Singularities of Space time

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1 Introduction

Einstein's theory of gravitation, the theory of general relativity (GR) predicts two remarkable and unexpected geometrical singularities:

- the origin of the universe in a singularity, the so-called "big bang", and
- the existence of "black holes", regions of space-time which are cut-off from the rest of the world by a one-way "membrane" through which no physical information can flow outwards.

The state of matter is also singular in these configurations, because distances shrink to zero, and densities increase to infinity. These physical singularities of space–time are so extraordinary, even revolutionary, that it took quite some time, before the physics community could accept them as serious facts. It is the basic belief of physicists that all natural quantities are finite and well defined. Singularities occurring in physical theories are considered to be a consequence of an unsatisfactory mathematical formulation, or of an inherent incompleteness. In that sense GR predicts its own downfall, the limits of its validity.

On the other hand, it is intuitively clear that the action of gravity for large masses must have catastrophic consequences. Gravitation is an attractive, large–range force for all massive particles. If we add particle upon particle the gravitational force just adds up with growing particle number, and eventually it dominates over all counteracting pressures, such as thermonuclear pressure, Fermi pressure of a gas of degenerate electrons or neutrons, and the repulsive nuclear forces. In addition not only matter, but also antimatter, and every form of energy acts as a source for gravitational fields. Gravity in turn acts on every form of energy. Thus a large internal pressure can balance gravity to some extent, but if the pressure becomes large enough it will itself contribute to the gravitational force and help to bring the configuration down in a gravitational collapse. A state of infinite energy density is, of course, unphysical, and this singular state will eventually be removed by some – hopefully some-day appearing – theory of quantum gravity. But for matter as we know it, states of extremely high density are a natural consequence of gravity's work.

At first it was thought that the singularities are due to the idealisations of high symmetry made in order to be able to solve the complicated equations.

It was hoped that they would disappear, if slightly perturbed situations were considered. It was found over the last 30 years that these singularities are generic and rather stable against perturbations. The physicist's attitude is to investigate the space–time surrounding such a singularity, even though the real singularity is outside of the scope of his theories. In these remarks here, I shall follow this line. The emphasis will be on the big bang singularity, but some aspects of black holes will also be discussed.

2 Basic Facts about Black Holes

A few months after Albert Einstein published his fundamental paper on GR in 1916, Karl Scharzschild derived on exact solution to the field equation. This "Schwarzschild" geometry has become known as the prototype of a "black hole", but it describes quite generally the curved space–time outside of an isolated spherically symmetric mass distribution. For a Schwarzschild Black Hole of mass M there is a quantity $r_s = 2GMc^2$, the so–called "Scharzschild radius" (G is Newton's gravitational constant, c is the velocity of light).

Now, in GR masses curve the path of light rays, and at the Schwarzschild radius the light rays are curved so much that they can no larger escape outwards, but have to go round on this sphere of radius r_s. Light rays emitted inside this radius will be focussed down to the singularity at $r = 0$ by the space–time curvature. Light from the outside can pass through the Schwarzschild radius, but nothing can come out.

The Schwarzschild radius defines a horizon which shields the singularity from the outside world. Since any information can only be transmitted with the speed of light or less, the horizon acts like a one–way membrane through which energy and information can pass to the interior but not to the exterior.

The existence of horizons or causal boundaries is a remarkable consequence of GR. The singularity is on the other side of the horizon, and thus has no causal connection to an external observer. He cannot see it. It is suspected that this is true for all realistic singularities ("cosmic censorship").

An observer on the surface of a collapsing star will, except for tidal forces, not notice anything peculiar, when the Schwarzschild surface is crossed. Locally the space–time geometry is the same as it is elswhere, and for very large masses, tidal forces at the horizon are harmless.

For the external observer the stellar surface reaches the Schwarzschild horizon only after an infinitely long time interval. The star seems to "fruze" at the Schwarzschild radius. In practice, however, the star will suddenly become invisible, since the redshift will increase exponentially and any luminosity decrease exponentially. Thus, on a time scale $T \sim \tau s/c \sim 10^{-5}({}^{M}/M_{\odot})$ sec (i.e. one hundredthousandth of a second for a mass like the sun ($M_{\odot}$)) the star will really be a "Black Hole".

The Black Hole will only act gravitationally through its mass, and we need not worry about the singularity which is conveniently hidden by the horizon.

The spherical surface defined by the Schwarzschild radius can be considered as the surface of the Black Hole. Luckily, normal stars, as well as objects of every–day life are much larger than their Schwarzschild radius. For the sun, we find by inserting the numerical values ($M = 2x10^{33}g$) that

$$r_s = \frac{2GM}{c^2} = 3km$$

3 Can Black Holes be observed?

The most exciting prospect of black hole physics is that these weird objects can actually be observed in astro–physical settings. Although invisible in principle the black hole is a deep gravitational potential well, and matter falling into this potential will be heated up. The radiation emitted by the material accreting onto the black hole can be observed.

3.1 Black Holes as X–ray Binaries

Satellite observations of the X–ray sky have revealed the existance of several binary system, where an optical star is circled by the invisible X–ray source. Eclipses and spectroscopic Doppler shifts in these systems can be measured, and then the parameters of the system can be determined with some precicion. In some cases it has turned out that the estimated mass of the compact X–ray source is larger than the maximum mass of a neutron star. The most famous candidate is the source Cyg X–1 for which a most likely mass between 9 and 16 $M_\odot$ has been derived. So Cyg X–1 is very probably a black hole. But the conclusion is only indirect via the argument of the large mass. A similar argument can be given for the other black hole candidates. The astronomers would be much happier, of course, if a definite feature in the radiation pattern –as e.g. a peculiar temporal structure – could be used to identify a black hole without doubt. No such characteristic is known at present.

3.2 Black Holes in the Centers of Galaxies

Detailed observations of the central regions of active galaxies and quasars have revealed a high density of mass and energy, and high velocities in small volumes. A most recent example is M87, where observations with the Hubble Space Telescope have shown a rotating gas mass within the central 500 light years, rotating with a velocity of 750 km s^{-1}. The most likely model is a central black hole of $\sim 10^9 M_\odot$ surrounded by the gas.

Even at the center of our Milky Way detailed infrared observations of the motion of stars indicate that within a radius of ~ 0.3 light years (i.e. $10^{17.5}$cm) a mass of $10^6 M_\odot$ is contained ($r_s \simeq 3x10^{11}$ cm). It seems a safe conclusion that a black hole sits there, because any other model – a dense star cluster, a giant star – is unstable, and will end up as a black hole within a million years.

4 The First Second of the Universe

The present picture of the universe rests on two fundamental observational facts: The expansion of the system of galaxies discovered during the twenties by Hubble, and the uniform cosmic background radiation in the microwave range (CMB) at a temperature of 2.7 Kelvin, measured first in 1964 by Penzias and Wilson (fig.1). These discoveries of modern astronomy suggest a very simple structure for the early universe. Objects moving away from each other now, must have been closer together some time ago. The density of matter and radiation must therefore have been higher in earlier epochs. The uniform CMB can be interpreted as the cooled remnant of a denser, hotter, and very uniform early phase. The simple mathematical models which have been derived as solution of Einstein's GR (the Friedmann-Lemaitre (FL) cosmological models) describe the uniform expansion as the drifting apart of the particles of a uniform fluid characterized by density ρ and pressure p. There are several types of solutions: The distance between 2 particles (galaxies) increases without end for FL-models, where the mean density is equal or below a critical value.

The space at a constant time is flat (for the critical density model, $k = O$) or hyperbolically curved (for a density below critical, $k = -1$). The distance between two galaxies reaches a maximum, than decreases again, if the density is above the critical value ($k = +1$). 3–space is spherically curved in this type. Astronomical measurements of distant supernovae are consistent with a subcritical density, and an accelerated expansion induced by a positive cosmological constant (graph $\Omega_\Lambda > 0$ in fig.2).

Can be singularity be avoided, by a less symmetric cosmological model? The singularity theorems proven by Hawking and Penrose between 1965 and 1970 imply that a singularity is unavoidable even in asymmetric space–times. The proofs rest on a positive energy condition which reads "$\rho + 3p$ is nonnegative" for the FL–models. $\rho + 3p > 0$ is certainly satisfied for matter known to us. It is not known, whether the singularity theorems exist for more general matter models, where $\rho + 3p$ may become negative. We cannot be sure therefore that the cosmological singularity is really unavoidable. So far a nonsingular model with a reasonable matter content has not been found.

The FL–models imply the concept of a beginning of the world some 15 billion years ago in a primeval explosion, the "big bang" of infinite density, temperature, and with infinite initial momentum. Time and space also were created in the singular initial state, at least according to the classical big bang model. After the first second of expansion the universe had cooled down and was thinned out sufficiently, so that the physics is well–understood.

For the epochs after the first second till now the cosmologists have worked out a "standard model" which explains within the context of the simple FL models the uniform expansion, the CMB, the abundance of the light elements. An open problem which is at the center of present–day research activities is the structure formation and the dark matter problem.

This "standard model" neglects, however, completely the problem of the origin of the universe. The singular beginning is excluded, and only shortly after the big bang certain conditions are suggested for the structure of matter and radiation such that the presently observed state of the cosmos is reached. It is very probable that rational statements about the beginning can only be made, once a supertheory encompassing gravitation and quantum theory has been found. At the moment this question can only be discussed in a dogmatic way –papal behaviour can clearly be seen in many quantum cosmologists– and the rational investigation is limited to the question: What are the initial conditions that must be posed to arrive at the universe as we see it? The fewer such conditions, and the more general the initial state, the better. In the following some speculations concerning such initial conditions will be discussed.

5 Elementary Particle Physics and Cosmolgy

Very shortly after the big bang thermal energies far exceeded the energies that can be attained in terrestrial accelerators. Especially the highest energy behaviour of the elementary particles could be tested then. No observers were present, but this early phase is a great playground for theorists.

According to the ideas of particle physicists the hierarchical structure of the particle interactions – the weak, electromagnetic, and strong interactions – should disappear at high enough energies or for sufficiently small separations. Above this unification energy the different interactions should merge into one.

For the weak and electromagnetic interaction this threshold is experimentally accessible at $10^3 GeV$. The strong interactions unify at an energy of about $10^{15} GeV$. These great unification (GUT) would occur about 10^{-34} seconds after the big bang. (fig. 3)

5.1 Baryonsynthesis

The CMB as a thermal radiation of 2.7k and the matter content at present are such that for each matter particle there are about 1 billion photons. In the very early universe, when the hot radiation field was essentially composed of particles and antiparticles, this ratio means that every billion plus one particles was matched by one billion antiparticles.

The primeval gas expanded and cooled down, particles and antiparticles aninhilated each other into photons, and the small surplus of one to one billion remained. This small asymmetry is one reason for our existence. Can this asymmetry be produced by particle processes from a completely symmetric initial state containing exactly equal numbers of particles and antiparticles?

It has been shown recently that during the electroweak phase transition a bound state, the "sphaleron" occurs which has the effect that any previous asymmetry of baryons and antibaryons is damped out. The task is to construct a convincing model that generates this asymmetry during the electroweak phase transition at an epoch 10^{-12} seconds after the big bang.

6 Inflationary Models

The "standard model" requires a very finely tuyned balance between the initial momentum of the expansion, and the action of gravity decelerating the motion of the cosmic objects. The mean density of the universe must be close to a critical value (corresponding to the excellent vacuum of 3 to 10 hydrogen atoms per m^3) and for early epochs it can only differ by a tiny fraction from it. Otherwise, too large a density would lead to an early recollapse without a chance for evolution, too low a density would lead to a rapid expansion such that the condensation of stars and galaxies would become impossible.

Another property of FL–models is the existence of horizons, of causally separated regions. Lightsignals can connect only points with a separation which is small compared to the size of universe. Thus, for instance, the microwave radiation from regions on the sky separated by $180°$ has not been in causal contact up to the epoch of decoupling of matter and radiation. But the temperature across the sky is uniform down to 10^{-5} relative amplitude variations. Within the standard model this uniformity is just a property of the initial conditions, it cannot be explained by physical processes. The inflationary models of the universe promise to provide explanations for these features of the standard model.

6.1 Inflation

The unified theories of particle physics contain scalar fields, the "Higgs fields", postulated to produce the transition from the highly symmetric GUT state to the less symmetric state with the hierarchical ordering of interactions. The energy of the ground state of these fields could be the dominant energy density in the GUT phase of the universe about 10^{-34} seconds after the big bang, and it could dominate the expansion. This is an exponential expansion proceeding at constant energy density, because for the false vacuum material the pressure is equal to the negative energy density. Expansion is equivalent to work against this negative pressure, i.e. it leads to an energy gain.

This produces an enormous acceleration of the expansion. From $10^{-34}s$ to $10^{-32}s$ after the big bang the normal FL-expansion would have increased the separation of two particles by a factor 100, whereas inflationary models give a factor 10^{29} or more. In a rather chaotic region of the early universe there might be a small relatively homogeneous patch with a high value of the Higgs field energy. This small patch could than expand dramatically and reach a size of $\sim 10cm$ at the end of the inflationary phase which is about right for our observable universe. If the inflationary factor is $\sim 10^{29}$, then our whole observable universe could have been a tiny part of one causal horizon at the beginning of inflation. Because the initial patch was small and relatively uniform, the resulting universe would, after the transformation of the field energy into radiation, be a uniform cosmos of the FL–type. Everything would be causally connected, since the initial seed region was, and the density would be close to critical, because the rapid expansion would force the space–time curvature to be nearly zero.

The expansion of the universe is explained as the acceleration by the energy density of the false vacuum. This inflationary scenario has the merit that the fine tuning features of the standard model become subject to a rational investigation.

Several ad hoc postulates of this scenario still are in need of a convincing physical and mathematical basis. A very basic difficulty is the role of the vacuum energy density responsible for inflation in the early universe. In contrast, the vacuum of the weak, electromagnetic, or strong interaction theory cannot have any gravitational effect. The typical energy densities are so large that the discrepancies to astronomical observations would be enormous.

Why should the vacuum be gravitationally inactive now, but dominant during inflation? If we could answer this question we would certainly have made some progress towards understanding the beginning of the universe.

7 Speculations about the initial state

Penrose has given some interesting thermodynamic arguments that make a rather uniform beginning plausible. The second law of thermodynamics, the increase of entropy (loosely speaking disorder) characterizes our world. The observable universe must therefore have evolved from a state of low entropy. Since matter and radiation were in thermal equilibrium, i.e. in a state of maximum entropy, the huge entropy reservoir contained in the gravitational field must have been unused. The initial state must have been well–ordered and not chaotic.

Hawking has tried to find a solution for the quantum amplitude of the initial state of the universe. In this playing around at the frontiers of speculation, he postulates a dependence of this amplitude on certain types of highly symmetric classical space–time solutions, the so–called "no boundary" postulate.

Rather doubtful manipulations lead to solutions which correspond to the 4–dimensional surface of a 5–dimensional sphere (a sphere's surface in 3–space is two–dimensional; you just have to imagine two more dimensions).

Time is found as one of the spatial coordinates, counted in imaginary numbers. The representations of simple examples in real space imply a hot big bang starting from a small, finite radius. The question, where this initial configuration comes from, cannot be posed in the normal categories of space and time. Time is simply a coordinate which describes the evolution of the classical world, but it plays no role in the quantum amplitudes of the possible initial states. This can be illustrated by the system of geographic latitude and longitude on the earth's surface. They cannot describe a point above the earth.

These attempts to make rational guesses for the initial state of the cosmos cannot be judged ("They are not even wrong" as Wolfgang Pauli would have said), without a theory of quantum gravity. Unfortunately such a theory is not in sight at present.

- **Fig. 1** : The measurement of the COBE satellite (boxes) of the CMB fit extremely well a thermal spectrum (line) at a temperature of 2.73 K. Deviations from the thermal black–body spectrum are smaller than the measurement errors Alternative theories, such as the Steady State Cosmology are not able to produce such a perfect black–body spectrum.

- **Fig. 2:** The separation of two comoving galaxies changes propartial to R(t). This is plotted for different cosmological models. Recent supernova measurements indicate that the graph with $\Omega_\Lambda > 0$ ist the correct one for our universe. This would mean that, at present, the expansion is accelerating.

- **Fig. 3:** The very early universe is a testing ground for the high energy behaviour of particle theories. As thermal energies become higher and higher the theories get more speculative. Above the Planck energy of 10^{19} GeV a quantum universe must be described. There are only a few people, who know this can be done.

Energy(GeV)		**time(seconds)**	
10^3	———————————	10^{-12}	electroweak phase transition
10^{15}		10^{-34}	GUT phase transition Inflation
10^{19}		10^{-43}	Planck scale
			Quantum Gravity
	Pope Dalai Lama Stephen Hawking		

Suggested further reading:

- Silk, J. "The Big Bang" (W.H. Freeman 1989)
- Ellis, F.R.S., Hawking S. "The Large–Scale Structure of Space–Time" (Cambridge Univ. Press 1973)
- Börner, G. "The Early Universe" (Springer 1993)
- Weinberg S. "The First 3 Minutes" (.............)
- Wald, R.M. "Space, Time and Gravity" (Uiv. Chicago, Press 1977)

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