Core Collapse Supernova Mechanism, Nuclear Statistical Equilibrium (NSE) and Radioactive Abundances

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Introduction

- Two types of supernovae:
  - Thermonuclear explosions of accreting white dwarfs (Type Ia)
  - Core collapse supernovae (Type II, Ib, Ic)
- Classification:
  - Type I is defined by a lack of hydrogen lines in its spectrum
  - Type II has those lines
Core collapse supernova

• Starting point: stars heavier than 8 solar masses
• Power: \(10^{53} \text{ erg s}^{-1} = 10^{46} \text{ J s}^{-1}\); released by neutrinos
• End product:
  • Neutron star
    - Radius: ca. 10 km
    - Density: nuclear matter
    - 90% Neutrons, 10% Protons
  • Black hole (if the progenitor star had a mass roughly heavier than 25 solar masses)
Evolution of a massive star

- Each time a fuel runs out, the star contracts and heats up, until the next burning stage is reached
- Overall, the life of a massive star is a continued contraction
Evolution of a massive star

- The evolution accelerates greatly after helium burning
- Reason: energy loss due to neutrino cooling
- Neutrinos are produced for example by pair annihilation:

\[ e^- + e^+ \rightarrow \nu + \bar{\nu} \]

<table>
<thead>
<tr>
<th>Stage</th>
<th>Timescale</th>
<th>Fuel or product</th>
<th>Ash or product</th>
<th>Temperature (10^9 K)</th>
<th>Density (gm cm(^{-3}))</th>
<th>Luminosity (solar units)</th>
<th>Neutrino losses (solar units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>11 Myr</td>
<td>H</td>
<td>He</td>
<td>0.035</td>
<td>5.8</td>
<td>28,000</td>
<td>1,800</td>
</tr>
<tr>
<td>Helium</td>
<td>2.0 Myr</td>
<td>He</td>
<td>C, O</td>
<td>0.18</td>
<td>1,390</td>
<td>44,000</td>
<td>1,900</td>
</tr>
<tr>
<td>Carbon</td>
<td>2000 yr</td>
<td>C</td>
<td>Ne, Mg</td>
<td>0.81</td>
<td>2.8 \times 10^5</td>
<td>72,000</td>
<td>3.7 \times 10^5</td>
</tr>
<tr>
<td>Neon</td>
<td>0.7 yr</td>
<td>Ne</td>
<td>O, Mg</td>
<td>1.6</td>
<td>1.2 \times 10^7</td>
<td>75,000</td>
<td>1.4 \times 10^6</td>
</tr>
<tr>
<td>Oxygen</td>
<td>2.6 yr</td>
<td>O, Mg</td>
<td>Si, S, Ar, Ca</td>
<td>1.9</td>
<td>8.8 \times 10^6</td>
<td>75,000</td>
<td>9.1 \times 10^7</td>
</tr>
<tr>
<td>Silicon</td>
<td>18 d</td>
<td>Si, S, Ar, Ca</td>
<td>Fe, Ni, Cr, Ti</td>
<td>3.3</td>
<td>4.8 \times 10^7</td>
<td>75,000</td>
<td>1.3 \times 10^11</td>
</tr>
<tr>
<td>Iron core</td>
<td>\sim 1 s</td>
<td>Fe, Ni, Cr, Ti</td>
<td>Neutron star</td>
<td>&gt; 7.1</td>
<td>&gt; 7.3 \times 10^9</td>
<td>75,000</td>
<td>&gt; 3.6 \times 10^{15}</td>
</tr>
</tbody>
</table>
The star before the collapse

- Onion shell structure (see right)
- The inner core consists of iron-group elements (Fe, Ni, Cr, Ti, ...)
- No further nuclear fusion is possible in the iron core
- Electrons supply most of the pressure, that stabilizes the star
- Photodisintegration of iron nuclei destabilizes the iron core; e.g.:
  \[ \gamma + ^{56}\text{Fe} \rightleftharpoons 13\alpha + 4n \]
- Because of the high temperature and pressure, these reactions are in equilibrium (NSE)
Nuclear Statistical Equilibrium (NSE)

• For a realistic calculation, one must treat the entire set of nuclei

• Typical reactions:
  • \((Z, A) + \gamma \rightleftharpoons (Z - 1, A - 1) + p\)
  • \((Z, A) + \gamma \rightleftharpoons (Z, A - 1) + n\)
  • \((\gamma, \alpha), (n, \alpha), (p, \alpha), (n, p)\)

• Chain of reactions, which are all in equilibrium:
  \((Z, A) + \gamma \rightleftharpoons \ldots \rightleftharpoons Zp + (A - Z)n\)

• From thermodynamics we know:
  \[dU = TdS - p\, dV + \sum_i \mu_i dN_i\]

• This gives in equilibrium:
  \[\sum_i \mu_i dN_i \rightleftharpoons 0\]

and hence:
\[\mu(Z, A) = Z\mu_p + (A - Z)\mu_N.\]
Nuclear Statistical Equilibrium (NSE)

\[ \mu(Z, A) = Z \mu_p + (A - Z) \mu_n. \]  

(1)

- The nuclei obey Boltzmann statistics for each species \(i\):

\[ n_i = g_i \left( \frac{m_i kT}{2\pi \hbar^2} \right)^{3/2} \exp \left( \frac{\mu_i - m_i c^2}{kT} \right) \]

where \(g_i\) is the nuclear partition function:

\[ g_i = \sum_r (2I_r + 1) \cdot e^{-E_r/kT} \]

\[ \Rightarrow \mu_i = m_i c^2 + kT \cdot \ln \left[ \frac{n_i}{g_i} \left( \frac{2\pi \hbar^2}{m_i kT} \right)^{3/2} \right] \]

(2)

- By inserting (2) into (1) for each species, one obtains the Saha equation:

\[ n(Z, A) = \frac{g(Z, A) A^{3/2}}{2^A} n_p^Z n_n^{A-Z} \left( \frac{2\pi \hbar^2}{m_u kT} \right)^{3/2(A-1)} \exp \left( \frac{E_B(Z, A)}{kT} \right) \]

with the nuclear binding energy:

\[ E_B(Z, A) = c^2 [Z m_p + (A - Z) m_n - M(Z, A)] \]
NSE: Solving the Saha equation

\[ n(Z, A) = \frac{g(Z, A) A^{3/2}}{2^A} n_p n_n Z n^{A-Z} \left( \frac{2\pi \hbar^2}{m_u kT} \right)^{3/2(A-1)} \exp \left( \frac{E_B(Z, A)}{kT} \right) \]

- In order to solve the Saha equation, one must specify \( n_p \) and \( n_n \) via:
  - baryon conservation: \( \sum_i n_i A_i = n = \frac{\rho}{m_u} \)
  - charge conservation: \( \sum_i n_i Z_i = n Y_e \)

- In NSE, the composition of the matter is determined by \( \rho, T \) and \( Y_e \).
- \( Y_e \) is identical to the proton-to-nucleon ratio and can be obtained from the prior evolutionary history.
The beginning of the collapse

- When the iron core reaches a mass of about 1.5 solar masses, it becomes unstable:
  - Photodisintegration of iron nuclei in NSE uses up energy
  - Neutrino cooling
  - Electron capture by protons:
    - Pressure gets reduced even more
    - Neutronization of the matter
    - $e^- + p \rightarrow \nu_e + n$
  - Self-enhancing process
Core collapse mechanism Pic. 2

- Neutrino trapping: diffusion time > collapse time
- NSE shifts to heavier and more neutron-rich nuclei
NSE abundance distributions

- Before the collapse:

- During neutrino trapping:
Core collapse mechanism Pic. 3

- Nuclear density: \( \rho_0 \approx 10^{14} \text{ g/cm}^3 \)
- Matter cannot get compressed further
- \( \rightarrow \) Bounce and Shock Formation
Core collapse mechanism Pic. 4

- Shock dissociates iron nuclei into free protons and neutrons
- This uses up ca. 8-9 MeV per Nucleon
- Neutrino burst at shock breakout:
  \[ e^- + p \rightarrow \nu_e + n \]
- Eventually, the shock wave stalls
- No explosion due to the prompt shock!
How can the shock get revived?

- Possible solution: “delayed neutrino-heating mechanism”
- Where do the neutrinos come from?
  - $\nu_e$ can diffuse out of the proto-neutron star
  - Neutrino cooling:
    - Inverse $\beta$-decay: $e^- + p \rightarrow n + \nu_e$
    - $e^+ + n \rightarrow p + \bar{\nu}_e$
    - Pair annihilation: $e^- + e^+ \rightarrow \nu + \bar{\nu}$
    - Bremsstrahlung: $N + N \rightarrow N + N + \nu + \bar{\nu}; \ N = p, n$
- Neutrinos carry most of the energy released during collapse, 10% of that energy would be enough to reanimate the shock
- Neutrino heating:
  - $\nu_e + n \rightarrow e^- + p,$
  - $\bar{\nu}_e + p \rightarrow e^+ + n$
Core collapse mechanism Pic. 5

- Competition between neutrino heating and cooling
- Neutrino heating causes convective overturn and increases pressure behind the shock → “hot bubble”
- Persistent neutrino heating drives the shock outwards again → Explosion!
Problems with the “delayed neutrino-heating mechanism”

- In spherically symmetric simulations, only stars with 8-10 solar masses and a ONeMg core explode.
- However, multi-dimensional models including convection could solve this puzzle.
Nucleosynthesis

Abundances come from the following processes:

- During the lifetime of the star:
  - Nuclear fusion
    - Almost all nuclei of the iron group get dissociated during core collapse
- While or after the supernova explosion:
  - Explosive burning
  - p-process
  - possibly r-process (?)
Nucleosynthesis: explosive burning

• Explosive oxygen and silicon burning
  • Between 3-4 billion K (oxygen) and 4-5 billion K (silicon)
  • Products are similar to the normal burning
  • “alpha-rich freeze out”:
    – oxygen burning produces many alpha particles
    – At low densities and at high expansion times, the alpha particles cannot reassemble
    – Radioactive abundances: $^{44}$Ti, $^{56,57}$Ni, etc.

• Explosive neon and carbon burning
  • Between 2-3 billion K
  • Primary products are similar to the normal burning
  • Neutron-rich isotopes, e.g.: $^{60}$Fe (radioactive)
Nucleosynthesis: p-process

- Early ejecta are rather proton-rich (\(Y_e \approx 0.57\))
- \(\rightarrow\) p-process
  - Rapid proton capture (rp-process)
  - \(\bar{\nu}_e + p \rightarrow n + e^+\) (vp-process)
Nucleosynthesis: r-process?

- **r-process** = rapid neutron capture
- **r-process** could produce heavy neutron-rich nuclei with $A \approx 80 - 240$
- Later ejecta may be neutron-rich ($Y_e < 0.5$)
- $\alpha$-particles do not capture neutrons, because $^5\text{He}$ is unstable
- But: wind may be 2-4 times too dense to produce the very heavy nuclei...
Summary

- Prompt shock caused by core collapse is not able to trigger an explosion.

- Possible solution: Delayed neutrino heating

- Core collapse supernovae are possible sites for the r-process.

- At high temperatures and high densities, nuclear reactions are in equilibrium similar to the chemical equilibrium. This is called NSE.
Thank you for your attention!
Sources

- Woosley, Heger, Weaver: Reviews of Modern Physics 74 (2002), 1015,