Could intermediate-mass AGB stars produce star-to-star abundance variations in globular-cluster red giants?

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Abstract. We performed detailed nucleosynthesis calculations in hydrogen and helium burning shells as well as for hot bottom burning at the base of convective envelope in a low-metallicity ($Z = 10^{-4}$) intermediate-mass star during thermal-pulsing AGB evolution. Based on complete stellar models, up-to-date simple analytical expressions were used to describe the model star and its evolution. Our study concentrated on surface abundances of light elements, such as C, N, O, Na and Al, and their isotopes in order to test a hypothesis of Cottrell and Da Costa (1981) frequently invoked to explain star-to-star abundance variations in globular-cluster red giants. It is shown that this hypothesis of primordial contamination of intracluster matter by nuclear products from early generation AGB stars fails to reproduce the observed O depletion and Al enhancement. We propose an alternative mechanism which combines some primordial composition anomalies with atmospheric abundance effects produced by deep mixing in globular-cluster red giants. We also suggest observational tests for the verification of our model.

Key words: stars: abundances – stars: evolution – stars: giant – stars: interiors – globular clusters: general

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1. Introduction

In globular-cluster red giants there are star-to-star abundance variations of some light elements such as C, N, O, Na and Al, the N abundance being usually increased and correlating whereas those of C and O are decreased and anticorrelate with enhanced abundances of Na and Al (see the review paper of Kraft 1994, and references therein; Norris & Da Costa 1995). Two alternative hypotheses exist which explain these abundance variations. The first one, called "primordial", shifts responsibility for the heterogeneous contamination of the intracluster matter to some primordial process which had taken place before the now observed red giants were formed (supernova explosions, winds from W-R and/or AGB stars etc., see, e.g., Brown & Wallerstein 1993). The second one sometimes referred to as the "deep mixing scenario" ascribes the origin of the discussed abundance anomalies to nuclear burning in deep interiors of the observed stars themselves accompanied by some mixing process (of as yet unknown nature), dredging up products of the nuclear burning to the stellar surface (Sweigart & Mengel 1979; Denissenkov & Denissenkova 1990; Smith & Tout 1992; Langer et al. 1993; Denissenkov & Weiss 1996, hereafter Paper I; Wasserburg et al. 1995).

From a quantitative point of view much work has been done to support the mixing scenario. A suitable place for the nuclear burning was found to be the stellar layers directly above the hydrogen burning shell (Sweigart & Mengel 1978). Here reactions of the CNO-cycle can considerably rearrange relative CNO-abundances without reducing the hydrogen abundance such that the overall structure and evolution of the star remain unaffected when allowing additional mixing to penetrate these layers. With regard to Na and Al enhancement, they can be produced by the reaction ^{22}Ne(p,γ)^{23}Na of the NeNa-cycle (Denissenkov & Denissenkova 1990) and by transformation of ^{25}Mg and ^{26}Mg isotopes into Al in the MgAl-cycle (Langer et al. 1993), respectively, in the same layers where O is depleted due to the ON-cycling. To illustrate the advantages of the mixing scenario, Denissenkov and Weiss (Paper I) demonstrated that additional mixing (modelled by large-scale diffusion) between the base of the convective envelope and the hydrogen burning shell (HBS) in red giants can account for the so-called global anticorrelation of [O/Fe] versus [Na/Fe][1] which was established by Kraft et al. (1993) to be a common feature for more than 70 cluster and halo giants studied to-date (cf. Fig. 1). In addition a systematic decline in C abundance with increasing stellar luminosity seen in the globular cluster M 92 (Langer et al. 1986) can be reproduced.

As regards the hypothesis of primordial nucleosynthesis, to our knowledge there were no quantitative analyses done so far despite of the fact that a number of possible sources, which seemed to be able to pollute the intracluster medium as it is required by observations, were proposed in the past. In particular, one of the sources was discussed by Cottrell & Da Costa (1981, hereafter CD). To explain Na and Al enhancements and their correlation with CN band strength (i.e. presumably with overabundance of N) in the globular clusters 47 Tuc and NGC 6752, CD proposed that nuclear processing in intermediate-mass stars (4-8 $M_{\odot}$, hereafter IMS) of an earlier stellar generation during their thermal-pulsing asymptotic giant branch (TP-AGB) evolution might be responsible for the observed abundance variations in red giants. In this scenario the primordial abundance variations arose as a result of the pollution of intracluster gas by strong winds from IMS. The polluted material was then incorporated into contracting low-mass stars which are now observed as red giants.

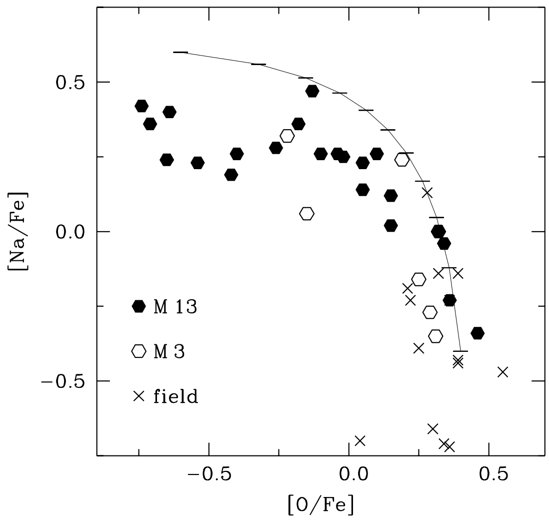

Fig. 1. The "global anticorrelation" of [O/Fe] versus [Na/Fe] for field and globular-cluster red giants (Kraft et al. 1993) and a dilution model. The curve shows the abundances of O and Na in the material the q-th part (by mass) of which has O depleted and Na enhanced by a factor of 10, and $(1-q)$-th part has unchanged chemical composition: [O/Fe]=0.4, [Na/Fe]=-0.4. The tick-marks on the curve correspond to the progressively increasing values of the dilution coefficient $q = 0, 0.1, 0.2$, etc., beginning from the right-bottom corner

The TP-AGB stars have an inert carbon-oxygen core and are in the double-shell nuclear burning phase of their evolution where they experience recurrent thermal pulses (TPs) of the helium burning shell (HeBS) (Schwarzschild & Härm 1967; Sanders 1967; Iben 1975a; Sugimoto & Nomoto 1975; Fujimoto, Nomoto & Sugimoto 1975). The

[1] We use the standard spectroscopic notation, [C]= lgC$-$ lgC$_{\odot}$

temperature at the base of the HeBS in IMS can reach very high values ($\gtrsim 3 \cdot 10^8$ K) sufficient for neutrons to be liberated via the reaction ^{22}Ne(α,n)^{25}Mg which may initiate s-process nucleosynthesis (Iben 1975b). A large abundance of ^{22}Ne for the above reaction is supplied by the chain ^{14}N(α,γ)^{18}F($\beta^+\nu$)^{18}O(α,γ)^{22}Ne, ^{14}N being a dominant isotope in the ashes left behind by the HBS. A convective zone is developed during the TP. It occupies almost the whole intershell region between the HeBS and the H-He interface and redistributes products of the helium burning and those of the s-process nucleosynthesis over this region. Immediately after the TP is finished the base of the extended convective envelope of the star, which is normally located close to but above the HBS, quickly reaches downwards passing through the H-He interface and penetrating even deeper into the region where the He convective shell was located, and dredges up a mixture of products of H and He burning and of s-processing to the stellar surface ("third dredge-up"). Between successive TPs hydrogen burning can occur at the base of convective envelope of the more massive stars (the so-called hot bottom burning, hereafter HBB) producing additional changes of the star's surface chemical composition.

It should be noted that this "classical" picture may be not correct in its substantial details. Evolutionary calculations by Blöcker (1995) showed that the total number of TPs in IMS was reduced to a few when the "superwind" (Renzini 1981) was taken into account. The inclusion of the "superwind" on the early AGB is demanded by the relations between initial and final stellar masses derived by Weidemann (1987). As a result, the IMS may even not achieve a stationary regime on the TP-AGB and the described chain of evolutionary events does not take place. Of course, this would reject CD's hypothesis from the very beginning. In our paper *we assume* that the IMS can experience a sufficiently large number of the TPs (say, a hundred) and the "third dredge-up" does occur. Indeed, there are some considerations arguing that in low-metallicity TP-AGB stars the winds may be not as strong as in AGB stars of higher metallicity (Jura 1986). We address the problem whether CD's hypothesis can be accepted from the nucleosynthesis point of view and in the light of the recent composition analyses of globular-cluster red giants, supposing the "classical" picture were true.

The "primordial" hypothesis of CD was based on an earlier idea by Iben (1976) who had supposed that in a low metallicity environment free neutrons would be captured preferably by light isotopes such as ^{22}Ne and ^{25}Mg produced abundantly in He-burning reactions and by their progenies rather than by Fe. This could result in enhancements of Na and Al. Such an environment could be found within the intershell region during TPs in globular-cluster intermediate-mass AGB stars of an earlier generation. Following each TP material presumably enriched in Na and Al due to a neutron flux exposure and having also ^{12}C abundance increased by the 3-α-reaction is dredged-up by convection. The HBB in the convective envelope can later transform a fraction of ^{12}C into ^{14}N. As IMS are known to be the main polluters of interstellar medium owing to their large mass-loss rates, and because of their lifetimes being short as compared to those of red giants, CD considered them as a possible source of star-to-star abundance variations in globular-cluster red giants and, in particular, as a source of the correlation of CN band strength with Na and Al overabundances.

Two more things are worth being pointed out here. Firstly, at the time CD wrote their paper neither the ^{22}Ne(p,γ)^{23}Na nor the ^{26}Mg(p,γ)^{27}Al reaction was known as potential sources of Na and Al. Perhaps that was a main reason for CD to search for (or to think of) a primordial non-H-burning origin of those elements. Secondly, CD were not aware of the existence of the "global anticorrelation" of [O/Fe] versus [Na/Fe] which has been revealed later (Kraft et al. 1993). It is obvious even without doing detailed calculations that it would be very difficult to decrease the O abundance considerably in IMS. Despite of this evident deficiency of CD's hypothesis, it is still very often referred to (Smith 1992; Kraft 1994). That is why we decided to consider in this paper the nucleosynthesis in IMS of low metallicity with a special emphasis on production and/or destruction of such light elements as C, N, O, Na and Al.

2. Model and method of calculations

2.1. Physical picture

Our task is to follow the changes with time of the surface abundances in IMS of low metallicity evolving along the AGB. We are mainly interested in calculating the final abundances of C, N, O, Na and Al after a large number of TPs. Since we are going to address the problem whether the "primordial" hypothesis of CD can *at least qualitatively* account for the abundance variations in globular-cluster red giants we find it possible to make use of a very simple model for doing this particular work. Saying *qualitatively* we mean that only estimates of predicted surface abundance changes rather than their exact values are of main interest for us.

During TP-AGB evolution material from the convective envelope in IMS passes successively through the following stages of nuclear/mixing processing. It detaches from the base of convective envelope (BCE) when the latter retreats outwards following core mass growth, then finds itself in a scorching heat of the HBS where the CNO-, NeNa- and MgAl-cycles do their work on rearranging C, N, O, Ne, Na, Mg and Al abundances and after that settles onto the surface of the C-O core and waits for a next stage of processing. The next stage comes regularily about every three thousand years, lasts 10-30 years and is called (helium) convective shell burning (CSB). The convective shell appearing during the TP mixes what remained from the preceding pulse together with the fresh ashes from the HBS. Helium burning nuclear reactions and neutron capture nucleosynthesis take place predominantly at the base of the convective shell where the highest temperatures are achieved. After the pulse episode envelope convection penetrates into the intershell region and takes away part of the material which has now experienced double shell nuclear processing. In the envelope it is diluted with unprocessed material and, at last, can be subject to HBB-processing during the interpulse period of the star's evolution. What we now need to calculate the nucleosynthesis in the framework of the above outlined physical picture are characteristic densities, temperatures and time-scales for the three events of nuclear processing (i.e. for HBS, CSB and HBB), as well as some mixing parameters.

2.2. Parametrized model structure and evolution

We used a simple parametrized model because detailed calculations of the advanced TP-AGB evolution are very time-consuming and can be done only for a restricted set of parameters.

As a model star suitable for our study we chose one of $5\,M_\odot$ having initially $Z = 10^{-4}$ and $Y = 0.25$. With a code described elsewhere (Wagenhuber & Weiss 1994) we followed its evolution from the main sequence through the beginning of the 9th TP on the TP-AGB. That enabled us to use detailed runs of structural parameters from those realistic stellar models to test and calibrate our simplified model developed for nucleosynthesis calculations. Densities at the base of the helium convective shell, $\rho_{\rm CSB}$, in the hydrogen burning shell, $\rho_{\rm HBS}$, and at the base of the convective envelope, $\rho_{\rm BCE}$, were considered to be fixed during nucleosynthesis calculations. This assumption does not seem to be far from reality, at least for the first two quantities (this is confirmed by our exact models; see also Uus 1970). We found $\rho_{\rm CSB} = 4.5 \cdot 10^3$ g cm^{-3} and $\rho_{\rm HBS} = 35$ g cm^{-3} from the exact models. Regarding $\rho_{\rm BCE}$, which determines a time-scale for the HBB (the proton nuclear lifetime at the BCE is inversely proportional to $\rho_{\rm BCE}$), we performed calculations with several different values for it and found no *qualitative* changes in the final abundances (see below).

The dependence of the temperature at the base of the convective shell during the TP peak on the core mass $M_{\rm C}$ was taken to be

$$T_{\rm CSB} = [T^0_{\rm CSB} + 285(M_{\rm C} - 0.96)] \cdot 10^6\,{\rm K},$$

where putting $T^0_{\rm CSB} = 310$ recovers the well-known approximate formula proposed by Iben and Truran (1978). Temperatures in the HBS and at the BCE were assumed to grow proportionally to $T_{\rm CSB}$, their initial values $T^0_{\rm HBS}$ and $T^0_{\rm BCE}$ being considered as two additional parameters.

The algorithm to calculate other stellar characteristics and, in particular, those relevant to the mixing events was essentially the same as that described in a similar study by Renzini and Voli (1981). Our approach seems to be acceptable despite of the fact that we used approximate relations which had been obtained for AGB model stars of higher metallicity because Z cannot affect the structure and evolution of the part of the star below the HBS, the hydrogen burning processing itself being parametrized in our model.

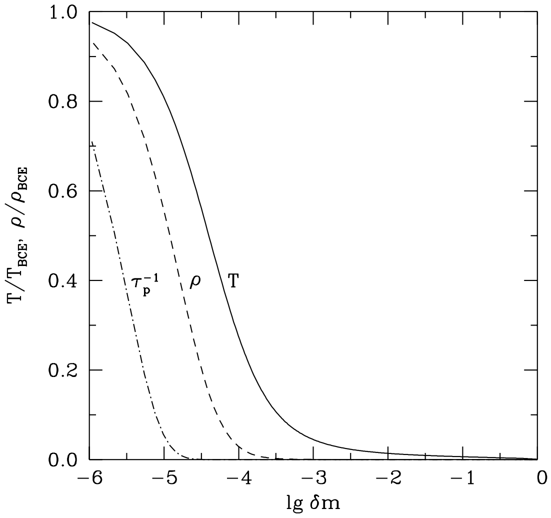

Fig. 2. Temperature, density and reverse nuclear lifetime of protons against captures by ^{14}N nuclei normalized to their values at the base of convective envelope (BCE) as functions of the logarithm of the relative mass coordinate $\delta m = (M_r - M_{\rm BCE})/(M - M_{\rm BCE})$ in the envelope of an AGB model star

Our further approximations were the following: We did not take into account the structural evolution of the helium convective shell during the development of a pulse, considering temperature and density within the shell to change linearly with mass coordinate and to be independent of time. Our test calculations which used realistic T and ρ distributions from the exact models have shown that the linear approximation works well.

Temperature and density profiles in the convective envelope are characterized by an extremely steep increase while approaching to the BCE. In fact, only a tiny region adjacent to the BCE has temperatures high enough for nuclear reactions to proceed. The relative mass fraction of this region $\delta m_{\rm HBB}$ is of the order of 10^{-5}, as can be seen from Fig. 2 where we plotted the dimensionless temperature, density and reverse nuclear lifetime of protons against captures by ^{14}N as functions of δm in the convective envelope (see also Lattanzio 1992). The HBB is known to be very sensitive to the choice of the mixing-length parameter α (the ratio of the mixing length to the pressure scale height in an adopted mixing-length theory of convection) in the convective envelope. The larger α, the higher the temperature at the BCE is, and the more efficient HBB-processing of the envelope material occurs (Iben 1976; Boothroyd et al. 1993). Recently an important influence of the HBB on the AGB evolution has been revealed (Blöcker & Schönberner 1991; Lattanzio 1991; Boothroyd & Sackmann 1992). If envelope convection is so powerful (a large α is chosen) that the BCE penetrates deeply enough into the HBS, the luminosity of the star increases considerably at almost constant core mass due to the very efficient energy transport by convection, and the usual core-mass-luminosity relation established for AGB stars by Paczyński (1970) and Iben (1977) holds no longer. This will influence the mass-loss rate since it strongly depends on luminosity (Blöcker 1995) and will affect the determination of exact luminosity limits within which an AGB star can acquire carbon-star characteristics (Boothroyd et al. 1993). Concerning HBB-nucleosynthesis, the temperature at the BCE as well as the rate with which the envelope mass is reduced due to mass-loss are of major importance. Taking into account (many) uncertainties still existing in the treatment of HBB, we ignored the possible breakdown of the classical core-mass-luminosity relation and considered the quantities $\rho_{\rm BCE}$, $T^0_{\rm BCE}$ and $\delta m_{\rm HBB}$ as free parameters controlling the efficiency of the HBB nuclear processing. Let us stress here once more that we aim to find out which changes of the chemical composition might be expected if the whole chain of nuclear/mixing processing events as proposed by CD (i.e. HBS, CSB, HBB, and "third dredge-up") took place. Supposing those changes disagreed with available observations, we could immediately discard CD's hypothesis without doing TP-AGB evolutionary calculations which are more complicated, time-consuming and sometimes controversial as compared to our nucleosynthesis estimates.

Reactions in the HBS were allowed to operate until the hydrogen abundance decreased to 10^{-15}. The CSB was stopped every time when the amount of synthesized carbon reached the value

$$X_{12}^{\rm max} = 0.23 + 0.017\exp\{7.756(M_{\rm C} - 1.16)\},$$

as proposed by Renzini and Voli (1981). The envelope material was being processed via HBB during every interpulse period of duration

$$\Delta t_{\rm ip} = (3.14\cdot 10^{18}{\rm s})\Delta M_{\rm C} X_{\rm env}/L_{\rm H}$$

(as estimated by Iben and Truran (1978)), where $X_{\rm env}$ is the hydrogen abundance in the envelope, and $L_{\rm H}$ the luminosity of the HBS.

According to observations and to some theoretical considerations AGB stars have to suffer from a so-called "superwind" in the course of their evolution (Renzini 1981) when almost the whole envelope is rapidly ejected in the form of a planetary nebula. The superwind phase limits the total number of TPs which an AGB star can experience to a few, dependent on initial stellar mass and on the adopted mass-loss rate law (Blöcker 1995). There are however some indications that AGB stars of low metallicity can survive longer (Jura 1986).

As a representative maximum number of TPs for the star to go through on the TP-AGB we used the value $N_{\rm max} = 100$ which was once decreased to 20 for testing purposes and reduced to other values when the mass-loss rate was artificially enhanced to study the influence of a superwind on the nucleosynthesis yield. The mass-loss rate in our nucleosynthesis calculations was expressed using the Reimers formula (Reimers 1975)

$$\frac{{\rm d}M}{{\rm d}t} = -4\cdot 10^{-13}\eta\frac{L}{gR} \quad (M_\odot\,{\rm yr}^{-1}),$$

where the luminosity L, the surface gravity g, and the radius R are in solar units, and η is a parameter. Fortunately, a choice of an appropriate mass-loss rate formula does not seem to influence the nucleosynthesis considerably because when the stellar wind can be described as a regular one the rate of mass-loss can be quite well approximated by Reimers' law (Renzini 1981), and when it turns into a superwind the time for removing the residual envelope becomes so short that any nuclear processing can be considered as being switched off completely. We modelled enhanced mass-loss by taking $\eta = 10$ and 50 instead of the standard value $\eta = 1.4$ as recommended by Kudritzki & Reimers (1978).

To summarize, our "standard set" of parameters to start nucleosynthesis calculations with is given in the second column of Table 1, the next columns containing other parameter sets for which calculations were also done.

2.3. Nuclear reactions network

To calculate the abundance changes within the HBS and in the course of CSB and HBB processing we used the same nuclear reactions network. Our particle list includes protons, α-particles, neutrons and 23 isotopes of light elements from carbon to silicon coupled by 63 nuclear reactions. We considered four branches of the CNO-cycle, NeNa- and MgAl-cycles simultaneously with all relevant helium burning reactions as well as a full set of neutron liberating and some neutron capture reactions. We did not follow s-process in detail because we were not interested in studying heavy element nucleosynthesis in this paper. Instead of that we used an abundance averaged neutron capture cross section for elements heavier than Si and considered them as one neutron sink seed nucleus. In special test calculations we decreased and increased this averaged cross section by a factor of 100 and found almost no quantitative changes in the final abundances of light isotopes.

Table 1. Parameter sets for which nucleosynthesis calculations were done

Abbrev.	HBS+CSB+ +HBB	no HBB	NMAX20	TCSB250	HBB50	HBBR1	VHBB	EHBB	DMHBB5	ETA10	ETA50
η	1.4	+	+	+	+	+	+	+	+	10	50
$N_{\rm max}$	100	+	20	+	+	+	+	+	+	85	16
$\rho_{\rm BCE}$	10	+	+	+	+	1	+	+	+	+	+
$T^0_{\rm CSB}$	310	+	+	250	+	+	+	+	+	+	+
$T^0_{\rm HBS}$	95	+	+	+	+	+	100	120	+	+	+
$T^0_{\rm BCE}$	70	+	+	+	50	+	90	100	+	+	+
$\delta m_{\rm HBB}$	10^{-5}	0	+	+	+	+	+	+	$5\,10^{-5}$	+	+

Values in the second column represent the "standard set" of parameters. In the next columns the sign "+" stands for the same value as in the standard set. Temperatures are given in units of 10^6 K, $\rho_{\rm BCE}$ in g cm^{-3}. Identical values for $\rho_{\rm CSB} = 4.5 \cdot 10^3$ g cm^{-3} and $\rho_{\rm HBS} = 35$ g cm^{-3} were used in all calculations.

Rates for most nuclear reactions between charged particles were taken from Caughlan & Fowler (1988). A few exceptions were the following: New reaction rates for the NeNa-cycle were recently published by El Eid & Champagne (1995). We used them. New measurements of the ^{17}O(p,α)^{14}N and ^{17}O(p,γ)^{18}F reaction rates by Landré et al. (1990) were taken into account with the uncertainty factors $f_1 = 0.2$ and $f_2 = 0.1$ as recommended by Boothroyd et al. (1994). For the reaction ^{12}C(α,γ)^{16}O the rate of Caughlan et al. (1985) was adopted which is in better agreement with a microscopic study of this reaction (Descouvemont 1989). Most of the neutron capture cross sections were taken from Bao & Käppeler (1987). Exceptions were the reactions ^{12}C(n,γ)^{13}C, ^{16}O(n,γ)^{17}O and ^{20}Ne(n,γ)^{21}Ne for which we used their up-dated cross sections 0.02 mb (Käppeler et al. 1989), 0.002 mb (Beer & Voß 1991) and 0.119 mb (Winters & Macklin 1988), respectively.

The initial chemical composition was essentially the same as in Paper I: Firstly, abundances of all isotopes heavier than ^{4}He were scaled down to 1/190th ($\approx Z/Z_\odot$) of their "solar values" given by Anders & Grevesse (1989). Secondly, the "α-isotopes" ^{16}O, ^{20}Ne, ^{24}Mg and ^{28}Si were enhanced by a factor 2.5. And, thirdly, the initial abundances of ^{23}Na and ^{27}Al were taken to be underabundant by a factor of 2.5 compared to their scaled solar values, which in particular implies that [^{22}Ne/^{23}Na]=[^{25,26}Mg/^{27}Al]=0.4. It is such an isotope mixture that is believed to exist in the material out of which low-mass stars in metal-poor globular clusters have been formed (see, e.g., Wheeler, Sneden & Truran 1989). Additionally, the effect of the 1st and 2nd dredge-up was taken into account by changing the abundances of ^{12}C, ^{13}C and ^{14}N according to the complete stellar models.

The numerical method used for solution of the nuclear reactions network has no substantial changes as compared to that described in Denissenkov (1990). An exception is the assumption of equilibrium neutron abundance at every mesh point inside the helium convective shell. This is motivated by the fact that a mean lifetime of neutrons against captures by isotopes in the CSB is shorter than the convective mixing turnover time.

3. Results and discussion

In Fig. 3 we show the chemical composition of material which has first passed through the HBS and immediately after that experienced the helium CSB without mixing with the envelope. The well-known HBS redistribution among CNO-isotopes whose main features are enhancements of the ^{13}C and ^{14}N abundances at the expense of ^{12}C and ^{16}O is accompanied by an almost total disappearence of ^{22}Ne and a slight increase in ^{23}Na due to the NeNa-cycling as well as by a substantial reduction of abundances of all magnesium isotopes in the MgAl-cycle. It should be emphasized that ^{27}Al has been reduced rather than overproduced in the HBS. This is caused by the large temperature assumed ($T^0_{\rm HBS} = 95 \cdot 10^6$ K). In a low metallicity environment temperature during hydrogen burning reaches such high values because the efficiency of the CNO-cycle in consuming hydrogen is proportinal to Z. At a low Z the model star tends to increase T in order to process a specified amount of H to maintain its luminosity (Paper I).

The helium CSB elevated considerably the abundances of ^{12}C, ^{16}O, ^{19}F, ^{21}Ne, ^{22}Ne, ^{25}Mg and ^{26}Mg, other isotopes being affected to a lesser extent. Neutron capture reactions contributed only to the further synthesis of ^{23}Na, ^{29}Si and ^{30}Si. It should be noted that a large additional enhancement of the sodium abundance in the convective shell as compared to its value after the HBS processing is entirely attributed to neutron captures by ^{22}Ne seed nuclei. This is in agreement with the prediction by CD. Concerning ^{27}Al, neutron capture nuclear reactions do not produce it at all! This is mainly due to the low cross section $\sigma = 0.084$ mb for the reaction ^{26}Mg(n,γ)^{27}Mg to be followed by ^{27}Mg($\beta^-\bar{\nu}$)^{27}Al and also because of the absorption of free neutrons by the abundant ^{22}Ne isotope.

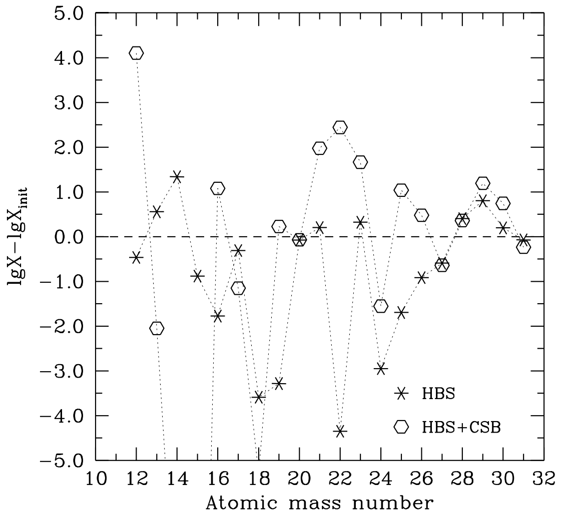

Fig. 3. Abundances in the material passed through the hydrogen burning shell (asterisks) and immediately after that having experienced (helium) convective shell burning (hexagons) without mixing with the envelope. X denotes relative mass fraction, $X_{\rm init}$ being the initial value. Atomic mass number 26 stands for ^{26}Mg rather than for ^{26}Al

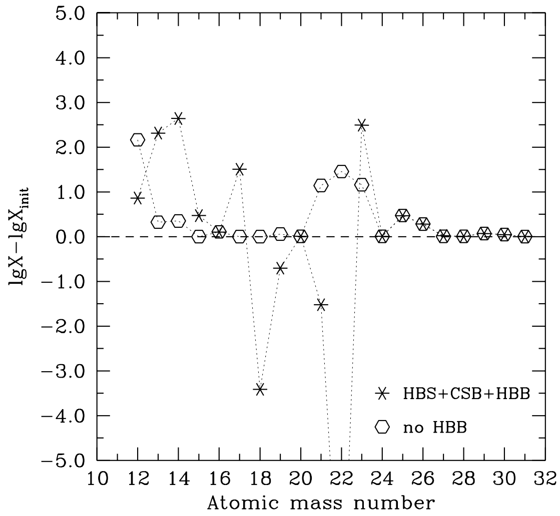

Fig. 4. Abundances after 100 pulses in the material passed through the whole chain of nuclear processing and mixing (i.e. HBS, CSB and HBB) and those which would be the result if no HBB occured (for abbreviations see Tab. 1)

It is interesting to note that during hydrogen burning some neutron capture nucleosynthesis takes place, too. Temperatures in the HBS are found to be sufficiently high for neutrons to be released via the reaction ^{13}C(α,n)^{16}O. Moreover, it turns out that the enhancement of ^{29}Si and ^{30}Si during hydrogen burning is due to neutron captures. If we had not included the reaction ^{13}C(α,n)^{16}O into our HBS nucleosynthesis calculations the ^{21}Ne abundance immediately after hydrogen burning would have been two orders of magnitude lower. The reaction ^{13}C(α,n)^{16}O results in a depletion of ^{13}C by about 0.5 dex and in an increase of ^{16}O by the same magnitude in the HBS as well as in a very small but visible decrease in the abundance of "isotopes" with atomic mass number 31 which is included only for the purpose of simulating a leakage of neutrons toward isotopes heavier than ^{30}Si. Let us stress once more that in our model there is actually double shell neutron capture nucleosynthesis going on, which for the first time occurs in the HBS where neutrons come from the reaction ^{13}C(α,n)^{16}O and for the second time in the helium CSB, neutrons now being supplied by the reaction ^{22}Ne(α,n)^{25}Mg.

In Fig. 4 the standard final abundances, i.e. the abundances in the material passed through the whole chain of nuclear/mixing processing stages described with our standard set of parameters (Tab. 1) are shown. In the same figure the final abundances which would be result if no HBB occured are also plotted. A comparison between these two sets of calculations reveals the role played by the HBB in modulating abundances of isotopes lighter than ^{24}Mg. We see that in the atomic mass range between 12 and 23 the relative abundance distribution after 100 TPs with HBB is similar to that which existed in the material passed only through the HBS (Fig. 3). The HBB nucleosynthesis is on the whole less efficient than the HBS processing because of a small value of $\delta m_{\rm HBB}$. This explains why hydrogen burning at the BCE does not affect isotopes heavier than ^{23}Na as well as a restricted work of the NeNa-cycle whose net result is a further increase of the sodium abundance at the expense of ^{21}Ne and ^{22}Ne.

Let us return to the double shell neutron capture nucleosynthesis discussed above. In the final abundance distribution one can find hardly any overproduction of ^{29}Si and ^{30}Si which is in a great contrast with their clear enhancements observed in Fig. 3. This can be understood in the following way: Neutron capture reactions, which are the only source of the ^{29}Si and ^{30}Si enhancements, can effectively synthesize isotopes heavier than ^{23}Na only during the first several pulses, until the abundances of ^{21}Ne and ^{22}Ne become sufficiently large. After that the abundant isotopes ^{21}Ne and ^{22}Ne begin to absorb neutrons quickly thus producing sodium and not allowing heavier isotopes to be synthesized in great amounts (cf. Truran & Iben 1977). This may even result in a sodium overproduction without being accompanied by considerable enhancements of s-process elements. This is sometimes considered as an argument against the interpretation that the Na excesses in globular-cluster red giants were produced in prior generation intermediate-mass TP-AGB stars. We are going to address the problem of heavy element nucleosynthesis in IMS of low metallicity in a forthcoming paper.

Figs. 5-8 show what happens with the final abundances when individual parameters in the standard set are varied. We see that *qualitatively* the results remain unchanged, which means that our calculations are rather stable and reliable. The most considerable changes are caused by variations of $\rho_{\rm BCE}$ and especially $T^0_{\rm BCE}$ which control the efficiency of HBB. In particular, the only case when we obtained a considerable increase of the ^{27}Al abundance was the model with extremely hot bottom burning (EHBB) (Fig. 6). In this case the aluminum enhancement is entirely attributed to hydrogen burning at the BCE and is accompanied with less Na overproduction and a stronger ^{24}Mg depletion. In order to decide whether such a reduction of ^{24}Mg abundance agrees with the observed chemical composition of red giants in globular clusters one first needs to estimate the fraction of the material ejected by intermediate-mass AGB stars which was captured by low mass stars during their formation (see the next section).

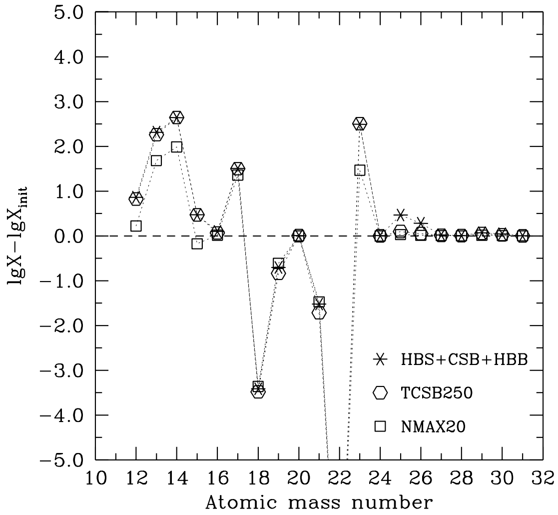

Fig. 5. Comparison of the final abundances calculated for the standard set of parameters with those obtained when $T^0_{\rm CSB} = 250 \cdot 10^6$ K has been assumed (model TCSB250) and when the maximum number of TPs followed has been decreased to 20

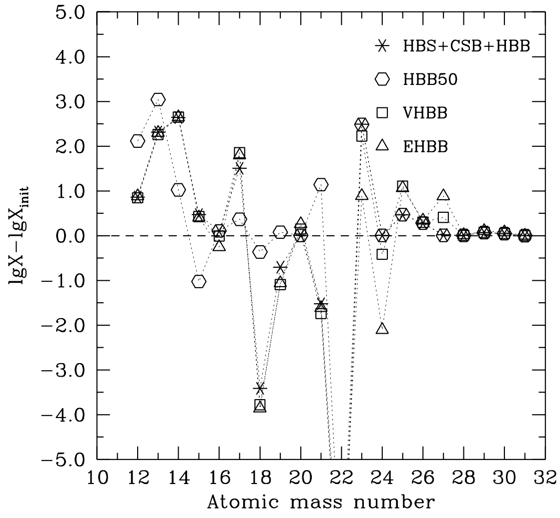

Fig. 6. Dependence of the final abundances on the initial temperature at the base of convective envelope, i.e. on the efficiency of HBB. The standard value $T^0_{\rm BCE} = 70 \cdot 10^6$ K has first been decreased to $50 \cdot 10^6$ K (model HBB50) and then increased to $90 \cdot 10^6$ K (model VHBB) and to 10^8 K (model EHBB). In the latter two cases temperature in the HBS has been correspondingly increased, too (see Tab. 1)

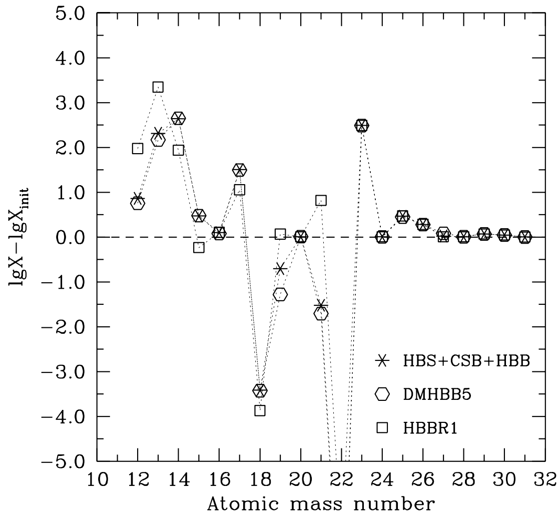

Fig. 7. The relative mass fraction of the zone with HBB in the convective envelope has been taken $5 \cdot 10^{-5}$ (model DMHBB5) instead of the standard value 10^{-5}, and $\rho_{\rm BCE} = 1\,{\rm g\,cm^{-3}}$ has been assumed (model HBBR1)

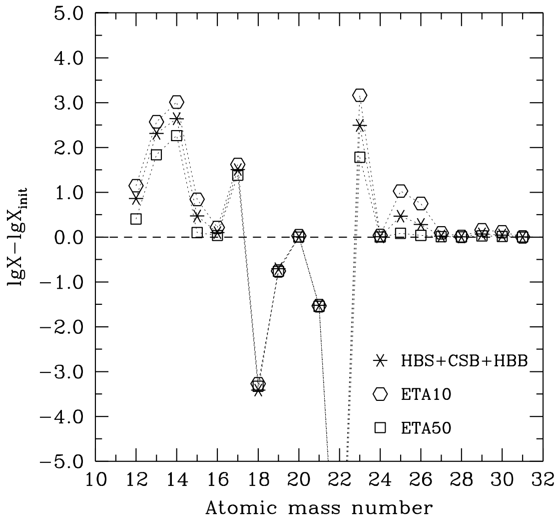

Fig. 8. The influence of enhanced mass-loss rate on the final abundances (for abbreviations see Tab. 1)

We also verified the hypothesis proposed by Iben (1976) that a ^{14}N spike could result in large abundances of ^{22}Ne, ^{23}Na, ^{24,25,26}Mg, ^{27}Al and ^{28,29,30}Si. This spike is expected to be formed in the intershell region if one takes into account convective overshooting at the BCE when it extends downwards after a pulse is finished. Convective overshooting forms a smeared-off ^{12}C profile at the point of the deepest inward penetration of the BCE. When the HBS reignites it first burns hydrogen in a small region adjacent to the H-He interface where the average carbon abundance has been considerably increased ($\bar{X}_{12} \approx 0.1$) by overshooting material from underlying C-rich layers, and thus the ^{14}N spike is produced. During a subsequent CSB phase the ^{14}N spike gets mixed with the material having lower ^{14}N abundance, which raises somewhat the average shell ^{14}N abundance and, as a consequence of the reactions $^{14}\mathrm{N}(\alpha,\gamma)^{18}\mathrm{F}(\beta^{+}\nu)^{18}\mathrm{O}(\alpha,\gamma)^{22}\mathrm{Ne}(\alpha,\mathrm{n})^{25}\mathrm{Mg}$, favours releasing more neutrons. We modelled the ^{14}N spike in our calculations and found no *qualitative* changes in the final abundances.

4. Conclusions

By means of detailed nucleosynthesis calculations which used thermodynamic conditions appropriate to the HBS, helium CSB and HBB in TP-AGB stars of intermediate masses and initial chemical composition characteristic for low-metallicity globular clusters we have determined the final abundances of light elements, such as C, N, O, Na, Mg and Al, and their isotopes which could be present in the material out of which globular-cluster red giants were formed. According to CD, those abundances might survive in atmospheres of red giants and be a cause of star-to-star abundance variations observed in many globular clusters. To be more exact, CD proposed that Na and Al enhancements in globular-cluster red giants had been produced in IMS of an earlier generation by neutron capture reactions during TPs of the HeBS on the TP-AGB. From this point of view *CD's primordial hypothesis cannot be longer considered as acceptable* because our calculations have convincingly demonstrated that Al cannot be synthesized in the HeBS. Concerning Na, we do have found considerable overproduction by neutron captures on ^{22}Ne seed nuclei in the intershell region. At the same time, it has been revealed that the Al abundance can be increased somewhat in the convective envelope during the interpulse periods if one admits extremely hot bottom burning (EHBB) at the BCE.

However, even for the EHBB one fails to obtain the oxygen abundance as low as it is demanded by observations (Fig. 1). Let q denote the relative mass fraction of the material whose abundances were changed from their initial values $y_{\rm i}$ to the final ones $y_{\rm f}$ by some primordial process and which after that was admixed to the material having initial chemical composition. The average abundance y of any isotope in the mixture will be

$$y = (1-q)y_{\rm i} + q y_{\rm f},$$

where $0 \leq q \leq 1$. If the nuclear/mixing processing events in the convective envelopes of intermediate-mass AGB stars are considered to be the only primordial source of abundance variations in globular clusters, the defined dilution coefficient can be estimated as

$$q = 0.85\,\frac{F_{\rm M}(>4M_\odot) - F_{\rm M}(>8M_\odot)}{1 - F_{\rm M}(>0.8M_\odot)},$$

where $F_{\rm M}(>m)$ is the fraction by mass of stars more massive than m, and the numerical coefficient 0.85 gives the mass fraction returned by IMS into the interstellar medium (Weidemann 1987). If the initial mass function $\xi(\lg m)$ (IMF) is specified, the quantity $F_{\rm M}(>m)$ can be calculated as

$$F_{\rm M}(>m) = \frac{\int_m^{m_{\rm u}} m\,\xi(\lg m)\,d\lg m}{\int_{m_{\rm l}}^{m_{\rm u}} m\,\xi(\lg m)\,d\lg m},$$

where $m_{\rm l}$ and $m_{\rm u}$ are the lower and upper mass limits. Unfortunately, only the IMF based on observations of stars in the solar neighborhood is known (Scalo 1986). It results in $q = 0.07$. There is nothing better for us but to take the estimate $q \approx 0.1$. It should be emphasized that this estimate for q implies that IMS as well as contracting low-mass stars were distributed uniformly over the cluster and that the envelope material ejected by IMS had been mixed quite well with intracluster material before it was captured by low-mass stars.

The observed oxygen abundances in red giants in the globular cluster M 13 vary by more than an order of magnitude (Fig. 1). If this spread in O is considered to be of primordial origin it means that the stars with the lowest [O/Fe] values were formed from the material with $q \approx 1$ (see the curve in Fig. 1, which has been obtained under the assumption $y_{\rm f}/y_{\rm i} = 0.1$ for oxygen). This conclusion only slightly depends on the assumed ratio $y_{\rm f}/y_{\rm i}$. Even if $y_{\rm f}/y_{\rm i} = 0$ is taken, we get $y/y_{\rm i} = (1-q)$ and the observed ratio $y/y_{\rm i} \leq 0.1$ requires $q \geq 0.9$. It seems unlikely that about 30 per cent of low-mass stars in M 13 (Fig. 1) were formed almost entirely from the material ejected by IMS (or by other primordial sources). Consequently, the most serious argument against CD's (and any other) primordial hypothesis is the wide spread in the O abundance anticorrelating with the Na enhancement in M 13. However it can be easily explained in the "deep mixing scenario" (Paper I).

In Table 2 the logarithms of the relative abundances $\lg(y/y_{\rm i})$ in the material which was polluted by ejecta from intermediate-mass AGB stars are given for different values of q. We see that the standard case "HBS+CSB+HBB" disagrees with observations because it can reproduce neither the O depletion, nor the Al enhancement. The case of "EHBB" also results in a too small O decrease unless we assume the dilution coefficient to be close to unity. But the choice of $q \approx 1$ has an undesirable side-effect, a strong reduction of the ^{24}Mg abundance which is not observed (Kraft et al. 1993).

On the basis of the calculations performed we propose another hypothesis which can be considered as a modification of CD's one. Let us suppose that both primordial abundance anomalies produced by intermediate-mass TP-AGB stars (with HBB), and some deep mixing process contribute to the star-to-star abundance variations in globular clusters. In this scenario the primordial part is assumed to be responsible for some initial enrichment of low-mass stars with N, Na, Al and ^{25,26}Mg isotopes, as can be infered from our calculations (Tab. 2, "EHBB", $q = 0.1, 0.2$). During ascending the red giant branch deep mixing in low-mass stars produces O depletion and at the same time additionally increases the Al abundance at the expense of primordially enhanced ^{25,26}Mg, sodium being also increased some more at the expense of ^{22}Ne. Langer & Hoffman (1995) and Denissenkov & Weiss (Paper I) pointed out that in order to explain extremely high Al abundances found in some globular clusters (Shetrone 1994; Norris & Da Costa 1995) one needed initial ^{25,26}Mg abundances increased by about 0.5 dex as compared to the initial ratio [^{25,26}Mg/Al]=0.4. Such an increase in ^{25}Mg could be provided by the HBB in IMS followed by ejection and dilution of the envelope material in the interstellar medium with the coefficient $q \approx 0.2$ (Tab. 2, "EHBB") which is not too far from the above estimated value $q \approx 0.1$. This composite hypothesis predicts that (1) red giants in globular clusters can exist (they have primordial abundance anomalies but no deep mixing) which show only large N and moderate Na and Al overabundances without being O deficient; (2) strong O depletion has to be always accompanied by large Al enhancement and vice versa; (3) if we go along the red giant branch down to the subgiant branch and farther to the main sequence turn-off, large N and moderate Na and Al overabundances can be preserved as primordial by their origin whereas strong O depletion and extremely large Al enhancement may disappear (if deep mixing takes place only on the red giant branch). These conclusions can be tested by observations.

Table 2. The logarithms of the relative abundances $\lg(y/y_{\rm i})$ in the material which was polluted by ejecta from intermediate-mass AGB stars

	"HBS+CSB+HBB"			
isotope	$q = 0.1$	$q = 0.2$	$q = 0.9$	$q = 1$
^{14}N	1.65	1.95	2.60	2.64
^{16}O	0.01	0.02	0.09	0.10
^{22}Ne	-0.05	-0.10	-1.00	-9.32
^{23}Na	1.50	1.80	2.45	2.49
^{24}Mg	0.0	0.0	0.01	0.01
^{25}Mg	0.08	0.14	0.44	0.47
^{26}Mg	0.04	0.07	0.26	0.28
^{27}Al	0.0	0.0	0.02	0.02
	"EHBB"			
isotope	$q = 0.1$	$q = 0.2$	$q = 0.9$	$q = 1$
^{14}N	1.66	1.95	2.60	2.65
^{16}O	-0.02	-0.04	-0.22	-0.25
^{22}Ne	-0.05	-0.10	-1.00	-8.33
^{23}Na	0.22	0.37	0.85	0.89
^{24}Mg	-0.05	-0.10	-0.97	-2.10
^{25}Mg	0.31	0.49	1.02	1.06
^{26}Mg	0.05	0.09	0.32	0.34
^{27}Al	0.22	0.37	0.85	0.89

The quantity q is the dilution coefficient. The value $q = 1$ corresponds to the final abundances in our nucleosynthesis calculations $\lg(y_{\rm f}/y_{\rm i})$ (cf. Fig. 6).

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