Exoplanet Mass and Radius and the Physics of Planetary Interiors

Sara Seager
Massachusetts Institute of Technology
The goal is to constrain the interior composition of exoplanets by their mass and radius measurements.
Planet Mass-Radius 2000

The graph depicts a scatter plot of Earth masses versus Earth radii, highlighting exoplanets (squares) and solar system planets (triangles).

- **Exoplanets**
  - Squares
  - Data points spread across a range of masses and radii.

- **Solar System Planets**
  - Triangles
  - Concentrated at lower masses and radii compared to exoplanets.

The axes are labeled as follows:
- **Earth Masses** (horizontal axis)
- **Earth Radii** (vertical axis)
Exoplanet Mass-Radius Diagram

Aim to infer an exoplanet’s bulk composition from its M and R

Seager et al. 2007
We infer an exoplanet’s bulk composition from its $M$ and $R$. 

\[
\begin{align*}
\frac{dm(r)}{dr} &= 4\pi r^2 \rho(r) \\
\frac{dP(r)}{dr} &= \frac{-Gm(r)\rho(r)}{r^2} \\
\rho(r) &= F(P(r),T(r))
\end{align*}
\]
Equation of State

\[ \rho(r) = f(T(r), P(r)) \]

Describes relationship between density, temperature and pressure for a material in thermodynamic equilibrium

1) Ideal gas law: \( P = nkT; \rho = Pm_H\mu/kT \)
2) Polytrope: \( P = K\rho^{(n+1)/n} \)
3) Vinet EOS

\[ P = 3K_0\eta^{2/3} \left[ 1 - \eta^{-1/3} \right] \exp \left( \frac{3}{2} (K'_0 - 1) \left[ 1 - \eta^{-1/3} \right] \right) \]

\[ \eta = \rho/\rho_0 \]
High Pressure Physics Experiments
Mass-radius relationships appear to have a common functional form.

Seager et al. 2007
Equation of State

\[ \log_{10} R_s = -0.209 + \frac{1}{3} \log_{10} M_s - 0.0804 \times M_s^{0.394} \]

Seager et al. 2007
Overall, the EOSs approximately follow $\rho = \rho_0 + cP^n$
A "modified polytrope"

Thomas-Fermi-Dirac EOS

No simple EOS formulation for pressures between Vinet and TFD

Seager et al. 2007
The mass-radius relationships for cold terrestrial \( M \) mass planets follow a generic functional form because the EOS are well approximated by a modified polytrope. 

Seager et al. 2007
Diversity of super Earths?

Water planets?

Current ground-based transit surveys. Some planets are too big!

Neptune-size but not necessarily ice giants

Diversity of super Earths?

Seager et al. 2007
Mass-Radius Relation Summary

Mass-radius relationships are well understood but are not adequate to study individual objects.
Exoplanet Interiors

Introduction

Mass-Radius Relationships

Two Transiting Super Earths

Kepler and Beyond
Exoplanets can be composed of three (or four) materials: rock (and iron), ice, and gas.

Rogers and Seager 2010b; Chambers 2010
Ternary Diagrams

http://csmres.jmu.edu/geollab/fichter/SedRx/readternary.html
Ternary Diagrams

http://csmres.jmu.edu/geollab/fichter/SedRx/readternary.html
Ternary Diagrams

http://csmres.jmu.edu/geollab/fichter/SedRx/readternary.html
Ternary Diagrams

- Silicate is 20% mass fraction
- Water is 40% mass fraction
- Iron is 40% mass fraction

Zeng and Seager 2008
CoRoT-7b

The first transiting super Earth
R = 1.68±0.09 R⊕
M = 4.8±0.8 M⊕
P = 0.85 days
a = 0.017 AU
T ~ 2500 K
Leger et al. 2009
Queloz et al. 2009

Degeneracy in internal composition is a permanent limitation no matter how small the observational uncertainties
Rogers and Seager 2010b

Ternary diagrams: see Valencia et al. 2007
Zeng & Seager 2008
CoRoT-7b

$R = 1.68 \pm 0.09 \, R_{\text{Earth}}$

$M = 4.8 \pm 0.8 \, M_{\text{Earth}}$

CoRoT-7b is likely made of material less dense than Earth’s, assuming no water content
GJ 1214b

The second transiting super Earth
R = 2.68±0.13 R⊕
M = 6.6±0.8 M⊕
P = 1.58 days
a = 0.014 AU
T ~ 550 K

GJ 1214b by Charbonneau et al. 2009
Interpretation by Rogers and Seager 2010a
Quaternary Diagrams

Most super Earths or exo-Neptunes found in the near future are likely to have gas envelopes.

This adds a further degeneracy to the interior composition interpretation.

Adds more complexity to the models because of the free parameters in the gas layer.

Rogers and Seager 2010b
GJ 1214b

The second transiting super Earth
R = 2.68±0.13 R⊕
M = 6.6±0.8 M⊕
P = 1.58 days
a = 0.014 AU
T <~ 550 K

GJ 1214b by Charbonneau et al. 2009
Interpretation by Rogers and Seager 2010a
Water Phase Diagram

Pressure (Pa)

Temperature (K)

Solid

Liquid

Vapor

X

XI

VIII

IX

II

Ic

Ih

VI

VII

M

E

V

10^{12}

10^9

10^6

1

0

200

400

600

800

100

200

300

400

500

600

700

800

900

1000

1100
Super Earth Interiors Summary

Interior composition interpretation is highly degenerate based on mass and radius measurements.

Parameter space can be quantitatively constrained, and critical interpretation can still be made, e.g., the likely absence of liquid water on GJ 1214b.
Exoplanets Interiors

Introduction
Mass-Radius Relationships
Two Transiting Super Earths
Kepler and Beyond
Goal: to determine the frequency of Earth-size planets in Earth-like orbits about sun-sized stars
Borucki et al. 2010

Telescope Summary
0.95 m
105 sq-degree FOV
Centered in the Cygnus-Lyra region
No moving parts in science payload
Heliocentric Earth-trailing orbit
Telemetry limited
Bandpass 423-897 nm
Too faint for RV followup
About $\frac{1}{2}$ false positives

See Borucki et al. 2010, astroph yesterday
Astrophysical False Positives

- Eclipsing binary with grazing orientation
- Small star crossing in front of another star
- Eclipsing binary diluted by the light of a third star (“blend”): the trickiest case

Adapted from G. Torres
Number of Planet Candidates

Borucki et al. 2010
Planet Candidates vs. Semi-Major Axis

Borucki et al. 2010
Five multi-planet candidate systems were announced yesterday.

Steffen et al. astroph yesterday
Multi-Planet Transits

Harbinger for huge advancements in exoplanet science

Planet formation
- why so coplanar?
- frequency of coplanar systems?

Orbital evolution
- orbital resonances

Planet characterization
- masses (from transit timing variations) and radii for planets in the same system

\[ \begin{align*}
V &= 13.9 \\
R &= 0.58 \, R_J \\
P &= 27 \, d
\end{align*} \]

\[ \begin{align*}
R &= 0.3 \, R_J \\
P &= 27.406 \, d
\end{align*} \]

\[ \begin{align*}
R &= 0.3 \, R_J \\
P &= 13.478 \, d
\end{align*} \]
Kepler Summary

Kepler will more than double the number of exoplanets, extending the orbital separation of transiting planets out to 1 AU.

The data release is a game-changer for exoplanet science in terms of quantity vs. detailed physical properties.

The plan is to connect radii vs. period with planet interiors and population synthesis models to constrain planet formation models.
Ultimately we want to connect planet atmosphere and interior models with planet formation and population synthesis models and large observational data sets like Kepler.
Summary

• Mass-Radius Measurements
  – For almost 100 exoplanets
  – Mass-radius relationships are well understood

• Characterization of Individual Super Earths
  – Only two data points (mass and radius) translates to a permanent degeneracy in interior composition
  – Quantitative constraints are possible

• Kepler Data
  – Historic announcement of 5 multi-planet transiting candidates
  – Hundreds of new planet candidates

• The Way Forward
  – connect mass, radius, and/or period data, planet population synthesis models, and planet interior and atmosphere models
  – With the aim of understanding planet formation, migration, and evolution