Solitons, granules, miniclusters: axions gone nonlinear

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Collaborators:

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Simplifications

In the context of structure formation, we usually assume that axions are

- classical

  disagreeing views:

  1. Sikivie et al.
  2. Lentz et al.

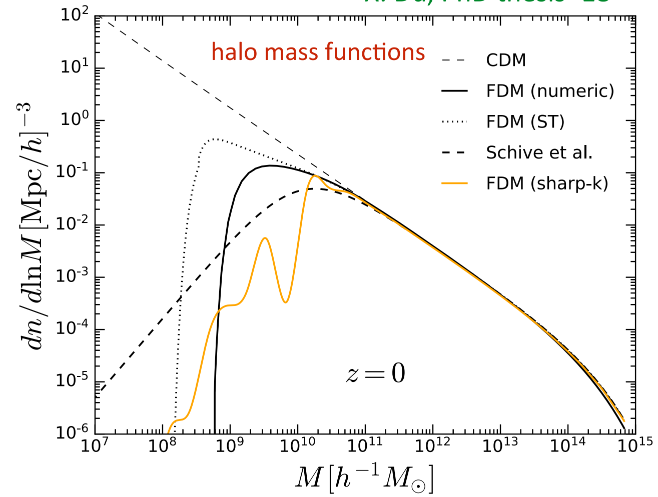
- nonrelativistic

- only gravitationally coupled

→ m is the only free parameter

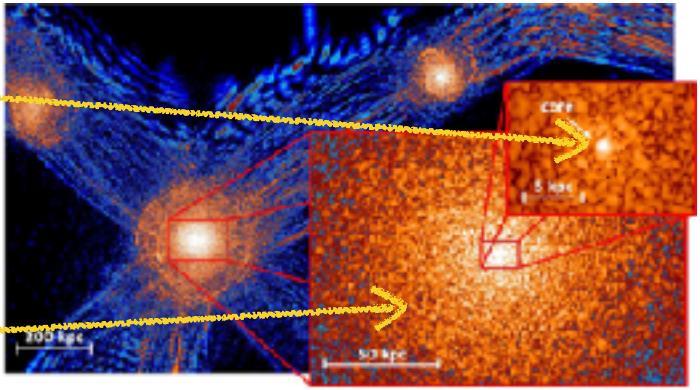
Axion DM phenomenology

X. Du, PhD thesis ´18



Ultralight axions

- Suppression of small-scale perturbations *(„WDM-like“)*
  - high-z luminosity functions (Bozek+ ’15, Schive+ ’16, Corasaniti+ ’17, Menci+ ’17)
  - Lyman-α forest (Iršič+ ’17, Armengaud+ ’17, Rogers+ ‘20) → m ≳ 10^{-20} eV
  - reionization (Bozek+ ’15; Schneider ’18; Lidz, Hui ’18)
- Formation of coherent solitonic halo cores *(„SIDM-like“)*
  - cusp-core etc., halo substructure (Marsh,Silk ’13, Schive+ ’14, Marsh,Pop ’15, Calabrese,Spergel ’16, Du+ ’16)
- Incoherent interference patterns and granularity on scales of λ_{dB} ~ 1 … 100 kpc
  - „quasi-particle relaxation“ → dynamical friction / heating / diffusion (Hui+ ’17, Bar-Or ´18, Marsh & JN ´18) *(„PBH-like“)*

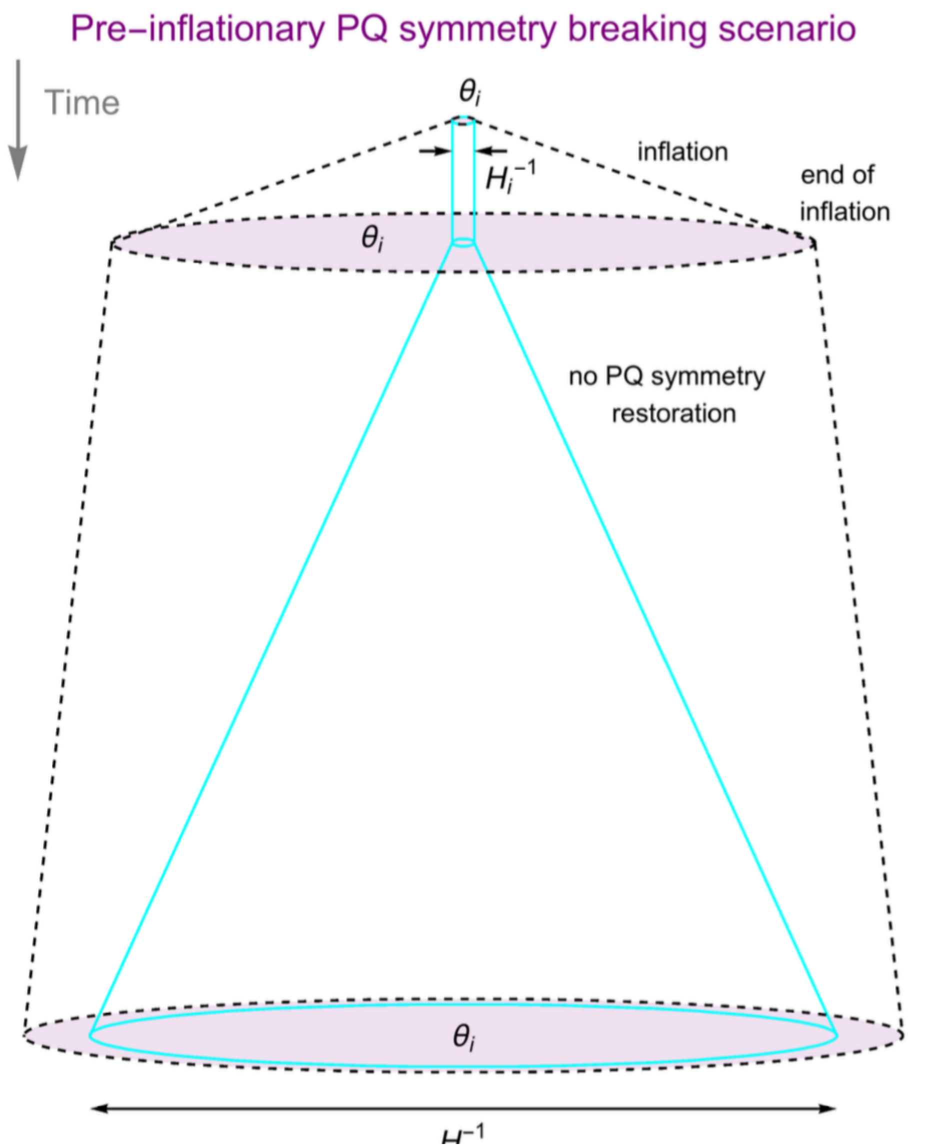


Schive+ ’14

QCD axions

- Formation of axion miniclusters (Hogan,Rees ’88; Kolb,Tkachev ’93/94; Zurek+ ’07)
  - relevant for direct detection experiments
  - potentially observable in fast radio bursts, tidal streams, microlensing (Tkachev ’15, Tinyakov+ ’16, Fairbairn+ ’17)
- Formation of axion stars (Tkachev ’86; Levkov+ ’18, Eggemeier & JN ‘19)

I. Ultralight axions



K. Saikawa

Ultralight Axion (Fuzzy) Dark Matter

- Ultralight axions (or axion-like particles) are a candidate for fuzzy dark matter (Hu, Barkana, Gruzinov '00)
- Assume:
  - Production by misalignment (non-thermal) → cold condensate
  - Small mass → large occupation numbers → classical field equations
  - $H \ll m$, k → Newtonian dynamics

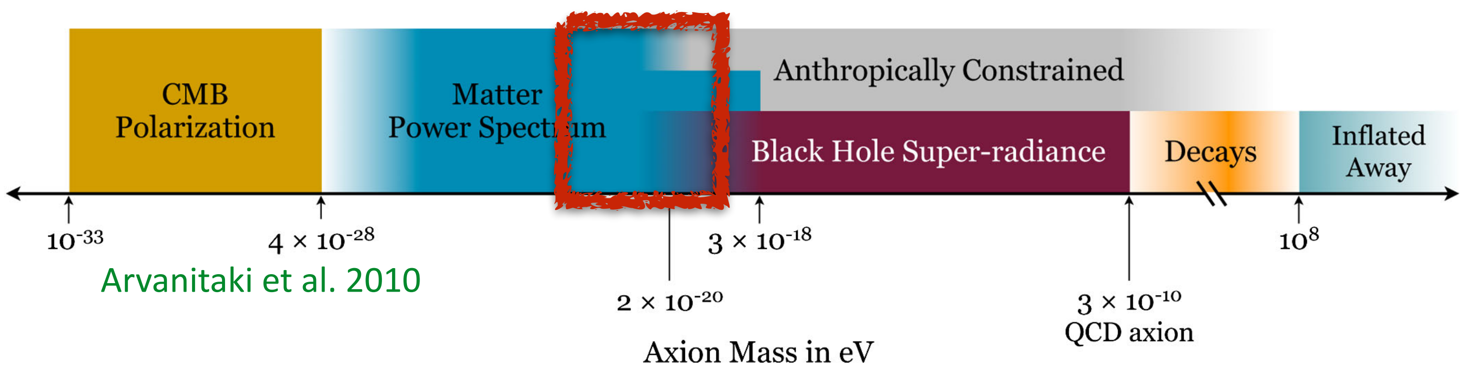
- „Quantum pressure" prevents gravitational collapse of structures ~ below the de Broglie wavelength:

$$v \sim (G\rho)^{1/2} r \quad \Rightarrow \quad \lambda \sim (mv)^{-1} \sim m^{-1}(G\rho)^{-1/2} r^{-1}$$

- „Quantum Jeans length":

$$r_J = \lambda = r$$

$$r_J = 2\pi/k_J = \pi^{3/4}(G\rho)^{-1/4} m^{-1/2}\,,$$
$$= 55 m_{22}^{-1/2}(\rho/\rho_b)^{-1/4}(\Omega_m h^2)^{-1/4}\mathrm{kpc} \qquad m_{22} = m/10^{-22}\mathrm{eV}$$



Arvanitaki et al. 2010

In the Newtonian limit, ULAs obey the Schrödinger-Poisson (SP) equations:

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2a^2m}\nabla^2\psi + mV\psi}$$

$$\nabla^2 V = 4\pi G a^2 \delta\rho = \frac{4\pi G}{a}\rho_0(|\psi|^2 - 1)$$

Scaling symmetry of the Schrödinger-Poisson equations:

$$\{t, \mathbf{x}, V, \Psi\} \to \{\lambda^{-2}t, \lambda^{-1}\mathbf{x}, \lambda^2 V, \lambda^2\Psi\}$$
$$\{\rho, M, K, W\} \to \{\lambda^4\rho, \lambda M, \lambda^3 K, \lambda^3 W\}\ .$$

Dimensional analysis for Newtonian boson stars / axion stars / solitonic cores:

- dynamical time: $t \sim M^{-1/2}\, R^{3/2} \sim \rho^{-1/2}$
- radius: $R \sim m^{-1}\, R^{-1}\, t \sim m^{-1/2}\, \rho^{-1/4}$
- mass: $M \sim \rho\, R^3 \sim m^{-3/2}\, \rho^{1/4} \sim m^{-2}\, R^{-1}$

Simulations with bosonic dark matter

Different scales / physics require different numerical methods.

1. **N-body with modified initial conditions**:
   CDM-like dynamics, linear / weakly nonlinear scales (Ly alpha forest, HMF)

   Dynamics of gravitationally interacting random waves equivalent to collisionless matter

2. **Madelung (fluid) formulation** (SPH, PM, or finite volume):
   same as above, includes „quantum pressure" effects, resolution requirements and validity unclear

$$\dot{\rho} + \nabla(\rho \mathbf{v}) = 0 \qquad \dot{\mathbf{v}} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla(Q + V)$$

$$\mathbf{v} = m^{-1} \nabla S \qquad Q = -\frac{\hbar^2}{2m^2} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}$$

„quantum pressure"

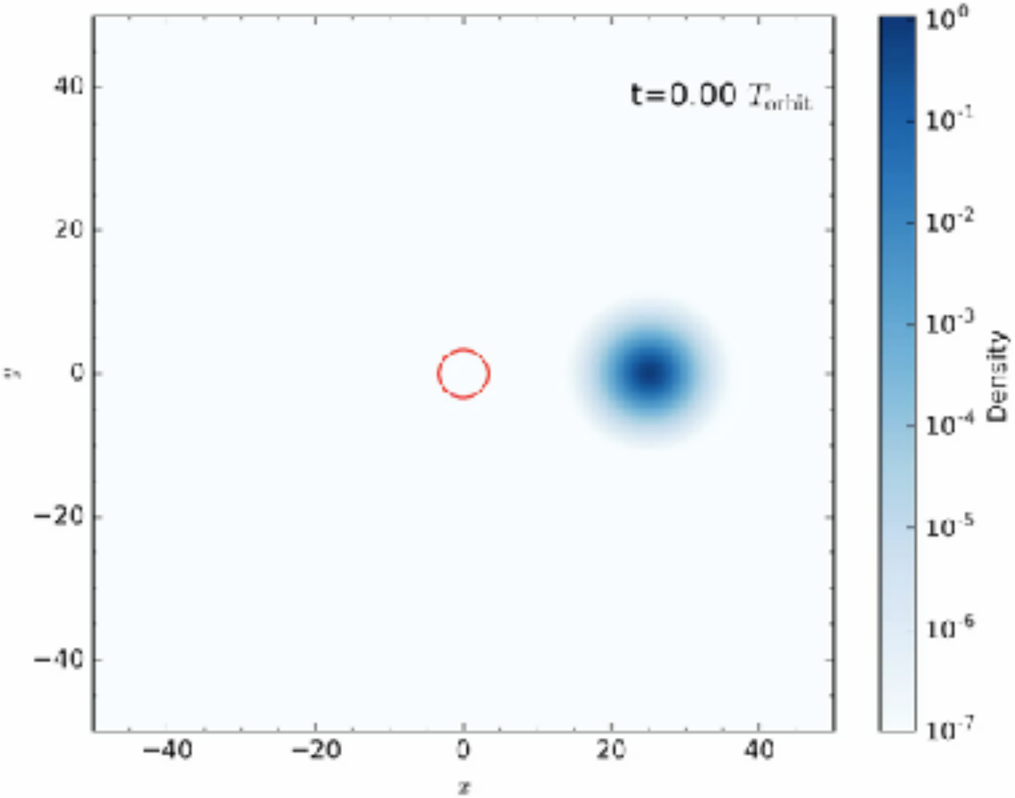
4. **Schrödinger formulation** (finite difference or pseudo-spectral):
   full wave-like dynamics, requires phase resolution, can only handle relatively small boxes, nonlinear scales

5. **Hybrid zoom-in method** (N-body on coarse grids, Schrödinger on finest grid):
   dynamics CDM-like on large scales, wave-like on small (nonlinear) scales

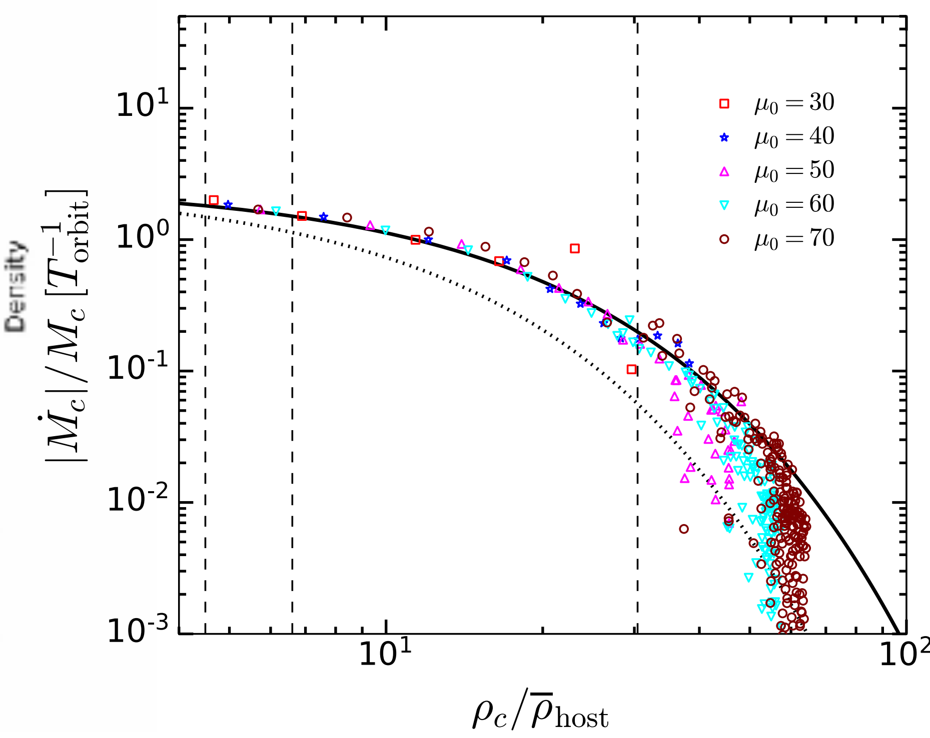
Tidal disruption of FDM substructure halos

(Du, Schwabe, JN+ ´18; PRD 97, 063507)

In addition to classical tidal stripping, FDM halos are unstable to tidal mass loss by „quantum tunnelling“ (Hui+ ’17). The mass loss rate depends only on the ratio of soliton and host density μ.



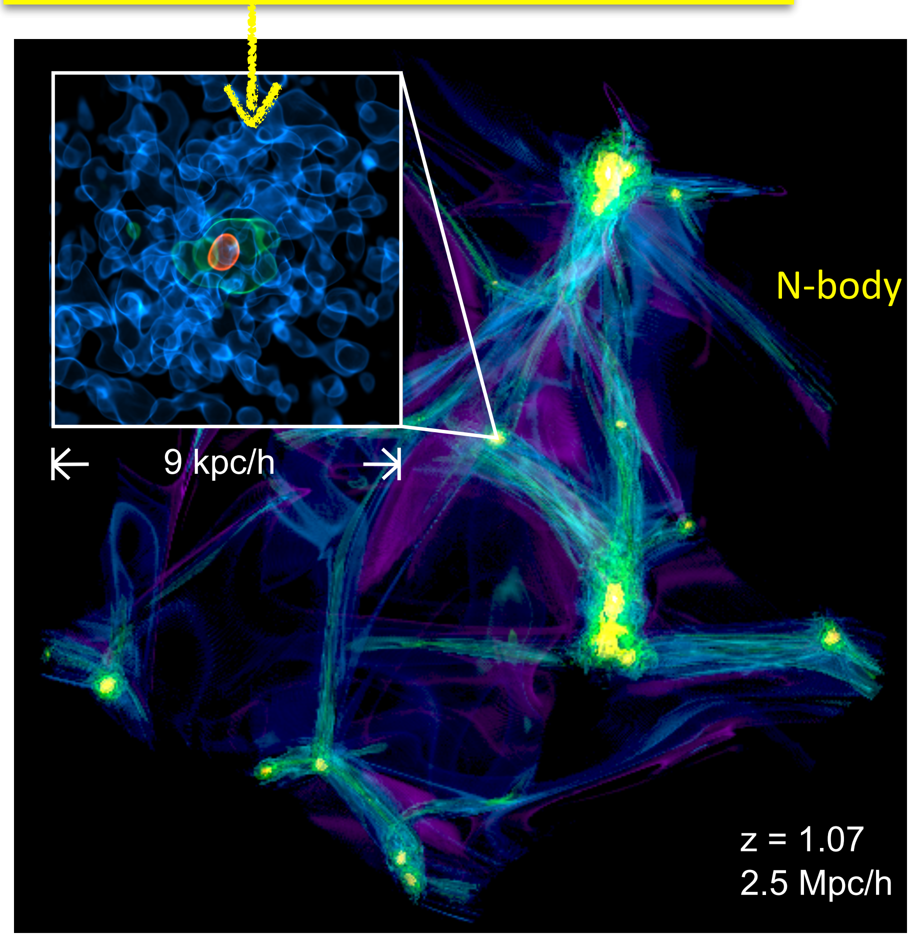
mass loss rate

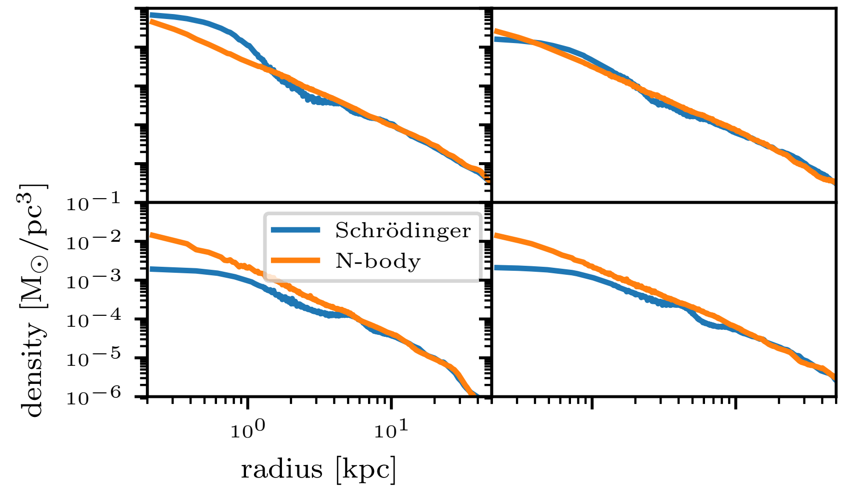


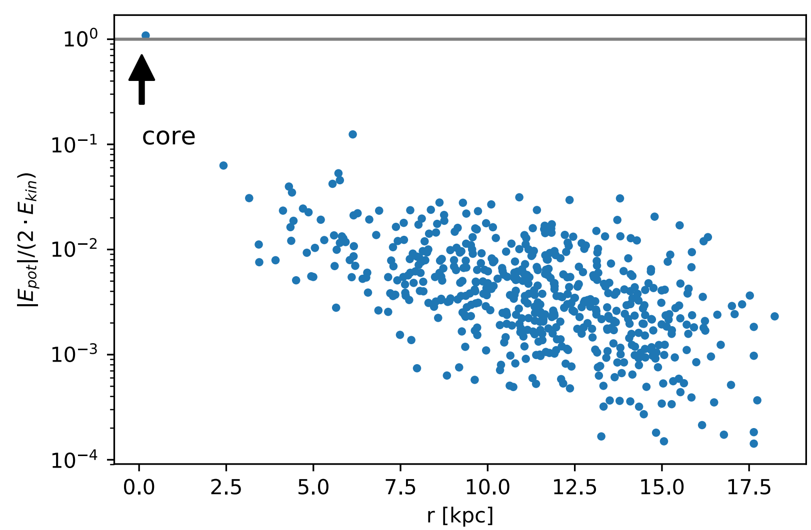
Simulations of halo formation with ultralight axion (fuzzy) dark matter

(Veltmaat, JN, Schwabe ’18, PRD 98, 043509)

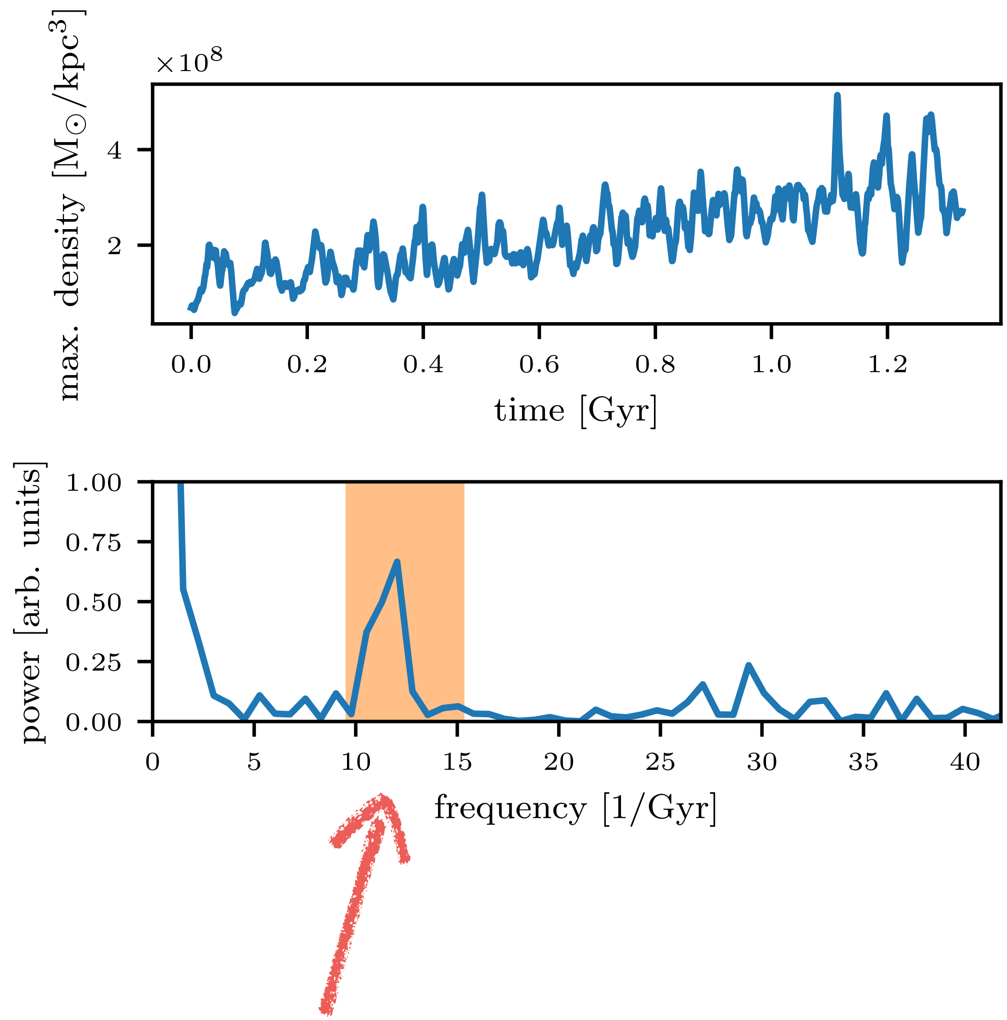
$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2a^2 m}\nabla^2\psi + mV\psi$$







Core oscillations

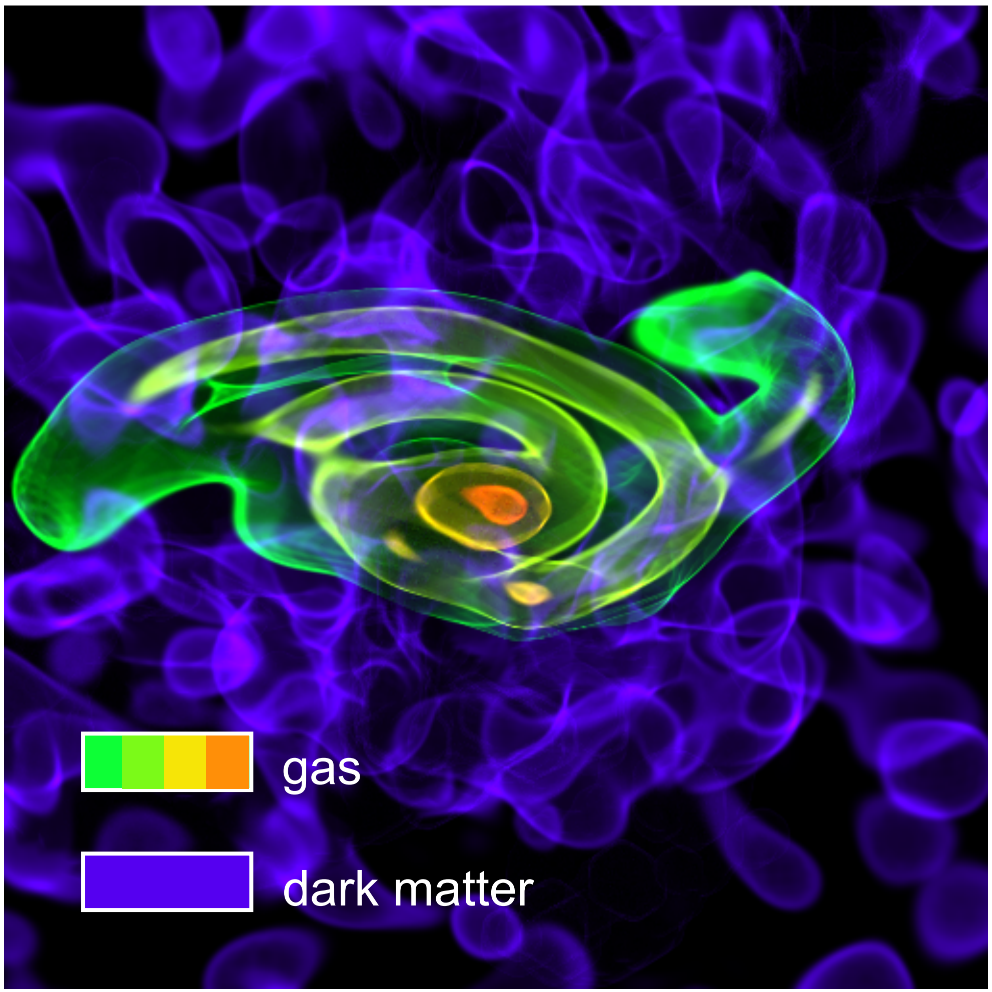


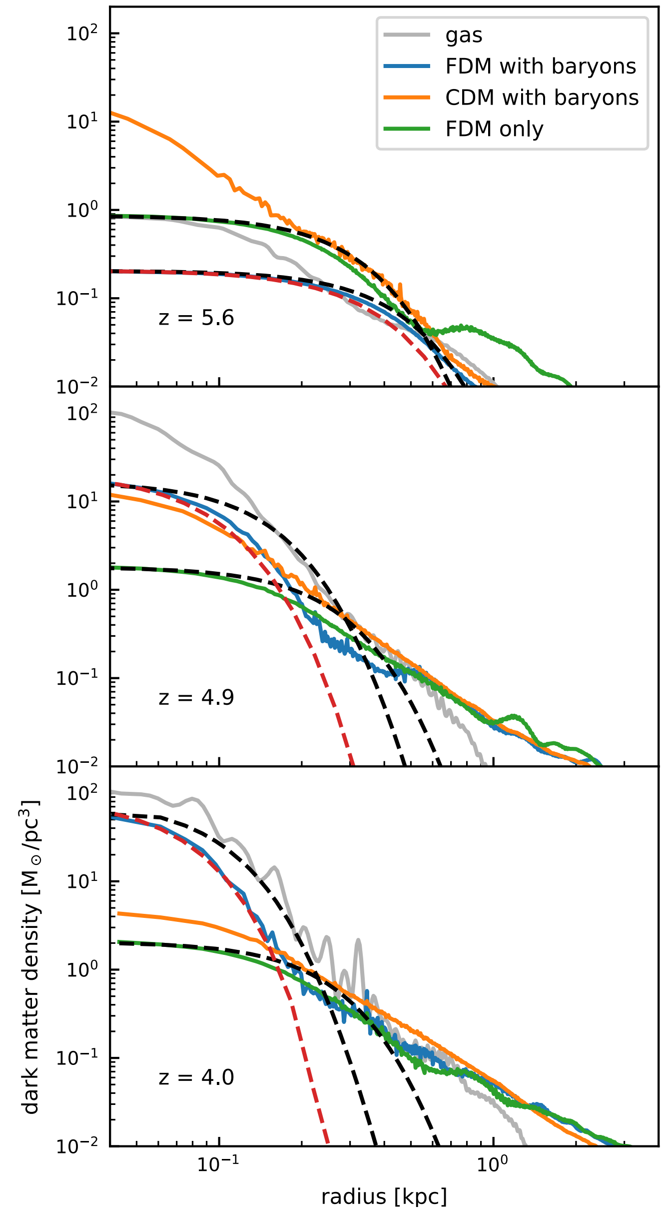
quasinormal soliton mode

FDM with baryons and star formation

(Veltmaat, Schwabe, JN, arXiv:1911.09614)

Deepened central potential → higher soliton mass, smaller radius from λ_{dB} with local velocity dispersion





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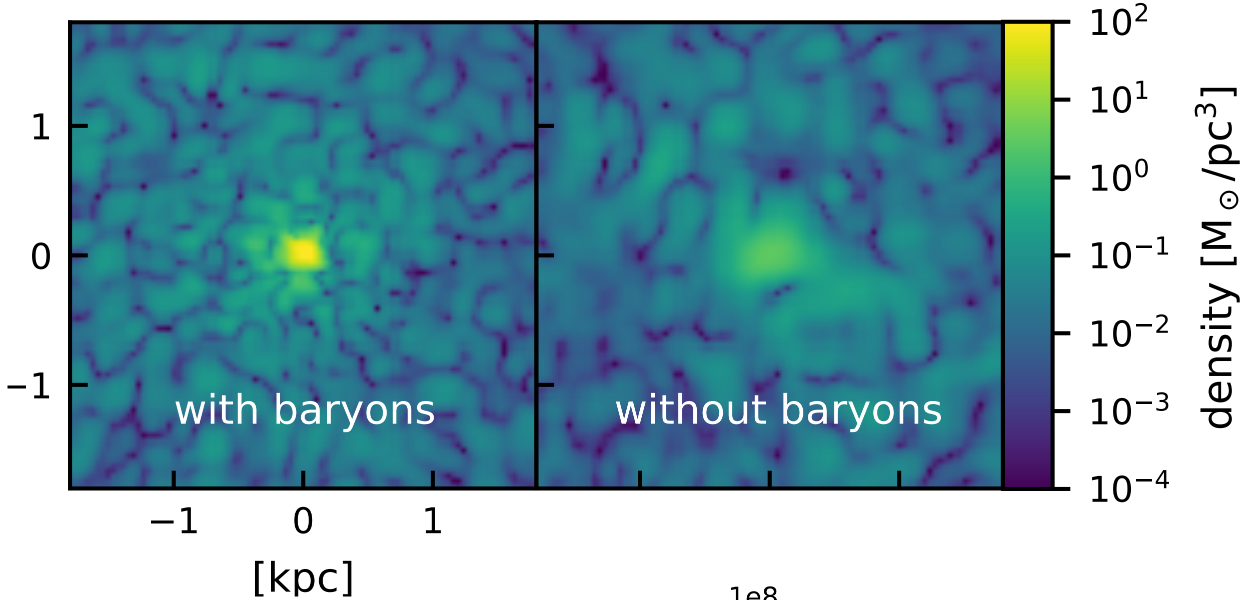
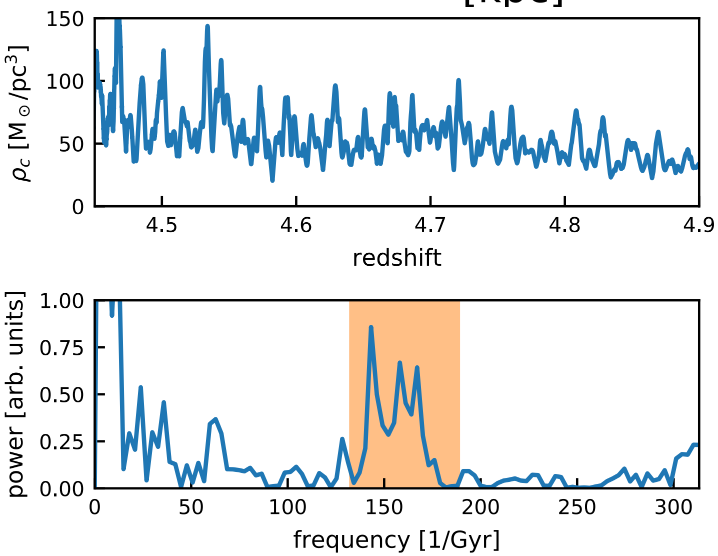
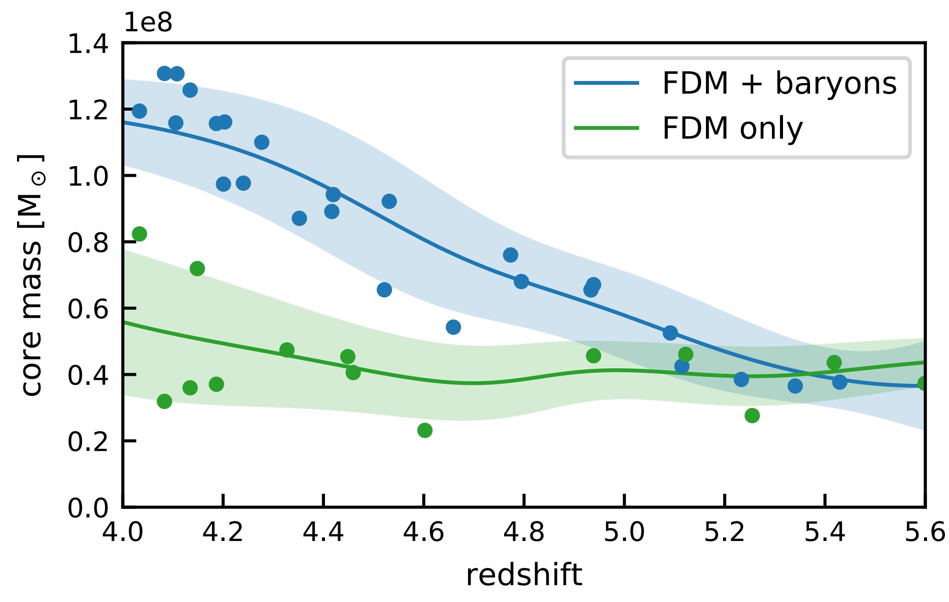






Gravitational relaxation from wave interference noise

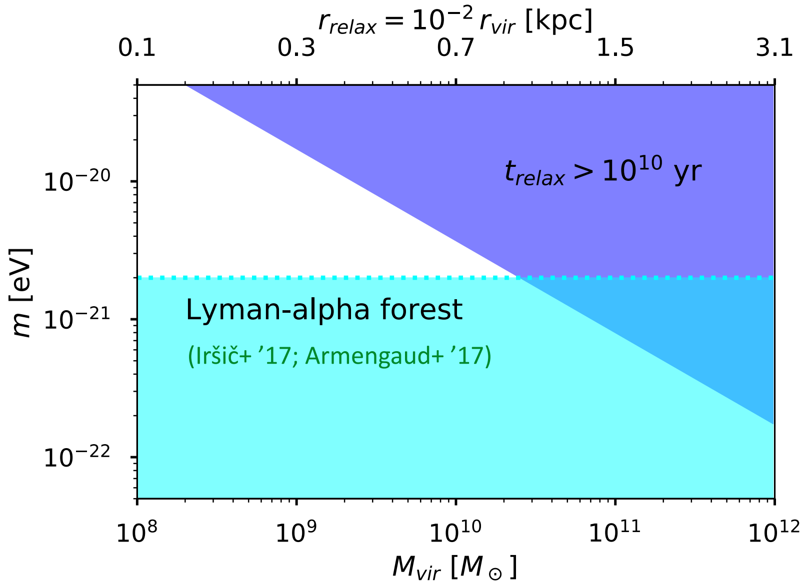
Wave nature of FDM produces O(1) density fluctuations on scale of λ_{dB}

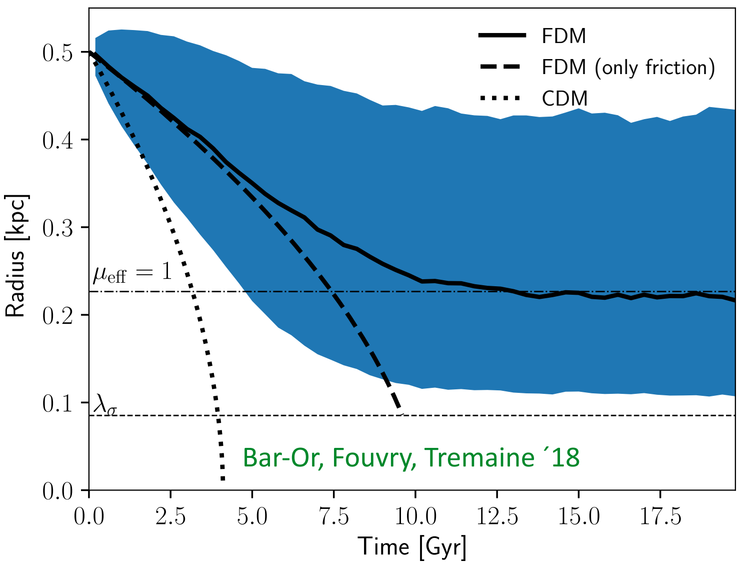
Gravitational scattering → relaxation / condensation time scale:

$$\tau \sim \frac{m^3 v^2 R^4}{h^3} \sim \left(\frac{R}{\lambda_{\mathrm{dB}}}\right)^3 \frac{R}{v}$$

from quasi-particle approximation (Hui+ ´17), shot noise diffusion (Marsh, JN ´18), or wave condensation (Levkov+ ´18).

Dynamical friction (cooling) vs. diffusion (heating) (Bar-Or+ ´18) → inspiral of SMBHs or globular clusters can be stalled by FDM.

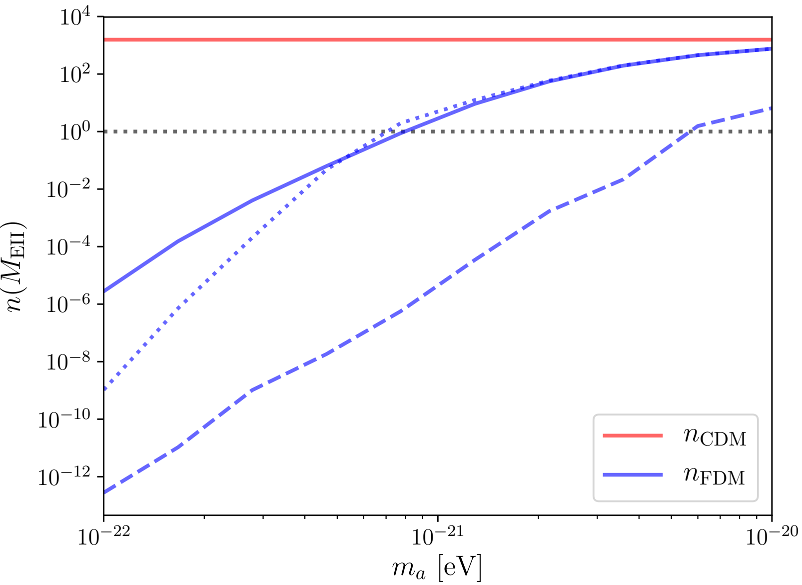


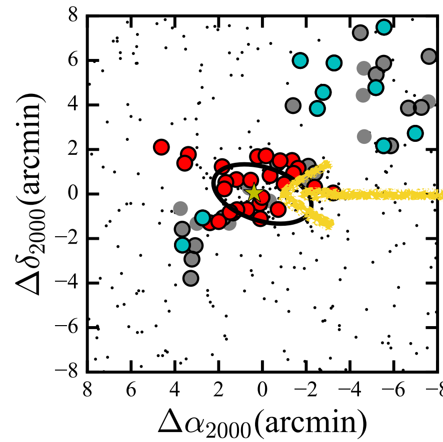


Gravitational heating constraints: Star cluster in UFD Eridanus II

(Marsh & JN ´19, PRL 123, 051103)

expected number of subhalos:



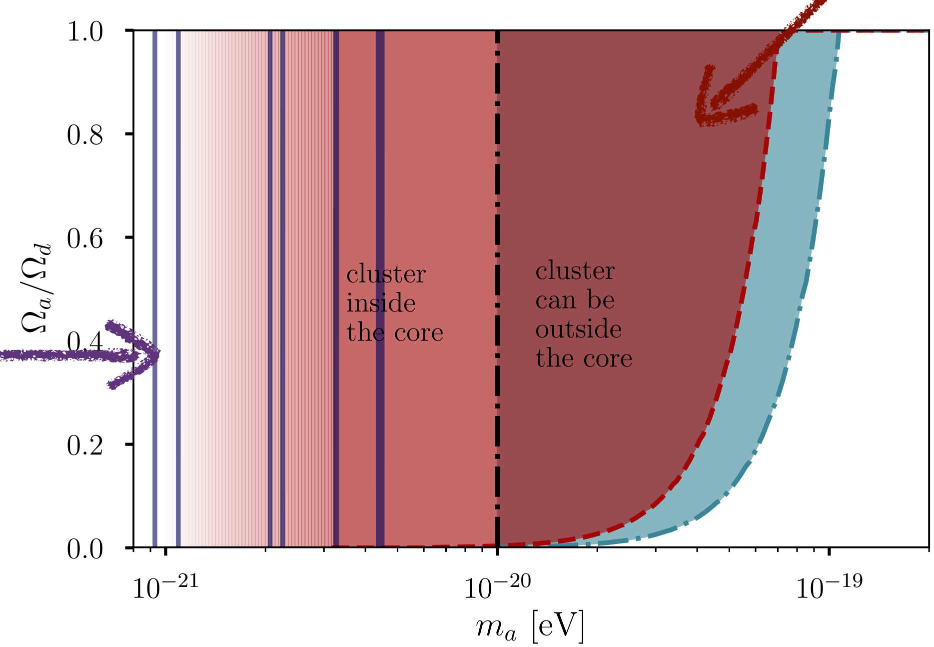


Li+ ´17

central star cluster

diffusion / heating

resonance



many uncertainties, need simulations!

Fuzzy dark matter as a superfluid

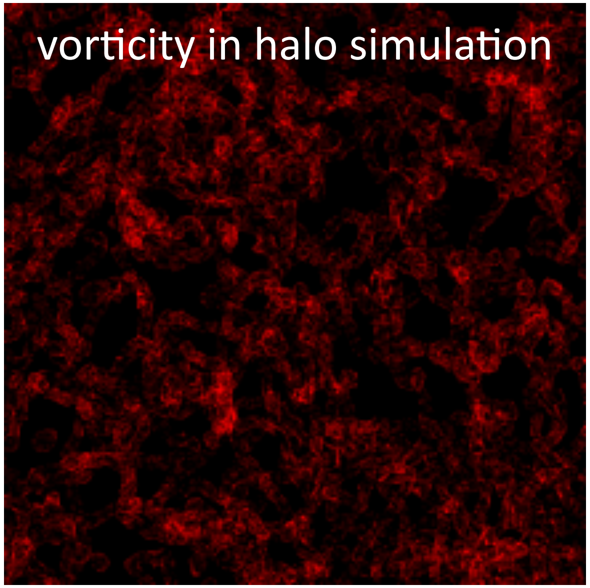
- The dynamics of FDM are described by a Gross-Pitaevskii (or nonlinear Schrödinger) equation:

$$i\dot{\psi} = \left[-\frac{1}{2m}\nabla^2 - g|\psi|^2 - Gm^2 \int \frac{|\psi(\mathbf{x}')|^2}{|\mathbf{x}-\mathbf{x}'|}\, d^3x' \right] \psi$$

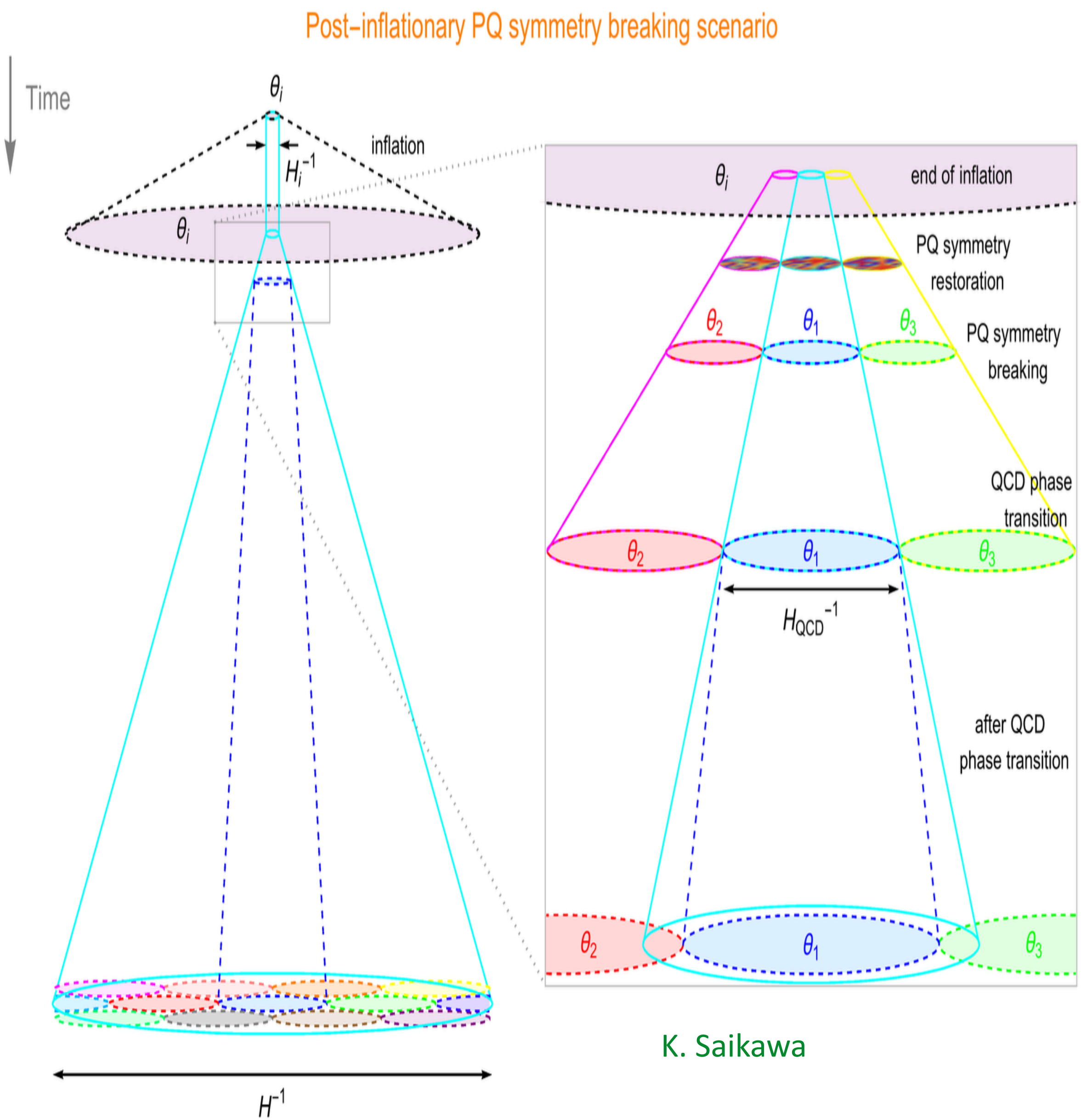
- Phenomenology similar to „quantum turbulence“ in superfluid He-II (Mocz+ ’17)?

- Differences to superfluid He-II:
  - dominant nonlinearity (gravity) is nonlocal → suppressed condensation / relaxation
  - local interaction is attractive → no stable vortex cores

- Questions:
  - do vortex lines form? (and if yes, do they matter?)
  - relaxation rate, core growth?



II. QCD axions



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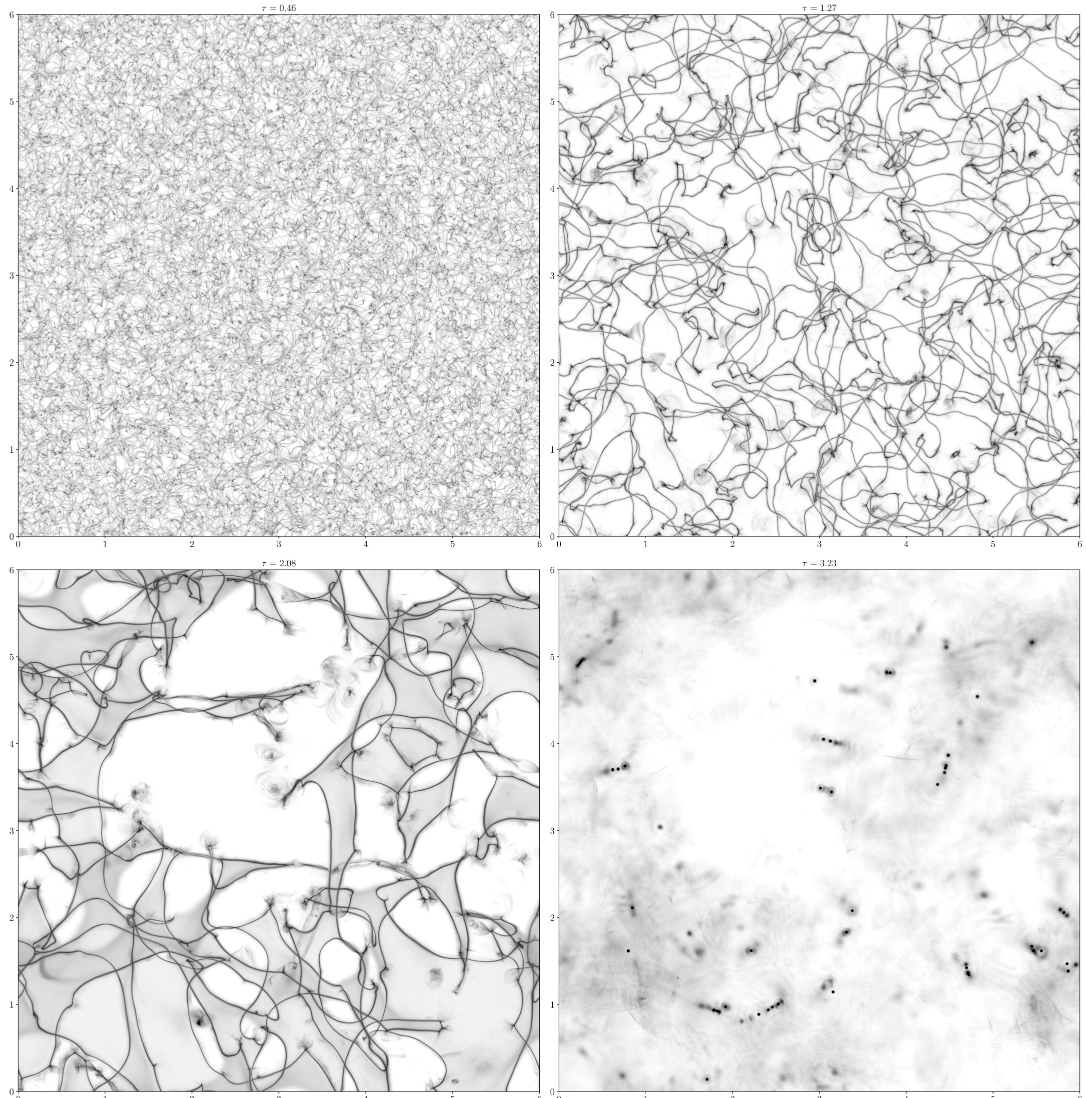
Formation of axion miniclusters

N-body simulations of nonlinear density perturbations after QCD phase transition

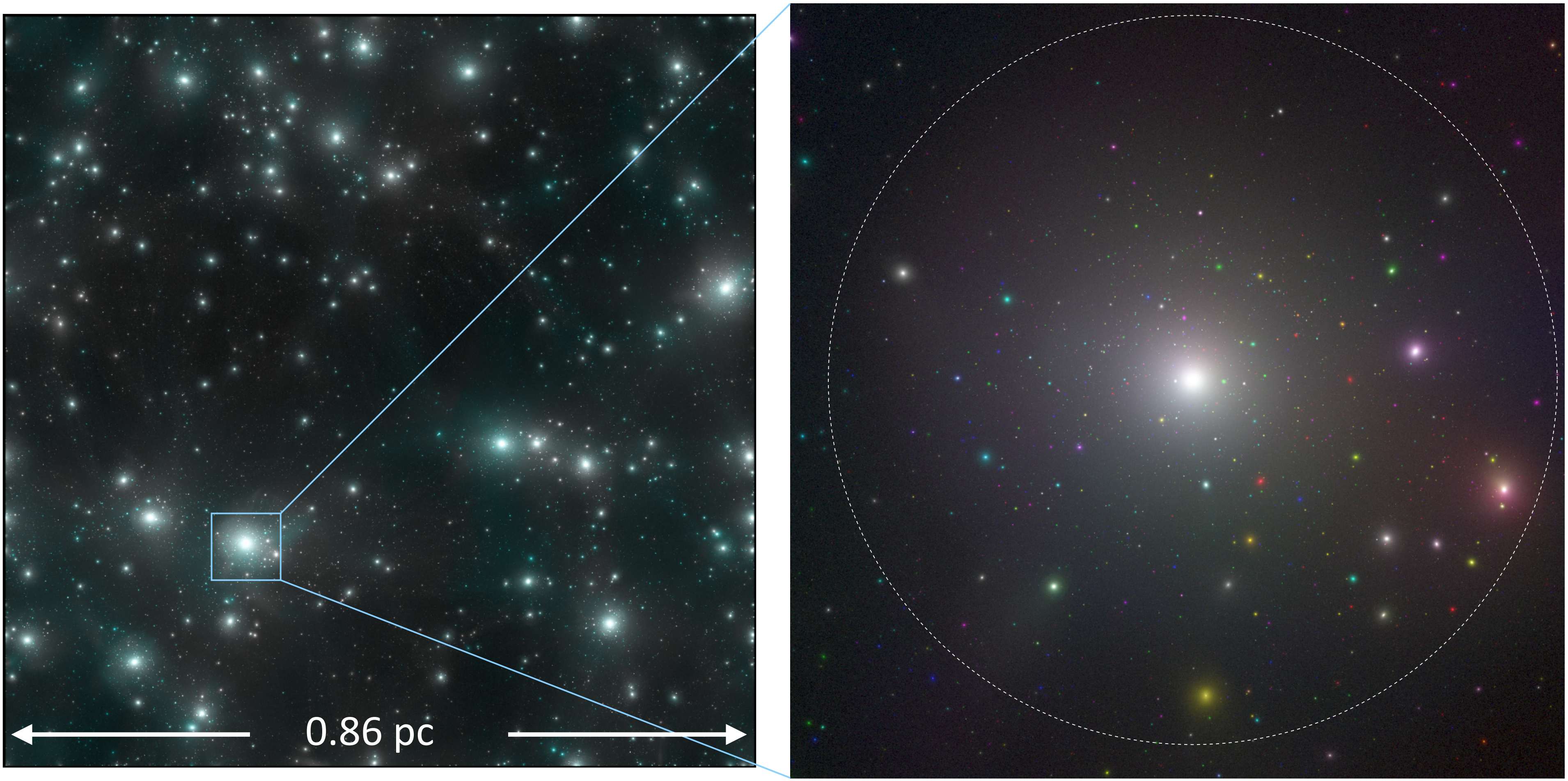
1. Initial conditions from simulations of the complex axion field (Vaquero, Redondo, Stadler ´18):



Formation of axion miniclusters

N-body simulations of nonlinear density perturbations after QCD phase transition

2. 1024^3 particle simulation of gravitational evolution (Eggemeier+ ’20, PRL 125, 041301):



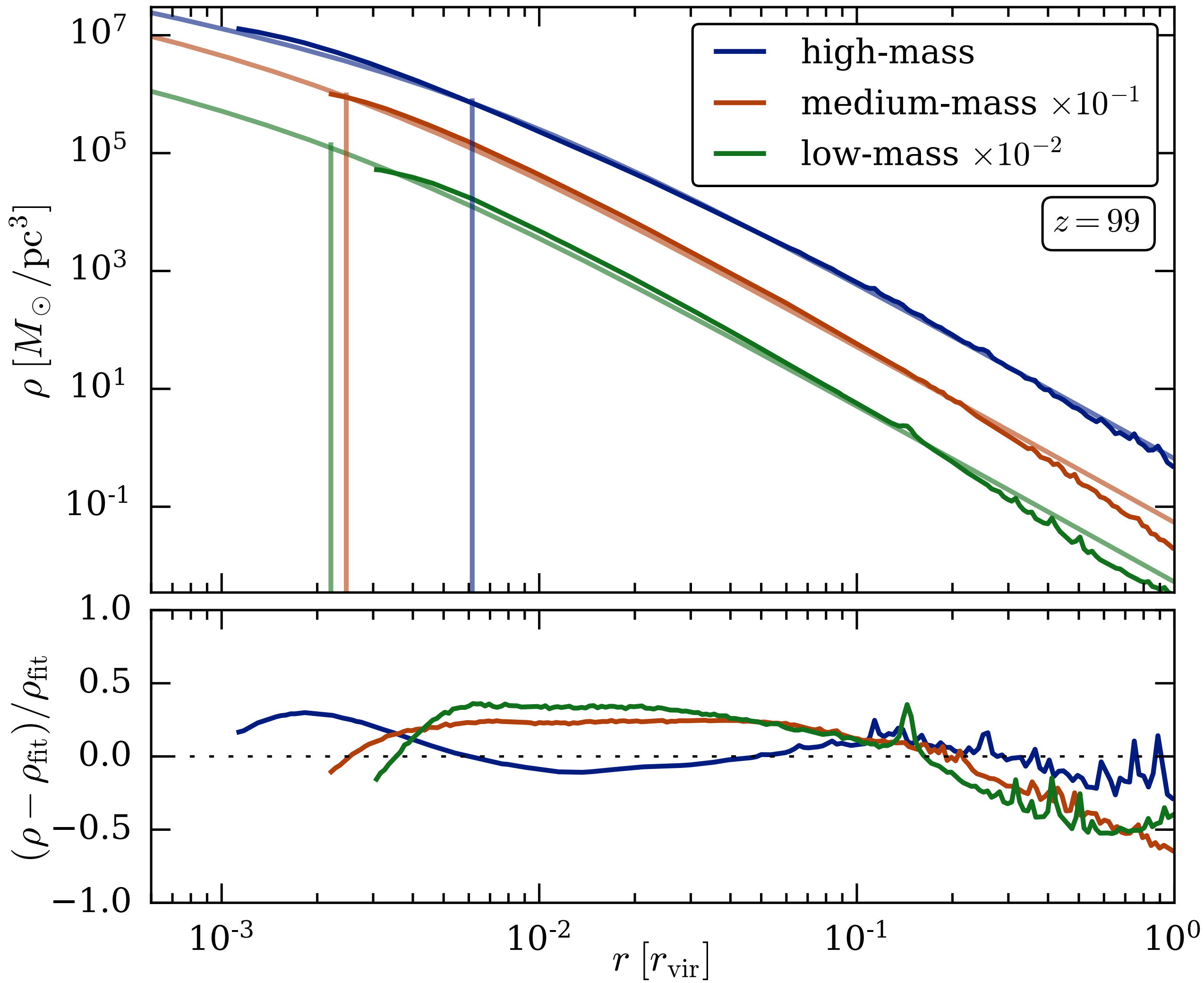
Small-box simulation challenge, part I

Formation of axion miniclusters

(Eggemeier+, ’20, PRL 125, 041301)

density profiles:

difference from NFW:

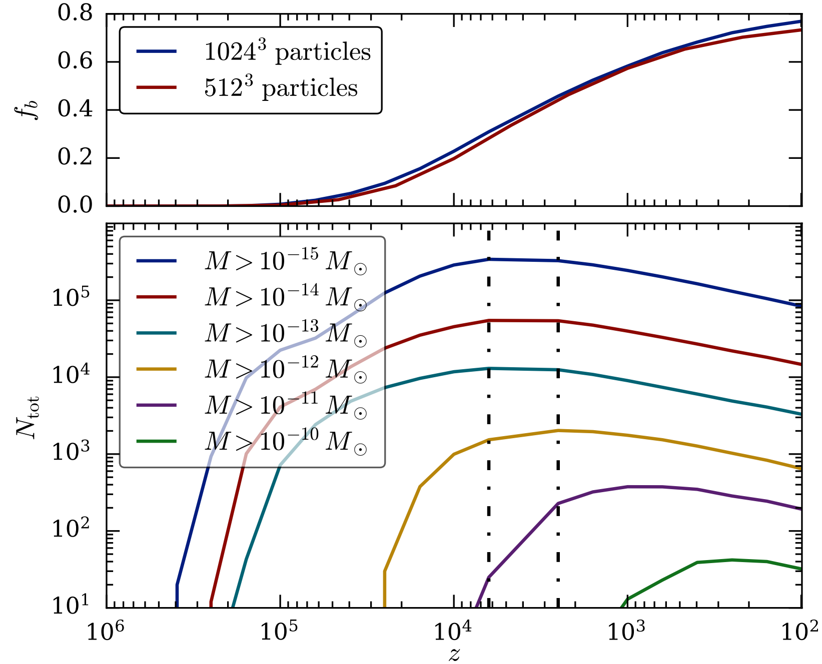


Formation of axion miniclusters

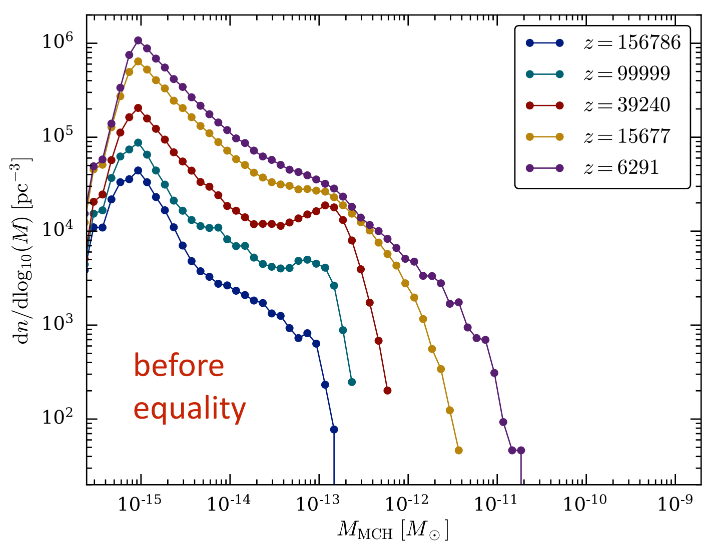
(Eggemeier+, ’20, PRL 125, 041301)

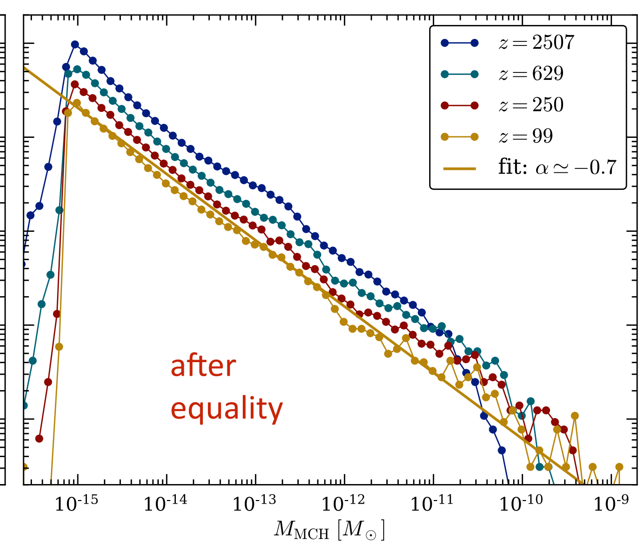
bound fraction:

number of miniclusters:



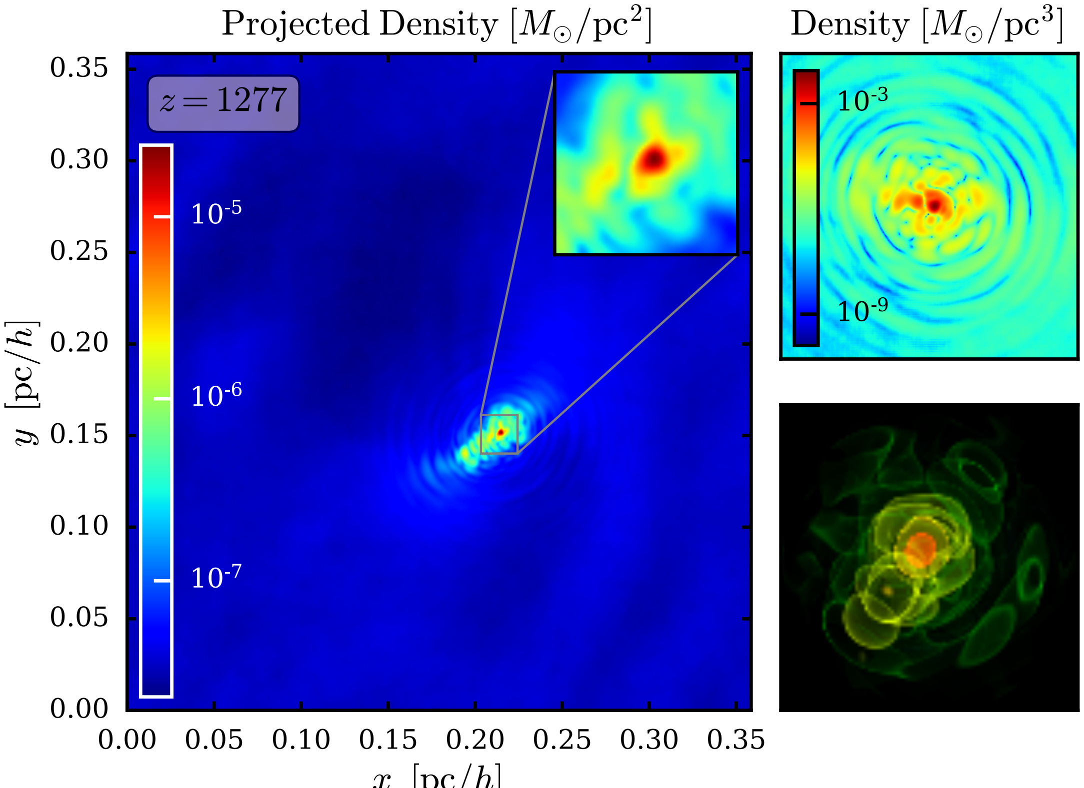
halo mass function:

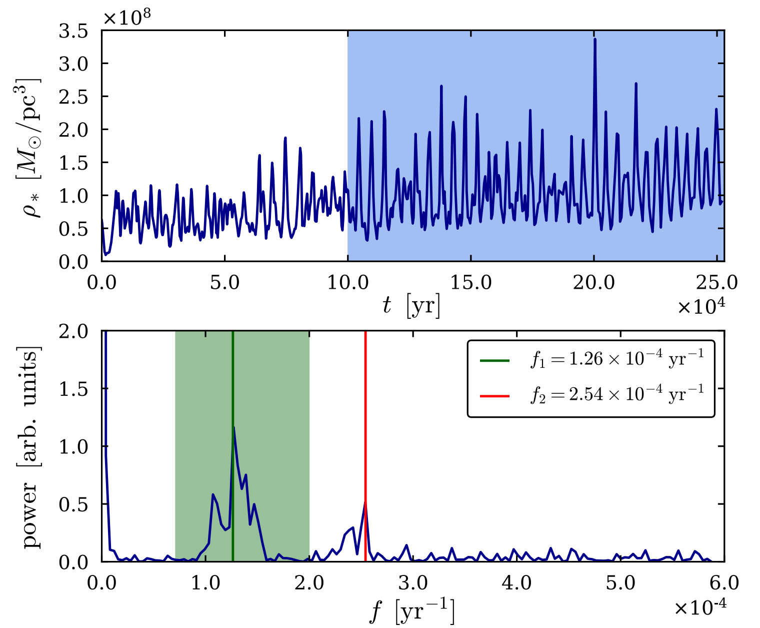




Axion star formation in miniclusters

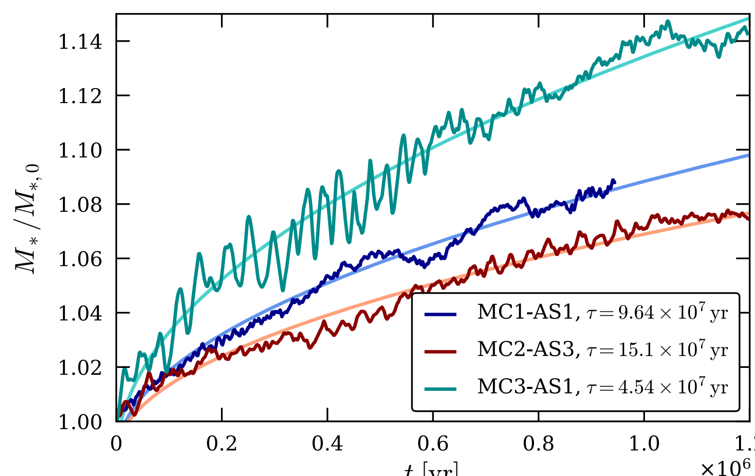
(Eggemeier, JN ’19, PRD 100, 063528)





Mass grows as $t^{1/2}$ in agreement with wave condensation (Levkov et al. 2018)

prediction: growth ~ $t^{1/8}$ after reaching virial temperature of minicluster

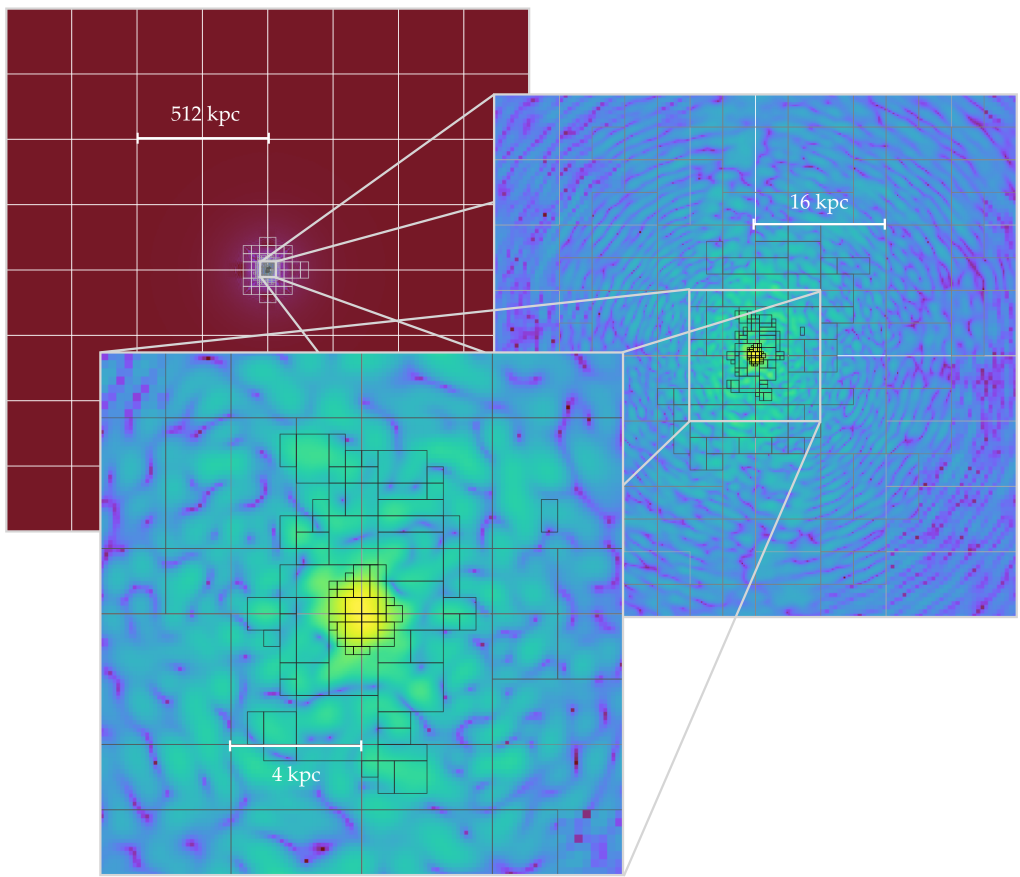


AxioNyx

(Schwabe, Gosenca, Behrens, JN, Easther ’20, PRD 102, 083518)

Public code for mixed FDM+CDM+hydro simulations

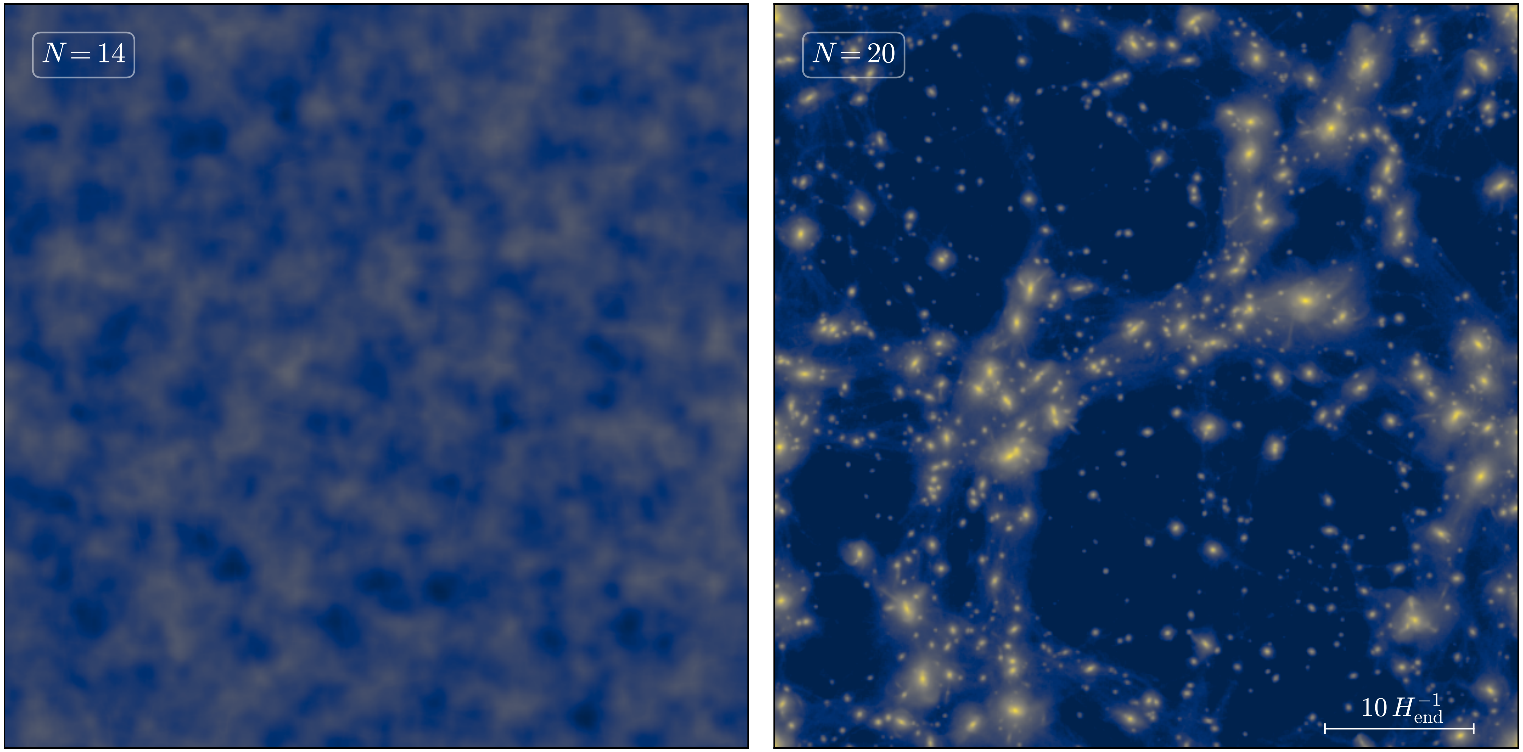
- based on AMReX
- 2nd and 6th order pseudo-spectral Schrödinger solvers



Small-box simulation challenge, part II: and the winner is …

- Early matter dominated phase after the end of inflation: inflaton condensate is cold, nonrelativistic, gravitationally unstable → SP equations work (Musoke, Hotchkiss, Easther '19)

- Formation of inflaton clusters and inflaton stars (JN, Easther '20; Eggemeier, JN, Easther in prep.):



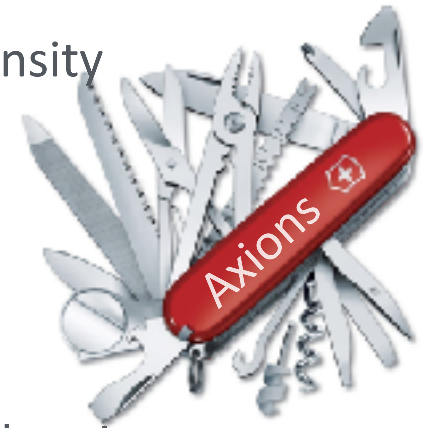
random final thoughts

- Physics of soliton (boson star) formation by classical wave condensation, mass saturation, core-halo relation etc. broadly understood

  important details still missing, e.g. is there always one soliton per halo? do they condense or remain as residuals of initial coherence? importance of oscillations?…

- You come for the solitons but stay for the granules

  lots of unexplored territory for relaxation effects by large density fluctuations, similarities to PBH constraints



- FDM is already an endangered DM species

  mass window closing, with Ly-alpha forest and gravitational heating on one side and BH superradiance on the other

  …but you can always retrain and work on QCD axion stars or inflaton stars!