MASS TRANSFER CYCLES IN CATACLYSMIC VARIABLES

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ABSTRACT

It is well known that in cataclysmic variables (CVs) the mass transfer rate must fluctuate about the evolutionary mean on timescales too long to be directly observable. We show that limit–cycle behaviour can occur if the radius change of the secondary star is sensitive to the instantaneous mass transfer rate. The only reasonable way in which such a dependence can arise is through irradiation of this star by the accreting component. The system oscillates between high states in which irradiation causes slow expansion of the secondary and drives an elevated transfer rate, and low states in which this star contracts.

Subject headings: accretion — instabilities — stars: binaries: close — stars: cataclysmic variables — X–rays: stars

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1. INTRODUCTION

Estimates of the accretion rates or absolute magnitudes in cataclysmic variables (CVs) (e.g. Patterson, 1984; Warner, 1987) typically show a large degree of scatter when plotted against the binary period P. Although there are large systematic errors in such estimates, the existence of an intrinsic scatter in any accretion–rate – period relation is strongly supported by the coexistence of various CV subtypes at similar orbital periods, as the sharp observed differences between these subtypes are generally ascribed to differences in the accretion rates. For example, dwarf novae probably have rather low accretion rates, while steady (novalike) systems have high rates, although both are found above the period gap. However, if the inferred scatter in the accretion rate were characteristic of the long–term mass transfer rate, features of the CV period histogram such as the period gap and minimum period probably could not be anything like as sharply defined as they are observed to be (e.g. Hameury et al, 1989). This is because the conventional view of these features ascribes them to the effects of thermal disequilibrium resulting from mass loss (e.g. Paczyński & Sienkiewicz, 1981; Rappaport, Verbunt & Joss, 1983; Spruit & Ritter, 1983). Real differences in the long–term average mass loss rates at a given period would cause the secondary stars to have widely varying degrees of disequilibrium, leading to very different minimum periods and period gaps for systems with high and low transfer rates, contrary to observation.

The resolution of this apparent disagreement is well known (e.g. Hameury et al, 1989): the evolutionary calculations predict only the *secular mean* transfer rate, i.e. the average over timescales $t_{\rm ev}$ long compared with the timescale $t_{\rm L}$ on which the Roche lobe moves through a stellar scaleheight H. The latter timescale is typically $> 10^4 - 10^5$ y. The "instantaneous" transfer rate can fluctuate strongly about the secular mean on timescales $< t_{\rm ev}$, which are nevertheless far too long to be directly observable. Consistency between observations of CVs and the theory of their secular evolution thus demands the existence of such fluctuations, some of the various CV subtypes evidently inhabiting specific parts of these fluctuation cycles. For example, dwarf novae above the period gap are probably CVs in states of low mass transfer.

Although the existence of these cycles is tacitly assumed, there has until now been no convincing suggestion of how the required limit–cycle behaviour might be caused. The aim of this Letter is to show how these fluctuations may arise by giving an explicit example of such a limit cycle. The properties leading to cyclic behaviour strongly suggest that the basic cause is radius variation of the secondary star resulting from irradiation by the accreting component. In later papers we will prove that these properties are actually required for a limit cycle, and give an explicit model for the effect of irradiation, which indeed has these properties.

The effects of strong irradiation on the evolution of low–mass X–ray binaries (LMXBs) have been considered recently under various simplifying assumptions (Podsiadlowski 1991; Harpaz & Rappaport 1991, 1994; Frank et al 1992; Hameury et al 1993; Ritter 1994). In this paper we consider the effects of anisotropic irradiation in CVs (cf Ritter, Zhang & Kolb 1995). It is clear (e.g. King, 1989, and references therein) that the irradiation and its effects must be much weaker here than in LMXBs.

It is important at the outset to distinguish between these fluctuations and *observed* variations in CVs such as the VY Scl subclass of novalike variables, which show low states at intervals of months or years. These much more rapid variations must arise from very short–term effects (such as the passage of starspots on the secondary star under the Lagrange point), and are unrelated to the much slower fluctuations we discuss here.

2. MASS TRANSFER VARIATIONS

We study the effect of variations in the radius R_2 of the lobe–filling star on the transfer rate. We represent the radius variation as

$$\frac{\dot R_2}{R_2} = K(R_2, \dot M_2) + \zeta_s \frac{\dot M_2}{M_2}. \qquad (1)$$

Here ζ_s is the adiabatic mass–radius exponent ($\simeq -1/3$ for a fully convective star) and $K(R_2, \dot M_2)$ represents all radius variations other than the adiabatic reaction to mass–loss (e.g. thermal relaxation, nuclear evolution, or irradiation of the star by the primary). We expect short–term hydrodynamic oscillations (timescales $\lesssim$ orbital period) to be superimposed on the long-term behaviour described by equation (1) (see e.g. King, 1989). However as remarked in the Introduction, we are interested only in longer timescales here, so we neglect these effects. Combining (1) with standard expressions for the motion of the Roche lobe and assuming $M_1 + M_2 =$ constant (e.g. King & Kolb, 1995) gives

$$\ddot M_2 = \frac{2\dot M_2}{\epsilon}\left(\frac{1}{t_J} + \frac{D_s \dot M_2}{M_2} + \frac{K}{2}\right) \qquad (2)$$

with $t_J = J/(-\dot J)$ the angular momentum loss timescale, $\epsilon = H/R_2$, and

$$D_s = \frac{5}{6} + \frac{\zeta_s}{2} - \frac{M_2}{M_1}. \qquad (3)$$

Typically $t_J \sim 10^8 - 10^9$ y and $\epsilon \simeq 10^{-4}$ (Ritter 1988). Thus we expect deviations from quasi–steady mass transfer ($\ddot M_2 \simeq 0$) to be corrected on rather short timescales $t_{\rm L} = \epsilon t_J \sim 10^4 - 10^5$ y. Given the form of K, the behaviour of the system is simple to describe: because of the sharp difference in timescales between the two equations (1, 2), the system will stay very close to the curve on which $\ddot M_2 = 0$, i.e.

$$-\dot M_2 = \left[1 + \frac{K t_J}{2}\right] \frac{M_2}{D_s t_J}, \qquad (4)$$

wherever this is an attractor.

The time t does not appear explicitly in the two first–order equations (1, 2) determining R_2 and $\dot M_2$. As is well known, such plane autonomous systems can be studied qualitatively by a phase–plane analysis. We shall give such an analysis in a subsequent paper and show that *limit–cycle behaviour can only occur if K depends on $-\dot M_2$*. The only plausible reason for such a dependence is irradiation by the companion, and we consider an analytic model of this effect in the next Section.

3. IRRADIATION

In the course of their secular evolution, the secondary stars in CVs gradually depart from thermal equilibrium. For $M_2 \lesssim 0.6 {\rm M}_\odot$ the effect of mass loss is to make them oversized and overluminous for their masses compared with the main sequence. The surface luminosity L exceeds the luminosity $L_{\rm nuc}$ generated by nuclear reactions in the core. In the absence of irradiation the main contribution to the radius variation function K in (1) is thermal relaxation, which takes place on the Kelvin–Helmholtz timescale of the luminosity deficit, i.e.

$$K(R_2) = -\frac{R_2 f}{GM_2^2}(L - L_{\rm nuc}). \qquad (5)$$

Here f is a dimensionless number depending on the detailed structure of the star: for example an $n = 3/2$ polytrope would have $f = 7/3$. The effect of irradiation on a star with a deep convective envelope, as in CV secondaries, is to block a portion $L_{\rm block}$ of the luminosity, reducing the effective surface luminosity and causing the star to swell, i.e.

$$K(R_2, \dot{M}_2) = -\frac{R_2 f}{GM_2^2}(L - L_{\rm block} - L_{\rm nuc}). \quad (6)$$

The behaviour of the mass transfer rate and stellar radius is now specified by the $R_2, -\dot{M}_2$ dependences of $L, L_{\rm block}, L_{\rm nuc}$. The nuclear luminosity is clearly independent of $\dot{M}_2$, and by homology we can write $L_{\rm nuc} = L_{\rm nuc,0}(R_2/R_0)^{-(\nu+3)}$, where $\nu \simeq 5$ for low–mass stars such as the secondaries in CVs and R_0 is the corresponding thermal equilibrium radius. The form of the surface and blocking terms is a much more complex problem. Only one face of the secondary is irradiated; the star adjusts its surface conditions to the incoming flux in the very thin superadiabatic layer above the convection zone where energy transport is radiative because convective transport is inefficient. It thus maintains different surface temperatures on the illuminated and unilluminated faces, and the blocking is much less than if the same impinging luminosity is assumed spread spherically around the star (Ritter, 1994). There is nevertheless a small but significant blocking effect. We model this crudely here in order to gain a qualitative understanding of the resulting limit cycle, and defer a full discussion to a later paper (Ritter, Zhang & Kolb, in prep.; see also Ritter, Zhang & Kolb 1995). Moreover, our analysis is local in the sense that we formally consider systems with a fixed secondary mass and neglect any effects the secular evolution could have on the resulting limit cycle.

We write $L = L_0(R_2/R_0)^2$, and absorb the effect of the higher surface temperature on the irradiated face into $L_{\rm block}$. This term must depend on the effective irradiating flux F (corrected for albedo, etc), which is itself proportional to $-\dot{M}_2$. We can deduce a qualitatively correct form for this dependence by a physical argument. For vanishing irradiating flux the blocking luminosity must vanish, so $L_{\rm block} \to 0$ as $F \to 0$. Once the effective irradiating flux exceeds a value of order the stellar flux F_* no more luminosity can be blocked; thus we require $L_{\rm block} \to sL$ for $F >> F_*$, where $s \simeq 1/2$ is the irradiated fraction of the secondary. This regime is stable, in that on the curve (4) the transfer rate decreases as the radius increases. If the form of K were such that this was true for all F, we would have to conclude that the radius of a highly irradiated star was always smaller than an essentially unirradiated star, which is implausible for reasonable radiative efficiencies. We conclude that an unstable regime must exist, in which R_2 and $-\dot{M}_2$ decrease together. We expect further that the curve (4) should imply a unique stellar radius for a given irradiating flux (i.e. given transfer rate). This forces the unstable regime to lie below the stable high transfer–rate regime: the curve (4) must have the shape of of a right–hand bracket “)”, rather than a letter “C”. A simple form satisfying these criteria is

$$L_{\rm block} = sL\tanh\Big(k\frac{F}{F_*}\Big),\ k > 0. \qquad (7)$$

The full treatment by Ritter et al (in prep) indeed arrives at expressions which can be approximated by (7) with k varying between 0.3 and 0.9. Inserting (7) into (6) and defining $r = R/R_0$ we find

$$K(R_2, \dot{M}_2) = \qquad (8)$$

$$-\frac{R_0 L_0 f}{G M_2^2}\Big[\Big(1 - s\tanh(k\frac{F}{F_*})\Big)r^3 - \frac{1}{r^{\nu+2}}\Big].$$

The ratio F/F_* is specified by the transfer rate and irradiation efficiency $\eta < 1$ as

$$\frac{F}{F_*} = \qquad (9)$$

$$0.25\frac{m_1\eta_{0.1}}{R_9 l_2}\Big(\frac{m_2}{m_1+m_2}\Big)^{2/3}\Big(\frac{-\dot{M}_2}{10^{-8}{\rm M}_\odot\ {\rm y}^{-1}}\Big),$$

where m_1 and l_2 are M_1 and L_0 in solar units, and $\eta_{0.1}$ and R_9 are η and the white dwarf radius in units of 0.1 and 10^9 cm respectively.

This simple form of K embodies the essential qualitative features of CV evolution with irradiated secondaries.

4. LIMIT–CYCLE BEHAVIOUR

The curve $\ddot{M}_2 = 0$, controlling a possible limit cycle, is given by (4). In Fig. 1 we plot this curve in the r-$(-\dot{M}_2)$-plane for typical system parameters and the form (8) for K. The branch at high transfer rate is an attractor, as expected, while the branch with $\partial(-\dot{M}_2)/\partial R_2 > 0$ is a repellor. This leads to limit–cycle behaviour in which the system moves repeatedly through the points ABCD in the Figure; Fig. 2 shows the corresponding behaviour of $-\dot{M}_2$ and R_2, obtained by a simple numerical integration of the governing equations (1) and (2).

The nature of the limit cycle is as follows. In the high state (A – B in Fig. 1) the stellar radius is close to the Roche lobe and the transfer rate is rather greater than the secular mean driven by angular momentum loss; the star is irradiated by a flux somewhat larger than its own. This blocks most of its intrinsic luminosity, and expands the star on the Kelvin–Helmholtz timescale specified by $L_{\rm nuc}(A) - L(A)s$. This expansion in turn competes with the expansion of the Roche lobe which occurs because the transfer rate is above the secular mean, and sustains the high mass transfer rate needed to provide the irradiation. Ultimately (point B) the stellar expansion becomes too slow compared with that of the Roche lobe to supply a large enough transfer rate to maintain the high state. (This is inevitable as the transfer rate is above the secular mean.) This in turn reduces the irradiating flux and the blocking effect, slowing the expansion and reducing the transfer rate still further. The system drops rapidly (timescale $\sim t_{\rm L} = \epsilon t_J \simeq 10^4 - 10^5$ y) to a low, almost detached state (point C). By (1) and (2) the system point will begin to move to smaller R_2 once $-\dot{M}_2$ is close to zero (cf Fig. 1). The radius in the detached state shrinks exponentially: writing $r = R_2/R_0$, with $\Delta r = r - 1$ and $\Delta r << 1$ we have

$$\Delta r(t) \simeq \Delta r(t=0)\exp\Big(-\frac{f(5+\nu)}{t_{\rm KH,0}}t\Big) \quad (10)$$

where $t_{\rm KH,0} = GM_2^2/L_0R_0$ denotes the thermal equilibrium Kelvin–Helmholtz timescale. Ultimately (at point D) the lobe catches up with the star and the transfer rate begins to rise rapidly again. Thus the time spent in the low state is

$$t_{\rm det} \simeq \frac{t_{\rm KH,0}}{f(5+\nu)} \ln\Bigl(\frac{\Delta r(B)}{\Delta r(A)}\Bigr). \qquad (11)$$

The relative radial amplitude $A_r/R_0 = \Delta r(B) - \Delta r(A)$ is roughly $2(\Delta r(A) - \Delta r(Z))$, where $r(Z)$ is the radius where the curve $\ddot{M}_2 = 0$ formally meets the r-axis. With the transition to the high state the system reaches point A again, with a radius larger by a few scaleheights than at D, and the cycle recommences. Neglecting the nuclear luminosity the time spent in the high state can be expressed as

$$t_{\rm high} \approx \frac{A_r}{R_0}\Bigl(\frac{1}{t_J} + \frac{1}{t_{\rm KH,0}}\Bigr)^{-1}. \qquad (12)$$

With a typical radial amplitude $A_r \sim 10^{-2} R_0$ the times $t_{\rm det}$ and $t_{\rm high}$ are $\lesssim 10^7$ y.

5. CONCLUSIONS

We have shown that weak irradiation of the secondary star can drive mass transfer cycles in CVs. The system alternates between high states in which irradiation drives expansion of the star and an above–average transfer rate, and low states in which the star contracts. With the form (8, 9) of K the system effectively detaches in this state.

In future papers we shall present a phase–plane analysis of the effect of radius variations of the secondary on the transfer rate, and show that only variations sensitive to the transfer rate itself can give limit cycles. This is effectively a proof that irradiation drives the inferred mass transfer fluctations. A separate paper will discuss in detail the response of the secondary to irradiation. While these results strongly suggest that irradiation is responsible for the large spread in observed transfer rates around the evolutionary mean, it still remains to identify the observed subtypes with various phases of these cycles, and to show that the predicted cycles give the correct proportions of these subtypes as a function of orbital period.

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Fig. 1.— Resulting limit cycle in the r - $\dot{m}$ - plane, with $r = R/R_0$ and $\dot{m} = -\dot{M}_2/\dot{M}_0$, $\dot{M}_0 = M_2/(D_s t_J)$. The critical curves where $\ddot{m} = 0$ (short-dashed, see Eq. (4)) and $\dot{r} = 0$ (long-dashed) intersect at the unstable fixed point of Eqs. (1, 2), around which the system point circles on a limit cycle A → B → C → D → A (full line). In our example the system moves rapidly from its initial position (marked by "×") towards the limit-cycle track.

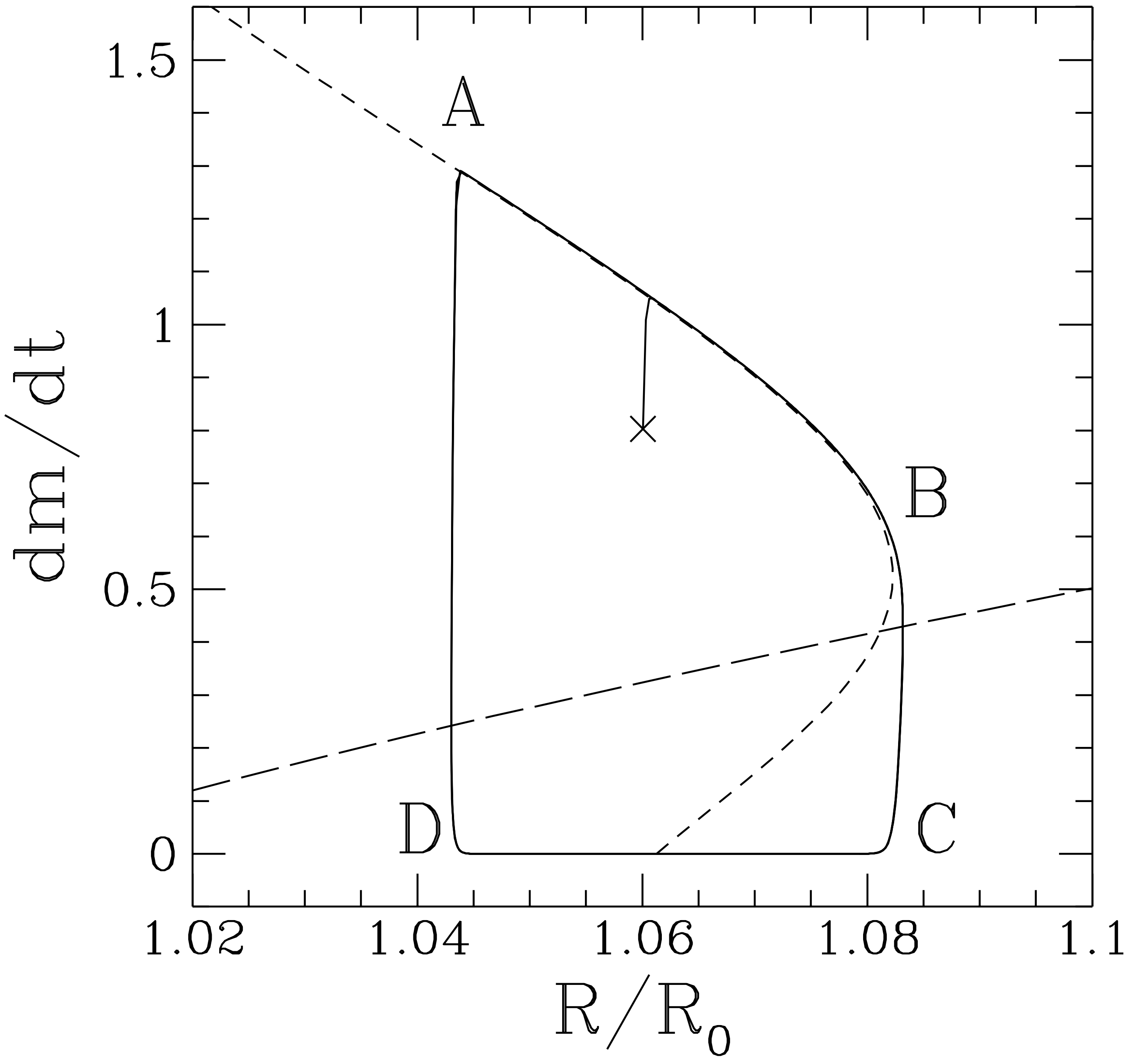

Fig. 2.— Time dependence of the radius $r = R/R_0$ and the mass transfer rate $\dot{m} = \dot{M}/\dot{M}_0$ for a system moving along the limit cycle shown in Fig. 1.