one draws two immediate conclusions:
(i) Galaxy disks were assembled while still gaseous – their stars were all formed *in situ*. Of course, this does not preclude disk growth through gas infall after the formation of many of the stars.
(ii) galaxy disks cannot have been violently disturbed since formation of the bulk of their stars, otherwise they would no longer be thin.
Another critical observation is that the outer rotation curves of most spirals are approximately flat and appear to supported primarily by the gravity of their dark halos. This suggests that the properties of disks may be determined by

those of their dark halos.

The standard model for disk formation was set out by Fall & Efstathiou (1980) in an extension of the ideas of White & Rees (1978). After a dark halo comes to equilibrium, much of its baryonic material is supposed to remain as diffuse gas with a distribution similar to that of the dark matter. Subsequently this gas radiates its binding energy but retains its angular momentum and so flows inwards until it settles into a rotationally supported disk. Fall & Efstathiou showed that an extended dark halo is required, and that little angular momentum can be lost if disks similar to observed spirals are to form. Their scheme is the basis of most recent modelling of spiral galaxy formation (e.g. Kauffmann 1996a; Dalcanton et al 1997) but has not yet been shown to work in any numerical simulation of hierarchical galaxy formation. The difficulty is that inclusion of feedback from young stars is critical. Without it gas cools into small dense clumps at early times, and these lose most of their angular momentum as they merge at the centres of massive dark halos; the resulting disks are then too small to represent real galaxies (Navarro & Benz 1991; Navarro et al 1995; Navarro & Steinmetz 1997).

Let me work through a simple example to show how this scheme can be used to derive scaling relations for galaxy disks. If we model a halo as a singular isothermal sphere of circular velocity V_c, then its mass, kinetic energy and angular momentum within $r_{200} = V_c/10H(z)$ can be written as

$$M_t = \frac{V_c^3}{10GH(z)}, \quad E = M_t V_c^2/2, \text{ and } J_h = \sqrt{2}\lambda M_t r_{200} V_c, \tag{6}$$

where $H(z)$ is the Hubble constant at the redshift when the halo is identified and its central disk is made. Assume a fraction F of the mass of the halo is in the form of gas with the *same* specific angular momentum as the dark matter. Assume further that this gas sinks to the centre conserving its angular momentum and forms an exponential disk of mass M_d, central surface density S_0 and scale radius r_d. If we neglect the contribution of the self-gravity of the disk to its rotation curve then we find

$$M_d = \frac{FV_c^3}{10GH(z)}, \quad r_d = \frac{\lambda V_c}{10\sqrt{2}H(z)} \text{ and } S_0 = \frac{10FV_cH(z)}{\pi\lambda^2 G}. \tag{7}$$

These relations have some immediate consequences. If the stellar mass-to-light ratio of disks is assumed to be a constant value $\Upsilon(z)$ at each redshift, then the first relation gives a Tully-Fisher-like relation, $L \propto V_c^3$, which is *independent* of λ. On the other hand r_d and S_0 depend strongly on λ; slowly rotating halos produce compact and high surface brightness disks. This is encouraging because the exponent of the observed Tully-Fisher relation is not far from 3, and furthermore this relation appears to hold independent of galaxy surface brightness (de Blok & McGaugh 1996; Tully, this meeting). In addition, the proportionality constant seems reasonable. If, following McGaugh & de Blok (1997), we adopt $\Upsilon_B = 2.5h$, then the zero-point of the observed T-F relation, $L_B = 1.5 \times 10^{10} h^{-2} L_\odot$ at $V_c = 200$km/s (e.g. Strauss & Willick 1995), agrees with the prediction provided

$F = 0.02H_0/H(z)$, i.e. $F \approx 0.02$ if disks are assembled near $z = 0$ and $F \approx 0.05$ if disks are assembled near $z = 1$.

The predicted characteristic sizes of disks also seem reasonable. For a "typical" halo with $\lambda = 0.05$ and $V_c = 200$km/s the predicted scale radius is $r_d = 7H_0/H(z)\ h^{-1}$kpc, or $R_d \approx 7h^{-1}$kpc for assembly near $z = 0$ and $R_d \approx 3h^{-1}$kpc for assembly near $z = 1$. Notice that the redshift dependence in these equations is quite strong. It does not appear possible to make substantial numbers of big disks at high redshifts. Thus if damped Lyα absorbers in QSO spectra at $z \sim 3$ are indeed equilibrium disk systems with circular velocities of order 200km/s, then they must be quite small, $r_d \sim 1$ to 2 kpc, if they are to be explained in a hierarchical clustering model. Notice also, as mentioned above, that there cannot be much transfer of angular momentum from gas to dark matter during disk formation, otherwise the resulting disks will be too small for *any* assumed redshift of assembly.

The strong λ-dependence of r_d and S_0 together with the broad λ-distribution resulting from hierarchical clustering implies that galaxy disks are predicted to have a wide range of sizes and surface brightnesses at any given luminosity or circular velocity. A recent discussion of the observational data by Dalcanton et al (1997) suggests that this may indeed be the case. "Disks" formed from the low λ tail of the distribution are predicted to be so compact, however, that they should perhaps be identified with observed spheroids. In these objects the baryonic component should dominate strongly over the dark matter, and this may, perhaps, lead to violent instabilities which prevent thin disk formation.

A final important issue concerns the tightness of the observed T-F relation. This obviously implies some considerable uniformity in the formation of disk galaxies. As we have seen, the broad spin distribution does not, of itself, induce scatter. Variations in assembly time can do so through the $H(z)$ dependence of M_t. In combination with the size constraints already discussed, this suggests that most disks were assembled well after $z = 1$. Variations in the actual structure of halos of given mass and assembly epoch must also be sufficiently small to avoid excessive scatter in the $M_t - V_c$ relation. For the halos simulated by Navarro et al (1996, 1997) this relation is indeed tight enough. Finally, small scatter is required in the fraction F of the halo mass which condenses into a disk, in the disk mass-to-light ratio Υ, and in the disk contribution to the observed V_c values (which will vary with λ). The observed colours of disk galaxies are quite uniform, suggesting that Υ may not vary too much, and recent observations favour small Υ values, thus helping to satisfy the last condition (e.g. McGaugh & de Blok 1997). Since the required F-values are smaller than observed in galaxy clusters (e.g. White and Fabian 1995), the uniformity of F suggests that some feed-back process lowers the condensation efficiency in a way which depends only on V_c. Substantial feed-back appears necessary to account for the apparent global inefficiency of galaxy formation (e.g. White & Rees 1978; White & Frenk 1991) and a variation with V_c can induce a metallicity-luminosity relation (Larson 1972; Dekel & Silk 1986). In particular, feedback from star formation in CDM-like cosmologies can plausibly explain the observed metallicities both of present-day disks and of high redshift damped Lyα systems (Kauffmann 1996a)

A more careful analysis of many of the ideas in this section, together with applications to specific hierarchical cosmologies can be found in Mo et al (1997) and Dalcanton et al (1997). The latter paper compares its predictions in some detail with the observed sizes and surface brightnesses of disk galaxies.

4 Scaling Relations for Ellipticals

The properties of elliptical galaxies, particularly those of ellipticals in rich clusters, show some remarkable regularities. Most have very nearly elliptical isophotes and a luminosity profile which is well described by de Vaucouleurs' empirical fitting function. There is a tight relation, known as the fundamental plane, between the characteristic size of a galaxy, its total luminosity, and its central velocity dispersion. In addition there are tight relations between the luminosities of ellipticals and their colours and metallicities. The simplest interpretation requires
(i) that all ellipticals are made of old stars,
(ii) that they all formed in a similar way,
(iii) that the initial mass functions of their stellar populations (and so their M/L ratios at given age) are similar or at least vary only slowly with mass, and
(iv) that their metallicity increases (and so their colour reddens) systematically with mass.
The fundamental plane then reflects the virial relation $M \sim R\sigma^2$ with a slight tilt arising from the systematic variation of M/L with mass. Recent data on the evolution of ellipticals support this interpretation in that they are consistent with the fading in luminosity expected for a passively evolving population of equilibrium galaxies (see other contributions to this volume). An indication that the real picture may be more complex comes, however, from dynamical analyses which suggest that much of the mass within the luminous regions of ellipticals may in fact be pregalactic dark matter (e.g. Rix et al 1997).

More than twenty years ago Toomre (1977) remarked that star formation is observed only in galaxy disks, and further that the final state of pairs of interacting spirals must be something resembling an elliptical galaxy. In view of this he suggested that *all* star formation might occur in disk systems, and that ellipticals might *all* be formed by the merger of stellar disks. Although remaining controversial, these suggestions have gained much theoretical and observational support since they were made. Direct simulations of mergers between systems resembling disk galaxies have shown that they do indeed evolve into objects with a structure very like that of ellipticals (e.g. Barnes 1988). Furthermore, a number of transition cases have been found which seem to demonstrate empirically that merging spirals end up as ellipticals (e.g. Schweizer 1990). Finally it is still true that substantial star formation has been seen only in galaxy disks, or in starbursts either in the nuclear regions of gas-rich galaxies or in interacting disk systems.

The strongest objections to Toomre's proposal have come:
(i) from the tight systematic relations between E-galaxy properties – tight cor-

relations seem intuitively surprising if ellipticals are produced by the stochastic accumulation of smaller units,
(ii) from the fact that ellipticals are denser and more strongly bound than spirals – their progenitors must then have been more compact and more tightly bound than present-day disks, and
(iii) from the fact that most disk galaxies have central bulges which resemble ellipticals in many of their properties – how could mergers produce a central "elliptical" without disturbing the surrounding disk.
Semi-analytic models of hierarchical galaxy formation generally adopt the hypothesis that all star formation occurs in quiescent or interacting disks, and can address the above objections directly because they keep track of how and where disks grow and of how they merge together. It is therefore possible to trace the formation history of each elliptical galaxy, and to ask how it depends on luminosity and environment. The first detailed models of this kind were able to reproduce the characteristic luminosities and colours of ellipticals, the distribution of bulge-to-disk ratios of spirals, and the environmental segregation between ellipticals and spirals (Kauffmann et al 1993; Baugh et al 1996a). Objects with little or no disk are predicted to occur primarily in clusters and to have old stellar populations. They form by the merger of disks which were assembled well before $z = 1$ and so were compact (equ. 7). Present-day disks form late by accretion of new gas onto small "ellipticals" produced by the merging of earlier generations of disks.

In Figure 1 (adapted from Kauffmann 1996b) I illustrate when star-formation and merging are predicted to occur for cluster ellipticals in an $\Omega = 1$ CDM cosmogony normalised to give the correct abundance of rich clusters. The modelling scheme assumes that all objects with disk-to-bulge ratios less than 0.67 are classified as ellipticals, and for this plot the elliptical population in clusters of mass $10^{15}\mathrm{M}_\odot$ is analysed. The solid histogram shows the formation times of the stars which end up in these ellipticals. More than 40% form before $z = 3$, about 60% before $z = 2$, and more than 80% before $z = 1$. Very few stars have formed in these objects over the last few billion years. Thus cluster ellipticals are predicted to be red and to show little scatter in their colour-luminosity relation. More detailed study shows that ellipticals in high-z clusters are predicted to form their stars earlier on average than present-day ellipticals, and as a result the scatter in the luminosity-colour relation remains small out to redshifts of order unity (Kauffmann 1996b). The dashed histogram in Fig. 1 shows when these ellipticals underwent their last major merger. This is predicted to be quite late – more than 70% were assembled after $z = 1$. Somewhat later star-formation and merger times are predicted for ellipticals in groups rather than clusters, and similar patterns are predicted in other cosmogonies – formation is somewhat earlier in low density universes and somewhat later in $\Omega = 1$ cosmogonies with less small-scale power than CDM (e.g. mixed dark matter).

A natural prediction of hierarchical cosmogonies is that small things form first. As Figure 2 demonstrates, however, this effect is barely detectable for ellipticals in clusters. In this plot the mean stellar age of ellipticals is shown as a

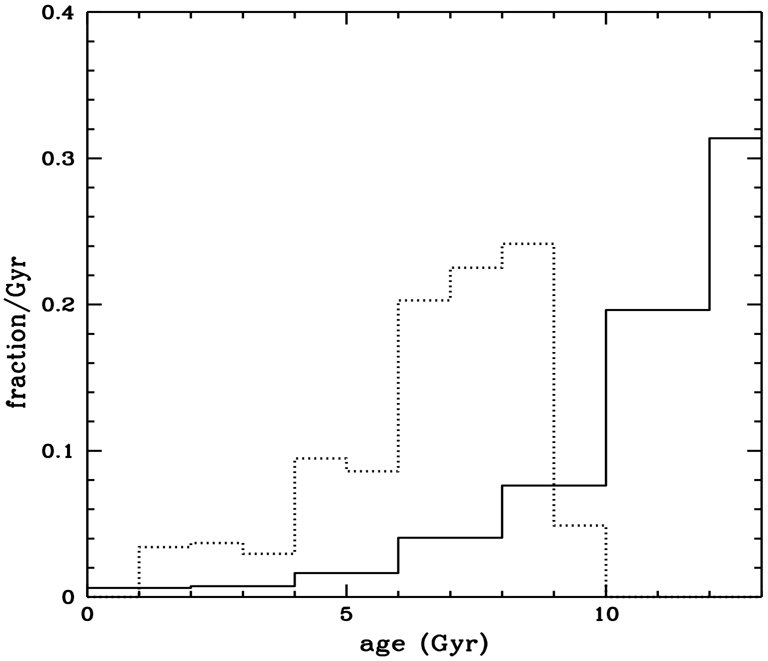

Fig. 1. The solid histogram gives the distribution of formation times for the stars in elliptical galaxies in a $10^{15}\mathrm{M}_{\odot}$ cluster. The semi-analytic model assumes a standard CDM cosmogony normalised to $\sigma_8 = 0.67$. The dotted histogram gives the distribution of the times when these elliptical galaxies underwent their last major merger (data taken from Kauffmann 1996b).

function of their total stellar mass for the same cosmogony analysed in Fig. 1. Ellipticals of all masses are made of old stars, and the decrease in age with increasing mass is less than the (small) age scatter between galaxies. If metallicity effects are ignored, the colours of ellipticals are predicted to be essentially independent of luminosity and to have small scatter (e.g. Kauffmann 1996b; Baugh et al 1996a). The inclusion of chemical enrichment effects can plausibly produce the observed colour-luminosity relation because: (a) more massive ellipticals are predicted to form from the merging of more massive disks, and (b) as a result of feed-back effects, the metallicity of disks is predicted to increase strongly with their mass (e.g. Kauffmann 1996a). The first effect is illustrated in Fig. 2 which gives the ratio of mean progenitor mass to final mass as a function of final mass. The mean progenitor mass is defined by tagging each star with the mass of the disk galaxy in which it formed, and then averaging this mass over all the stars in the final elliptical. The stars in a $2.5 \times 10^{12}\mathrm{M}_{\odot}$ elliptical typically formed in disks which were more than ten times as massive as those which merged to make a 10^{10}M$_\odot$ elliptical. It will be possible to check whether the resulting metallicity-luminosity relation reproduces the observed colour-luminosity plots as soon as reliable population synthesis models are available for a wide range of metallicities.

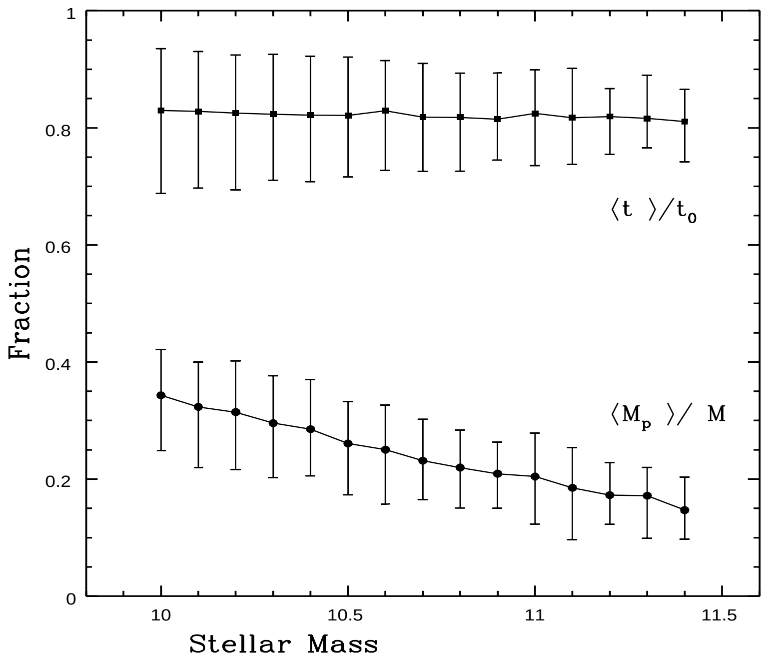

Fig. 2. The upper points give the mean ages of cluster ellipticals (in units of the age of the Universe) as a function of their stellar mass for the same model plotted in Fig. 1. The lower points give the mean progenitor masses of these same ellipticals in units of their total mass. In both cases the error bars join the upper and lower 5% points of the galaxy to galaxy scatter in these quantities.

According to these models the stellar population of present-day ellipticals formed in compact disks with scale radii $\sim$ 1kpc and circular velocities $\sim$ 200km/s. These disks were among the most rapidly star-forming objects at $z \sim 2.5$, and should presumably be identified with the galaxies recently discovered by Steidel and collaborators (Steidel et al 1996; Giavalisco et al 1996). The observed objects have roughly the correct size, abundance, star formation rate and internal characteristic velocity, but it is not yet clear whether they are indeed disk-like. Very recent studies of the abundance of such objects during

$0 < z < 5$ suggest that the overall rate of unobscured star formation in the Universe actually peaked at $1 < z < 2$, that star formation in this mode could possibly account for *all* the observed stars in galaxies, and that most stars formed after $z \sim 1$ (Madau et al 1997). Such late star formation is one of the most robust and controversial predictions of hierarchical models (e.g. White 1989; Cole et al 1994) but observational verification is difficult since the conversion from observed UV flux to star-formation rate is uncertain by at least a factor of 2. Thus one cannot tell whether all stars formed in the observed unobscured mode or only 30% to 50% of them, for example the stars in present-day disks. A direct proof of recent elliptical formation could come from a survey of the Universe at, say, $z = 2$, which showed the current population to be absent at that epoch. Recent deep redshift surveys selected at I and K allow complete samples of early-type galaxies to be identified to $z \sim 1$. V/V_{max} tests applied to these samples show unambiguously that the early-type population does not follow standard passive evolution models (Kauffmann et al 1996). In fact, roughly two thirds of the present population appears to be missing at $z = 1$; either the galaxies were actively forming stars or they were in several pieces at that time. If further deep surveys confirm this result, we may conclude that the bulk of galaxy formation has already been observed, and that we now have a crude quantitative understanding of the origin and evolution of the basic properties of galaxies.

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