Dust-induced torques in protoplanetary disks

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STARS, PLANETS, & GALAXIES, 2018

Harnack Haus, Berlin, April 14th, 2018

The Marie Skłodowska-Curie actions

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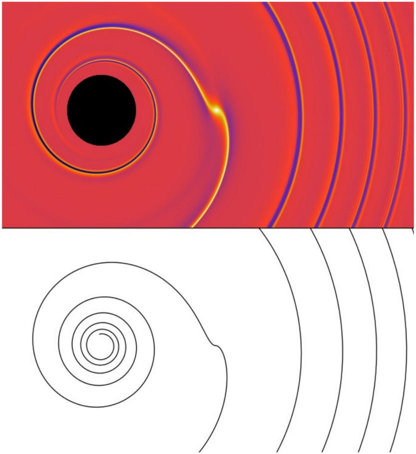
Introduction

Context, Motivation & Takeaway

- Planets interact gravitationally with the disk in which they form.
- Planet-disk interaction - important in shaping planetary systems.
- Studied for several decades in gaseous disks.
- Classical theory leads to fast inward migration problem.
- Dust is ubiquitous but its role on migration has not been assessed.
- This work is a first step in this important direction.

Dust-driven migration could play an important role in the formation history of planets.

Planet-disk interaction in gaseous disks



Top: Numerical simulation
Bottom: Position the wake (Ogilvie & Lubow 2002)

- PPDs support wave propagation.
- The effect of this propagation is a global-scale spiral wake.
- In the limit of a *cold* disk the system behaves like the Saturn rings.
- The spiral wake can be interpreted as the constructive interference of waves launched at *some locations* in the disk (Ogilvie & Lubow 2002).

Planet migration

- The waves induced by the planet carry energy & angular momentum.
- These quantities are usually extracted from the planet (by the gas) and deposited / redistributed in the disk (dissipation).
- This mechanism is the driver of planetary migration.

Angular momentum

$$L = m_p \Omega_{\rm p} r_{\rm p}^2$$

Torque

$$\Gamma \equiv \frac{dL}{dt} = 2 m_p r_{\rm p} \left(\Omega_{\rm p} + \frac{2}{r} \frac{d\Omega}{dr}\bigg|_{r_{\rm p}} \right) \frac{dr}{dt}$$

Planet migration

$$\frac{dr}{dt} = \left(\frac{2}{m_p r_p \Omega_{\rm p}} \right) \Gamma$$

Torques, however, are not necessarily linked to waves.

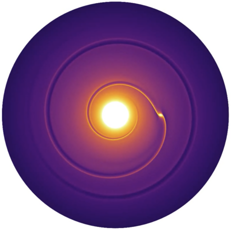
Migration rate

is proportional to the torque

Classical sources of torque

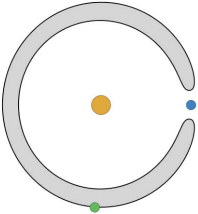
Two main (classical) sources of planet torques in PPDs:
(Goldreich & Tremaine 1979, 1980, Lin & Papaloizou 1979, ..., Kley & Nelson 2012)

Lindblad torque (-)



- Typically dominant over the co-orbital torques.
- Does not depend on disk physics.
- Main parameter is $q = m_{\mathrm{p}}/M_{\star}$.

Coorbital torque (+)



- Locally strong.
- Strongly depends on disk physics.
- Sensitive to gradients of intensive quantities (temperature, density, vortensity, ...).

Fast inward migration problem

Planet migration is usually fast.

In order to reproduce the observed distribution of exoplanets,

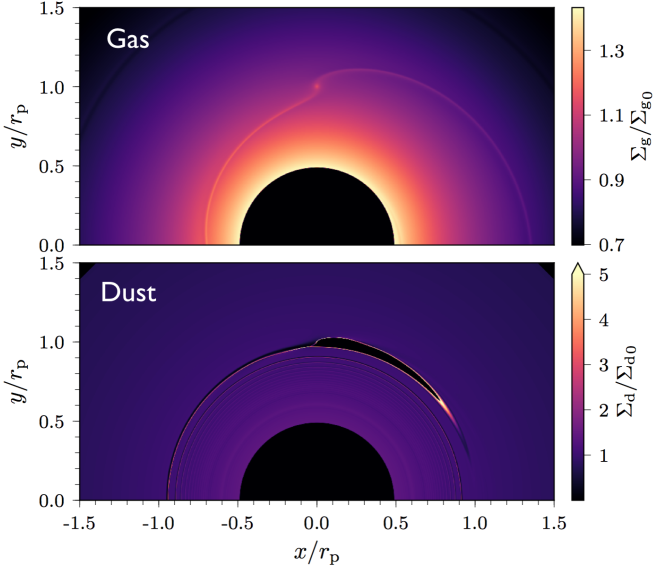
mechanisms able to slow down the migration rate are needed.

(e.g., Ida & Lin 2004, Cossou et a. 2014, Morbidelli & Raymond 2016)

Understanding coorbital torques is crucial to correctly describe the migration history of planets.

Problem

Dust dynamics in the presence of a planet



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Planet migration in dusty disks

- PPDs are composed of gas and a small fraction of dust, pebbles and rocky fragments.
- It is typically accepted that the dust-to-gas mass ratio is $\sim 1\%$.
- So, is dust relevant for migration?

Let's make some estimations to answer this question...

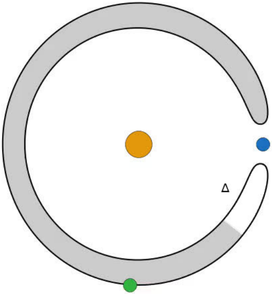
Estimating coorbital dust torques

Torque estimation
How much torque can an asymmetric coorbital region exert on a planet?

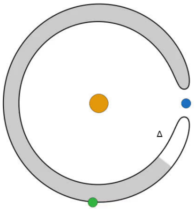
- Let's assume an axi-symmetric disk and a horseshoe described by a density distribution of the form

$$\Sigma_{\rm d} = \Sigma_{\rm d,0}\Theta(\varphi;\Delta)$$

Θ is the step function



Estimating coorbital dust torques



The torque can be computed as:

$$\Gamma_{\mathrm{d}} = \int -\Sigma_{\mathrm{d}} \frac{\partial \phi_{\mathrm{p}}}{\partial \varphi} d^3 x$$

$$\Gamma_0 \equiv \Sigma_{\mathrm{d},0} q \Omega_p^2 r_{\mathrm{p}}^4,$$

$$\Gamma'_{\mathrm{d}}(\Delta) \equiv 2 \int_{-2\pi}^{-\Delta} \int_{1}^{1+x'_{\mathrm{s}}} \frac{r'^2 \sin\varphi \quad d\varphi dr'}{[1 + r'^2 - 2r' \cos\varphi + (\beta h)^2]^{3/2}}$$

$$|\Gamma'_{\mathrm{d}}| \leq |\Gamma'_{\mathrm{d}}|_{\max} \equiv |\Gamma'_{\mathrm{d}}|(\pi) \simeq \frac{2x'_{\mathrm{s}}}{\beta h}$$

x'_{s}: width of horseshoe; β: smoothing length; h: disk aspect ratio

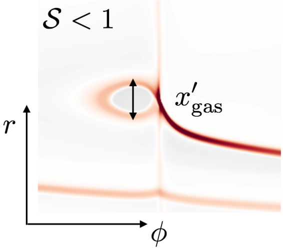
Dust horseshoes

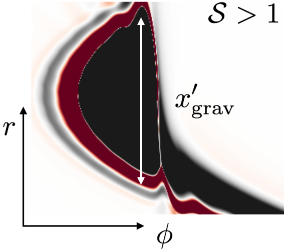
- Dust is not directly affected by pressure forces

$$\frac{d\mathbf{v}}{dt} = -\frac{\Omega_{\mathrm{K}}}{\mathcal{S}}\left(\mathbf{v} - \mathbf{v}_{\mathrm{g}}\right) - \nabla\phi_* - \nabla\phi_p$$

Stokes number $\mathcal{S}$ governs dust-gas coupling

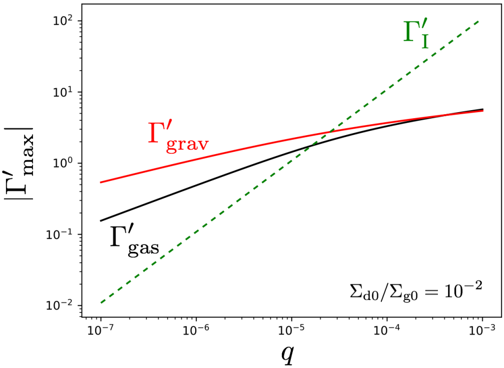
At fixed planetary mass, the horseshoe region width depends on $\mathcal{S}$, leading to different scalings.





Estimating coorbital torques: gas vs dust

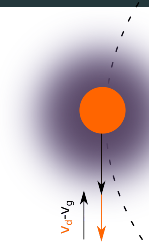
For low planet mass, gas torque is $\Gamma'_{\mathrm{I}} \propto q$ (e.g. Tanaka et al. 2002):



This is interesting! For low q dust torques can dominate over gas torques.
But when do we expect dust horseshoe orbits to be modified?

sub-Keplerian motion and dust dynamics

Dust momentum (steady-state):



$$v_r \partial_r v_r - \frac{v_\varphi^2}{r} = -\frac{\Omega_\mathrm{K}}{\mathcal{S}} \left(v_r - v_{r,\mathrm{g}} \right) - \Omega_\mathrm{K}^2 r$$

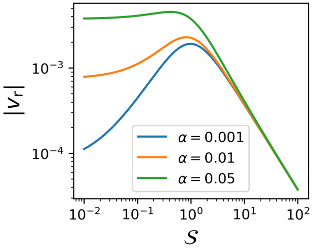
$$v_r \partial_r v_\varphi + \frac{v_r v_\varphi}{r} = -\frac{\Omega_\mathrm{K}}{\mathcal{S}} \left(v_\varphi - v_{\varphi,\mathrm{g}} \right)$$

Dust velocity field:

(Takeuchi & Lin, 2002)

$$v_r = \frac{1}{1+\mathcal{S}^2} v_{r,\mathrm{g}} - \frac{\mathcal{S}}{1+\mathcal{S}^2} \eta v_\mathrm{K}$$

$$v_\varphi = v_{\varphi,\mathrm{g}} - \frac{1}{2} \mathcal{S} v_r$$



$\eta = -h^2 d \ln P / d \ln r$; sub-Keplerian gas rotation

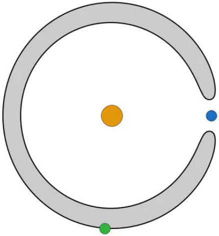
When does drift modify dust horseshoe orbits?

An important timescale is the horseshoe *libration* time:

$$\tau_{\rm hs} \equiv \frac{2\pi}{\Omega_p - \Omega_{\rm hs}} \simeq \frac{4\pi}{3x_{\rm s}}\Omega_p^{-1}$$

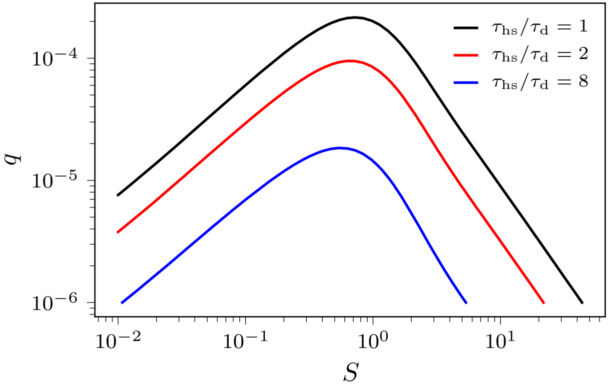
Dust crosses the horseshoe region in a time:

$$\tau_{\rm d} \equiv \left|\frac{x_{\rm s}}{v_r}\right| = \frac{1+\mathcal{S}^2}{\alpha_{\rm P}\mathcal{S} - 1.5\alpha}\left(\frac{x_{\rm s}}{r_{\rm p}}\right) h^{-2}\Omega_{\rm p}^{-1}$$



For $\tau_{\rm d}$ shorter than $\tau_{\rm hs}$
significant modifications to the horseshoe orbits are expected.

Drift vs Horseshoes



Dust horseshoe orbits are modified by dust drift below the black line.
This is the region of parameter space where dust torques may matter.

Model, Simulations & Results

Physical model

We solve a two-fluid system:

Model

$$
\begin{aligned}
\frac{\partial \Sigma_{\rm g}}{\partial t} + \nabla \cdot (\Sigma_{\rm g} \mathbf{v}_{\rm g}) &= 0 \\
\frac{\partial \Sigma_{\rm d}}{\partial t} + \nabla \cdot (\Sigma_{\rm d} \mathbf{v}_{\rm d}) &= 0 \\
\frac{\partial \mathbf{v}_{\rm g}}{\partial t} + \mathbf{v}_{\rm g} \cdot \nabla \mathbf{v}_{\rm g} &= -\nabla \phi - \varepsilon \frac{\Omega_{\rm K}}{\mathcal{S}} (\mathbf{v}_{\rm g} - \mathbf{v}_{\rm d}) - \frac{\nabla P}{\Sigma_{\rm g}} - \nabla \cdot \mathbf{T} \\
\frac{\partial \mathbf{v}_{\rm d}}{\partial t} + \mathbf{v}_{\rm d} \cdot \nabla \mathbf{v}_{\rm d} &= -\nabla \phi - \frac{\Omega_{\rm K}}{\mathcal{S}} (\mathbf{v}_{\rm d} - \mathbf{v}_{\rm g}) \\
P(r) &= c_s^2 \Sigma_{\rm g}
\end{aligned}
$$

For $\epsilon = \Sigma_{\rm d}/\Sigma_{\rm g} \ll 1$, we can neglect dust feedback onto gas.

Numerical domain

FARGO3D code (Benitez-Llambay & Masset 2016) - 2D-cylindrical mesh

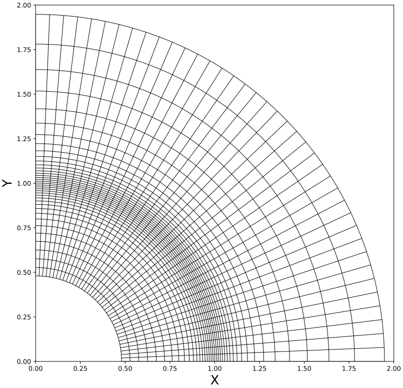
Domain extent is important

- Lindblad resonances (in & out)

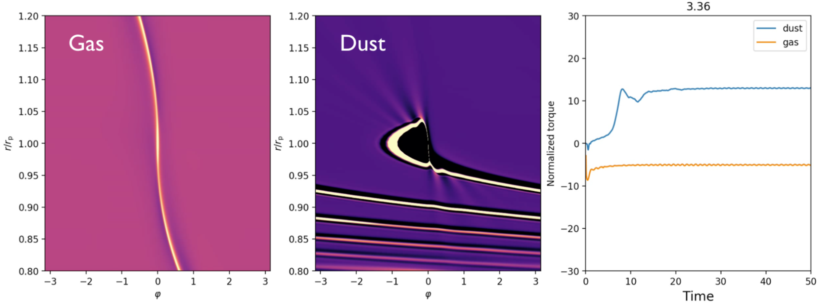
Resolution is important!

- Horse-shoe region
- Pressure scale ($\mathcal{O}(H)$)
- Mesh density function

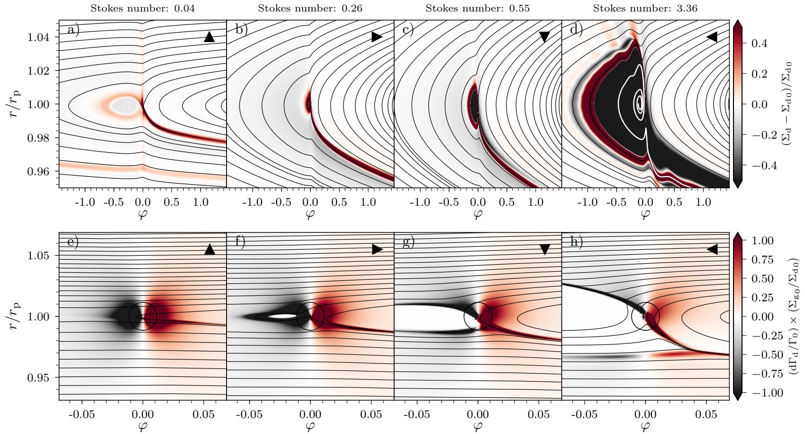
$$\psi(r) = \frac{1}{r} + \frac{\xi}{(r - r_\mathrm{p})^2 + \xi^2}$$



Results: Let's look at one example



Different regimes: gas vs gravity dominated



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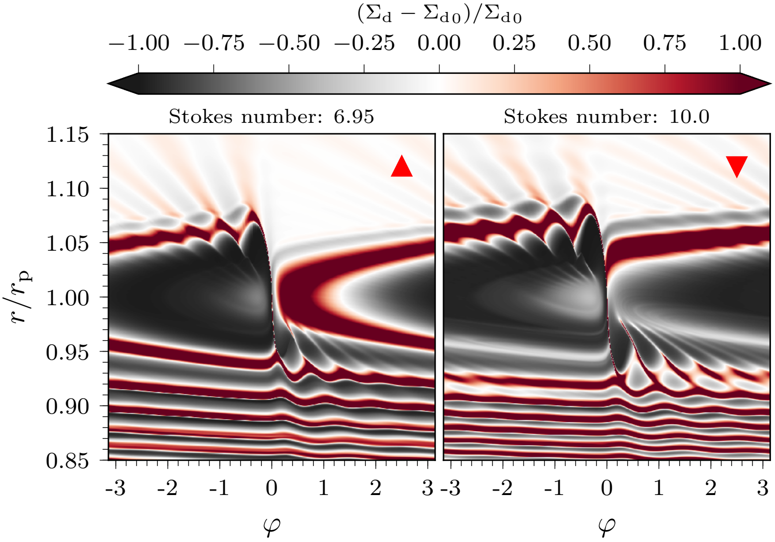
Gap regime



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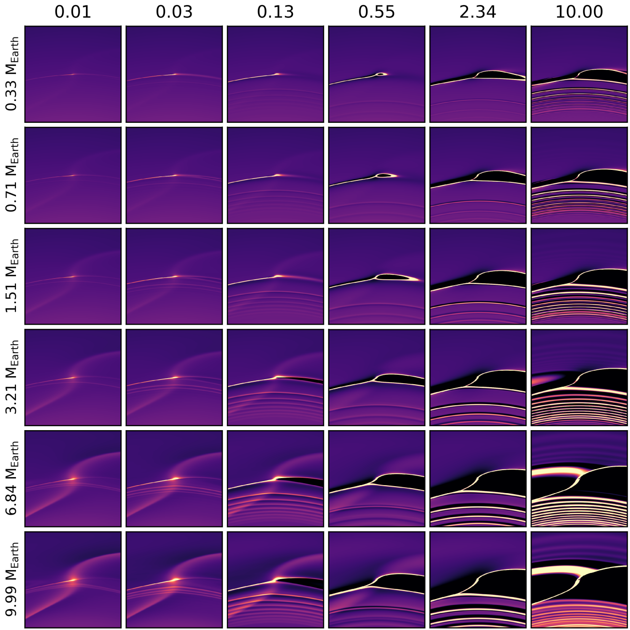
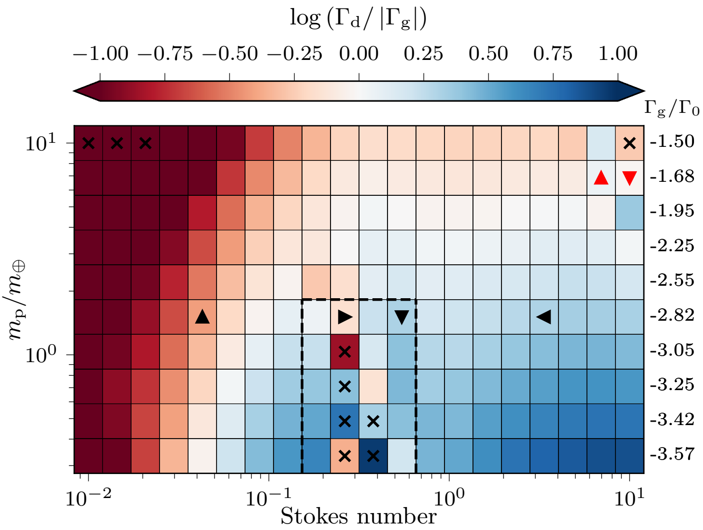

0.01
0.03
0.13
0.55
2.34
10.00
0.33 M_{Earth}
0.71 M_{Earth}
1.51 M_{Earth}
3.21 M_{Earth}
6.84 M_{Earth}
9.99 M_{Earth}

Torque map



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Final remarks

- Global fast outward migration of small cores is naturally obtained.
- Dust-driven migration could be dominant mode for small bodies.
- Dust-torque is larger than thermal torque (Benitez-Llambay et al. 2015). Need to model both of these self-consistently.
- Dust torques scale with disk metallicity. Implications for giant planets in high-metallicity systems.

Takeaway: Dust torques could play an important role in the formation history of planets, including those in our Solar System. A complete assessment requires realistic models for the dust distribution close to the planet (dust feedback, dust accretion, planet migration, other planets).