Chapter 1

PRINCIPLES OF SEMI-DETACHED BINARY EVOLUTION

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Abstract

Basic principles which govern the evolution of semi-detached binaries are reviewed. The main emphasis is on the evolution of short-period compact binaries, i.e. cataclysmic variables (CVs) and low-mass X-ray binaries (LMXBs). The main topics addressed are: computing the mass transfer rate, stationary and nonstationary mass transfer, stability of mass transfer, mechanisms which can drive or enhance mass transfer, the reaction of low-mass stars to mass loss, including a simple quantitative model, the phenomenon of the converging evolutionary tracks and finally applications of these concepts to CVs and LMXBs (stability of mass transfer, standard evolution of CVs, population synthesis, non-standard evolution of CVs and LMXBs).

1.1 Introduction

By definition, close binaries are double stars which experience during their evolution at least one phase of mass transfer, i.e. of semi-detached evolution. Because at the first occurrence of mass transfer it is the more massive and thus the more evolved primary star which loses mass to its companion, mass transfer is, in general, unstable, either thermally (if the donor's envelope is predominantly radiative) or adiabatically (if the envelope is predominantly convective). Thereby the time scale of unstable mass transfer is so short (compared to the nuclear time scale) that chances of observing a binary in such an evolutionary phase are very small (see e.g. the review by van Beveren, this volume).

The situation is completely different if, on the other hand, mass transfer is thermally and adiabatically stable. In that case, mass transfer must be driven, either by the expansion of the donor star (usually due to nuclear evolution) or by the shrinkage of the orbital separation (due to loss of orbital angular momentum). Because the associated time scales are long, numerous systems, belonging to quite different classes of binaries in a phase of stable semi-detached evolution, are known. The most important of these classes are 1) the Algol systems (where mass transfer to an essentially unevolved companion results from the nuclear evolution of a giant or a subgiant donor), 2) the cataclysmic variables (CVs) (where the donor star is usually an unevolved low-mass star, the accretor a white dwarf and mass transfer driven mainly by loss of angular momentum, occasionally also by nuclear evolution), and 3) the low-mass X-ray binaries (LMXBs) (where the accretor is either a neutron star or a black hole, the donor star either a low-mass star near the main sequence or a subgiant and mass transfer is driven mainly by angular momentum loss, and nuclear evolution if the donor is evolved). Mass transfer, i.e. semi-detached evolution, is of particular importance for understanding and modelling compact binaries, i.e. CVs and LMXBs. Because of the very high gravitational potential at the surface of the accretors ($\sim$ 100 keV per nucleon at the surface of a white dwarf and $\sim$ 100 MeV per nucleon at the surface of a neutron star and even more in the case of a black hole) even very low mass transfer rates give rise to accretion luminosities which far exceed the intrinsic luminosity of the donor star or the accretor. Thus, observationally, CVs and LMXBs are totally dominated by the accretion light source which is in many cases an accretion disk around the accretor. Whereas LMXBs, due to their very compact accretor, are sufficiently bright (in X-rays) to be detectable with current X-ray detectors throughout the galaxy and even beyond, CVs are relatively faint objects (in the visual and in X-rays), and few of them would be known were it not for the fact that their brightness is highly variable on a wide range of time scales. This variability, which is itself a direct consequence of mass transfer is observed e.g. in the form of flickering (one of the observational hallmarks of CVs), dwarf nova eruptions and classical nova outbursts. Dwarf nova outbursts, in turn, are a consequence of a thermal instability of the accretion disk which appears if the mass accretion rate is too low to keep the disk temperature above the ionization temperature of hydrogen ($\sim$ 6000 K) (see e.g. Meyer-Hofmeister and Ritter (1993) for a review). Nova outbursts, on the other hand, are the result of a thermonuclear runaway at the base of the accreted hydrogen-rich layer on the surface of the white dwarf (for a review see e.g. Fujimoto 1982, Truran 1982). An entirely analogous process, namely a thermonuclear runaway in the He layer which accumulates on the surface of an accreting neutron star, gives rise to the Type I X-ray bursts seen in many LMXBs (for a review e.g. Lewin, van Paradijs and Taam 1995).

The current great interest in compact binaries derives not only from the rich variety of accretion phenomena which they display but also from the fact that CVs and LMXBs represent very late stages of binary evolution. If it were possible to understand and model the properties of the observed populations of CVs and

LMXBs adequately, one could hope to gain information about their parent systems and details of the evolution leading to such end states by applying population synthesis techniques (for details see e.g. de Kool, this volume). Modelling the population of binaries which are in a phase of semi-detached evolution requires precise knowledge of the mass transfer rate at all times during the evolutionary phase in question. This is necessary because the mass transfer rate determines three main properties of such systems, namely 1) their lifetime, 2) their luminosity (particularly if the accretion luminosity dominates as in CVs and LMXBs) and with it their observability, and 3) whether and how frequently secondary phenomena associated with the mass transfer such as dwarf nova and classical nova outbursts (in CVs) or X-ray bursts (in LMXBs) which give rise to large amplitude brightness variations do occur.

Given the importance of knowing the mass transfer rate in a semi-detached binary, we must ask which properties of a binary determine it and how can we compute it. This is the main topic of this article. The main emphasis will be on basic principles and simple models which provide some understanding of the underlying physics rather than on computational techniques or detailed numerical results, both of which are well documented in the literature (e.g. Kolb and Ritter 1990; 1992, and references therein). Although most of the following is tailored to the evolution of CVs and LMXBs (with a low-mass main sequence donor), many of the topics to be discussed apply to other types of semi-detached binaries as well.

1.2 Computing the mass transfer rate

In the following we shall discuss the evolution of semi-detached binaries in the framework of the Roche model. By adopting this model (for a detailed description see e.g. Kopal 1959, 1989) we make a number of implicit assumptions which restrict its applicability. These assumptions are 1) that the binary orbit is circular, 2) that the components are point masses or, if they are extended, that they are corotating with the orbital motion and their mass distribution is spherically symmetric, and 3) that forces other than gravity and the centrifugal force are not important. From these restrictions it is already clear that in a number of interesting cases mass transfer cannot be dealt with adequately in the framework of the Roche model. Examples are e.g. Be-star X-ray binaries (because of their usually very eccentric orbits) or high-mass X-ray binaries (where the radiation pressure from the donor star or from the accreting compact component can be important). Also in the case we are mostly interested in here, namely semi-detached binaries with a compact component and in a phase of sustained mass transfer, applicability of the Roche model is not straightforward. Whereas the requirements of a circular orbit and corotation (of the mass donor) are usually well fulfilled because the strong tidal interaction yields just that on a relatively short time scale, the mass distribution of the lobe-filling star is clearly non-spherical. Therefore, the Roche potential can only be an approximation at best. The justification for nevertheless using the Roche potential is, on the one hand, its simplicity (calculation of a selfconsistent potential is much too complicated for most practical purposes) and, on the other hand, that stars are sufficiently centrally condensed objects to make the Roche potential an acceptable approximation. Yet in the light of recent findings by Rasio (this volume, see also Rasio and Shapiro 1995 and references therein) using the Roche model needs some further justification. As has been shown by Rasio and coworkers the total potential of a binary consisting of two sufficiently extended stars in hydrostatic equilibrium is such that the orbit is either dynamically or tidally (secularly) unstable, causing the system to coalesce. If, on the other hand, only one of the components is extended, i.e. filling its critical potential lobe, and the other one is a compact star, the orbit is dynamically stable. It is also tidally stable if the lobe-filling star is not very much more massive than the accretor (see Rasio, this volume). Since in the systems we shall be concerned with the donor star has a mass which is less than or at most comparable to that of the accretor, such systems are also tidally stable. Thus, for the special type of binaries we are interested in here, the Roche model is applicable and the Roche potential an acceptable approximation of the true potential.

The centrifugal force and the companion's gravity limit the size of each component to which it can grow without losing mass to its companion. This limit is usually expressed in terms of the critical Roche radius. In the following we shall assume the primary star (subscript 1) to be the detached component and the secondary (subscript 2) to be the donor star. The critical Roche radius of the secondary is given by

$$R_{2,R} = a\, f_2(q) \quad , \tag{1.1}$$

where a is the orbital separation, $q = M_1/M_2$ the mass ratio and the function f_2 is tabulated e.g. in Kopal (1959, 1989) or Mochnacki (1984). Analytical approximations for f_2 have been given e.g. by Paczyński (1971) and Eggleton (1983). In this simple picture mass transfer occurs whenever the secondary's radius R_2 exceeds the critical Roche radius. This is because matter outside $R_{2,R}$ is free to flow to the companion star on a hydrodynamical time scale. The problem here is not to determine whether or not mass transfer occurs, but rather how much. Among the various techniques which have been used in the past to calculate the mass transfer rate we shall discuss two, a simple and a more elaborate one.

The simplest prescription for computing the mass transfer rate relies on the fact that all but the most distended stars, i.e. giants or supergiants, have a well-defined surface, the photosphere, i.e. a well-determined radius. Because the mass of the donor star which is above the critical potential surface, i.e. outside $R_{2,R}$, cannot stay there for long but has to flow away towards the companion, a star cannot exceed its Roche radius by a large margin. Therefore one can compute the mass transfer rate by requiring $R_2 \leq R_{2,R}$. Of course, if $R_2 < R_{2,R}$ there is no mass transfer. In numerical calculations the mass transfer rate follows from enforcing $R_2 = R_{2,R}$ to within some accuracy. This is achieved by iterating for the mass M_2 at each time step. This technique, which is the one which has been most often used for computing binary evolution, usually results in a very noisy mass transfer rate $\dot{M}_2(t)$. Yet in a wide range of applications this does not really matter as long as $M_2(t)$ is reasonably smooth. However, in the case of compact binaries in which we are mostly interested and where major properties such as the accretion luminosity depend critically on the precise value of $\dot{M}_2$, a noisy mass transfer rate has to be avoided. This can only be achieved by a better, i.e. a more detailed and physical model of the mass transfer process. The model most often used in the recent past is based on a simplified hydrodynamical treatment for the mass overflow. Following Lubow and Shu (1975) it is assumed that the mass flow far from the inner Lagrangian point L_1 is subsonic and isothermal (in layers which are optically thin) or adiabatic (in layers which are optically thick) and that the flow reaches sound velocity near L_1. In the optically thin approximation which is adequate for our purposes, the mass transfer rate is given by (e.g. Ritter 1988; Kolb and Ritter 1990)

$$-\dot{M}_2 = \dot{M}_0 \, e^{-\frac{R_{2,R}-R_2}{H_P}} \quad , \tag{1.2}$$

where H_P is the pressure scale height in the secondary's atmosphere. $\dot{M}_0$ depends mostly on integral parameters of the donor, i.e. on M_2, R_2, its effective temperature and on the photospheric density and mean molecular weight. For low-mass stars on or near to the main sequence (as they prevail among CVs and LMXBs) $\dot{M}_0$ and H_P vary only very little with mass and have typical values of $\dot{M}_0 \approx 10^{-8} M_\odot \mathrm{yr}^{-1}$ and $H_P \approx 10^{-4} R$. Because $H_P \approx 10^{-4} R_2$, $\dot{M}_2$ is an extremely sensitive function of $R_{2,R} - R_2$. This means that one needs to compute the stellar radius as a very smooth function of time in order to obtain a reasonably smooth $\dot{M}_2(t)$. (For a discussion of numerical problems arising from using (1.2) see e.g. Kolb and Ritter 1990.) Although application of (1.2) may give some problems in numerical calculations, it is the ideal starting point for the discussion of our next topic, namely stationary and nonstationary mass transfer and the stability of mass transfer.

1.3 Stationary and nonstationary mass transfer; stability of mass transfer

We start by taking the time derivative of (1.2). Neglecting the derivatives $\ddot{M}_0$ and $\dot{H}_P$ which are smaller by about a factor $R_2/H_P \approx 10^4$ than the remaining term, we obtain

$$\ddot{M}_2 = \dot{M}_2 \, \frac{R_2}{H_P} \left[\frac{\dot{R}_2}{R_2} - \frac{\dot{R}_{2,R}}{R_{2,R}} \right] \quad . \tag{1.3}$$

Stationary mass transfer implies $\ddot{M}_2 = 0$, i.e.

$$\frac{\dot{R}_2}{R_2} = \frac{\dot{R}_{2,R}}{R_{2,R}} \quad . \tag{1.4}$$

This is equivalent to requiring $R_2 = R_{2,R}$ during phases of mass transfer. Thus the standard criterion for deriving the mass transfer rate (see above Sect.1.2) corresponds to the condition of stationary mass transfer. We note, however, that $R_2 = R_{2,R}$ as a prescription for computing $\dot{M}_2$ is inadequate if H_P/R_2 is not a small number (cf. Pastetter and Ritter 1989).

Mass loss from the secondary and mass transfer in the binary change the secondary's radius R_2 and the critical Roche radius $R_{2,R}$ with time. The temporal change of R_2 can be decomposed in the following way:

$$\begin{aligned} \frac{\dot{R}_2}{R_2} &= \left(\frac{\partial \ln R_2}{\partial \ln M_2}\right)_{ad} \frac{\dot{M}_2}{M_2} + \left(\frac{\partial \ln R_2}{\partial t}\right)_{th} + \left(\frac{\partial \ln R_2}{\partial t}\right)_{nuc} \\ &= \zeta_{ad} \frac{\dot{M}_2}{M_2} + \left(\frac{\partial \ln R_2}{\partial t}\right)_{th} + \left(\frac{\partial \ln R_2}{\partial t}\right)_{nuc} \quad , \end{aligned} \tag{1.5}$$

where ζ_{ad} is the adiabatic mass radius exponent, $(\partial \ln R_2/\partial t)_{th}$ and $(\partial \ln R_2/\partial t)_{nuc}$ the change of the stellar radius, respectively due to thermal relaxation and nuclear evolution. The thermal relaxation term arises because, in general, mass loss drives the star out of thermal equilibrium. We shall come back to this point below (see Eq.(1.11) and Sect.1.5). The temporal change of $R_{2,R}$ can be written as

$$\frac{\dot{R}_{2,R}}{R_{2,R}} = \zeta_R \frac{\dot{M}_2}{M_2} + \left(\frac{\partial \ln R_{2,R}}{\partial t}\right)_{\dot{M}_2=0} = \zeta_R \frac{\dot{M}_2}{M_2} + 2\left(\frac{\partial \ln J}{\partial t}\right)_{\dot{M}_2=0} \quad . \tag{1.6}$$

Here ζ_R is the mass radius exponent of the Roche radius. The term proportional to ζ_R arises because mass transfer changes the mass ratio and thus $R_{2,R}$ even at constant orbital angular momentum. On the other hand, $R_{2,R}$ can also change in the absence of mass transfer because the orbital angular momentum J can change. This gives rise to the term $(\partial \ln R_{2,R}/\partial t)_{\dot{M}_2=0} = 2(\partial \ln J/\partial t)_{\dot{M}_2=0}$.

Inserting now (1.5) and (1.6) into (1.3) we obtain

$$\begin{aligned} \ddot{M}_2 &= \dot{M}_2 \frac{R_2}{H_P} \\ &\left\{(\zeta_{ad} - \zeta_R)\frac{\dot{M}_2}{M_2} + \left(\frac{\partial \ln R_2}{\partial t}\right)_{th} + \left(\frac{\partial \ln R_2}{\partial t}\right)_{nuc} - 2\left(\frac{\partial \ln J}{\partial t}\right)_{\dot{M}_2=0}\right\} . \end{aligned} \tag{1.7}$$

From (1.7) we derive immediately the well-known expression for the stationary mass transfer rate. Setting $\ddot{M}_2 = 0$ yields

$$(-\dot{M}_2)_{st} = \frac{M_2}{\zeta_{ad} - \zeta_R} \left\{\left(\frac{\partial \ln R_2}{\partial t}\right)_{th} + \left(\frac{\partial \ln R_2}{\partial t}\right)_{nuc} - 2\left(\frac{\partial \ln J}{\partial t}\right)_{\dot{M}_2=0}\right\} \quad . \tag{1.8}$$

From (1.7) we can also derive a criterion for stability against runaway mass transfer. For that we ask for the sign of $\delta\ddot{M}_2$ if the stationary mass transfer rate (1.8) (for which $\ddot{M}_2 = 0$) is perturbed, i.e. if $\dot{M}_2 = (\dot{M}_2)_{st} + \delta\dot{M}_2$. Mass transfer is stable if the reaction to the perturbation opposes the perturbation, i.e. if $\delta\ddot{M}_2/\delta\dot{M}_2 < 0$. Assuming the perturbation to be so fast that the star has no time to react thermally but only adiabatically, i.e. that $\delta(\partial \ln R_2/\partial t)_{th} = 0$, *mass transfer is adiabatically stable* if

$$D_{ad} = \zeta_{ad} - \zeta_R > 0 \quad . \tag{1.9}$$

Note that in this context we use the term "adiabatic stability" instead of "dynamical stability" or "convective stability" which are sometimes found in the literature. In particular, we refrain from using "dynamical stability" in order to avoid confusion with the dynamical stability referred to in Sect.1.2 (see also Rasio, this volume).

If mass transfer in a binary is so slow that the donor remains essentially in thermal equilibrium, Eq.(1.5) can be replaced by

$$\frac{\dot{R}_2}{R_2} = \left(\frac{\partial \ln R_2}{\partial \ln M_2}\right)_e \frac{\dot{M}_2}{M_2} + \left(\frac{\partial \ln R_2}{\partial t}\right)_{nuc} = \zeta_e \frac{\dot{M}_2}{M_2} + \left(\frac{\partial \ln R_2}{\partial t}\right)_{nuc} \quad , \tag{1.10}$$

where ζ_e is the mass radius exponent of the secondary in thermal equilibrium. By comparing (1.5) and (1.10) we find that

$$\left(\frac{\partial \ln R_2}{\partial t}\right)_{th} = (\zeta_e - \zeta_{ad}) \frac{\dot{M}_2}{M_2} \quad . \tag{1.11}$$

This shows that, in general, the thermal relaxation term does not vanish because the adiabatic reaction to mass loss is different from the thermal one, i.e. $\zeta_e \neq \zeta_{ad}$.

Inserting (1.10) together with (1.6) into (1.3) yields the stationary mass transfer rate in thermal equilibrium

$$(-\dot{M}_2)_{st} = \frac{M_2}{\zeta_e - \zeta_R} \left[\left(\frac{\partial \ln R_2}{\partial t}\right)_{nuc} - 2\left(\frac{\partial \ln J}{\partial t}\right)_{\dot{M}_2=0}\right] \quad . \tag{1.12}$$

Perturbing mass transfer around the stationary value given by (1.12) yields *the criterion for thermal stability* against runaway mass transfer

$$D_{th} = \zeta_e - \zeta_R > 0 \quad . \tag{1.13}$$

So far we have implicitly assumed that in (1.5) $(\partial \ln R_2/\partial t)_{th}$ and $(\partial \ln R_2/\partial t)_{nuc}$, and $(\partial \ln J/\partial t)$ in (1.6) do not depend explicitly on the mass transfer rate. Though this is true for $(\partial \ln R_2/\partial t)_{nuc}$, it need not necessarily be the case for the other two terms. Allowing for such a dependence, we must replace Eqs.(1.5) and (1.6) by

$$\begin{aligned} \frac{\dot{R}_2}{R_2} &= \zeta_{ad} \frac{\dot{M}_2}{M_2} + \left(\frac{\partial \ln R_2}{\partial t}\right)_{nuc} \\ &+ \left(\frac{\partial \ln R_2}{\partial t}\right)_{th,\dot{M}_2=0} + M_2 \frac{\partial}{\partial \dot{M}_2} \left(\frac{\partial \ln R_2}{\partial t}\right)_{th} \frac{\dot{M}_2}{M_2} \end{aligned} \tag{1.14}$$

and

$$\frac{\dot{R}_{2,R}}{R_{2,R}} = \zeta_{R,J} \frac{\dot{M}_2}{M_2} + 2\left(\frac{\partial \ln J}{\partial t}\right)_{\dot{M}_2=0} + 2M_2 \frac{\partial}{\partial \dot{M}_2} \left(\frac{\partial \ln J}{\partial t}\right) \frac{\dot{M}_2}{M_2} \quad , \tag{1.15}$$

where now $\zeta_{R,J}$ is the mass radius exponent of the Roche radius computed for conservative mass transfer. Introducing the abbreviations

$$\zeta_{irr} = -M_2 \frac{\partial}{\partial \dot{M}_2} \left(\frac{\partial \ln R_2}{\partial t}\right)_{th} \tag{1.16}$$

and

$$\zeta_{CAML} = -2M_2 \frac{\partial}{\partial \dot{M}_2} \left(\frac{\partial \ln J}{\partial t} \right) \quad , \tag{1.17}$$

(1.14) and (1.16) inserted into (1.3) yield the following expression for the stationary mass transfer rate:

$$\begin{aligned}(-\dot{M}_2)_{st} &= \frac{M_2}{(\zeta_{ad} - \zeta_{irr}) - (\zeta_{R,J} - \zeta_{CAML})} \\ &\left[\left(\frac{\partial \ln R_2}{\partial t} \right)_{nuc} + \left(\frac{\partial \ln R_2}{\partial t} \right)_{th, \dot{M}_2 = 0} - 2 \left(\frac{\partial \ln J}{\partial t} \right)_{\dot{M}_2 = 0} \right] . \end{aligned} \tag{1.18}$$

By using in (1.16) the subscript "irr" in ζ_{irr} we indicate that the only conceivable way in which thermal relaxation of the secondary can depend explicitly on $\dot{M}_2$ is via irradiation of the secondary by accretion luminosity (see King et al. 1995; 1996; Ritter, Zhang and Kolb 1995a,b; Ritter, Zhang and Hameury 1996, and D'Antona, this volume). The subscript CAML in ζ_{CAML} stands for consequential angular momentum loss and expresses the fact that this term is non-zero only if there is angular momentum loss which arises as a consequence of mass transfer. Examples for such processes are angular momentum loss via a wind from the accretion disk or loss of angular momentum associated with nova explosions (see e.g. Livio, Govarie and Ritter 1991; Livio and Pringle 1994; King and Kolb 1995 for a discussion).

Stability analysis along the same line of arguments as we have used above yields the following generalized criterion for adiabatic stability:

$$D_{ad} = \zeta_{ad} - \zeta_{irr} - (\zeta_{R,J} - \zeta_{CAML}) > 0 \quad . \tag{1.19}$$

In the case that the secondary remains in thermal equilibrium, the generalized stability criterion is

$$D_{th} = \zeta_e - (\zeta_{R,J} - \zeta_{CAML}) > 0 \quad . \tag{1.20}$$

A further and very important conclusion we can draw from Eq.(1.3) is that phases of nonstationary mass transfer must be short. This is because of the large factor $R_2/H_P \approx 10^4$ by which the expression in the parenthesis on the right hand side of (1.3) is multiplied. From (1.3) one derives immediately that the time scale $\tau_{\dot{M}_2}$ on which $\dot{M}_2$ changes towards the stationary value and the momentary time scale of mass transfer $\tau_{M_2} = -M_2/\dot{M}_2$ are related via

$$\tau_{\dot{M}_2} = \frac{H_P}{R_2} \tau_{M_2} \left[(\zeta_{ad} - \zeta_R) \left(1 - \frac{(\dot{M}_2)_{st}}{\dot{M}_2} \right) \right]^{-1} \quad . \tag{1.21}$$

This shows that significant deviations from stationarity can persist only over short times, i.e. on the order of $H_P/R_2 \approx 10^{-4}$ mass transfer time scales. Because phases of nonstationary mass transfer are so short, we shall not discuss this topic any further here. The interested reader is instead referred to the extensive discussions in Ritter (1988) and D'Antona, Mazzitelli and Ritter (1989, 1990).

1.4 Driving and enhancing mass transfer

In systems which are adiabatically and thermally stable against mass transfer, i.e. for which $D_{ad} > 0$ and $D_{th} > 0$, mass transfer must be driven by some mechanism. The advantage of writing the mass transfer rate in the form (1.8), (1.12) or (1.18) is that these expressions tell us immediately which mechanisms can do that and which won't. As a general rule we can state that all mechanisms which give rise to terms in the numerator of the right hand side of these expressions can drive mass transfer, whereas those giving rise to coefficients in the denominator, i.e. ζ_{irr} or ζ_{CAML}, cannot do so. Rather the latter mechanisms can enhance mass transfer which is driven by some other mechanism in the first place by destabilizing mass transfer, i.e. reducing the value(s) of D_{ad} or D_{th}. Let us elaborate on these points a bit more.

From Eqs.(1.8), (1.12) or (1.18) we see that mass transfer can be driven by nuclear evolution via the expansion of the donor star, by loss of orbital angular momentum via the contraction of the orbit, and, in principle at least, by the secondary's expansion due to thermal relaxation. From Eq.(1.11) we see that this is possible only if $\zeta_{ad} > \zeta_e$, i.e. for predominantly radiative stars (see below, Sect.1.5). We should, however, keep in mind that the thermal relaxation term itself is only a consequence of mass loss and, therefore, even if positive, should not be counted among the driving terms. The driving mechanisms thus remaining are nuclear evolution and angular momentum loss. Both play an important role in the evolution of compact binaries. Nuclear evolution is the dominant driving mechanism in Algol binaries and in those LMXBs and (very few) CVs having a giant or a subgiant donor and an orbital period of typically $P \gtrsim 2^d$ (e.g. Taam 1983; Webbink, Rappaport and Savonije 1983). In systems containing a secondary on or near the main sequence nuclear evolution plays a role only if it is faster than or at least comparable to the time scale of angular momentum loss. This is usually the case if $M_2 \gtrsim 1M_\odot$ or $P \gtrsim 10^h - 15^h$ (e.g. Pylyser and Savonije 1988a,b). In systems with shorter orbital periods, i.e. in many LMXBs and most CVs, mass transfer must be driven by loss of angular momentum. Since the only well-understood angular momentum loss mechanism, i.e. gravitational radiation (see e.g. Kraft, Mathews and Greenstein 1962; Landau and Lifshitz 1975), cannot drive mass transfer rates much in excess of $\sim 10^{-10} M_\odot \ \mathrm{yr}^{-1}$ but, on the other hand, much higher rates are observed, at least one additional angular momentum loss mechanism must be involved. This is usually referred to as magnetic braking (for details see e.g. Verbunt and Zwaan 1981; Mestel and Spruit 1987; Li, Wu and Wickramasinghe 1994a,b; Li, Wickramasinghe and Wu 1995, and Spruit, this volume). Current estimates yield braking rates sufficient to drive mass transfer rates of up to $\sim 10^{-8} M_\odot \mathrm{yr}^{-1}$ which is adequate for LMXBs and CVs. It should, however, be kept in mind that there is as yet no theory of magnetic braking starting from first principles. The prescriptions used in current model calculations all contain at least one adjustable parameter, and one must admit that the necessary adjustments are made to fit observations as far as possible.

Let us now briefly come back to the mechanisms which enhance mass transfer by reducing either D_{ad} or D_{th}.

Consequential angular momentum loss (CAML) occurs to at least some degree in practically all systems via wind mass loss from the accretion disk (e.g. Livio and Pringle 1994), or from the near vicinity of the accretor (in CVs and LMXBs) and certainly in CVs as a consequence of nova explosions (e.g. Livio, Govarie and Ritter 1991; Schenker, Kolb and Ritter 1996). Parameterizing the loss of mass and angular momentum from a system by

$$\eta = -\frac{dM_1}{dM_2} \tag{1.22}$$

and

$$\nu = \frac{\partial \ln J}{\partial \ln(M_1 + M_2)} \quad , \tag{1.23}$$

where ν is the dimensionless specific angular momentum of the mass lost in units of the binary's specific angular momentum, (1.17), (1.22) and (1.23) yields

$$\zeta_{CAML} = -\frac{2\nu(1-\eta)}{1+q} \quad . \tag{1.24}$$

Since in an ordinary system $\zeta_{ad} - \zeta_R \approx 1$, we see from (1.24) that if a significant fraction of the transferred matter is lost (η not small) with $\nu \gtrsim 1$, CAML could have a very significant effect on the evolution of such binaries. In fact, in the context of CVs with nova explosions, CAML could even drive a system to unstable mass transfer, i.e. $D_{ad} < 0$ (see Kolb et al. 1996).

The fact that irradiating the donor star by accretion luminosity of the companion could have a dramatic influence on the long-term evolution of LMXBs and CVs was noted only rather recently by Podsiadlowski (1991) in the case of LMXBs and by Ritter, Zhang and Kolb (1995a,b) in the case of CVs. Among other things, irradiation can induce mass transfer cycles in LMXBs (see e.g. Frank, King and Lasota 1992; Hameury et al. 1993) and in CVs (King et al. 1995; 1996; Ritter, Zhang and Kolb 1995a,b; Ritter, Zhang and Hameury 1996). In the meantime this discovery has already generated a rich literature. For more details the reader is referred to the review by D'Antona (this volume). Depending on details of the stellar model and the irradiation model adopted, derivation of explicit expressions for ζ_{irr} can be rather involved. Therefore, we refrain from doing this here and refer the reader to the papers where this can be found (e.g. King et al. 1996; Ritter, Zhang and Kolb 1995a,b).

Let us now come back to Eqs.(1.8), (1.12) or (1.18) for the stationary mass transfer rate and ask whether these expressions are of any use in computing the mass transfer rate, say in numerical computations. Unfortunately, this is in general not the case. Although these equations are very instructive, they cannot normally be used for computing $\dot{M}_2$. The reason for this is that we are unable to calculate ζ_{ad}, ζ_e, $(\partial \ln R/\partial t)_{nuc}$ and, in particular, $(\partial \ln R/\partial t)_{th}$ explicitly for full stellar models. This can be done only if a particularly simple model for the structure of the donor star is adopted (which is what we shall do below in Sect.1.5). Without knowing these terms explicitly, Eqs.(1.8), (1.12) or (1.18) alone do not tell us how a system, i.e. $\dot{M}_2$, will evolve with time. For the purpose of numerical calculations one uses, therefore, one of the prescriptions discussed in Sect.1.2. However, in order to get some insight into how semi-detached binary evolution works it is worthwhile to adopt a simple model for the donor star and work out the necessary terms in Eq.(1.8) explicitly. This is what we shall do in the next two sections. If the chosen model is not too simple, this approach can even be used to perform numerical calculations of the long-term evolution of such binaries very efficiently and yet with acceptable accuracy. A case in point is the bipolytrope model (Rappaport, Verbunt and Joss 1983; Hameury et al. 1988; Kolb and Ritter 1992) which has been successfully used to model the evolution of CVs.

1.5 The reaction of low-mass stars to mass loss

For the following we restrict our discussion to low-mass stars, i.e. with a mass $0.1 M_\odot \lesssim M \lesssim 1 M_\odot$, on or near the main sequence.

1.5.1 The entropy profile in a star

As we shall see below, the reaction of a star to mass loss depends very much on the entropy profile in that star. Therefore, we begin with a brief discussion of the entropy distribution in low-mass stars. For that we assume that the star is in hydrostatic equilibrium, i.e. that

$$\frac{dP}{dM_r} = -\frac{G M_r}{4\pi r^4} \quad , \tag{1.25}$$

where P is the pressure, G the gravitational constant, r the radial coordinate and

$$M_r = \int_0^r 4\pi\, x^2\, \varrho(x)\, dx \tag{1.26}$$

the mass inside a sphere with radius r. $\varrho(r)$ is the density at the radius r. For simplicity we shall also assume that the equation of state is that of a fully ionized monoatomic ideal gas with a mean molecular weight μ and that radiation pressure can be neglected (an assumption which is well fulfilled in low-mass stars). In that case the entropy per unit mass is

$$S = \frac{\mathcal{R}}{\mu} \ln \frac{T^{1/\nabla_a}}{P} + \text{const.} \quad , \tag{1.27}$$

where $\mathcal{R}$ is the gas constant, T the temperature and

$$\nabla_a = \left(\frac{\partial \ln T}{\partial \ln P}\right)_s \tag{1.28}$$

is the adiabatic temperature gradient. Combining (1.25)–(1.27) we derive that

$$\frac{dS}{dM_r} = \frac{\mathcal{R}}{\mu} \frac{d\ln P}{dM_r} \left(\frac{\nabla}{\nabla_a} - 1\right) = \frac{\mathcal{R}}{\mu} \frac{G\, M_r}{4\pi\, r^4\, P} \left(1 - \frac{\nabla}{\nabla_a}\right) \quad , \tag{1.29}$$

where

$$\nabla = \frac{d\ln T}{d\ln P} \tag{1.30}$$

is the actual temperature gradient in the star. Eq.(1.29) allows us now to characterize the run of entropy through different stellar stratifications:

a) *the entropy profile in radiative layers:* Here $\nabla = \nabla_r < \nabla_a$, where ∇_r is the radiative temperature gradient, and therefore $dS/dM_r > 0$. In other words: entropy increases outwards in a radiative layer.

b) *the entropy profile in layers with effective convection:* Here $\nabla = \nabla_{ad} + \epsilon$, where in layers with effective convection $\epsilon \ll 1$, typically $\epsilon \approx 10^{-6} - 10^{-8}$. Thus for all practial purposes $\nabla = \nabla_{ad}$ and therefore $dS/dM_r = 0$, i.e. $S(M_r) = \text{const}$. This limit is known as adiabatic convection and is realized in convective layers sufficiently deep inside a star.

c) *the entropy profile in layers with ineffective convection:* Here $\nabla_a < \nabla < \nabla_r$ and $dS/dM_r < 0$, i.e. entropy decreases outwards. Inefficient convection is restricted to layers of low density and low optical depth and thus occurs always close to the stellar surface. Because $\nabla > \nabla_a$ this is also referred to as superadiabatic convection. In low-mass stars on or near the main sequence, the superadiabatic layer is very thin (in mass) and does not influence the reaction to mass loss significantly. The situation is quite different in very cool and distended stars such as giants and AGB stars where the superadiabatic layer can comprise a much larger fraction of the stellar mass.

1.5.2 The reaction to (rapid) mass loss: qualitative picture

In the following we shall examine the qualitative behaviour of a star upon mass loss. We assume that the star is initially in thermal equilibrium and has a mass M. We denote the associated entropy profile by $S_e(M_r; M)$, where the subscript e indicates that it refers to thermal equilibrium. Now we assume the star to suffer a sudden mass loss δM. Immediately after this mass loss, the entropy profile in the remaining star is $S(M_r; M - \delta M) = S_e(M_r \leq M - \delta M; M)$. In general, that entropy distribution is different from the one of a star with the mass $M - \delta M$ in thermal equilibrium $S_e(M_r; M - \delta M)$. Because even the qualitative reaction of a star to mass loss depends on details of the above entropy profiles we must treat stars with a predominantly radiative or convective envelope separately.

a) *stars with a radiative envelope:* The qualitative entropy profiles $S_e(M_r; M)$ and $S_e(M_r; M - \delta M)$ are sketched in Fig.1.1 as dashed lines, whereas the actual entropy profile immediately after mass loss, $S(M_r; M - \delta M)$ is shown as a full line. As can be seen, the entropy profiles $S_e(M_r; M - \delta M)$ and $S(M_r; M - \delta M)$ intersect at $M_r = M_x$, where in real cases M_x is deep inside the star. Furthermore, after mass loss the entropy in the outer layers of the star is lower than in thermal equilibrium, i.e. $S(M_r > M_x; M - \delta M) < S_e(M_r > M_x; M - \delta M)$. This means that the relevant parts of the star are

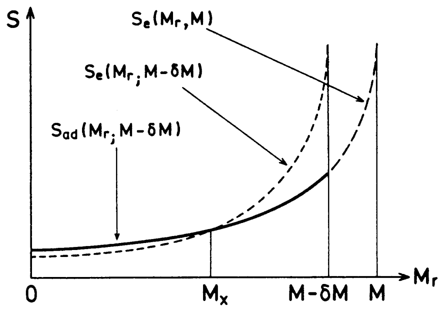

Figure 1.1: Sketch of the effect of (adiabatic) mass loss on the entropy distribution of main sequence stars with a radiative envelope. For details see Sect.1.5.2

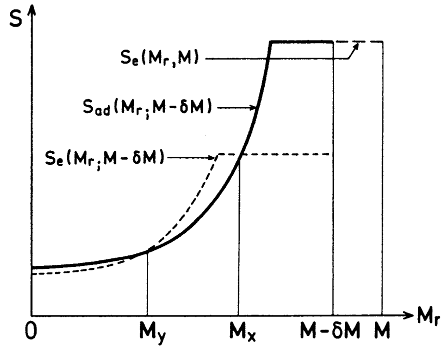

Figure 1.2: Sketch of the effect of (adiabatic) mass loss on the entropy distribution of main sequence stars with a convective envelope. For details see Sect.1.5.2

cooler than in thermal equilibrium with the result that in response to rapid mass loss the star contracts, at least with respect to the thermal equilibrium radius. Therefore

$$\zeta_{ad} = \left(\frac{\partial \ln R}{\partial \ln M}\right)_s > \zeta_e = \left(\frac{\partial \ln R}{\partial \ln M}\right)_e > 0 \quad . \tag{1.31}$$

Thus after a sudden mass loss, the star is too small. If allowed to relax thermally, it will therefore grow, i.e.

$$\left(\frac{\partial \ln R}{\partial t}\right)_{th} > 0 \quad . \tag{1.32}$$

b) *stars with a deep convective envelope:* The run of the entropy profiles for this case is sketched in Fig.1.2. As can be seen, the situation is slightly more complicated than above in that the profiles $S_e(M_r; M - \delta M)$ and $S(M_r; M - \delta M)$ intersect twice, at $M_r = M_x$ and $M_r = M_y$. In the layer which is most relevant for the star's response to mass loss, namely $M_r > M_x$, $S_e(M_r > M_x; M - \delta M) < S(M_r > M_x; M - \delta M)$. Thus these layers are too hot compared to the state in thermal equilibrium. As a consequence, the adiabatic reaction to mass loss from a star with a deep convective envelope is expansion, at least with respect to the thermal equilibrium radius, i.e.

$$\zeta_{ad} < \zeta_e \quad . \tag{1.33}$$

In fact, for stars which are either fully convective or have a sufficiently deep convective envelope $\zeta_{ad} < 0$. Thus after a sudden mass loss the star is too big. If allowed to relax thermally, it will therefore contract, i.e.

$$\left(\frac{\partial \ln R}{\partial t}\right)_{th} < 0 \quad . \tag{1.34}$$

1.5.3 The reaction of low-mass stars to mass loss: simple models

We shall start by computing the response of a simple polytrope upon mass loss. A polytropic stellar model is characterized by the polytropic relation

$$P = K\, \varrho^{1+\frac{1}{n}} \tag{1.35}$$

between the pressure P, the density ϱ, the polytropic constant K and the polytropic index n. In the following we shall consider a star with an adiabatic stratification, i.e. where P and T are related via the adiabate relation

$$P = K_a T^{1/\nabla_a} \quad . \tag{1.36}$$

Here K_a and ∇_a are respectively the adiabatic constant and adiabatic temperature gradient. The values of K_a and ∇_a depend on the equation of state. For a monoatomic ideal gas without radiation pressure we have, in particular, $\nabla_a = 2/5$, i.e. $n = 3/2$. The run of T as a function of P is sketched in Fig.1.3. By changing the stellar mass also the constants K and K_a change with time, whereas n and ∇_a remain the same. The change of the stellar structure with mass loss is also sketched in Fig.1.3. Changing K or K_a means that a particular mass element (at M_r = const.) moves non-adiabatically in the T–P–plane and that therefore gravo-thermal energy is liberated at a rate (per unit mass)

$$\epsilon_g = -c_p T \left(\frac{\dot{T}}{T} - \nabla_a \frac{\dot{P}}{P}\right) = c_p T\, \nabla_a \frac{\dot{K}_a}{K_a} \quad , \tag{1.37}$$

where c_p is the specific heat at constant pressure. From the mass radius relation of polytropes

$$R = \mathrm{const.} K^{\frac{n}{3-n}}\, M^{\frac{1-n}{3-n}} \tag{1.38}$$

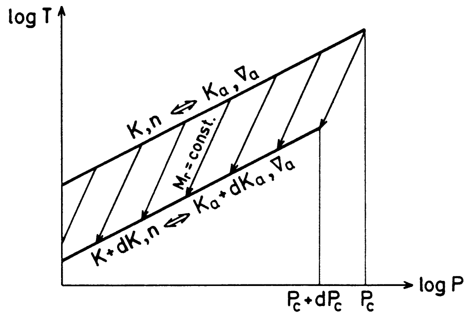

Figure 1.3: Sketch of the evolution in the $\log P$–$\log T$–plane of a fully convective star undergoing mass loss.

we obtain

$$\frac{d\ln R}{d\ln M} = \frac{1-n}{3-n} + \frac{n}{3-n}\,\frac{d\ln K}{dt}\left(\frac{d\ln M}{dt}\right)^{-1} \quad . \tag{1.39}$$

Using the equation of state of an ideal monoatomic gas, (1.35) and (1.36), yields

$$\frac{d\ln K}{dt} = -\frac{1}{n}\,\frac{d\ln K_a}{dt} = -\frac{1}{n}\,\frac{\epsilon_g}{c_p T\nabla_a} = -\frac{L_g}{E_{th}} = 2\,\frac{L_g}{E_g} = -\frac{5-n}{n}\,\frac{RL_g}{GM^2} \quad , \tag{1.40}$$

where

$$L_g = \int_0^M \epsilon_g\, dM_r \tag{1.41}$$

is the total gravitational luminosity,

$$E_{th} = \int_0^M c_v\, T\, dM_r \tag{1.42}$$

the thermal energy of the star, and

$$E_g = -\int_0^M \frac{GM_r}{r}\, dM_r = -2E_{th} \tag{1.43}$$

the gravitational binding energy which is related to E_{th} via the virial theorem. Defining now the mass loss time scale

$$\tau_M = -\frac{dt}{d\ln M} \tag{1.44}$$

and the Kelvin-Helmholtz time scale

$$\tau_{KH} = \frac{GM^2}{RL} \quad , \tag{1.45}$$

where L is the total luminosity of the star, Eqs.(1.40), (1.44) and (1.45) together yield the effective mass radius exponent of the star under mass loss

$$\zeta_{eff} = \frac{d\ln R}{d\ln M} = \zeta_{ad} + \frac{5-n}{3-n}\,\frac{\tau_M}{\tau_{KH}}\,\frac{L_g}{L} \quad , \tag{1.46}$$

where

$$\zeta_{ad} = \left(\frac{\partial \ln R}{\partial \ln M}\right)_K = \frac{1-n}{3-n} \tag{1.47}$$

is the adiabatic mass radius exponent defined earlier. We note that if $\nabla_a = 2/5$, i.e. $n = 3/2$, then $\zeta_{ad} = -1/3$.

A result very similar to (1.46) is also obtained if somewhat more elaborate stellar models than a simple polytrope are used. Writing instead of (1.46)

$$\frac{d\ln R}{d\ln M} = \zeta_{ad} + A\,\frac{\tau_M}{\tau_{KH}}\,\frac{L_g}{L} \quad , \tag{1.48}$$

D'Antona, Mazzitelli and Ritter (1989), elaborating on earlier work by Lauterborn and Weigert (1972) (who adopted a simple polytrope $n = 3/2$ with a radiative photospheric boundary condition), find $A = 7(a+1)/4$, where $a = (\partial \ln \kappa/\partial \ln P)_T$ with κ being the photospheric opacity parameterized in the form $\log \kappa = a \log P + b \log T +$ const. In the bipolytrope model (Rappaport, Verbunt and Joss 1983; Hameury et al. 1988; Kolb and Ritter 1992) both, ζ_{ad} and A, can be written as explicit functions of the model parameters, i.e. of the polytropic index of the radiative core n_1 and the relative radius of the convective core $Q = R_{core}/R$.

Eqs.(1.46) or (1.48) give us already important information about the properties of mass losing stars: If mass loss starts from a star in thermal equilibrium, then initially $L_g = 0$ and thus $\zeta_{eff} = \zeta_{ad}$, i.e. the inital response is always adiabatic no matter how slow mass loss is. Furthermore, the deviation from adiabatic evolution is the stronger the more time the star has to react thermally, i.e. the larger the ratio τ_M/τ_{KH}.

However, if we wish to get information about evolution under mass loss going beyond the qualitative statements above, we need to work out L_g. This will be our next topic and will provide us with some insight into how evolution with mass loss works.

1.6 Convergent evolution under mass loss

Starting point of our discussion, in which we shall follow arguments outlined in detail by Stehle, Ritter and Kolb (1996), is Eq.(1.48). Our aim is to transform this equation into a solvable first-order differential equation for the stellar radius $R(M)$. For this it is convenient to introduce the dimensionless variables

$$\begin{aligned} r &= \frac{R}{R_e} \quad , \\ \ell &= \frac{L}{L_e} \quad , \end{aligned} \tag{1.49}$$

and
$$\theta = \frac{T_{eff}}{T_{eff,e}} \quad ,$$

where the subscript e denotes quantities of the star in thermal equilibrium. With the definition of r, the left-hand side of (1.48) becomes

$$\frac{d\ln R}{d\ln M} = \frac{d\ln r}{d\ln M} + \zeta_e \quad , \tag{1.50}$$

where ζ_e is the thermal equilibrium mass radius exponent defined in Eq.(1.10). Next we wish to express τ_{KH} and L_g/L in (1.48) as functions of r. For this we assume that the changes of the stellar structure due to mass loss are homologous. Using homology relations (see e.g. Kippenhahn and Weigert 1990) we find that for stars with a (deep) convective envelope, i.e. near the Hayashi line,

$$\theta = r^{\mu} \quad , \tag{1.51}$$

where $0 \leq \mu \ll 1$. With (1.45), (1.49) and (1.51) we can write

$$\tau_{KH} = \tau_{KH,e} r^{-3}\theta^{-4} = \tau_{KH,e} r^{-(3+4\mu)} \quad . \tag{1.52}$$

Using again homology relations, the nuclear luminosity of the star can be written as

$$L_{nuc} = L_e r^{-(\nu+3)} \quad , \tag{1.53}$$

where $\nu = (\partial \ln \epsilon_{nuc}/\partial \ln T)_\varrho$ is the temperature exponent of the nuclear energy generation rate $\epsilon_{nuc} \sim \varrho T^{\nu}$. For low-mass stars with central hydrogen burning via the pp-chain $\nu \approx 5$. Because

$$L = L_e r^2 \theta^4 = L_e r^{2+4\mu} \tag{1.54}$$

we have

$$\frac{L_g}{L} = 1 - \frac{L_{nuc}}{L} = 1 - r^{-(\nu+5+4\mu)} \quad . \tag{1.55}$$

Inserting now (1.50), (1.52) and (1.55) into (1.48) we obtain

$$\frac{d\ln r}{d\ln M} = A\,\frac{\tau_M}{\tau_{KH,e}}\,\left\{ r^{3+4\mu} - r^{-(\nu+2)} - \frac{1}{A}\,\frac{\tau_{KH,e}}{\tau_M}\,(\zeta_e - \zeta_{ad}) \right\} \quad . \tag{1.56}$$

(1.56) is the equation we aimed at and which we shall now discuss. First, however, it is useful to recall what the three terms in the curly brackets on the right-hand side of (1.56) stand for. Apart from a factor r, the first term stands for the surface luminosity, the second one for the nuclear luminosity and the third one for the thermal disequilibrium which mass loss causes if $\zeta_e \neq \zeta_{ad}$.

The main properties of (1.56) can be summarized as follows:

a) In the limit of very rapid mass loss, i.e. $\tau_M/\tau_{KH,e} \to 0$, (1.56) yields, as expected, $d\ln R/d\ln M = \zeta_{ad}$, i.e. the star reacts adiabatically.

b) In the limit of very slow mass loss, i.e. $\tau_M/\tau_{KH,e} \to \infty$, (1.56) yields, as expected, $d\ln R/d\ln M = \zeta_e$, i.e. the star evolves in thermal equilibrium.

c) Let us define r_0 via

$$\left.\frac{d\ln r}{d\ln M}\right|_{r=r_0} = 0 \Leftrightarrow r_0^{3+4\mu} - r_0^{-(\nu+2)} = \frac{1}{A}\,\frac{\tau_{KH,e}}{\tau_M}\,(\zeta_e - \zeta_{ad}) \quad . \tag{1.57}$$

Now let us ask for the sign of $d\ln r/d\ln M$ if $r = r_0 + \delta r$. Linearizing (1.56) yields

$$\left.\frac{d\ln r}{d\ln M}\right|_{r_0+\delta r} = A\,\frac{\tau_M}{\tau_{KH,e}}\,\left\{(3+4\mu)\,r^{4+4\mu} + (\nu+2)\,r^{-(\nu+1)}\right\}\,\delta r \sim \delta r \quad . \tag{1.58}$$

What does (1.58) tell us? If $r > r_0$, r will decrease with mass loss; if $r < r_0$, it will increase. Thus, the evolution of r has the tendency to converge to a common attracting solution $r = r_A$. We note that in general $r_A \neq r_0$. This is because r_A must be a solution of (1.56), whereas r_0 is not a solution of (1.56) but of (1.57).

From a detailed investigation of (1.56) one can derive the following *global properties of the evolution of low-mass stars with mass loss:*

1.) $r_0 \approx r_A$ if $(\zeta_e - \zeta_{ad})\tau_{KH,e}/\tau_M$ does not change rapidly with mass. This is the case for fully convective stars and stars with deep convective envelopes for which both ζ_{ad} and ζ_e do not vary much with M.

2.) Evolutions with a fixed value for $\tau_M/\tau_{KH,e}$ which start from thermal equilibrium, i.e. $r = 1$, converge quickly to $r_A \neq 1$ on a time scale $\tau \lesssim 0.03\tau_{KH,e}$. We note that this remains valid also if $\tau_M/\tau_{KH,e}$ changes with time if it does so on a time scale which is longer than the time scale on which r tends to converge to r_A.

3.) Numerical integrations of (1.56) for fixed values of $\tau_M/\tau_{KH,e}$, $\zeta_e(M)$ and $\zeta_{ad}(M)$ taken from full stellar models (e.g. Hjellming 1988) show that $r_A > 1$ if $M \lesssim 0.6M_\odot$, i.e. if the convective envelope is very deep and $\zeta_{ad} < 0$, and $r_A < 1$ if $M \gtrsim 0.6M_\odot$, where the convective envelope is more shallow and $\zeta_{ad} > 0$. (Note that $r_0 = 1$ where $\zeta_e = \zeta_{ad}$, which is the case around $M = 0.8M_\odot$ and that $r_0 < 1$ if $\zeta_e < \zeta_{ad}$ and $r_0 > 1$ if $\zeta_e > \zeta_{ad}$.)

4.) r_0 and r_A are monotonically increasing functions of $\frac{1}{A}\,\frac{\tau_{KH,e}}{\tau_M}\,(\zeta_e - \zeta_{ad})$. Thus, under mass loss, the stellar radius deviates the more from $r = 1$, i.e. $R = R_e$, the larger the ratio $\tau_{KH,e}/\tau_M$, i.e. the faster the mass loss.

5.) The above results remain qualitatively correct even for stars which are not nearly fully convective, i.e. with masses typically in the range $0.6M_\odot \lesssim M \lesssim 1M_\odot$, i.e. for which $\zeta_{ad} > 0$ or even $\zeta_{ad} > \zeta_e$.

What does all this mean for the long-term evolution of semi-detached binaries? It means no less than that evolutionary tracks starting from different initial donor masses, and everything else being the same, converge to a common evolutionary

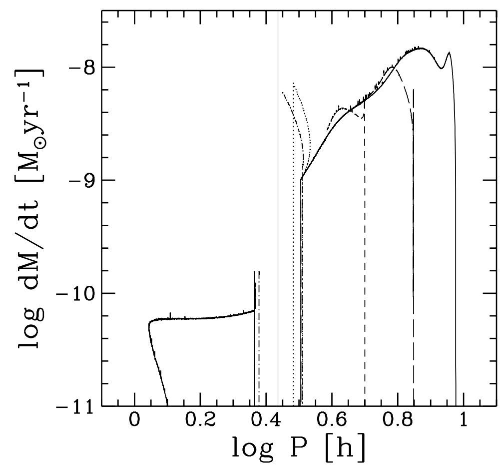

Figure 1.4: Evolutionary tracks in the log P–log $\dot{M}$–plane of CVs evolving according to the standard model computed by Kolb (1993). The calculations assume magnetic braking according to Verbunt and Zwaan (1981) and a constant primary mass of $M_1 = 1M_\odot$. The sequences differ in the initial secondary mass: $M_{2,i} = 1.2M_\odot$ (full line), $0.9M_\odot$ (long dashes), $0.6M_\odot$ (short dashes), $0.4M_\odot$ (dotted), and $0.36M_\odot$ (dash-dotted). The vertical line at $P \approx 2.76^h$ marks the critical period above which the secondaries initially have a radiative core.

track on a time scale which is short compared to the typical evolutionary time scale. And that, in turn, means that our simple model provides an explanation for a phenomenon first found in numerical calculations by Paczyński and Sienkiewicz (1981, 1983) and in numerous other calculations thereafter. As an illustration of this phenomenon we show in Fig.1.4 results from calculations by Kolb (1993).

Let us return briefly to Eq.(1.56) and examine why its solutions tend to converge. The reason for this is that the thermal imbalance caused by mass loss, i.e. a non-vanishing gravo-thermal luminosity L_g, can always be balanced by the star by changing its radius to a value different from R_e. That this is possible is, in turn, due to the fact that the surface luminosity $L \sim r^{2+4\mu}$ increases with r, whereas the nuclear luminosity $L_{nuc} \sim r^{-(\nu+3)}$ decreases strongly with increasing r. So it is the opposite sign of the power with which L and L_{nuc} scale with r which is responsible for this property.

1.7 Applications

In this section we shall review briefly how the concepts presented in the foregoing sections apply to the structure and evolution of CVs and LMXBs.

1.7.1 Stability of mass transfer

As far as we can tell from observations and from theoretical arguments (e.g. observed mass ratios, mass transfer rates, time scale arguments, outburst behaviour of CVs), LMXBs and all ordinary CVs are thermally and adiabatically stable against mass transfer, i.e. they fulfil Eqs.(1.9) and (1.13) (see e.g. Ritter 1986 for a more detailed discussion of CVs).

The fact that ordinary CVs are thermally stable together with the fact that the primary's mass must not exceed the Chandrasekhar mass $M_{CH} \approx 1.4 M_\odot$ yields an upper limit for the secondary's mass $M_2 \lesssim 1.75 M_\odot$ and, under the assumption that the secondary has not evolved beyond central hydrogen burning, a natural explanation for the observed long-period cutoff of the period distribution at $P \approx 16^h$ (see e.g. Ritter and Kolb 1995).

With the advent of ROSAT a new class of stellar X-ray sources, the supersoft sources (hereafter SSSs) has been discovered (see e.g. Hasinger 1994 for a review). Immediately after the discovery of the first SSSs, van den Heuvel et al. (1992) have proposed, based on the observed properties of the SSSs, that these are hot white dwarfs in a phase of stable or at least stationary (but transient) nuclear burning of hydrogen which they accrete from a $\sim 1.5 - 2.5 M_\odot$ secondary in a thermally unstable semi-detached binary system. Although it is now known that not all of the SSSs conform to this model, a number of them have in the meantime been identified with short-period binary systems (e.g. Hasinger 1994), consistent with the model of van den Heuvel et al. (1992). This particular subgroup of SSSs thus forms a new subclass of CVs. The distinguishing property of this subclass is that, contrary to ordinary CVs, mass transfer is thermally unstable.

1.7.2 Standard evolution of CVs

Standard evolution of CVs is described in the framework of the model of discontinuous magnetic braking (Rappaport, Verbunt and Joss 1983; Spruit and Ritter 1983). In this model, which is designed to account for the period gap among the observed CVs, i.e. for the deficiency of systems in the period range $2^h \lesssim P \lesssim 3^h$, it is postulated that mass transfer is driven by magnetic braking and gravitational radiation as long as the secondary retains a radiative core, and by gravitational radiation alone if the secondary is fully convective. We shall not go into details here but refer the reader to the reviews by King (1988) and Ritter (1991). Rather, we wish to point out that this model's success in explaining the period gap is a consequence of the fact that evolutionary tracks tend to converge (as shown in Sect.1.6), and that the angular momentum loss through magnetic braking does not depend sensitively on the mass of the white dwarf. These properties guarantee that the majority of the CVs born with an orbital period $P > 3^h$ enter the detached phase with secondaries which have all essentially the same mass and the same radius, i.e. the same radius excess (see Fig. 4). This, in turn, provides the coherence which is necessary to allow the detached phase of a single CV evolution to appear as the period gap of the CV population.

1.7.3 Population synthesis

Because population synthesis is discussed in detail in the review by de Kool (this volume), we shall restrict ourselves to a brief discussion of two studies which are of relevance here because they both deal with the synthesis of a population of semi-detached binaries. The first one is the synthesis of the population of CVs by Kolb (1993) based on evolutionary calculations by Kolb and Ritter (1992) and birth rate models by Politano (1988) and de Kool (1992). This study shows indeed that the detached evolutionary phase in an individual CV evolution manifests itself as a period gap in the synthetic, intrinsic and "observable" CV population.

The second study, by Rappaport, Di Stefano and Smith (1994), aims at modelling the population of the binary supersoft X-ray sources mentioned in Sect.1.7.1, i.e. of the thermally unstable CVs. In contrast to ordinary CVs, however, the main luminosity in these sources does not come from accretion but rather from nuclear burning on the white dwarf. Yet, also in this case, accurate calculation of the mass transfer rate is crucial because steady nuclear burning on an accreting white dwarf is possible only in a rather narrow range of accretion rates (see e.g. Fujimoto 1982).

1.7.4 Non-standard evolution

1.7.4.1 The evolution of AM Her stars

Observations with ROSAT have so far led to the discovery of about 35 new AM Her systems, thereby bringing the total number of currently known AM Her stars to about 52 (Beuermann 1995; Beuermann and Burwitz 1995). Whereas the period distribution of the 17 AM Her systems known before the ROSAT era (see e.g. Ritter 1990) was entirely consistent with the hypothesis that the AM Her stars evolve according to the standard evolutionary model of CVs, the distribution of the orbital periods of the AM Her systems of the post ROSAT era has led to a controversy about whether that is really true. The reason for this controversy is that relatively many of the AM Her stars discovered with ROSAT have an orbital period in the $2^h - 3^h$ range, i.e. they fall into the period gap. Based on these observations, Wu and Wickramasinghe (1993), Wickramasinghe and Wu (1994a), and Wu, Wickramasinghe and Li (1995) have concluded that the period distribution of AM Her stars is significantly different from the one of non-magnetic CVs and that, therefore, AM Her stars evolve differently from non-magnetic CVs. On the other hand, Kolb and de Kool (1993), and Wheatley (1995) conclude that, as far as the period gap is concerned, the difference between the two orbital period distributions is not statistically significant and that, therefore, no modification of the evolutionary model is necessary for AM Her stars. Unconvinced by these arguments, Wickramasinghe and Wu (1994b) have put forward an evolutoinary model, variously referred to as the non-magentic braking (NMB) or the reduced magnetic braking (RMB) model. This model postulates that in magnetic CVs, when the white dwarf's spin synchronizes with the orbital motion, magnetic braking either becomes significantly less efficient (RMB) or even vanishes totally (NMB) because the presence of the white dwarf's strong magnetic field reduces the number of open field lines along which the secondary's wind plasma can escape and carry away angular momentum (Li, Wickramasinghe and Wu 1994; Li, Wu and Wickramasinghe 1995; see also the review by Spruit, this volume).

It is clear from our findings in Sect.1.6 that in the RMB model the width of the period gap shrinks in proportion to the reduction of the angular momentum loss rate and that in the NMB limit a gap will not form at all. In this way the RMB or NMB models account for the "overpopulation" of AM Her stars in the $2^h - 3^h$ period range.

1.7.4.2 Classical novae

In CVs the hydrogen-rich gas which accumulates on the white dwarf must ignite repeatedly. Ignition can either be explosive, giving rise to novae, or non-explosive in the form of non-degenerate hydrogen shell flashes (e.g. Fujimoto 1982; MacDonald 1984). In either case, during such an outburst the binary is engulfed in a common envelope and suffers frictional angular momentum loss (FAML) (e.g. MacDonald 1986; Livio, Govarie and Ritter 1991). Because no detailed model for FAML is at hand as yet, Schenker, Kolb and Ritter (1996) used the parameterized prescription of Livio, Govarie and Ritter (1991) to study the influence of FAML on the long-term evolution of CVs. This study shows that the evolution of CVs remains qualitatively unchanged by FAML if the expansion velocity of the ejected nova envelope is not too small. However, in cases where the expansion velocities are sufficiently small (e.g. in slow novae or non-degenerate hydrogen shell flashes), FAML could become important enough to render a system adiabatically unstable (cf. Eq.(1.10)) (Kolb et al. 1996).

1.7.4.3 The evolution of LMXBs

The absence of a noticeable period gap in the period distribution of LMXBs, (see e.g. van Paradijs 1995) and the lack of short-period systems ($P \lesssim 2^h$) has led to the speculation that the evolution of LMXBs is in some significant way different from that of CVs. Currently the most promising idea as to why LMXBs could evolve differently from CVs is that strong irradiation of the low-mass secondary by the accretion luminosity (mostly in the form of hard X-rays) changes the secondary's structure fundamentally. Starting with the investigation by Podsiadlowski (1991), irradiation-induced mass transfer in LMXBs has become a hot topic. We shall not discuss this any further here but refer to the review by D'Antona (this volume).

1.7.4.4 Irradiation-induced mass transfer in CVs

Recently it was realized that irradiation-induced mass transfer could also be of importance for CVs and even lead to mass transfer cycles in which phases with high, irradiation-induced mass transfer alternate with phases with little or no mass transfer (e.g. Ritter, Zhang and Kolb 1995a,b; King et al. 1995). However, it can be shown that unless FAML plays an important role in addition to irradiation (cf. Eq.(1.20)), such mass transfer cycles do not occur if the secondary's mass is outside the range $0.7M_\odot \lesssim M_2 \lesssim 1M_\odot$ (Ritter, Zhang and Hameury 1996; King et al. 1996).

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