Tapping Gravitational Energy: Accreation onto Compact Stars

Neutron Star Radii and Equation of State

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Properties

- Mass: 1-2 Solar Masses
- Density: $\rho > 2\rho_0 = 2.7 \cdot 10^{14} \frac{\mathrm{g}}{\mathrm{cm}^3}$ in the core
- Rotation period: ms – hours, days
- Magnetic field: $10^8 - 10^{14}\,\mathrm{G}$
- Radius: ~10 km
- Gravitational acceleration: $10^{13} - 10^{14} \frac{\mathrm{cm}}{\mathrm{s}^2}$

Tolman-Oppenheimer-Volkov equation

$$\frac{dP(r)}{dr}=-\frac{G\varepsilon(r)M(r)}{c^2r^2}\left(1+\frac{P(r)}{\varepsilon(r)}\right)\left(1+\frac{4\pi r^3P(r)}{M(r)c^2}\right)\left(1-\frac{2GM(r)}{c^2r}\right)^{-1}$$

$$\frac{dM(r)}{dr}=4\pi r^2\rho(r)=\frac{4\pi r^3\varepsilon(r)}{c^2}$$

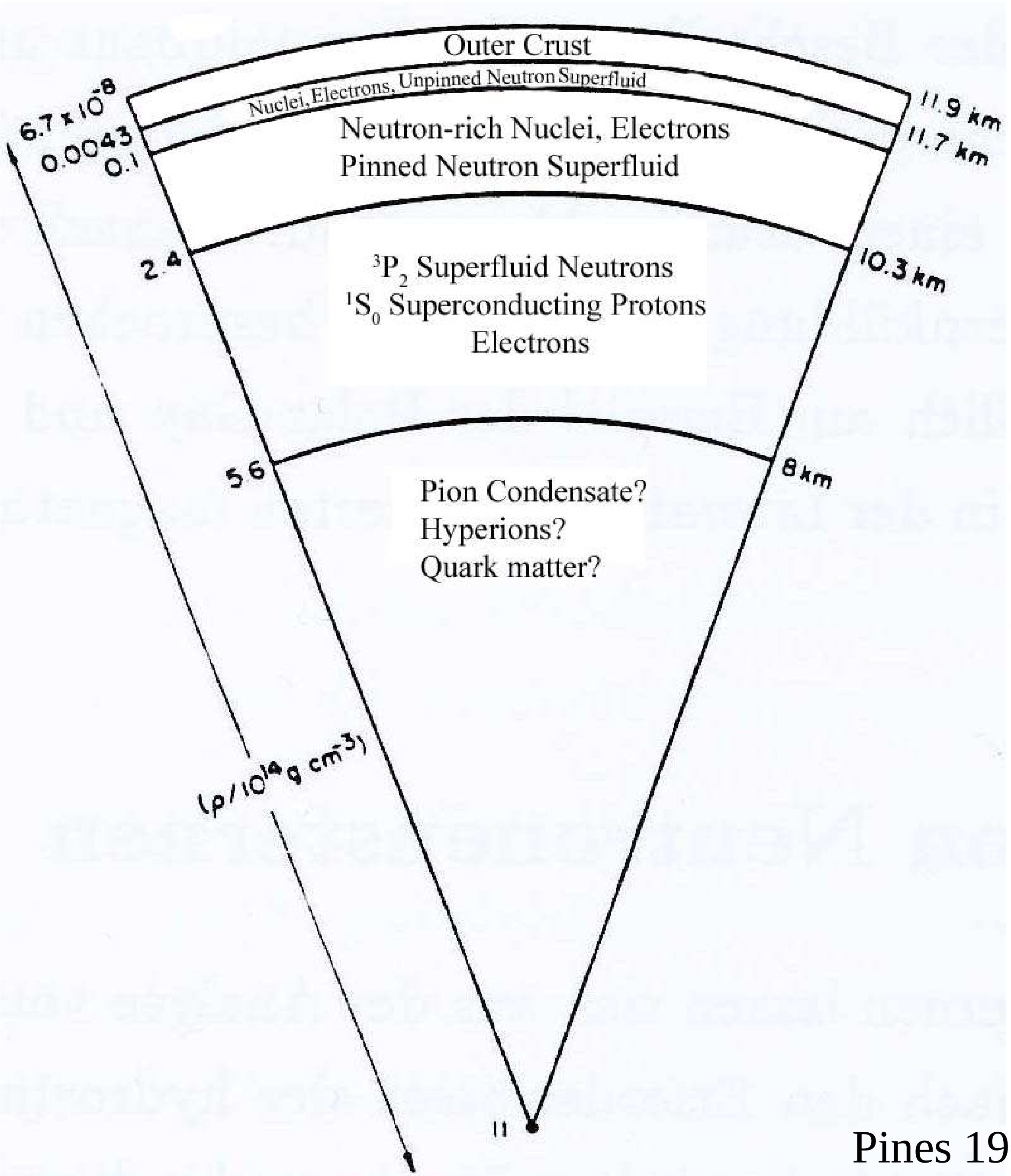
$$M(r)=4\pi\int_0^r R^2\rho(R)\,dR=4\pi\int_0^r R^2\,\frac{\varepsilon(R)}{c^2}\,dR$$

Equation of State

relationship between P(r) and ε(r) $\Rightarrow$ P(ε)

2 Assumptions:

- No rotation present, because rotation does not affect the fundamental EOS, it just changes the parameters we get up to about 20%
- Temperature is 0K. Reasonable because degeneration energy is much higher than the temperature

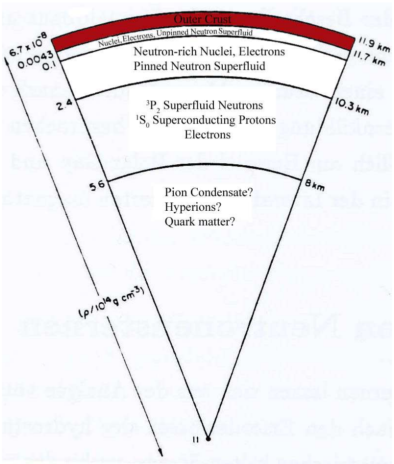

Outer Crust
Nuclei, Electrons, Unpinned Neutron Superfluid
Neutron-rich Nuclei, Electrons
Pinned Neutron Superfluid
$^3P_2$ Superfluid Neutrons
$^1S_0$ Superconducting Protons
Electrons
Pion Condensate?
Hyperions?
Quark matter?
6.7×10^{-8}
0.0043
0.1
2.4
5.6
11
$(\rho / 10^{14}\, g\, cm^{-3})$
11.9 km
11.7 km
10.3 km
8 km

Pines 1991

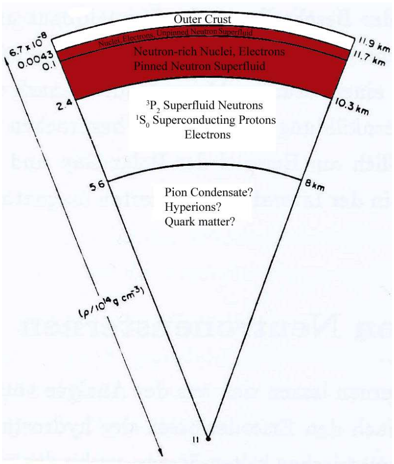
Atmosphere and Surface

- $\rho \leq 10^6 \, \frac{\text{g}}{\text{cm}^3}$
- Plasma of H, He, C, Fe in the atmosphere
- temperature and magnetic fields important for radiation
- atmosphere is not gray
- thin surface with large density gradient

outer crust

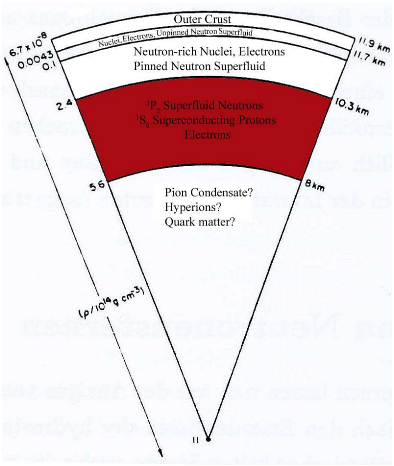


- $10^6 \frac{\text{g}}{\text{cm}^3} \le \rho \le 4.3 \cdot 10^{11} \frac{\text{g}}{\text{cm}^3}$
- relativistic degenerate electron gas
- heavy, neutron rich nuclei
- $^{56}\text{Fe} \rightarrow \ ^{62}\text{Ni} \ \rightarrow \ ^{118}\text{Kr}$
- β-equilibrium
- Coulomb lattice



inner crust

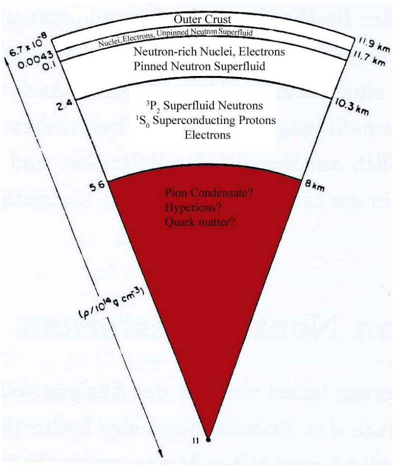
- $4.3 \cdot 10^{11}\,\dfrac{\mathrm{g}}{\mathrm{cm}^3} \le \rho \le (2-2.4)\cdot 10^{14}\,\dfrac{\mathrm{g}}{\mathrm{cm}^3}$
- neutrons drop off from the nuclei
- superfluid neutrons



neutron liquid

- $(2-2.4)\cdot 10^{14}\,\dfrac{\mathrm{g}}{\mathrm{cm}^3} \le \rho \le 2.8\cdot 10^{14}\,\dfrac{\mathrm{g}}{\mathrm{cm}^3}$
- superfluid neutrons
- superconducting protons

core region



- $2.8 \cdot 10^{14} \dfrac{\mathrm{g}}{\mathrm{cm}^3} \leq \rho$
- behavior of nuclear interactions not known
- muon condensation
- pion condensation
- hyperions
- quark matter

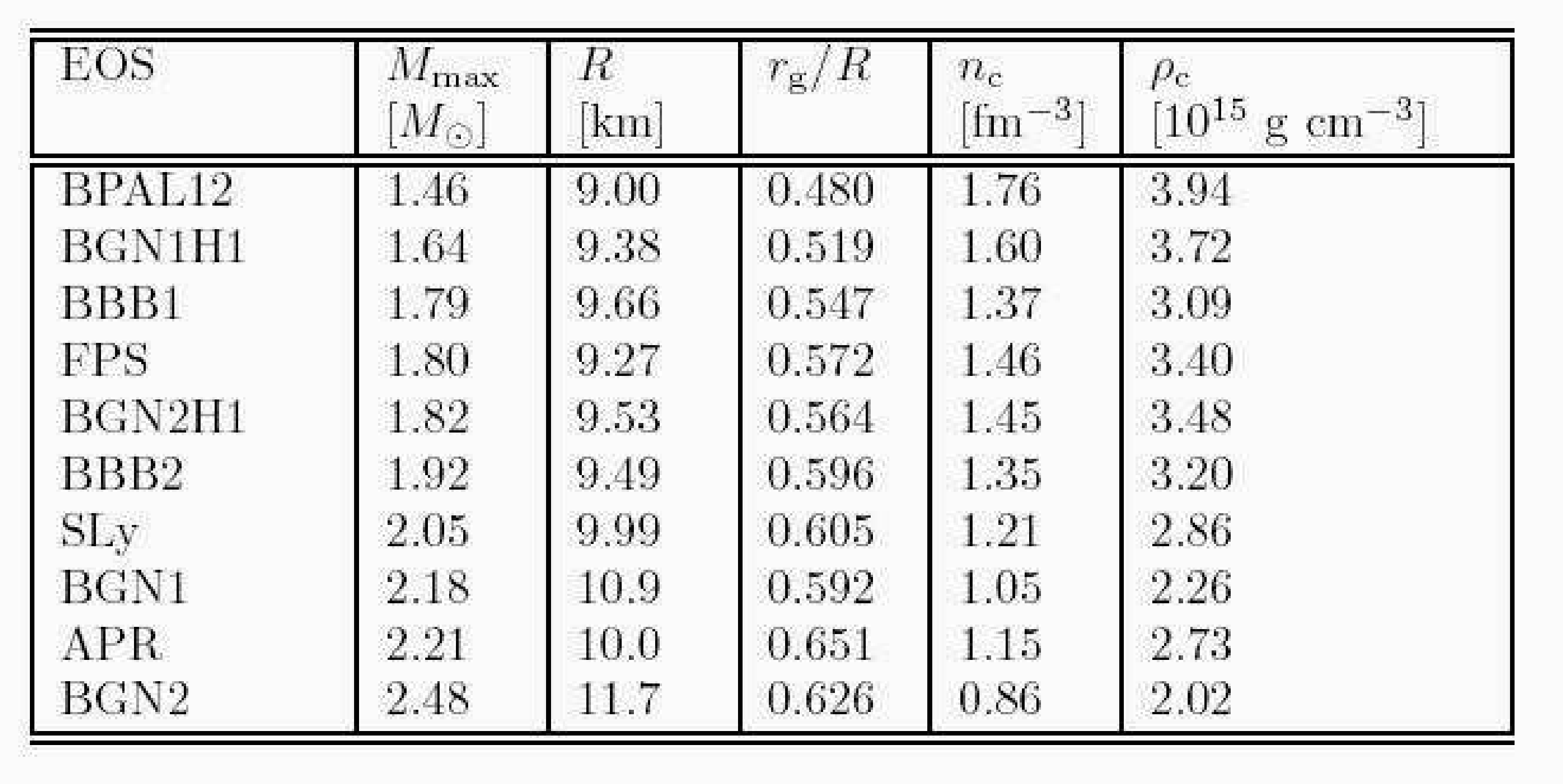
Maximum mass (in general)

Assumptions:

- General relativity is valid
- microscopic stability $\frac{dP}{d\rho} \geq 0$
- causality $\frac{dP}{d\rho} \leq c^2$
- known EOS below some density ρ_0

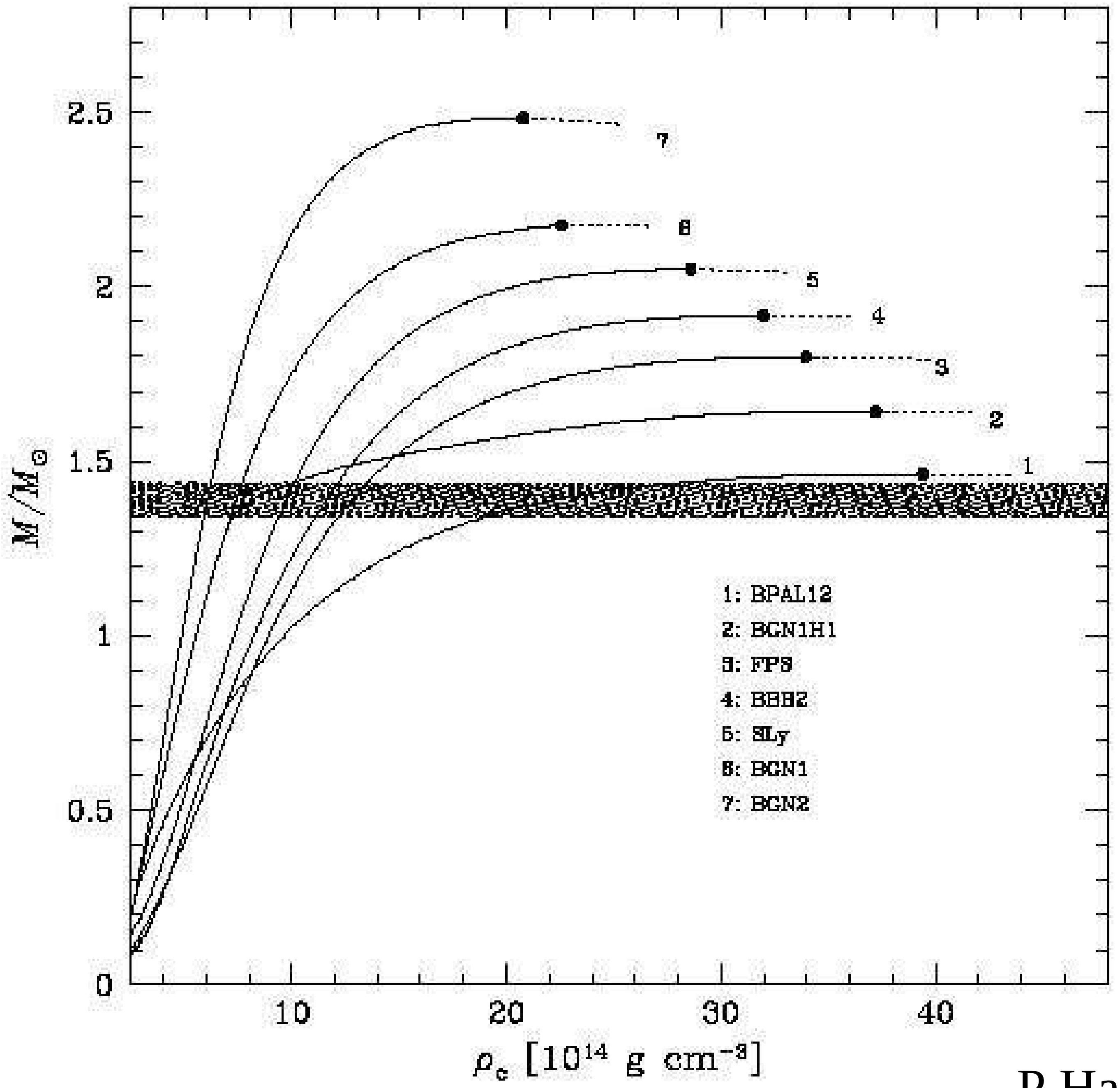
Result:

$$M_{Max} \approx 3.6\ M_{\odot}$$

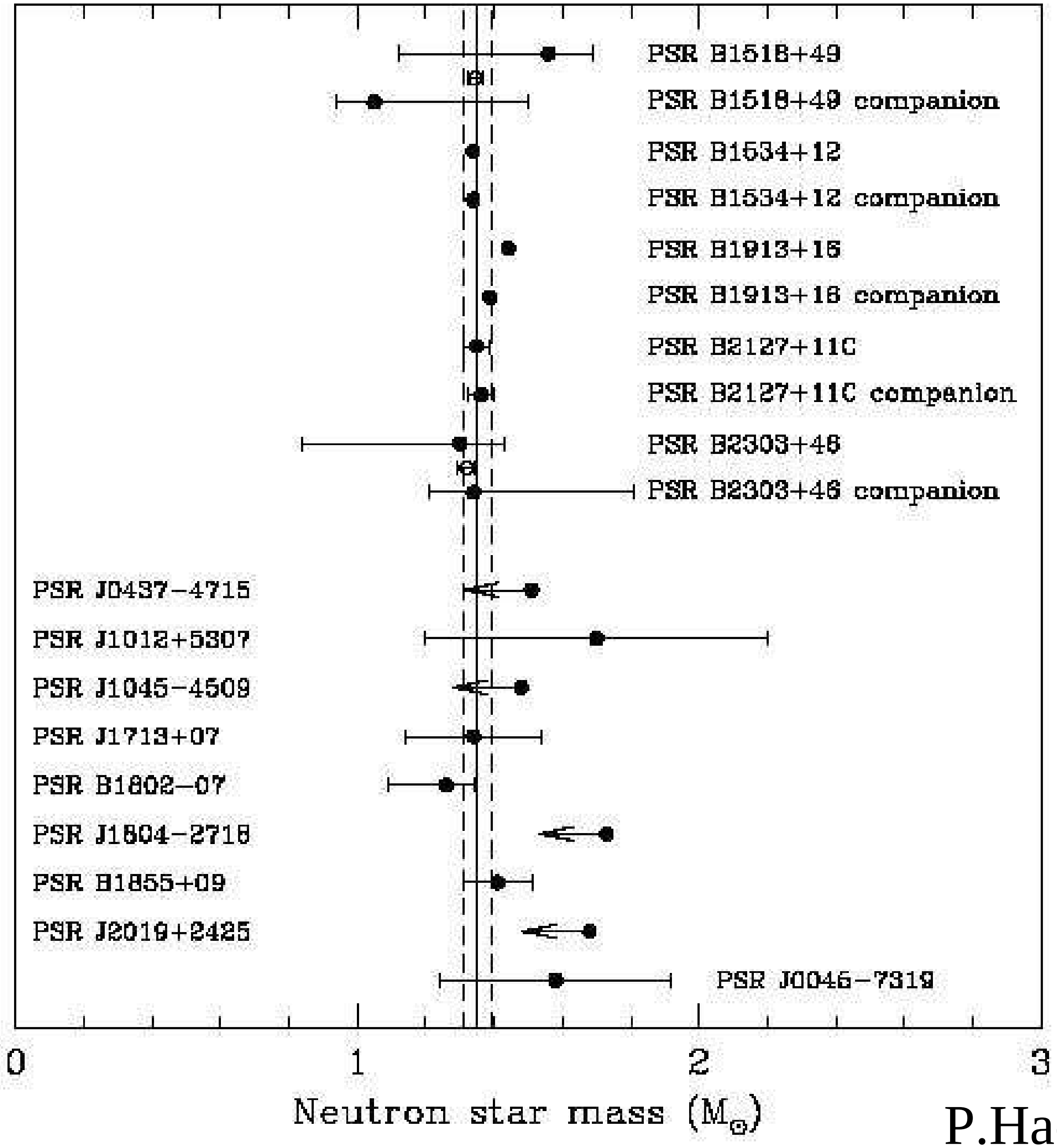


EOS	M_{max} $[M_\odot]$	R [km]	r_{g}/R	n_{c} $[\mathrm{fm}^{-3}]$	ρ_{c} $[10^{15}\ \mathrm{g\ cm}^{-3}]$
BPAL12	1.46	9.00	0.480	1.76	3.94
BGN1H1	1.64	9.38	0.519	1.60	3.72
BBB1	1.79	9.66	0.547	1.37	3.09
FPS	1.80	9.27	0.572	1.46	3.40
BGN2H1	1.82	9.53	0.564	1.45	3.48
BBB2	1.92	9.49	0.596	1.35	3.20
SLy	2.05	9.99	0.605	1.21	2.86
BGN1	2.18	10.9	0.592	1.05	2.26
APR	2.21	10.0	0.651	1.15	2.73
BGN2	2.48	11.7	0.626	0.86	2.02

P.Haensel, 2003



P.Haensel, 2003



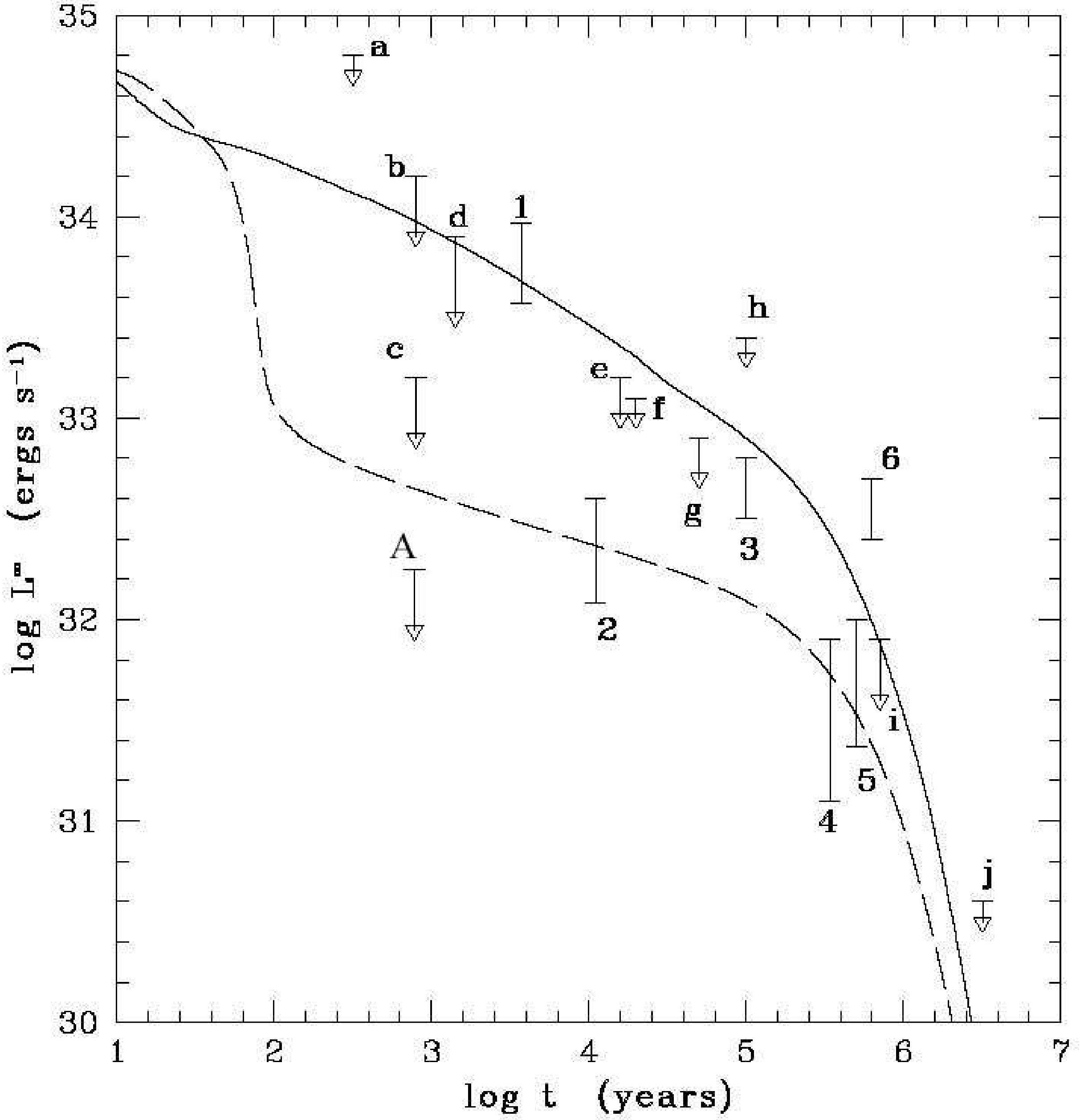
P.Haensel, 2003

Cooling of Neutron Stars

- neutrino emission
  - dominates at least for the first 50000 years
  - Neutron star is transparent for neutrinos, because the their mean free path is much larger than the radius of a neutron star
- photon emission
  - photons are emitted only from the surface
  - radiation should have black-body spectrum

process	reaction	emissivity (erg/s/cm³)
direct URCA process	$n \rightarrow p + e^- + \bar{\nu}_e$ $p + e^- \rightarrow n + \nu_e$	$\sim 10^{27} \times T^6{}_9$
π-condensate	$n + \pi^- \rightarrow n + e^- + \bar{\nu}_e$ $n + e^- \rightarrow n + \pi^- + \nu_e$	$\sim 10^{26} \times T^6{}_9$
quark URCA process	$d \rightarrow u + e^- + \bar{\nu}_e$ $u + e^- \rightarrow d + \nu_e$	$\sim 10^{26} \times T^6{}_9$
K-condensate	$n + K^- \rightarrow n + e^- + \bar{\nu}_e$ $n + e^- \rightarrow n + K^- + \nu_e$	$\sim 10^{24} \times T^6{}_9$
modified URCA process	$n + n \rightarrow n + p + e^- + \bar{\nu}_e$ $n + p + e^- \rightarrow n + n + \nu_e$	$\sim 10^{20} \times T^8{}_9$
coupled electron-neutrino processes	$\gamma + e^- \rightarrow e^- + \nu_e + \bar{\nu}_e$ $e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e$	$\sim 10^{20} \times T^8{}_9$
Neutron-Neutron and Neutron-Proton Bremsstrahlung	$n + n \rightarrow n + n + \nu + \bar{\nu}$ $p + n \rightarrow p + n + \nu + \bar{\nu}$	$\sim 10^{19} \times T^8{}_9$
election-ion-neutrino Bremsstrahlung	$e^- + (Z, A) \rightarrow e^- + (Z, A) + \nu_e + \bar{\nu}_e$	$\propto T^6{}_9$

Becker 1995



(1) RX J0822-4300
(2) the Velar pulsar
(3) PSR 0656+14
(4) Geminga
(5) RX J1856-3754
(6) PSR 1055-52

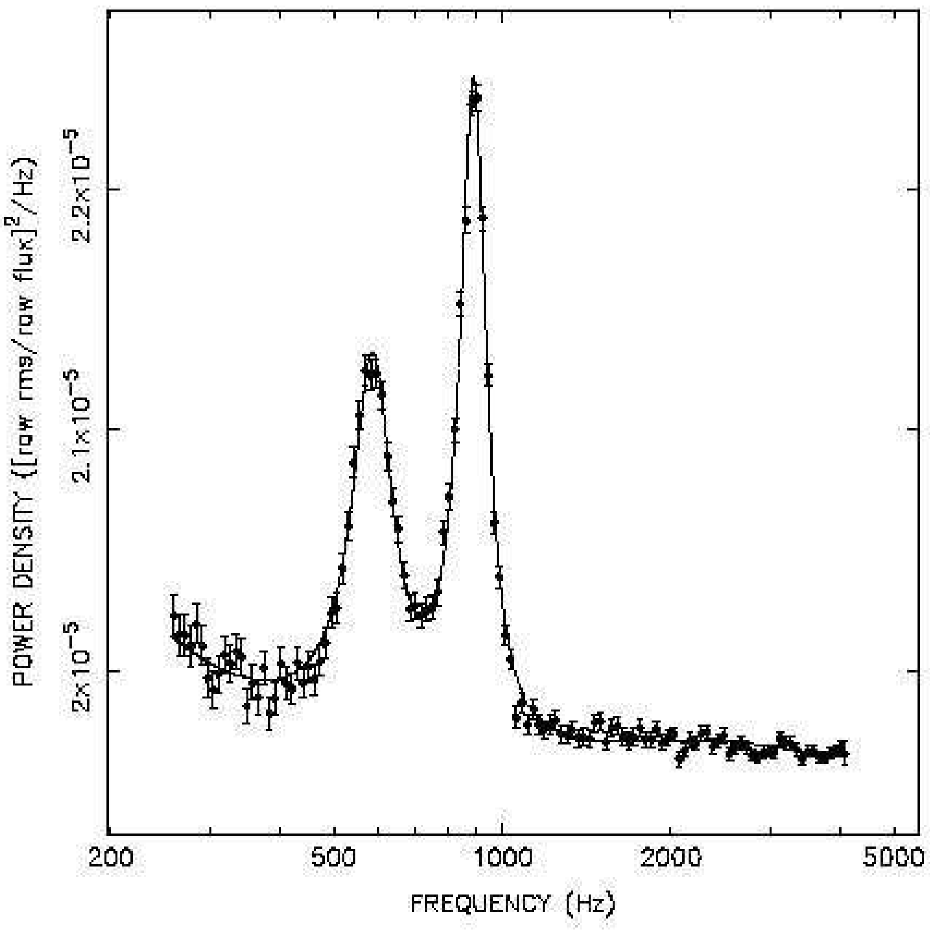
(a) Cas A point source
(b) Crab pulsar
(c) PSR J0205-6449
(d) PSR 1509-58
(e) PSR 1706-44
(f) PSR 1823-13
(g) PSR 2334+61
(h) PSR 1951+32
(i) PSR 0355+54
(j) PSR 1929+10

(A) 3C58

Tsuruta et al., 2002

Remember QPO‘s

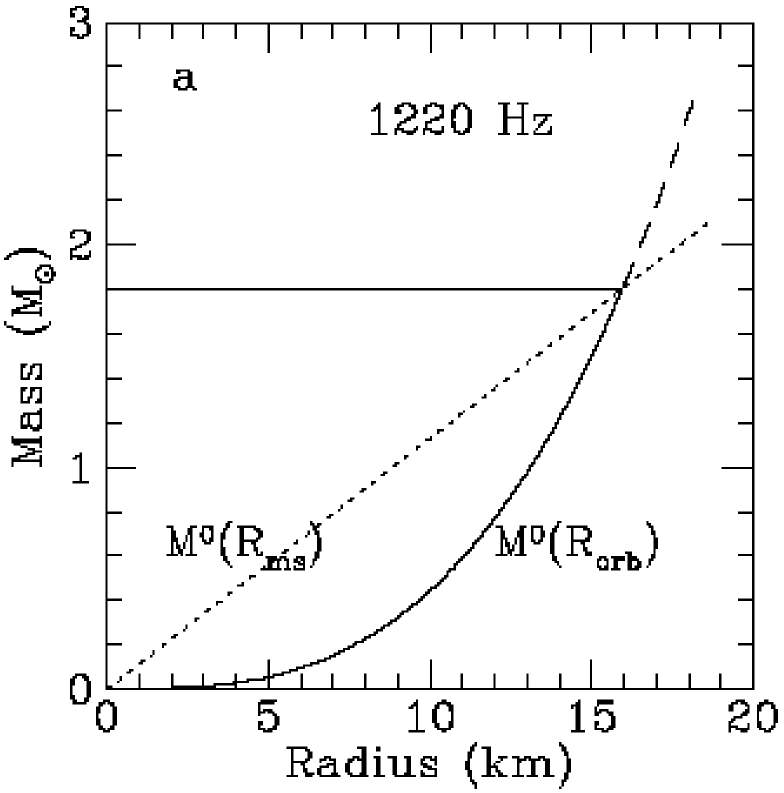
- two characteristic peaks
- assume sonic-point beat-frequency model
- frequency of the higher-frequency QPO is the Keplerian frequency at a special radius



Power spectrum of Sco X-1
(Van der Klis, 2000)

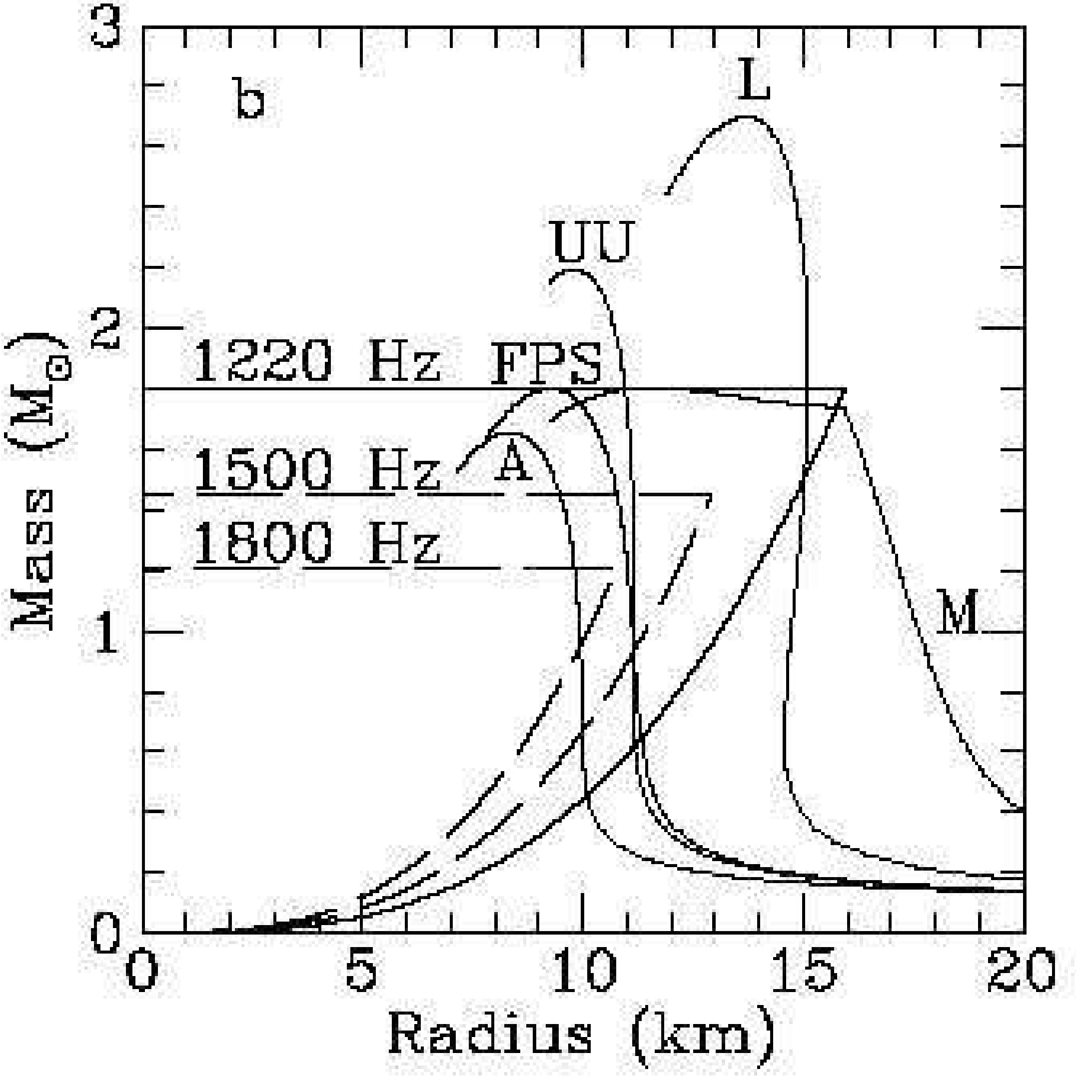
Getting maximum bounds for mass and radius of NS from QPOs:

- let 1220 Hz be the highest observed value of the higher-frequency
- the orbital radius R_{orb} of the clumps producing the QPO must be greater than the stellar radius
- R_{orb} has also to be greater than the radius R_{ms} of the innermost stable circular orbit, in order to produce wave trains that lasts tens of oscillation periods



Lamp, Miller, Psaltis, 1998

Comparison the allowed mass/radius regions with some EOS



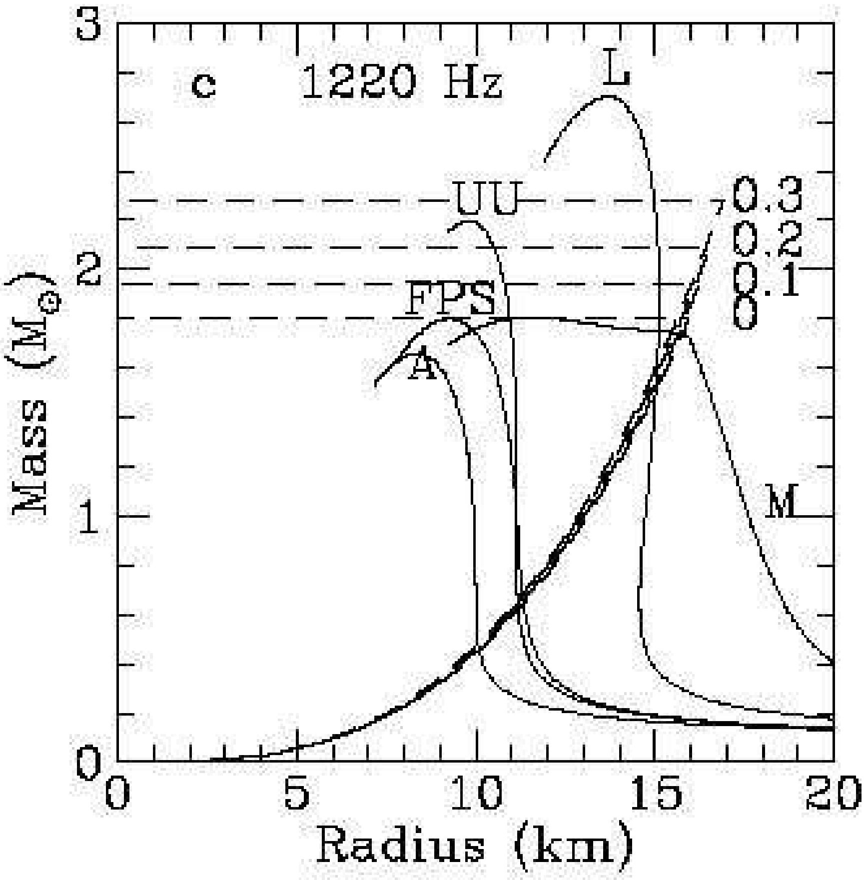
Lamp, Miller, Psaltis, 1998

regions allowed for rotating NS and comparing with EOS

- consider a fixed frequency
- parameter for importance of rotational effects:

$$j \equiv \frac{cJ}{GM^2}$$

- for spin frequencies ~300Hz j is about 0.1-0.3



Lamp, Miller, Psaltis, 1998

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