Accretion: Bondi/Hoyle and Shakura/Sunyaev

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Overview

1. Introduction and motivation
2. Spherical accretion
3. Disk accretion
4. General relativistic torus accretion
This talk is based on ch. 14 of Shapiro, Teukolsky: Black Holes, White Dwarfs and Neutron Stars (John Wiley & Sons)

Much more information can be found in Frank, King, Raine: Accretion Power in Astrophysics (Cambridge University Press)
1. Introduction and motivation
Accretion onto compact objects

- Accretion is inflow of matter onto compact objects by gravitational forces and transport of angular momentum

- Compact objects: White dwarfs, neutron stars, black holes

- Important, because accretion can be much more effective in converting rest-mass into radiation than e.g. fusion

- Complex physical process: (magneto-) hydrodynamics, radiative transfer, special or general relativity, chemistry and atomic physics

- In this talk: only simple scenarios under consideration
Astrophysical relevance

- Accreted matter will heat up $\Rightarrow$ EM radiation
- Effectively this is a conversion of gravitational binding energy (Newtonian viewpoint) or rest-mass (GR viewpoint) into radiation flux
- Primary radiation may be reprocessed (e.g. in AGN cores)
- More complex accretion models seek also to explain the core engine of jets in AGN or microquasars
Models of accretion

- Analytical models: Strong simplification of the accretion problem

- Possible assumptions:
  - Special relativistic or Newtonian mechanics
  - One particle species for the mechanics
  - No magnetic fields
  - Macroscopic charge neutrality
  - Hydrodynamic description with stationary flow
  - Further symmetries and accretion geometries
# Models of accretion

<table>
<thead>
<tr>
<th>Model</th>
<th>Assumptions</th>
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<tbody>
<tr>
<td>Zel’dovich/Novikov</td>
<td>Newtonian mechanics, collisionless gas (Vlasov equation), spherical symmetry</td>
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<tr>
<td>Bondi/Hoyle</td>
<td>Newtonian mechanics, adiabatic fluid (HD), spherical symmetry</td>
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<tr>
<td>Shakura/Sunyaev</td>
<td>Newtonian mechanics, viscous fluid (HD), simple radiative transfer, axisymmetry, disk geometry</td>
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<tr>
<td>Michel</td>
<td>GR mechanics, adiabatic fluid (HD), spherical symmetry</td>
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<tr>
<td>Other</td>
<td>Newtonian and GR models of accretion disks</td>
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2. Spherical accretion
Bondi/Hoyle accretion

- Newtonian mechanics, central potential for gravity
- Hydrodynamic flow, spherical symmetry, stationarity \(\Rightarrow\) 1D problem
- Self-gravitation of accreted matter is neglected
- Adiabatic, non-viscous fluid
- No radiative transfer
- Accretion sphere is stabilized by pressure gradients
Flow description

- 1D radial flow: characterised by density $\rho(r)$, velocity $u(r)$ and pressure $P(r)$

- HD equations for 1D adiabatic, non-viscous problems:

<table>
<thead>
<tr>
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<th>3D</th>
<th>Radial (1D)</th>
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<tbody>
<tr>
<td>Continuity</td>
<td>$\frac{\partial \rho}{\partial t} + \nabla (\rho u) = 0$</td>
<td>$\frac{1}{r^2} \frac{d}{dr} (r^2 \rho u) = 0$</td>
</tr>
<tr>
<td>Momentum</td>
<td>$\frac{\partial u}{\partial t} + (u \nabla) u = \frac{1}{\rho} \nabla P - \nabla \Phi$</td>
<td>$u \frac{du}{dr} = -\frac{1}{\rho} \frac{dP}{dr} - \frac{GM}{r^2}$</td>
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<tr>
<td>EOS</td>
<td>$P = K \rho^\Gamma$</td>
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• The *Bondi equations* are the integrated version of the radial hydro equations:

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<td>$4\pi r^2 \rho u = \dot{M}$</td>
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<tr>
<td><strong>Momentum</strong></td>
<td>$\frac{1}{2}u^2 + \frac{1}{\Gamma-1}a^2 - \frac{GM}{r} = \frac{1}{\Gamma-1}a_\infty^2$</td>
</tr>
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</table>

• Here we have used $u_\infty = 0$ and the sound speed: $a = \sqrt{dP/d\rho}$

• Given boundary conditions at infinity and at the surface of the CO, we can determine flow solutions

• A solution consists of $u(r)$ and $\rho(r)$
Spherical black hole accretion

- \( u(r) \) at small radii: free-fall speed

- Bondi equations can be solved and show the existence of a \textit{transsonic radius} \( \Rightarrow \textit{transsonic accretion} \)

- At the transsonic radius the accretion gets supersonic

- For general boundary conditions also subsonic accretion and wind (outflow) solutions of the Bondi equations exist
Radiation from Bondi/Hoyle accretion onto black holes

- Determining radiation efficiency is very important: We want to relate the accretion rate to luminosity.

- Because the Bondi/Hoyle scenario is adiabatic, we neglect back-reactions of the radiation on the flow.

- Dominant emission process is thermal bremsstrahlung, which depends on density and temperature.

- The emission is dominated by the flow near the event horizon (high density and temperature).

- Result: Bondi/Hoyle is inefficient accretion process for black hole boundary conditions:

\[ \epsilon = \frac{L}{\dot{M}c^2} \approx 6 \cdot 10^{-11} \frac{M}{M_\odot} \]
3. Disk accretion
Shakura/Sunyaev accretion

- Newtonian mechanics, central potential for gravity
- Hydrodynamic flow, axisymmetric disk geometry, stationarity $\Rightarrow$ 1D problem
- Disk is stabilized by centrifugal forces
- Self-gravitation of accreted matter is neglected
- Non-adiabatic, viscous fluid
- Simple radiative transfer
Disk accretion: General features

- Accreted matter may have angular momentum $\Rightarrow$ high angular velocity $u_\phi$ near the CO.

- Accretion disk consists of rings of nearly Keplerian orbits.

- $l_{Kepler} \propto \sqrt{r} \Rightarrow$ inward motion only possible through transport of angular momentum (slow).

- Thus matter has much time to radiate binding energy away.

- Much more efficient at generating luminosity than spherical accretion!

- Viscosity has two functions here:
  - Transport of angular momentum
  - Heat generation
Towards a model of thin accretion disks

- Thin disk means $h(r) \ll r$

- Approximate vertical equations by taking mean values or finite differences

- Shakura/Sunyaev thin disk model is a solution of a system of nine equations:
  - Definition of the surface density
  - 4 conservation equations (mass, angular momentum, energy, vertical momentum)
  - EOS (with gas and radiation pressure contributions)
  - Viscosity law
  - Opacity and radiative transfer from the center ($z = 0$) to the disk surface ($z = \pm h$)
Sketch of a thin accretion disk
Mass conservation

- Steady-state situation: Mass inflow (at infinity) per time is mass outflow (at the CO) per time

- Divide the disk into cylindrical rings; define the surface density of any ring by

  \[ \Sigma(r) \equiv 2h(r)\rho(r) \]

- Continuity equation demands that the flow through each cylinder at \( r \) is independent of \( r \):

  \[ 2\pi r \Sigma v_r = \dot{M} = \text{const} \]
Angular momentum conservation

- Motion on Keplerian orbits ⇒ accretion rate determines angular momentum inflow rate at $r$
  \[ \dot{J}^+ = \dot{M} \sqrt{GMr} \]

- Consumed rate of angular momentum at the CO:
  \[ \dot{J}^- \leq \dot{M} \sqrt{GMr_I} \]

- Let $f_\Phi$ be the viscous stress along the normal $e_\Phi$

- Torque is (force along $e_\Phi / \text{area}$) $\cdot$ (area) $\cdot$ $r$ and also equal to the difference between $\dot{J}^+$ and $\dot{J}^-$:
  \[ (f_\Phi)(2\pi r \cdot 2h)(r) = \dot{J}^+ - \dot{J}^- \]
Energy conservation

- Local viscous stresses generate heat, which we assume to be totally radiated away (and not stored)

- The stress \( f_\Phi \) determines the heat rate \( \dot{Q} \)

- Because \( f_\Phi = f_\Phi (r, \dot{M}) \), we get a relation

\[
\dot{Q} = \dot{Q}(f_\Phi) = \dot{Q}(r, \dot{M})
\]

- For the radiation flux at the surface this yields

\[
F(r) = \frac{1}{2} \cdot 2h\dot{Q}
\]
Vertical momentum conservation

- Determined by hydrostatic equilibrium

- Vertical Euler equation for $z \ll r$:

\[
\frac{1}{\rho} \frac{dP}{dz} = -\frac{GM}{r^2} \frac{z}{r}
\]

- Replacing differentials by finite differences yields

\[
\frac{P}{\rho h} \approx -\frac{GM}{r^2} \frac{h}{r} \Rightarrow h \approx \sqrt{\frac{P}{\rho} \sqrt{\frac{r^3}{GM}}} \approx \frac{c_s}{\Omega}
\]
Equation of state

- Assume a hydrogen gas in local thermodynamical equilibrium (LTE)
- Assume further a locally near blackbody radiation
- The pressure is related to temperature and density by

\[ P(\rho, T) = \frac{2\rho kT}{m_p} + \frac{1}{3} a T^4 \]

and is thus a sum of gas and radiation pressure
- Only in the innermost regions will radiation pressure dominate gas pressure
Viscosity law

- Several physical effects may be responsible for viscosity (turbulence, magnetic effects)

- We assume a simple empirical model following Shakura and Sunyaev:

\[ \eta \approx \rho v_{turb} l_{turb} \]

- This turbulent cell model sets limits on the generated force \( f_\Phi \) by \( v_{turb} \leq c_s \) and \( l_{turb} \leq h \):

\[ f_\Phi \leq P \]

- Introducing the parameter \( \alpha \) we define an \( \alpha \)-disk model by

\[ f_\Phi = \alpha P \]
Opacity and scattering

- To determine the transport of photons from the center to the disk surface, we need to (approximately) include absorption and scattering.

- The dominant absorption process is *thermal free-free absorption*:
  \[ \kappa_{abs} \propto \rho T^{-7/2} \]

- The dominant scattering process is *Thomson scattering*:
  \[ \kappa_{scatt} \propto 1 \]

- The total mean opacity is then
  \[ \frac{1}{\kappa(\rho, T')} = \left\langle \frac{1}{\kappa_{abs} + \kappa_{scatt}} \right\rangle \approx \frac{1}{\kappa_{abs}} + \frac{1}{\kappa_{scatt}} \]

- The last approximation is valid because for any \( r \) only one process, free-free absorption or Thomson scattering, is assumed to be important.
Radiative transport

- Transport from center to surface depends on the *optical thickness*: 
  \[ \tau = \int_0^h \kappa \rho \, dz \]

- For an \( r \) with \( \tau < 1 \) the disk is *optically thin*; then most photons escape freely, the flux is a sum over the local emissivity \( \Lambda \):
  \[ F(r) \approx \Lambda \, h \]

- For an \( r \) with \( \tau \geq 1 \) the disk is *optically thick*; then photons will scatter and be transported by diffusion. The flux on the surface is approximately given by
  \[ F(r) \approx \frac{acT^4}{\tau} \]
The full system of equations

- **Def. surface density** \( \Sigma = 2h\rho \)
- **Mass conservation** \( \dot{M} = 2\pi r \Sigma v_r \)
- **Angular momentum conservation** \( 4\pi r^2 h f_\Phi = \dot{J}^+(r) - \dot{J}^- \)
- **Energy conservation** \( F = h \dot{Q}(r, h) \)
- **Vertical momentum conservation** \( h = c_s(P, \rho)/\Omega(r) \)
- **EOS** \( P(\rho, T) = 2\rho kT/m_p + aT^4/3 \)
- **Viscosity law** \( f_\Phi = \alpha P \)
- **Opacity and scattering** \( \bar{\kappa}^{-1}(\rho, T) = \bar{\kappa}_{abs}(\rho, T) + \bar{\kappa}_{scatt}^{-1} \)
- **Radiative transport**
  \[ F(r) = \begin{cases} 
  F(r) = \Lambda(\rho, T) h & \tau < 1 \\
  F(r) = acT^4/\tau(\bar{\kappa}, \Sigma) & \tau \geq 1 
  \end{cases} \]

**Parameters** \( \alpha, M, \dot{M} \)

**Known relations** \( \dot{J}^+(r), \dot{J}^-, \dot{Q}(r, h), c_s(P, \rho), \bar{\kappa}_{abs}(\rho, T), \bar{\kappa}_{scatt}, \Lambda(\rho, T), \tau(\bar{\kappa}, \Sigma) \)

**Solution vector** \( \rho(r), h(r), \Sigma(r), v_r(r), P(r), T(r), f_\Phi(r), \bar{\kappa}(r), F(r) \)
The solution in $r$ can be divided into three regions:

- **Outer region**: Gas pressure dominates radiation pressure, opacity is free-free absorption
- **Middle region**: Gas pressure dominates radiation pressure, opacity is electron scattering
- **Inner region**: Radiation pressure dominates gas pressure, opacity is electron scattering

Depending on the choice of parameters, not all regions will exist.
Maximal radiation efficiency: Depending on inner boundary conditions \( \epsilon \approx 0.06 \) (Schwarzschild) or \( \epsilon \approx 0.40 \) (maximal Kerr)
4. General relativistic torus accretion
General relativistic accretion

- We are looking for a stationary solution of GR accretion disks, but will not include viscosity.

- The basic idea is instead: accretion is driven by pressure-gradient forces over a sharp *cusp* near the black hole (analogous to the $L_1$ lagrange point in binaries).

- Start with GR hydrodynamic equations on a chart:

\[
\begin{align*}
J^\mu &= \rho u^\mu \\
T^{\mu \nu} &= \rho h u^\mu u^\nu + p g^{\mu \nu} \\
\nabla_\mu J^\mu &= 0 \\
\nabla_\mu T^{\mu \nu} &= 0
\end{align*}
\]

- Barotropic EOS: $p = K \rho^\Gamma$
The torus (thick disk) model

- Assume a four-velocity of the form $u^\mu = (u^t, 0, 0, u^\Phi)$
- Allow thermal pressure to inflate the disk to a torus
- Project the conservation equation $\nabla_\mu T^{\mu\nu} = 0$ onto the spacelike 3-hypersurface
- Integrate to get the surfaces of constant pressure defined by an effective potential $W(r, \theta)$
- If we further assume a constant angular momentum

$$l = -u_\Phi/u_t = \text{const}$$

we can perform the integration analytically to get

$$W(r, \theta) = \ln u_t$$
Equipotential surfaces of a relativistic accretion torus
Summary

- Accretion is inflow of matter onto compact objects by gravitational forces and transport of angular momentum.
- Astrophysical importance: Conversion of gravitational binding energy into radiation flux.
- Spherically symmetric accretion is stabilized by pressure gradients.
- Disk accretion is stabilized by centrifugal forces.
- Disk accretion rate is determined by viscosity (= angular momentum transport).
- Complete Shakura/Sunyaev disk model includes conservation, EOS, viscosity and simple radiation transport.
- Relativistic accretion torus is not driven by viscosity, but by pressure over a sharp cusp near the black hole.