

**The
stellar
initial mass function
from
star formation
and in
galaxies**

Harnack Haus Berlin on
Stars, Planets and Galaxies
13.-18. April 2018

Pavel Kroupa

*Helmholtz-Institute for Radiation und Nuclear Physics (HISKP)
University of Bonn*

*Astronomical Institute,
Charles University in Prague*

*c/o Argelander-Institut für Astronomie
University of Bonn*

Pavel Kroupa: University of Bonn / Charles University

Issues which affect the determination of the IMF :

- 1) The IMF does not exist !

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Star-formation "hilfskonstrukt" $\neq$ galaxy-wide "hilfskonstrukt"

Pavel Kroupa: University of Bonn / Charles University

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The estimation of this "hilfskonstrukt" is compromised by

the stellar mass-to-light relation

dynamical evolution of birth clusters (very early phase and long-term)

binaries

the most massive star in birth clusters

IMF = probability density distr.function

or an optimally sampled distribution function ?

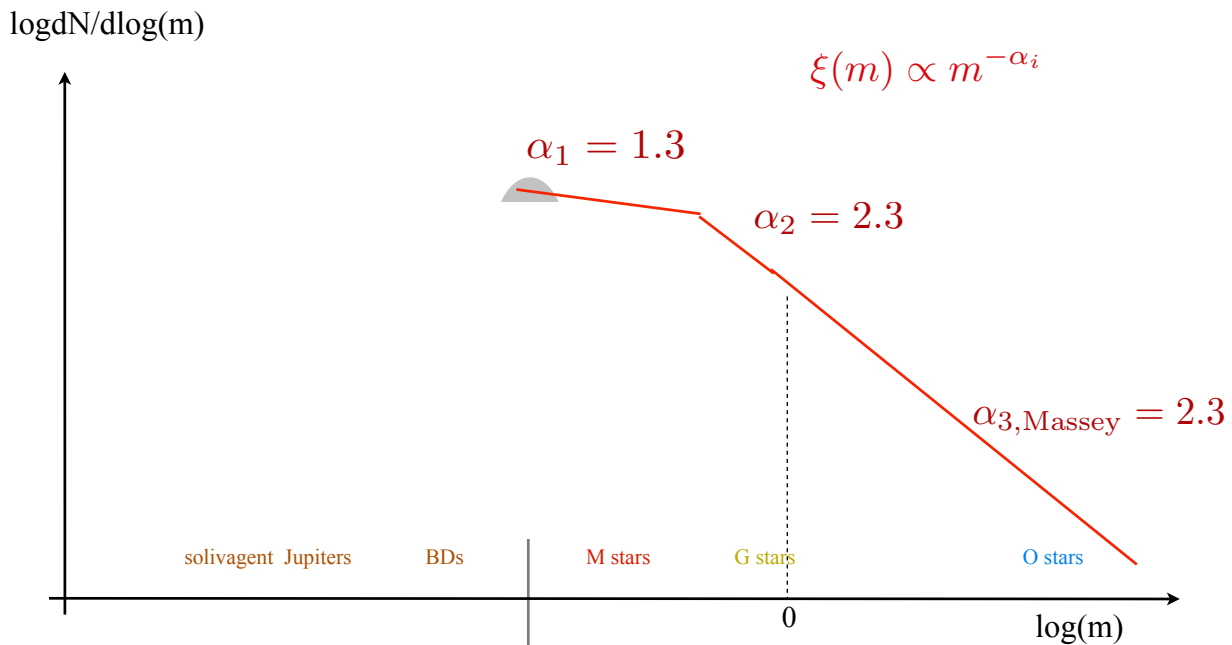
evidence for top-heavy IMF in extremely massive star burst "clusters"

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Pavel Kroupa: University of Bonn / Charles University



discontinuous four-part power-law IMF :



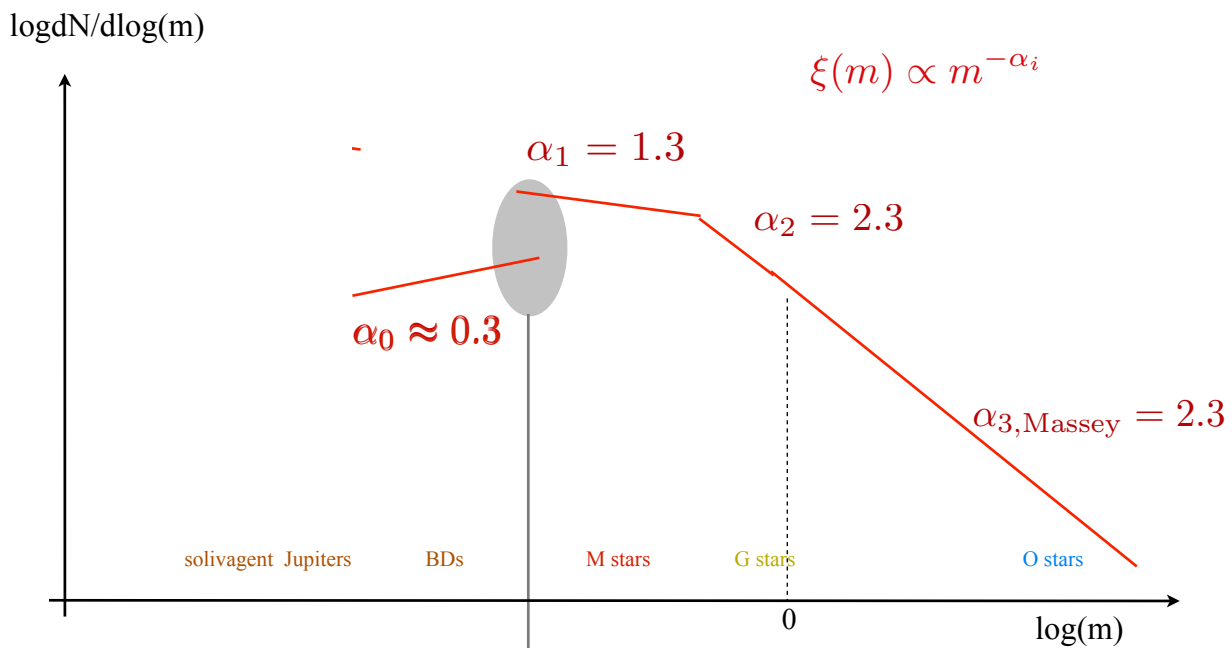
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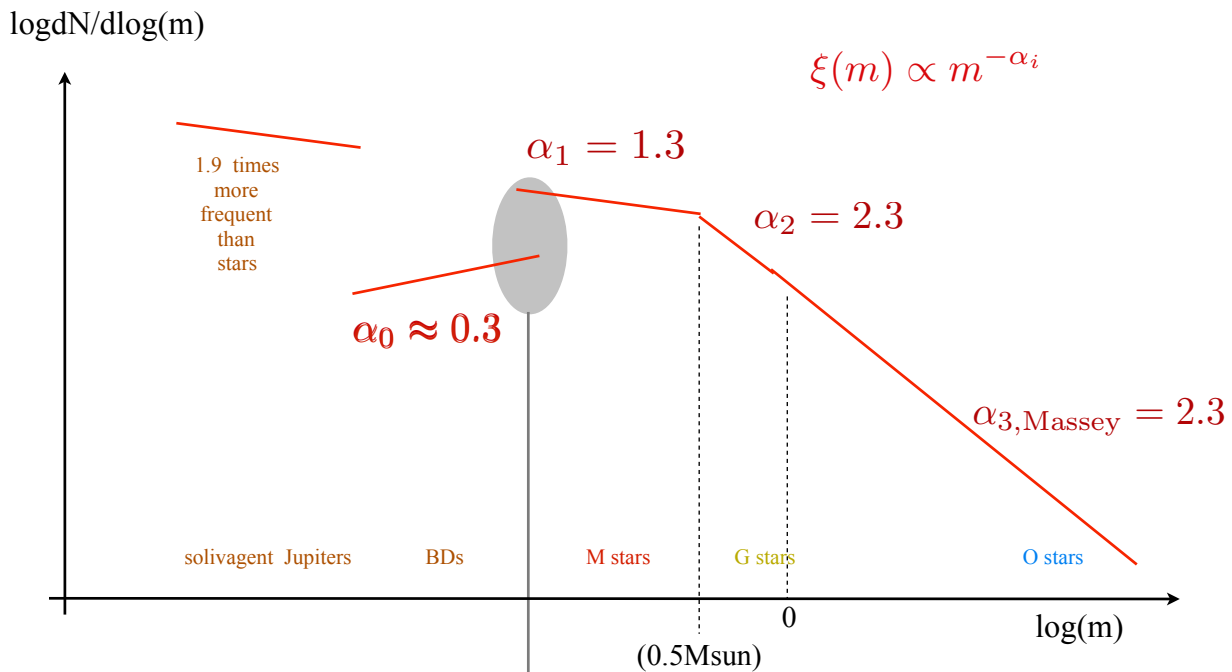
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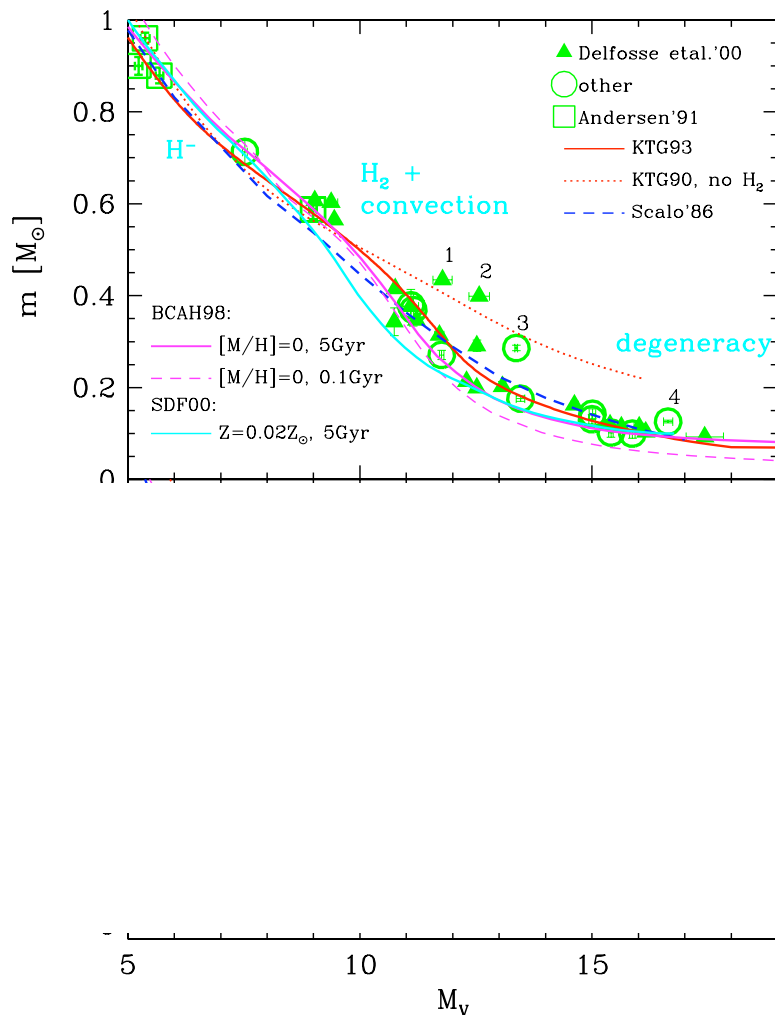
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The mass-luminosity relation of low-mass stars

Kroupa, Tout & Gilmore 1990;
Kroupa, 2002, *Science*

$$\Psi(M_V) = -\frac{dm}{dM_V} \xi(m)$$

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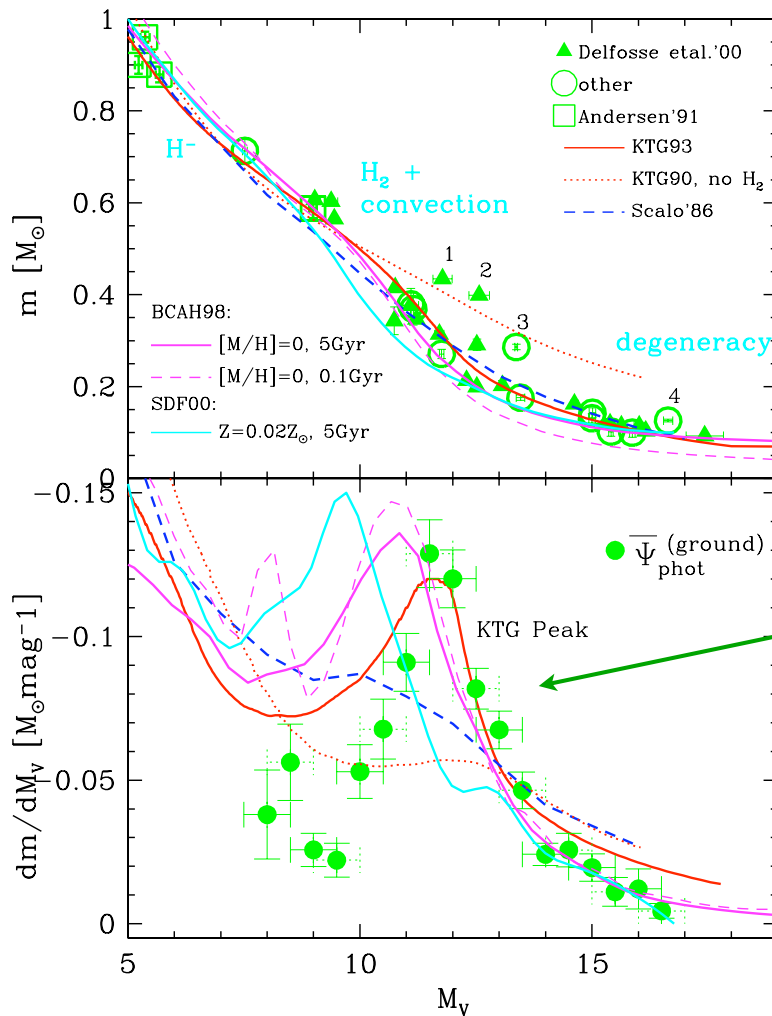
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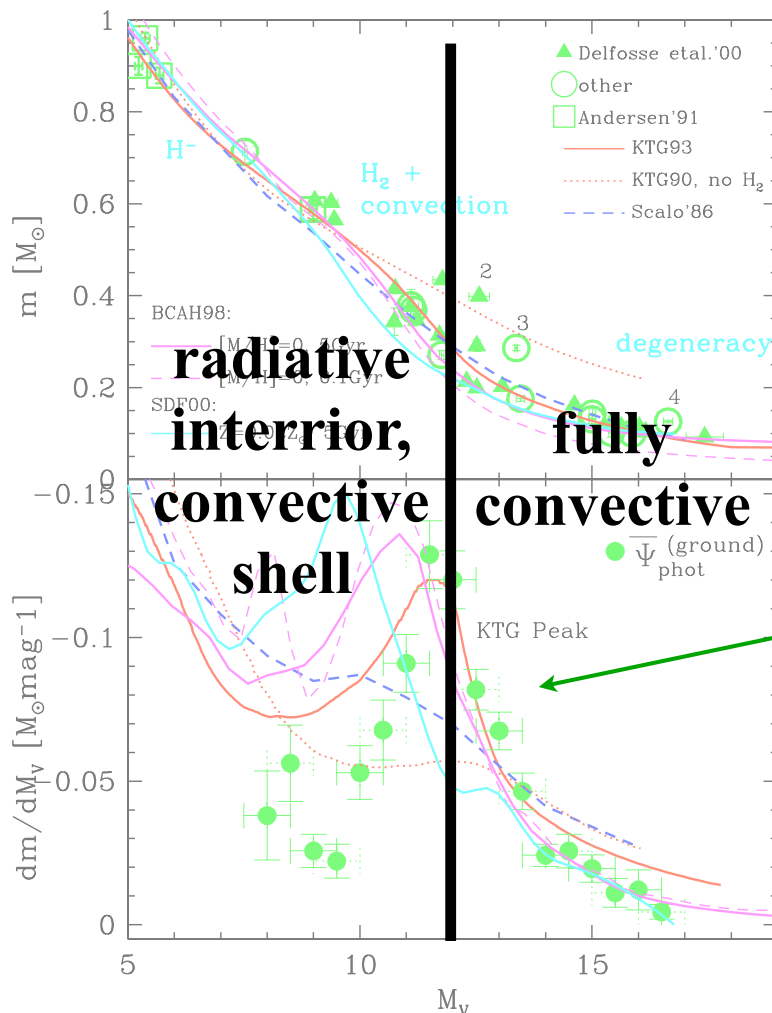
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1. position
2. width
3. amplitude
all agree !

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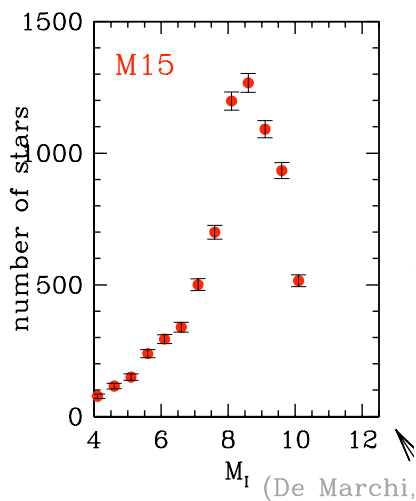
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Kroupa 2002

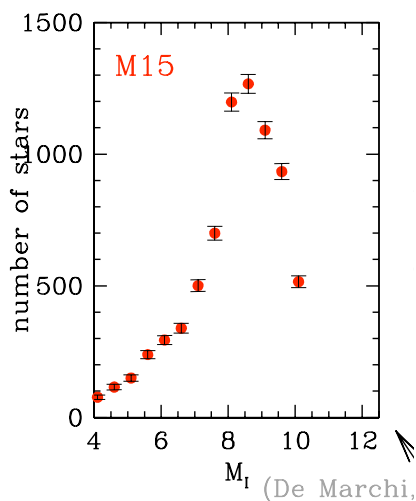
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By counting stars
we look into their
internal
constitution !

Peak in LF
is not due to MF
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because the formation
of pre-stellar cores is
independent of the physics of
stellar interiors.

Pavel Kroupa: University of Bonn / Charles University



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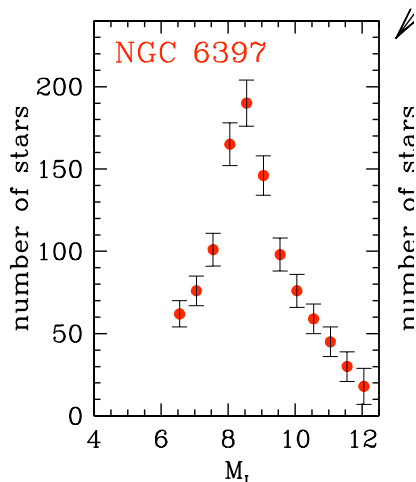
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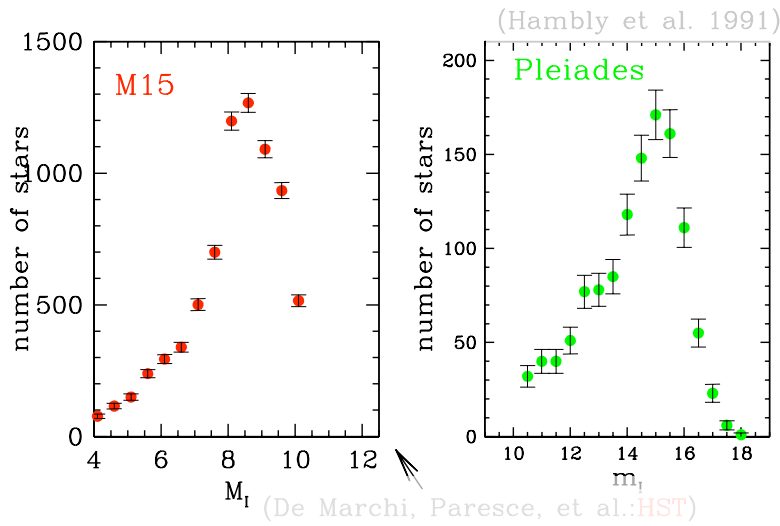
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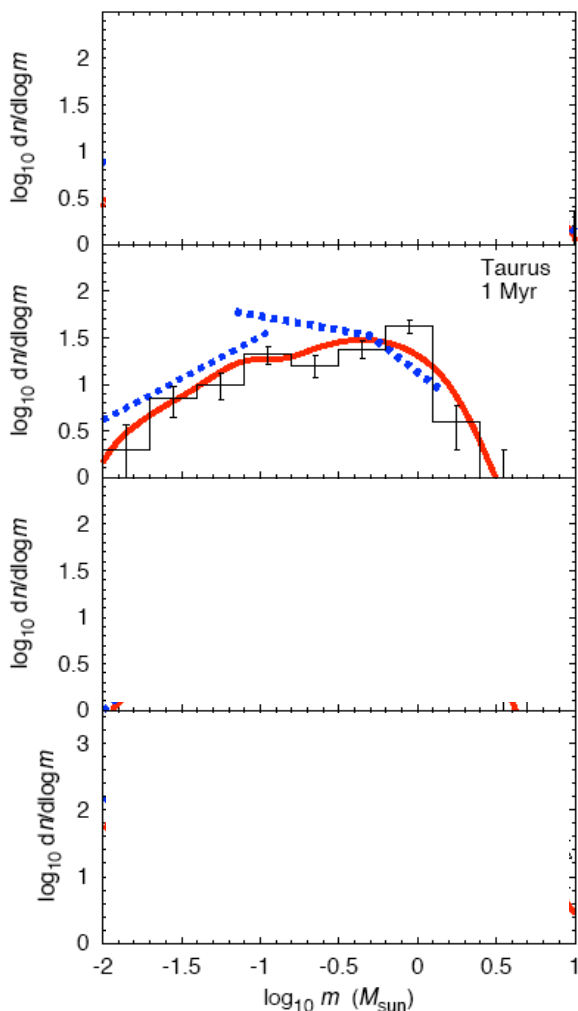
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(Thies & Kroupa 2007)



Standard (universal) IMF in
stellar regime.

Similar IMF in brown
dwarf regime $\alpha \approx 0.3 \pm 0.3$

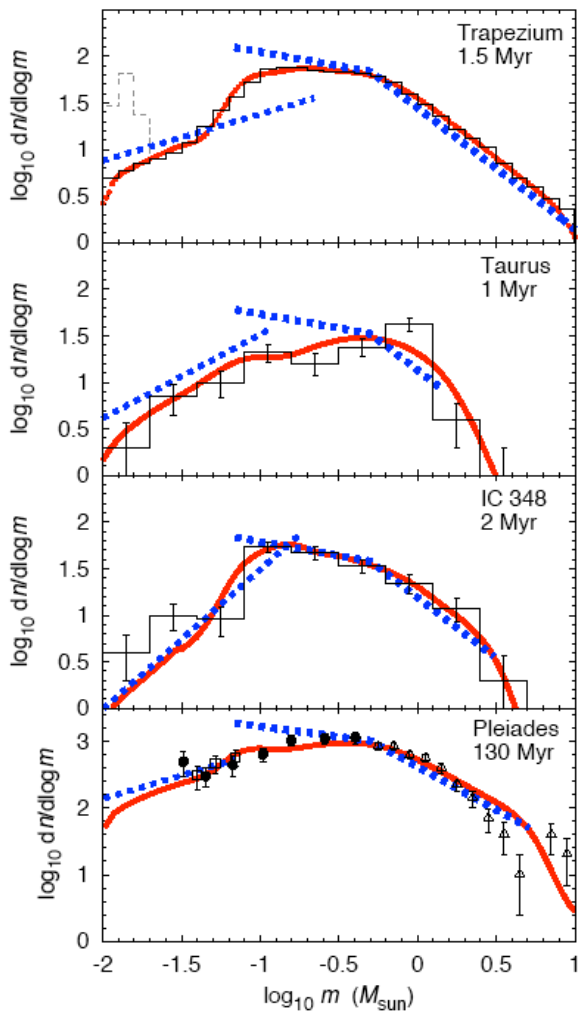
Discontinuity near $\approx 0.08 M_{\odot}$

$\mathcal{R} = 0.2 \pm 0.05$ BDs per star



One brown dwarf forms
per 5 stars !

Pavel Kroupa: AlfA, University of Bonn



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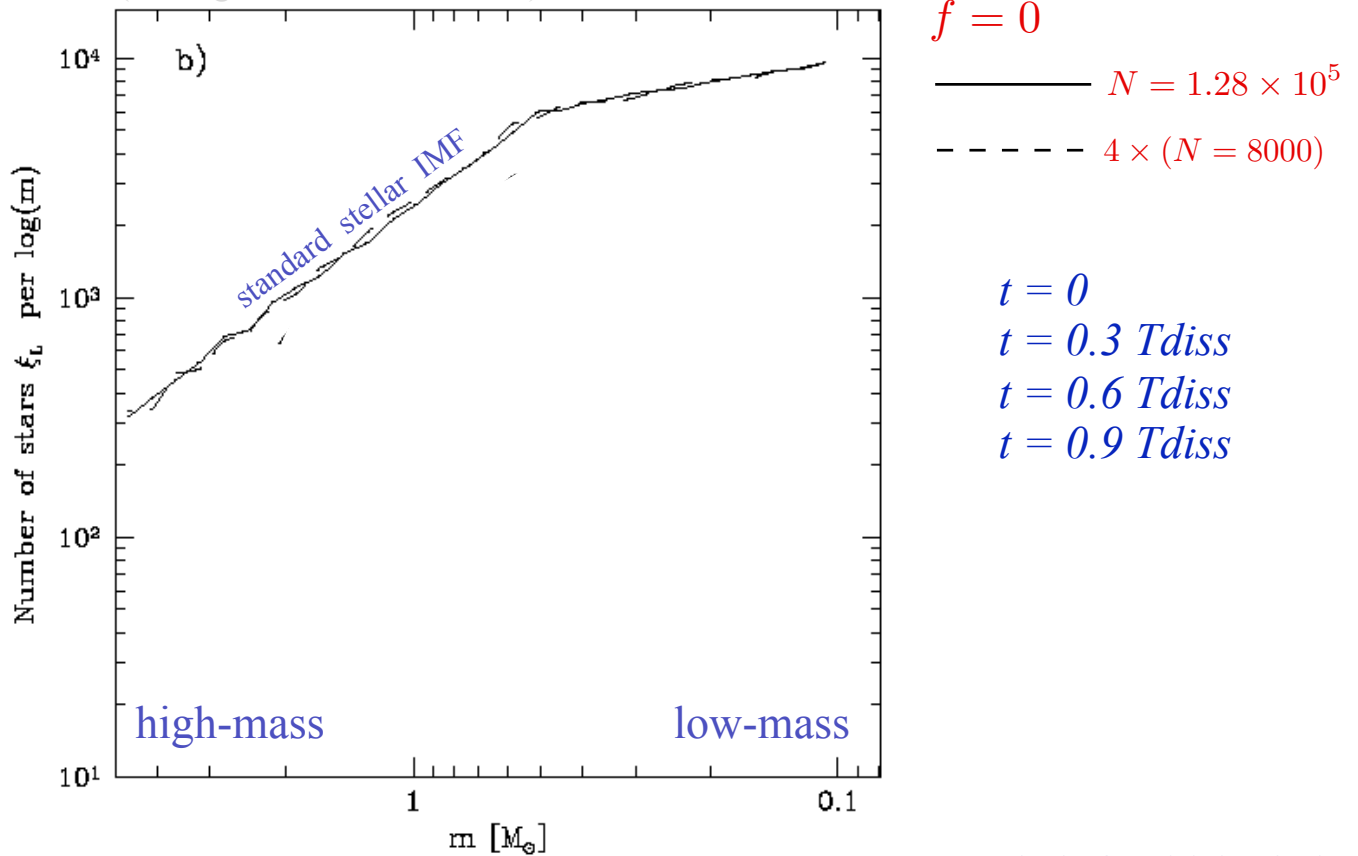
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MF(t) due to cluster evolution

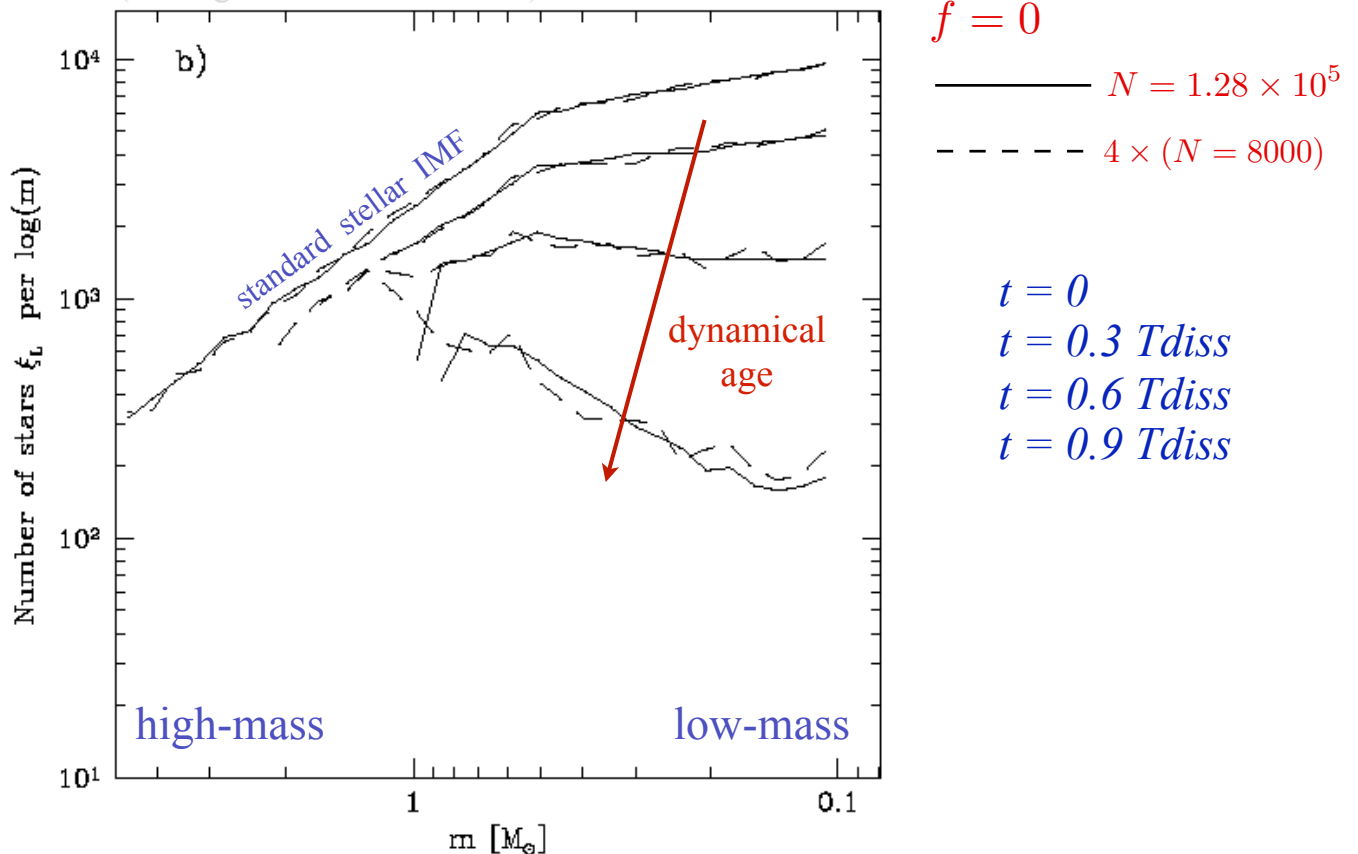
(Baumgardt & Makino 2003)



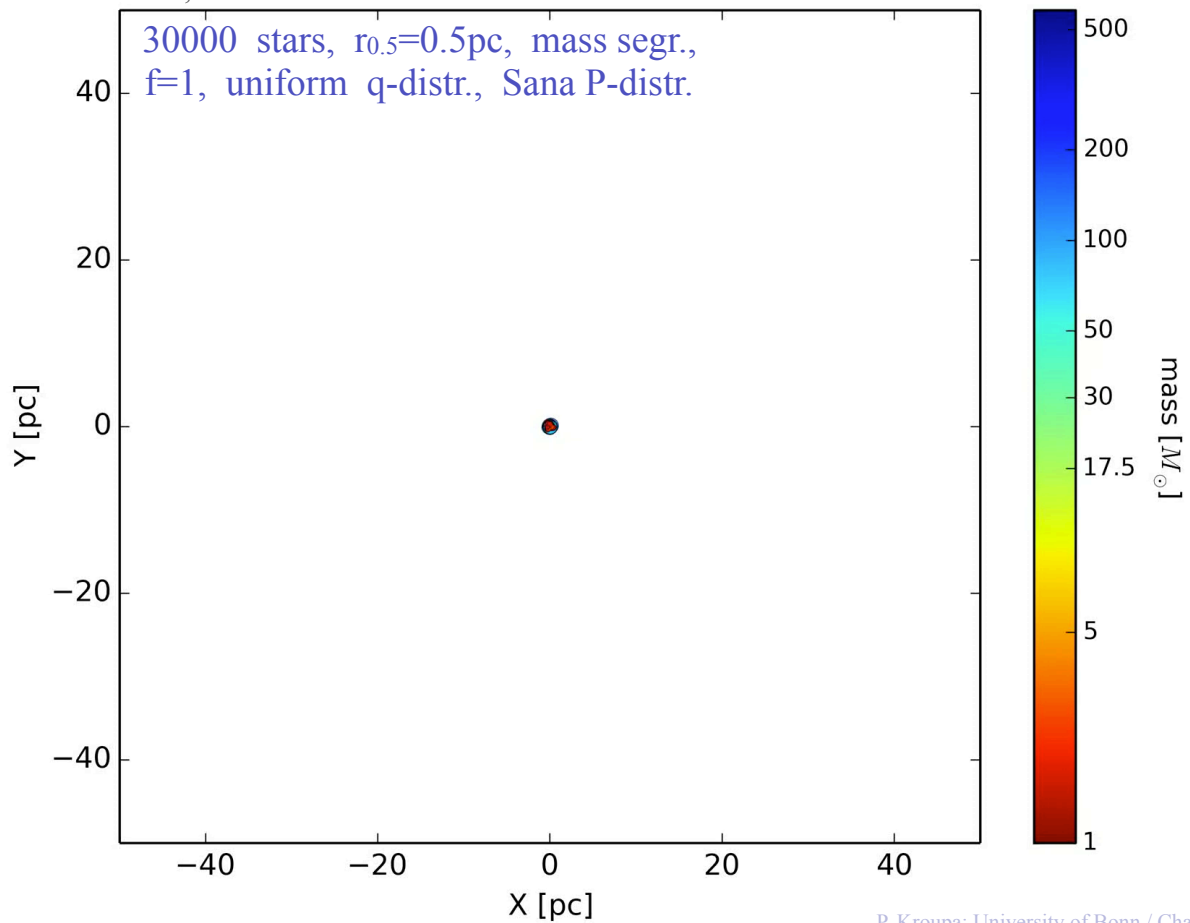
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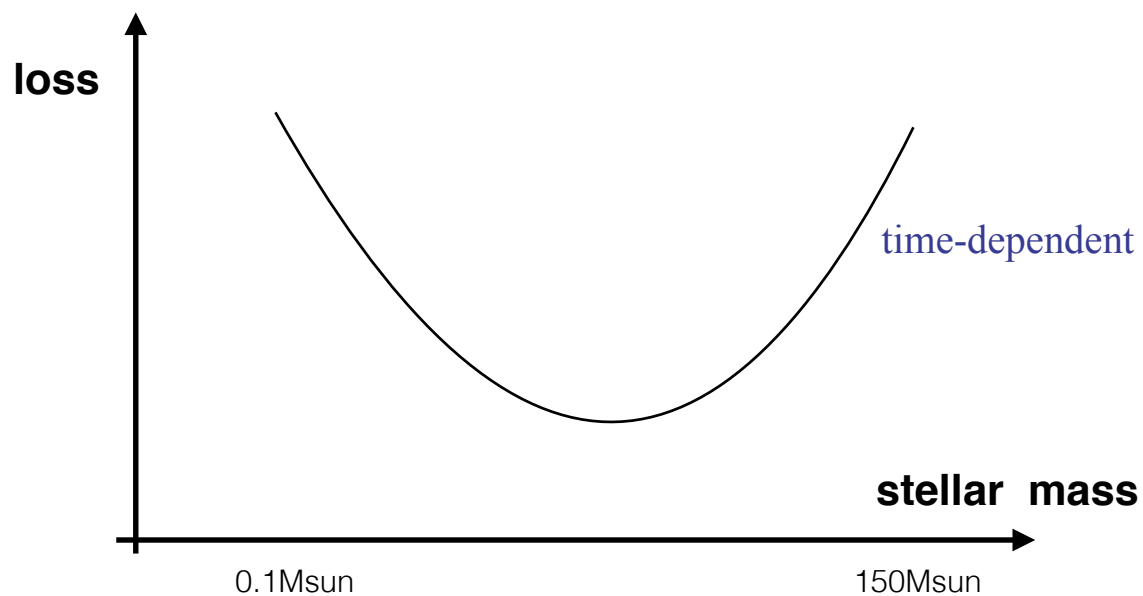
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Clusters depopulate themselves
off low-mass stars and high mass stars.



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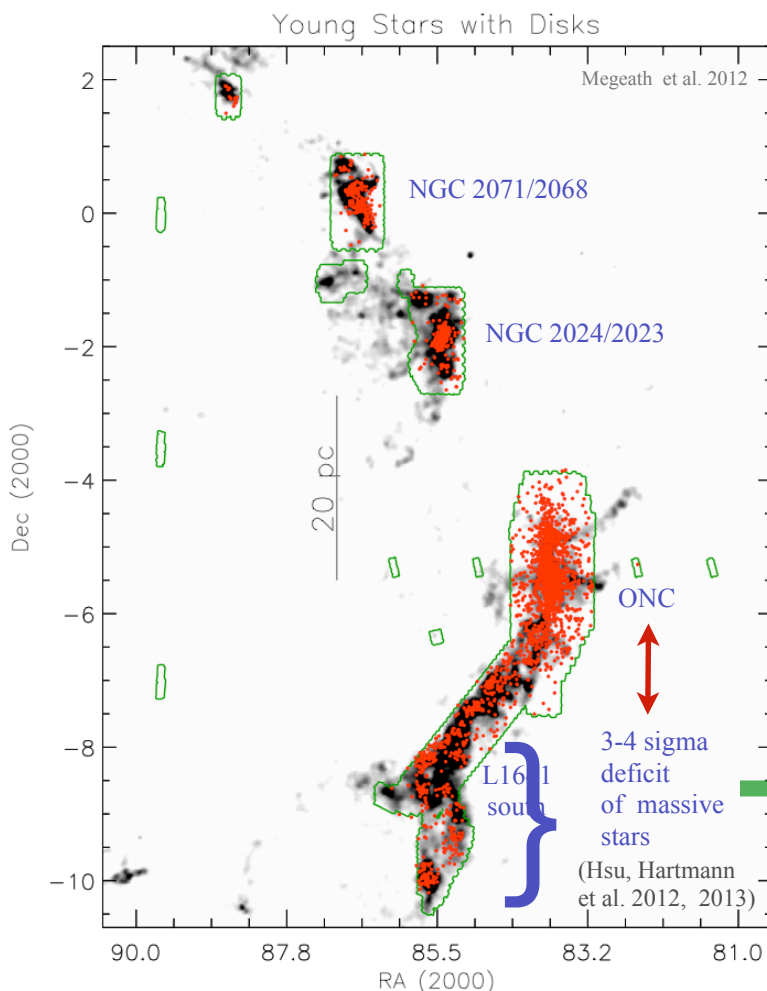
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Lack of O stars

Many small / low-mass groups or clusters do not yield the same IMF as one massive cluster

stochastic IMF in each group is ruled out.

What is an optimally sampled distribution function ?

Given the *mass reservoir* in stellar mass M_{ecl} ,
 starting with the most massive star,
 select the next most massive such that
 M_{ecl} is distributed over the distribution function
without Poisson scatter.

Kroupa et al. 2013
 Schulz, Pflamm-Altenburg & Kroupa 2015

The above ansatz can be extended to a discretized optimal distribution of stellar masses:
 Given the mass, M_{ecl} , of the population, the following sequence of individual stellar masses
 yields a distribution function which exactly follows $\xi(m)$,

Yan et
 al. 2017

$$m_{i+1} = \int_{m_{i+1}}^{m_i} m \xi(m) dm, \quad m_L \leq m_{i+1} < m_i, \quad m_1 \equiv m_{\text{max}}. \quad (4.9)$$

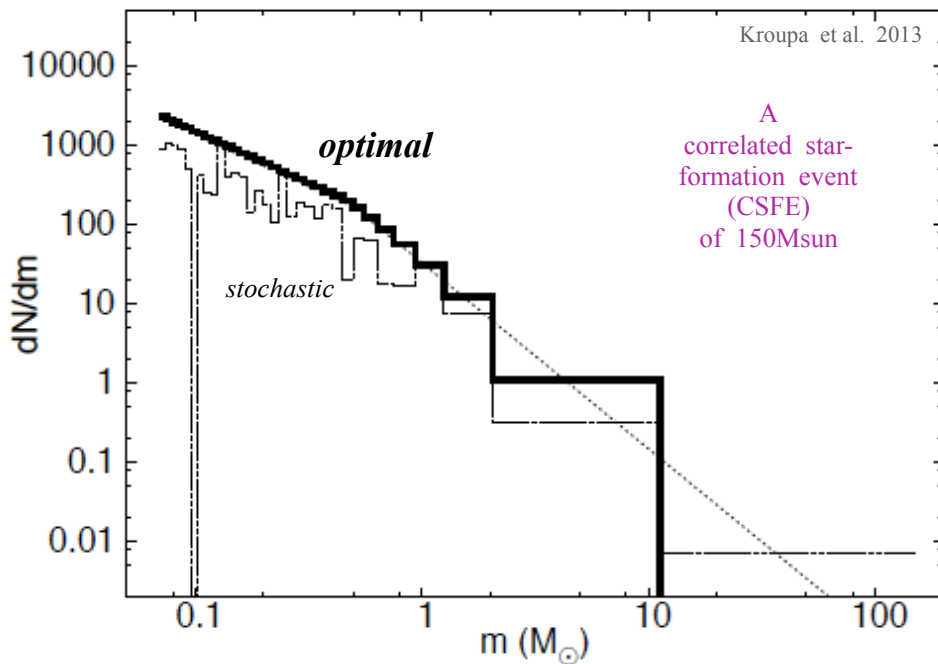
The normalization and the most massive star in the sequence are set by the following two equations:

$$1 = \int_{m_{\text{max}}}^{m_{\text{max}^*}} \xi(m) dm, \quad (4.10)$$

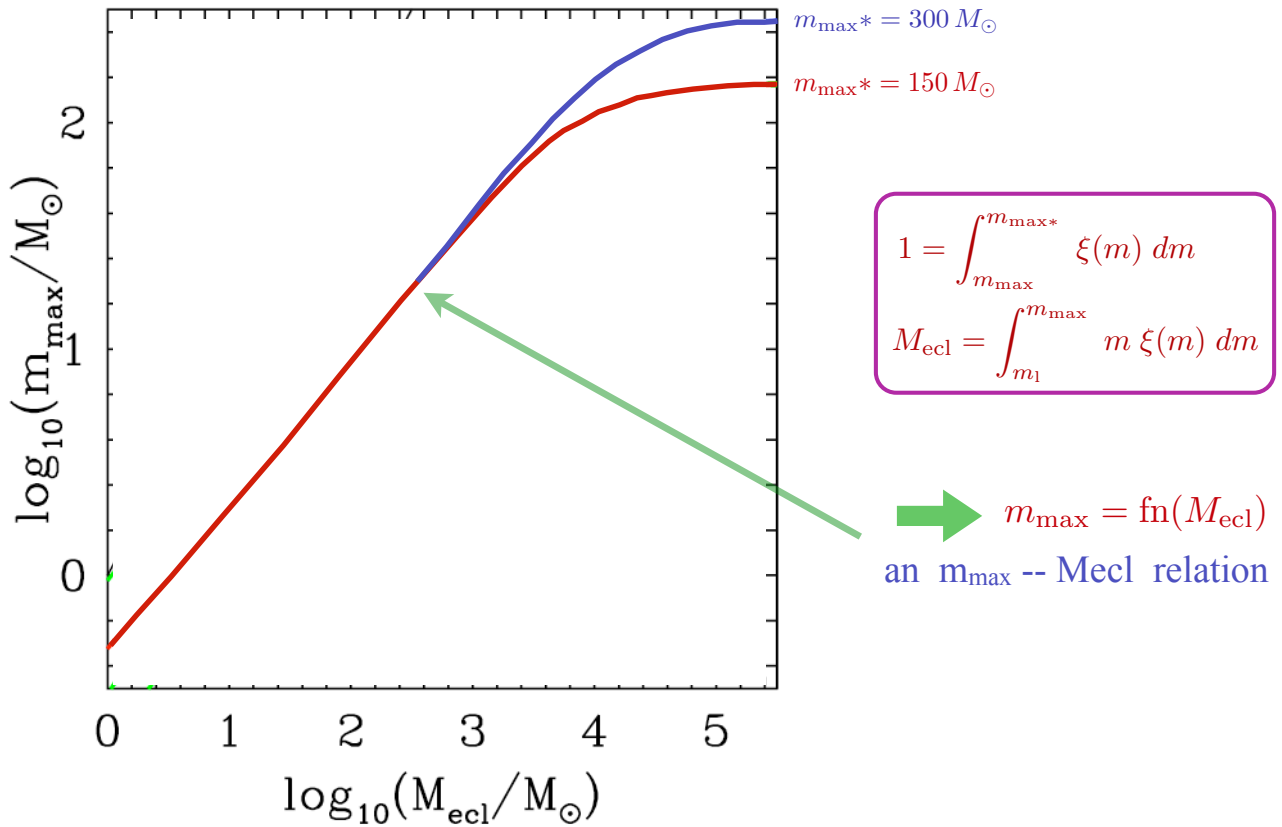
with

$$M_{\text{ecl}}(m_{\text{max}}) - m_{\text{max}} = \int_{m_L}^{m_{\text{max}}} m \xi(m) dm \quad (4.11)$$

as the closing condition. These two equations need to be solved iteratively. An excellent approx-



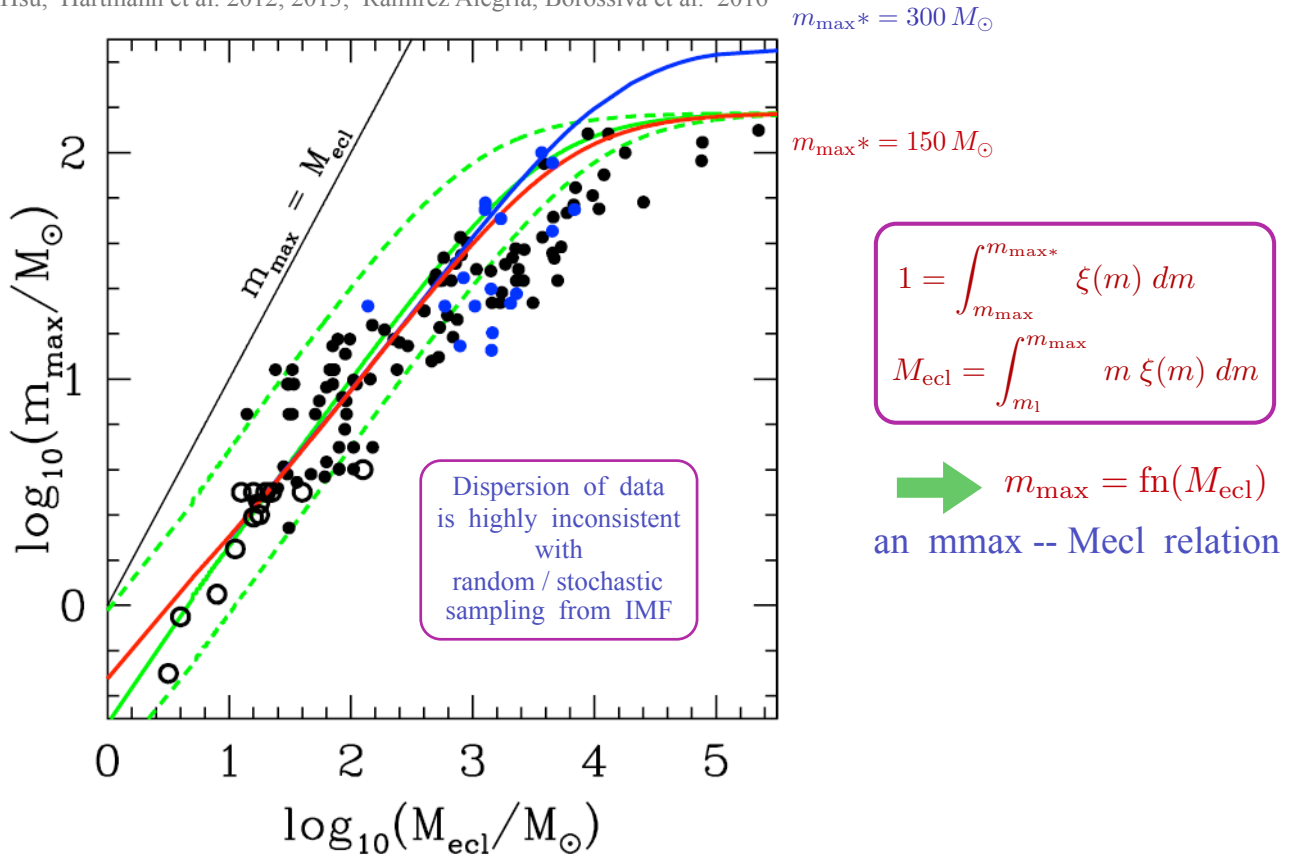
An important consequence of optimal sampling



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Weidner & Kroupa 2005, 2006; Weidner et al. 2010, 2013; Kroupa et al. 2013; Kirk & Myers, 2010, 2012; Hsu, Hartmann et al. 2012, 2013; Ramirez Alegria, Borossiva et al. 2016

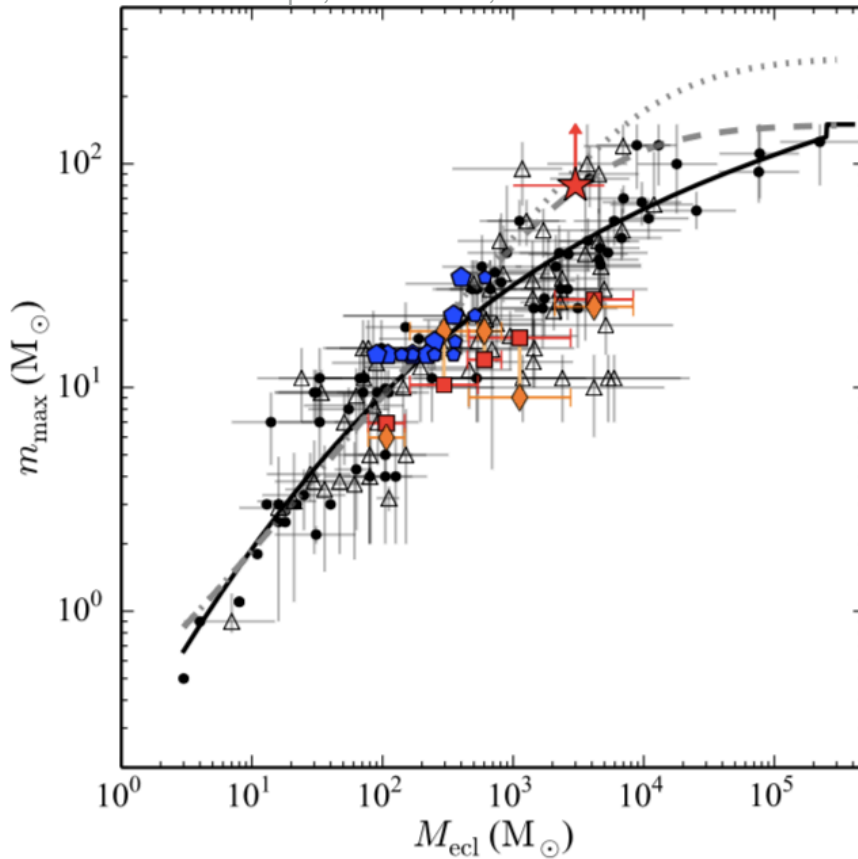


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Confirmation

Oh & Kroupa, submitted; Yan et al. 2017



"isolated" O stars in LMC

Stephens et al. 2017

Open clusters:

Ramírez Alegría et al. (2016).

Pavel Kroupa: University of Bonn / Charles University



Real young populations
are *not stochastic ensembles*
from invariant distr. functions

~~the star-formation hilfskonstrukt an invariant probability density distribution function~~

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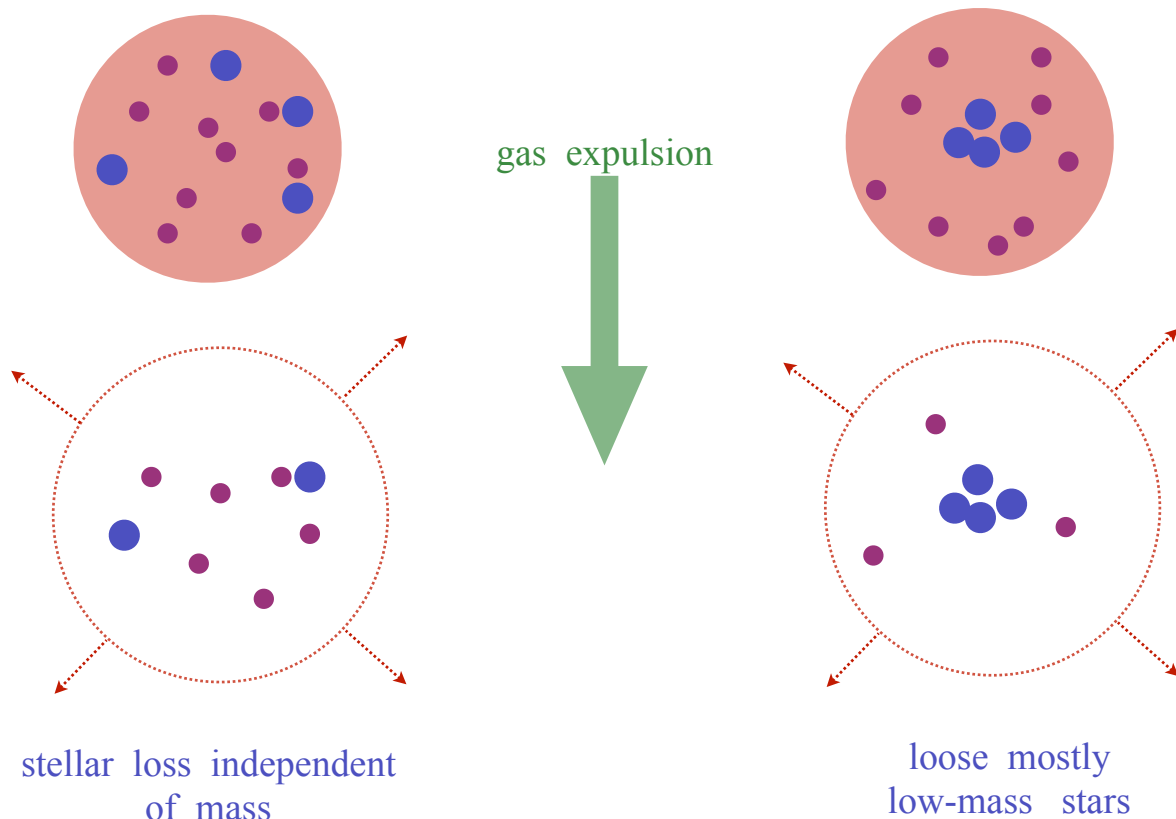
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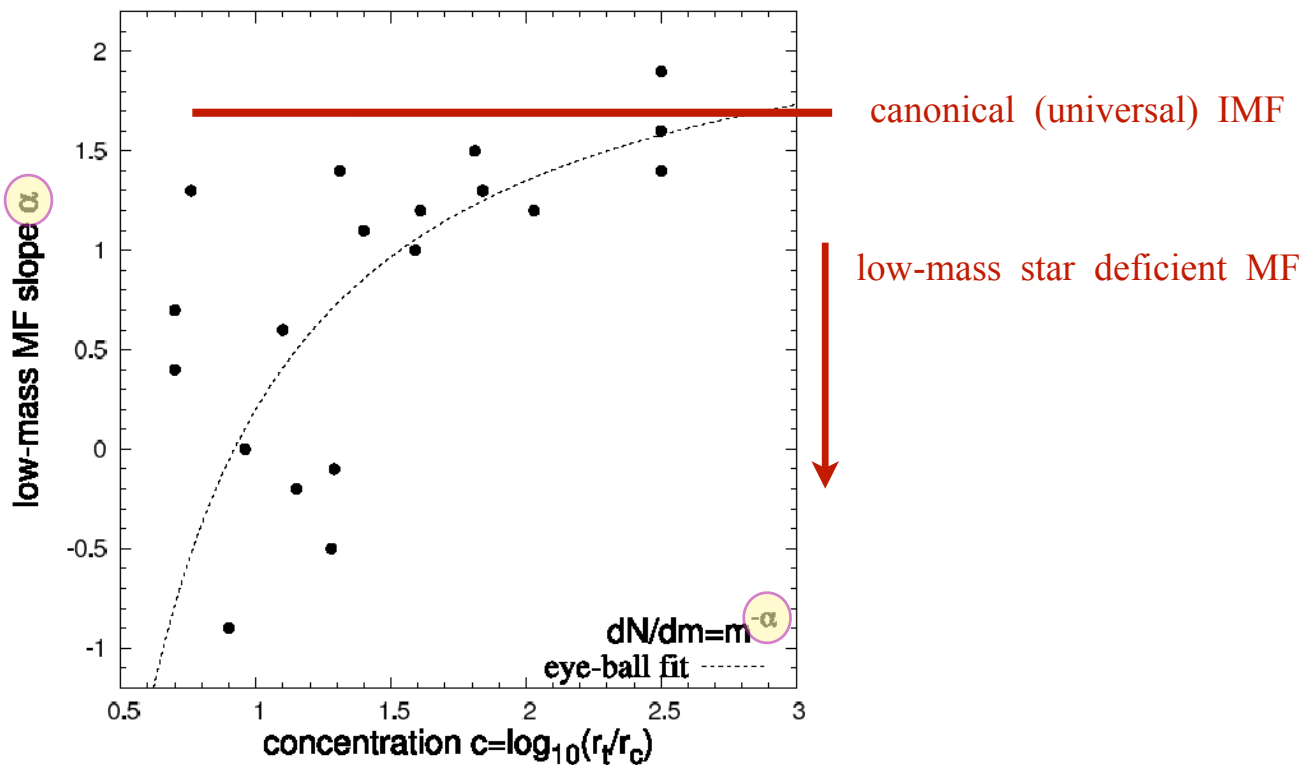
Nbody models of binary rich initially mass segregated clusters with residual gas expulsion after birth

(Marks, Kroupa & Baumgardt 2008)



*A sample of 20 Galactic GCs
with solid global MF measurements from
deep HST or VLT data.*

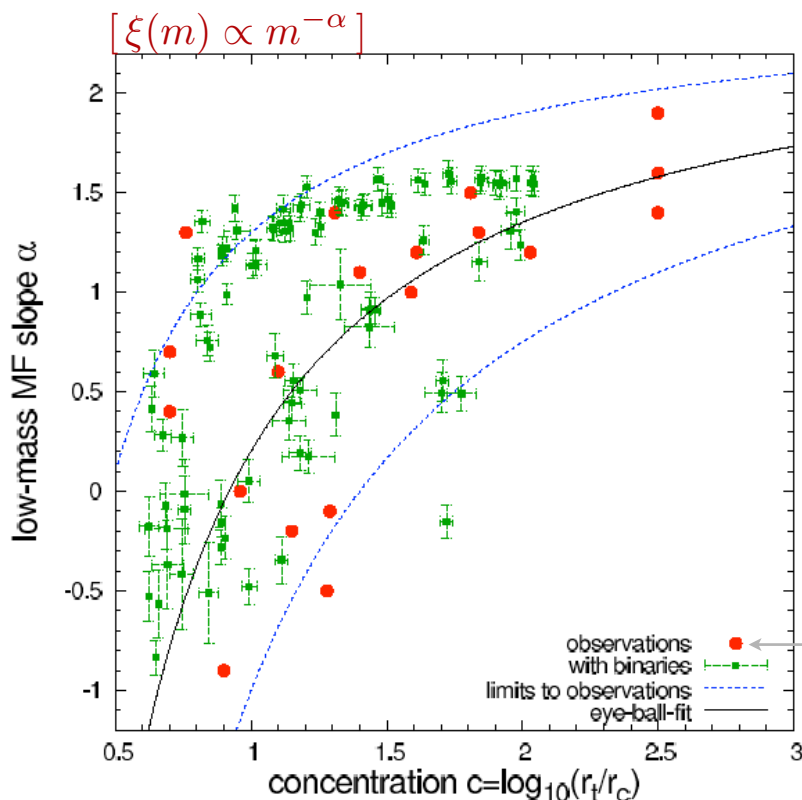
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for
residual gas expulsion
+
mass segregated clusters

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Top-heavy IMF in extreme-density environments :

THE STELLAR IMF DEPENDENCE ON DENSITY AND METALLICITY: Resolved stellar populations show an invariant IMF (Eq. 55), but for $SFRD \gtrsim 0.1 M_{\odot}/(\text{yr pc}^3)$ the IMF becomes top-heavy, as inferred from deep observations of GCs. The dependence of α_3 on cluster-forming cloud density, ρ , (stars plus gas) and metallicity, $[\text{Fe}/\text{H}]$, can be parametrised as

$$\begin{aligned} \alpha_3 &= \alpha_2, & m > 1 M_{\odot} \quad \wedge \quad x < -0.89, \\ \alpha_3 &= -0.41 \times x + 1.94, & m > 1 M_{\odot} \quad \wedge \quad x \geq -0.89, \\ x &= -0.14 [\text{Fe}/\text{H}] + 0.99 \log_{10} (\rho / (10^6 M_{\odot} \text{pc}^{-3})). \end{aligned} \quad (65)$$

Marks et al. 2012

Kroupa et al. 2013 (arXiv:1112.3340)

Star-counts: Correct star-counts in R136 for ejected stars
 → IMF in R136 top-heavy (Banerjee & Kroupa 2012)

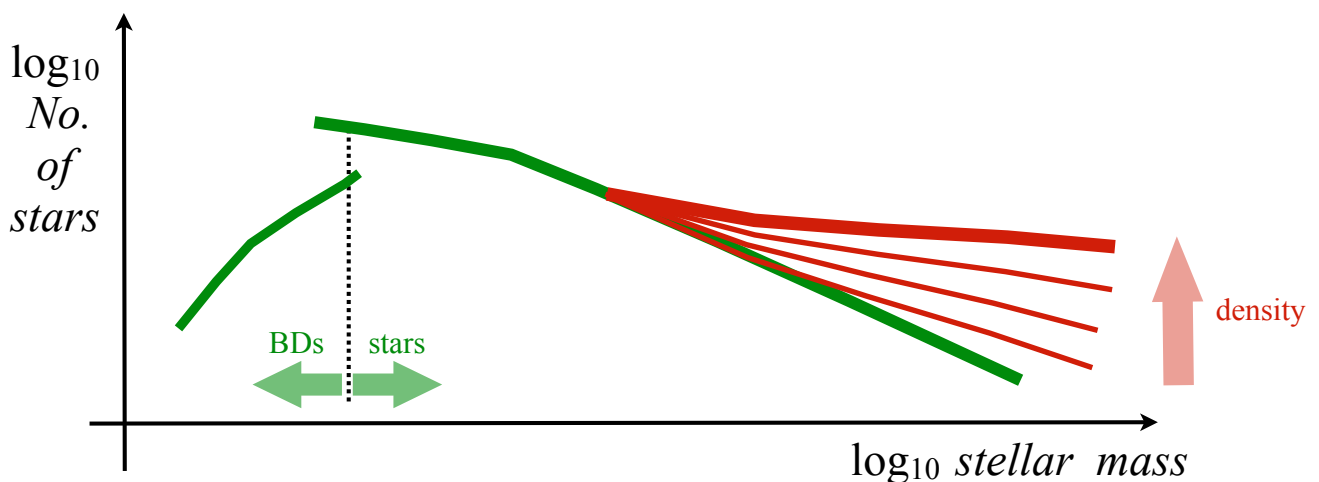
Excess of massive stars in whole 30Dor region
 (Schneider et al. 2018)

Top-heavy IMF in Magellanic Bridge cluster NGC796
 (Kalari et al. 2018)

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What this implies :



Using for $m < 1 M_{\odot}$

$$\alpha_{1,2} = \alpha_{1/2c} + 0.5 \left[\frac{\text{Fe}}{\text{H}} \right]$$

estimated from resolved MW populations
(Kroupa 2001)

where the canonical
solar-abundance values are

$$\alpha_{1c} = 1.3 \quad 0.07 < m/M_{\odot} < 0.5$$

$$\alpha_{2c} = 2.3 \quad 0.5 < m/M_{\odot} < 1.3$$

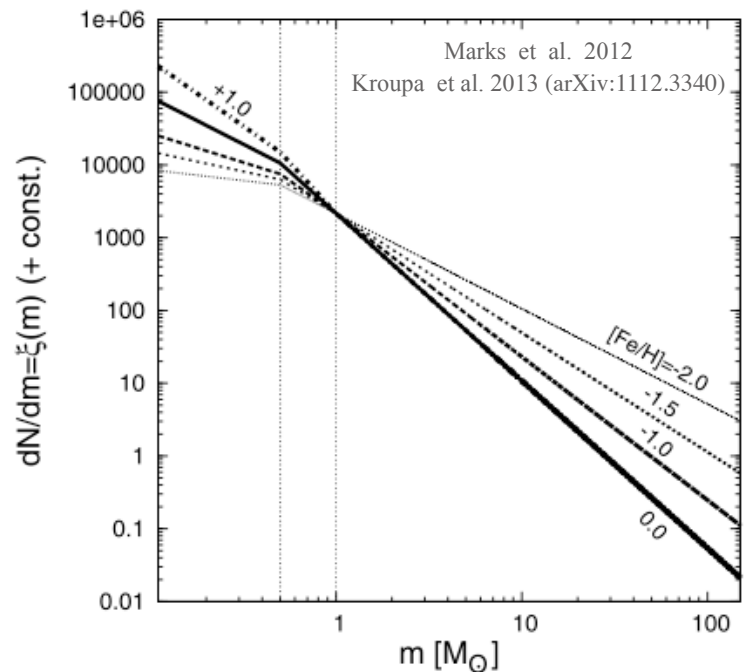


Figure 5. Suggested shape of the stellar IMF for different metallicities, $[\text{Fe}/\text{H}]$ (not taking into account the density dependence of the IMF). The IMFs are scaled such that their values agree at $m = 1 M_{\odot}$. Above $1 M_{\odot}$ the IMF slope is determined by the present work (Fig. 4, equation 11). Below $1 M_{\odot}$ the parametrization is determined by Kroupa (2001, equation 12), whose results suggest tentative evidence that more metal-rich environments produce relatively more low-mass stars. Note that only the metallicity dependence is shown, but not the dependence on mass (Fig. 2) or density (Fig. 3).

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Composite Stellar Populations

Stars form in a clustered mode (Lada & Lada 2003; Megeath et al. 2016).
Thus, the Integrated Galactic IMF follows from

$$IGIMF = \sum \text{ of IMFs (in all clusters)}$$



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$$\xi_{IGIMF}(m, t) = \int_{M_{ecl, min}}^{M_{ecl, max}(SFR(t))} \xi(m \leq m_{max}(M_{ecl})) \xi_{ecl}(M_{ecl}) dM_{ecl}$$

Kroupa & Weidner (2003); Weidner & Kroupa (2005, 2006)
Yan, Jerabkova & Kroupa (2017); Jerabkova et al. (2018)

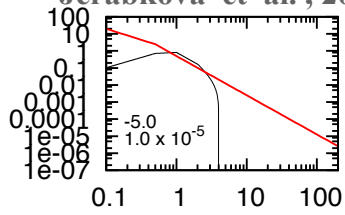


Natural explanation of the
mass-metallicity relation,
radial H α cutoff
vs UV extended disks
of galaxies
and many other problems in
understanding galaxies.

Jerabkova et al. 2018; Yan, Jerabkova et al. 2017; Fontanot et al. 2017;
and recent papers by Simone Recchi et al.; Sylvia Ploechinger et al.
and previous papers by Koeppen et al., Pflamm-Altenburg et al., Weidner et al.

The IGIMF for galaxies with different SFRs & [Fe/H]

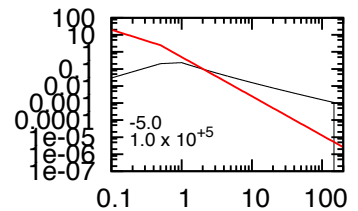
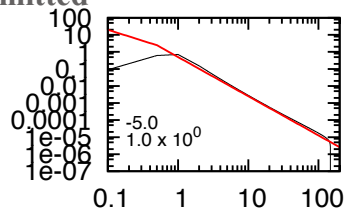
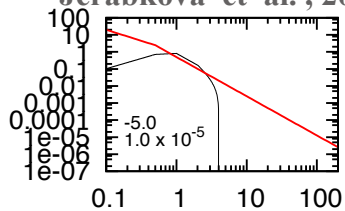
Jerabkova et al., 2018 submitted



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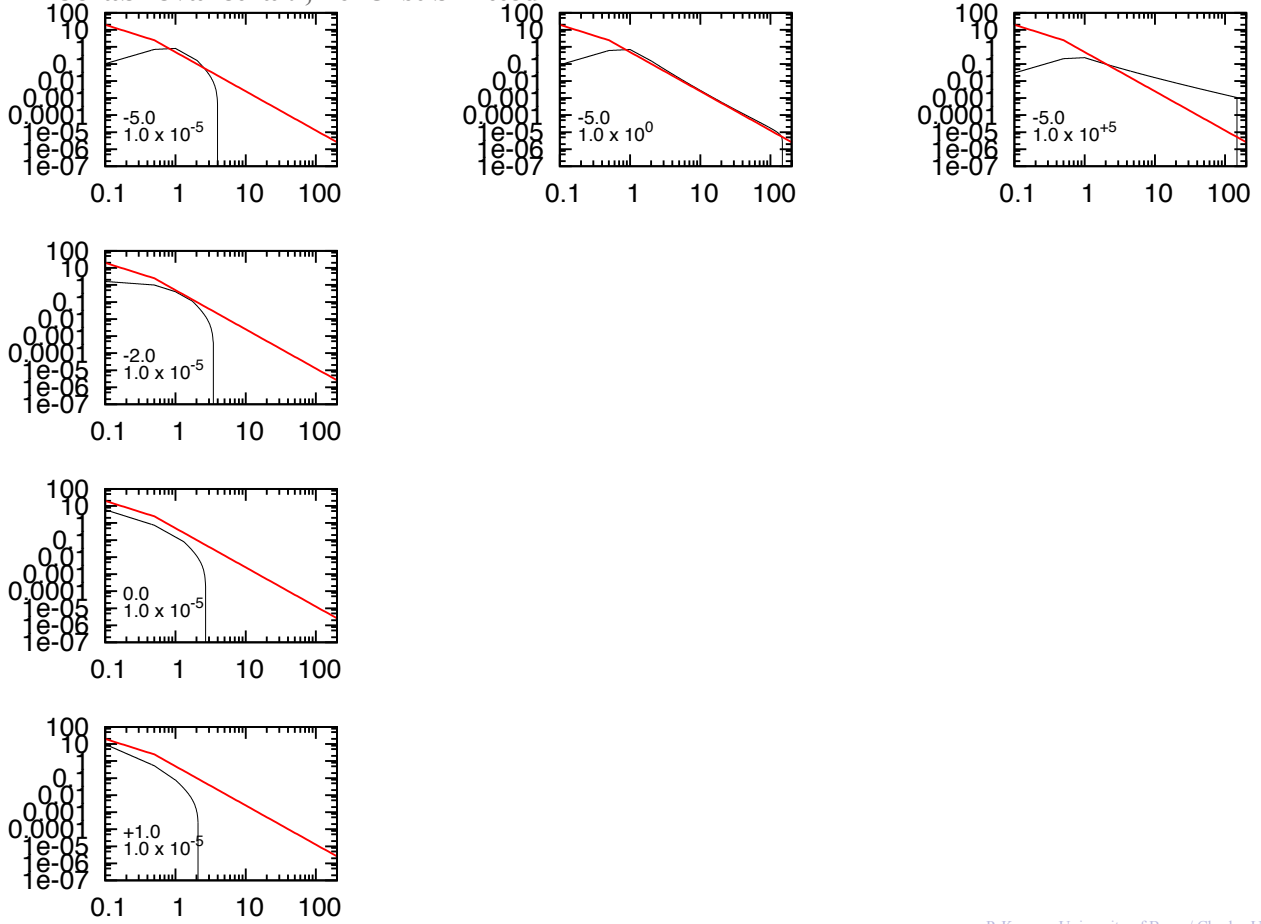
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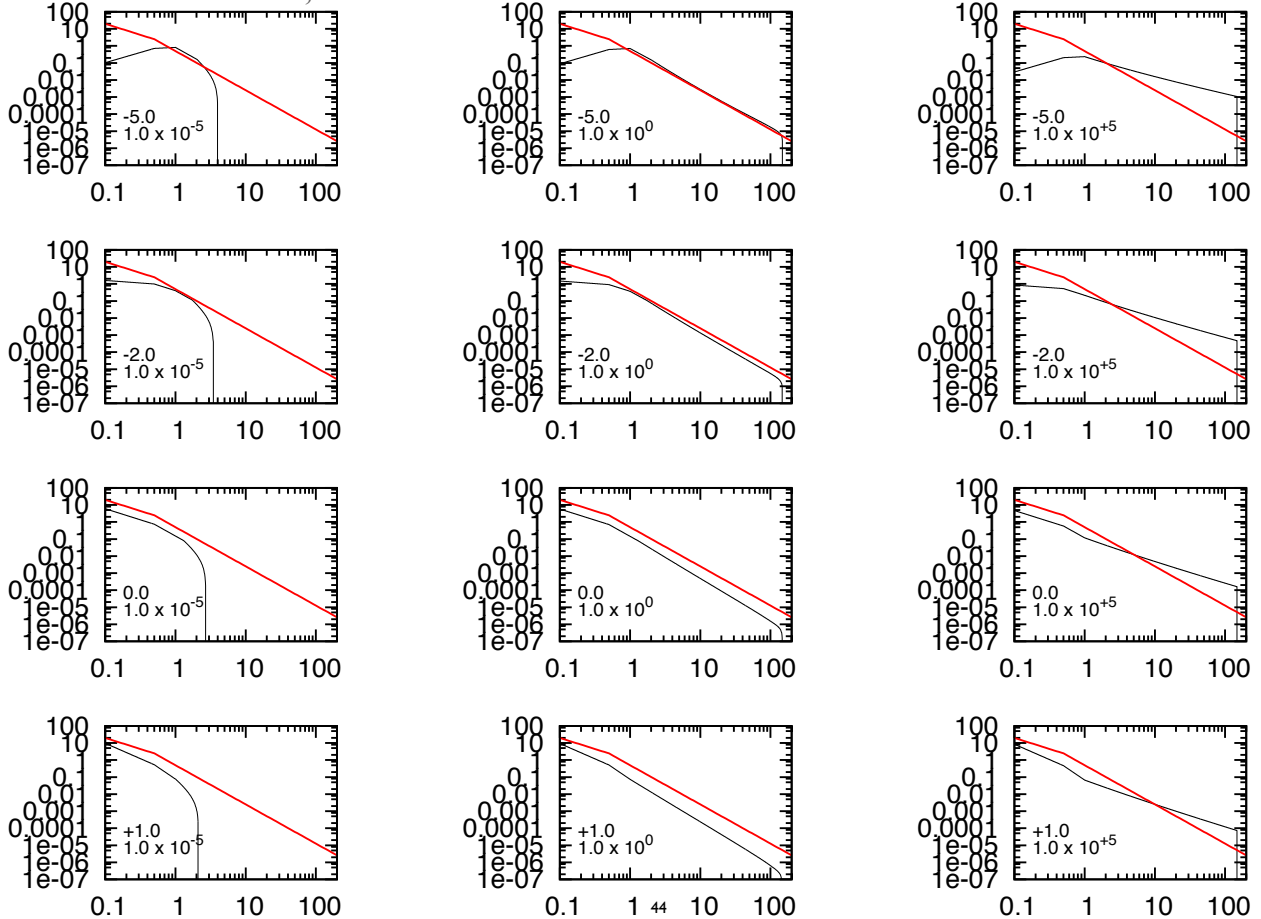
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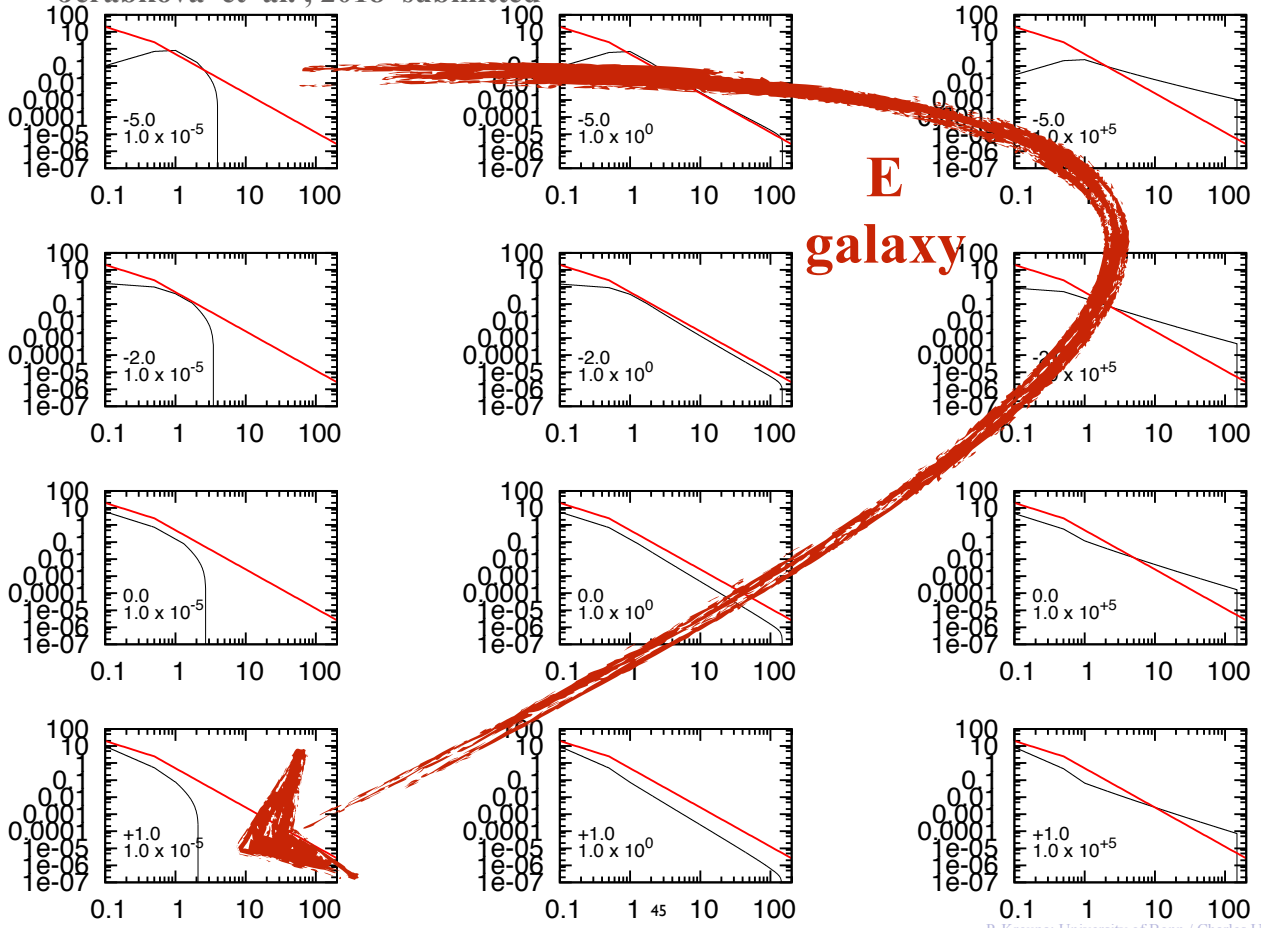
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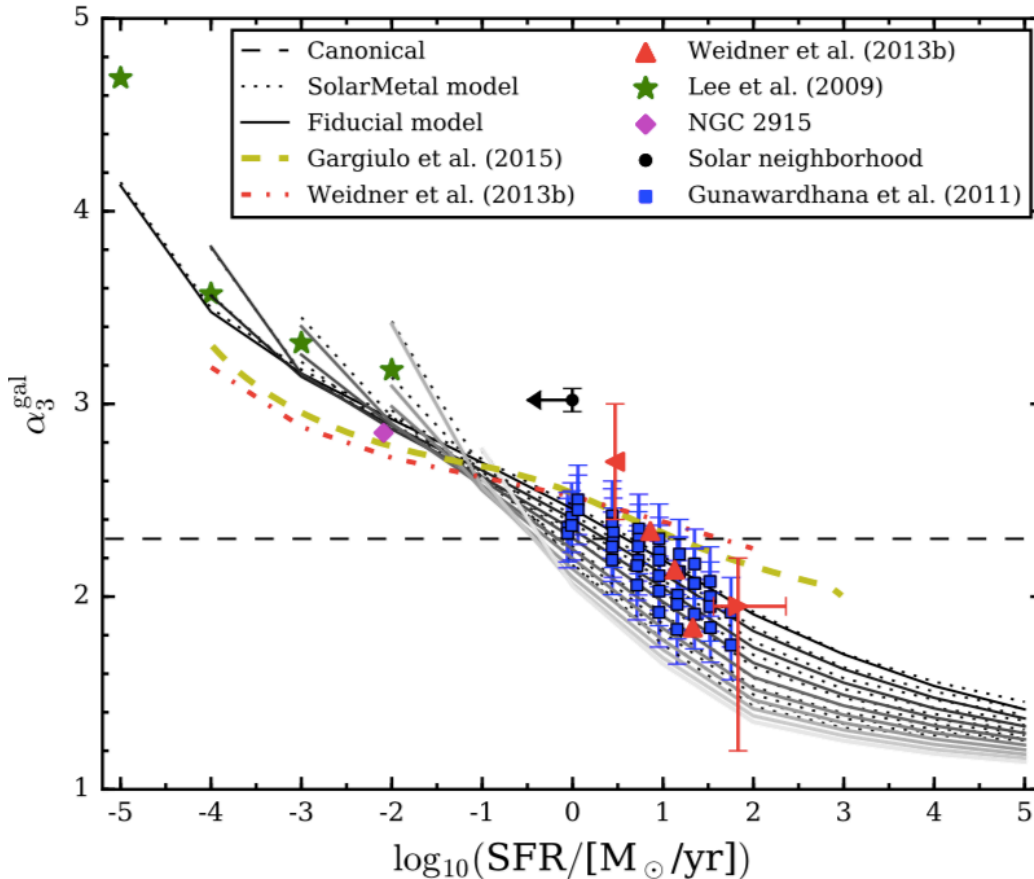
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Yan, Jerabkova & Kroupa 2017; see also Weidner et al. 2013



E galaxies
formed with
top-heavy IMFs

confirming Matteucci
(1994) !
and see recent
work by Vazdekis

Kennicutt et al. (1994) SFR relation (for a pure Salpeter power-law IMF betw. 0.1 and 100 Msun)



$$\text{SFR}(\text{total}) = \frac{L_{H\alpha}}{1.26 \times 10^{41} \text{ erg s}^{-1}} M_{\odot} \text{ yr}^{-1}. \quad (14)$$

$$SFR \propto L_{H\alpha} \text{ always for all galaxies}$$

$$L_{H\alpha} = \mu \ 3.0207 \times 10^{-12} \text{ erg } N_{\text{ion}}/\delta t, \quad (3)$$

$$N_{\text{ion},\delta t} = \int_{m_{\text{low}}}^{m_{\text{max}}} \xi \text{ IMF}(m) N_{\text{ion},\delta t}(m) dm, \quad (2)$$

$$\mu = 1 \quad (\text{Pflamm-Altenburg et al. 2007})$$

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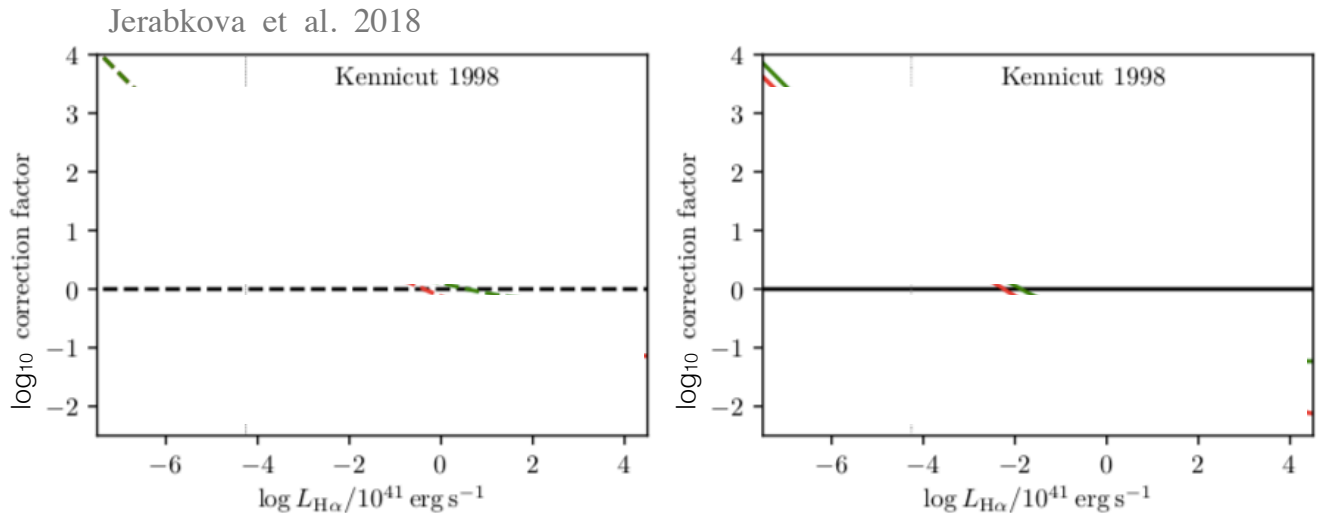


Fig. 8. Corrections to the Kennicutt SFR– $H\alpha$ relation. The true (IGIMF) SFRs are divided by the Kennicutt value, for the same $H\alpha$ flux and for the IGIMF models and metallicities shown in Fig. 7. The $H\alpha$ flux of LeoP is shown by the vertical dotted line in both panels. The two panels demonstrate that dwarf galaxies with $H\alpha$ fluxes near 10^{36} erg/s have a SFR which is more than 100 times larger than that suggested by the Kennicutt relation, while massive or profusely star-forming galaxies with $H\alpha \approx 10^{44}$ erg/s have SFRs which can be 10–100 times smaller than given by the traditional Kennicutt relation. This has major implications for understanding galaxy formation and evolution.

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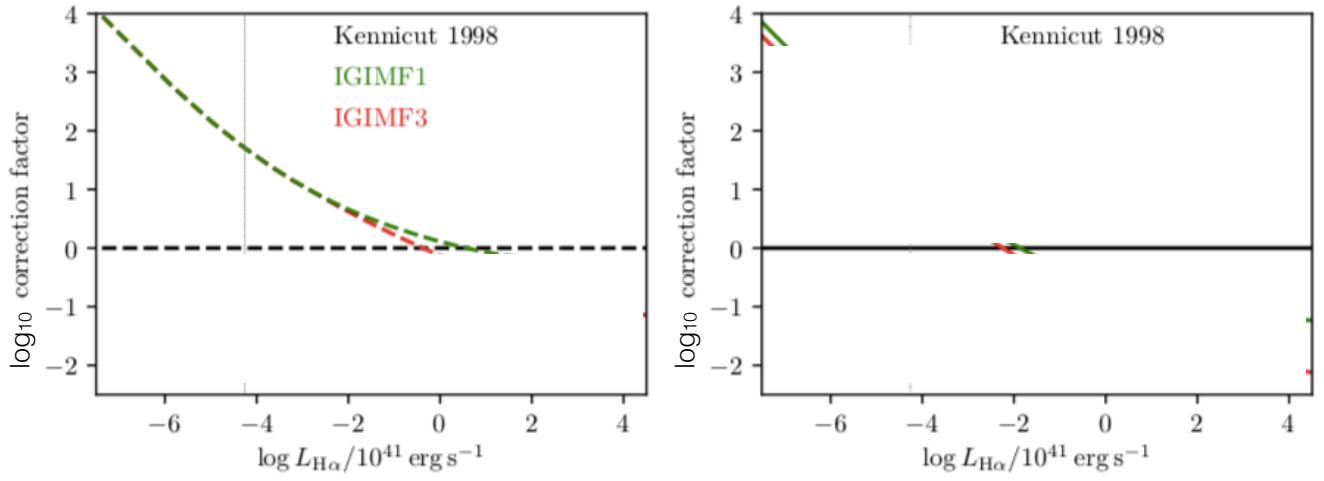


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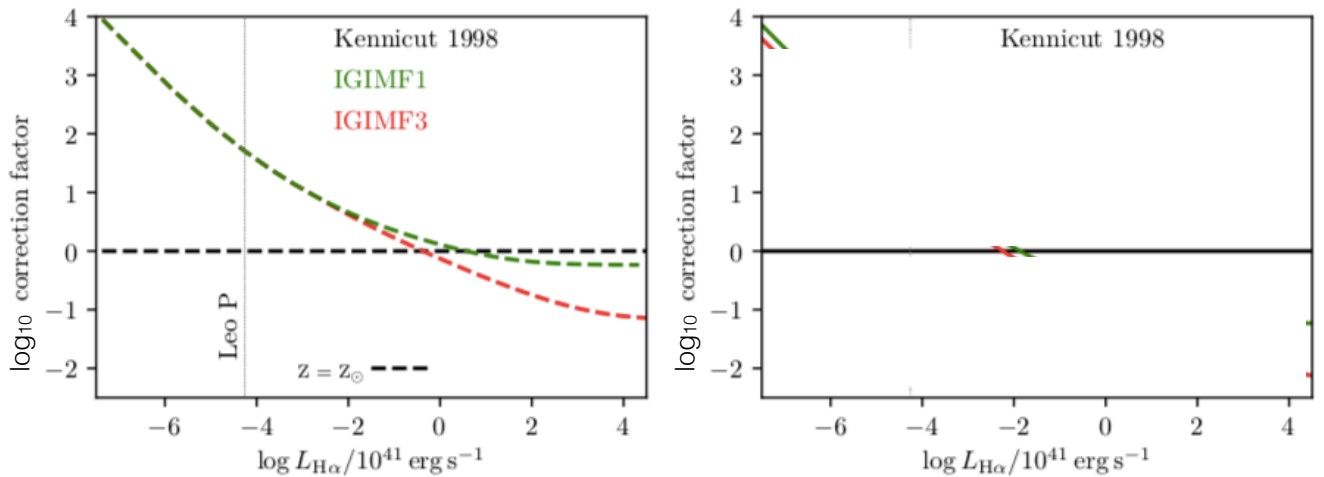


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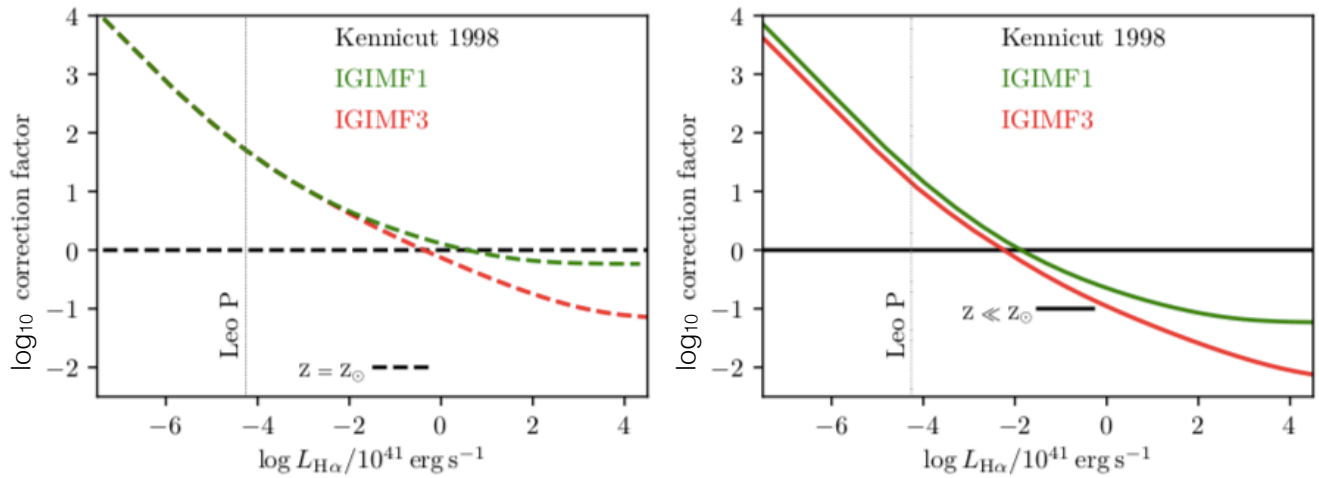
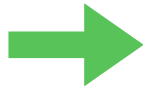


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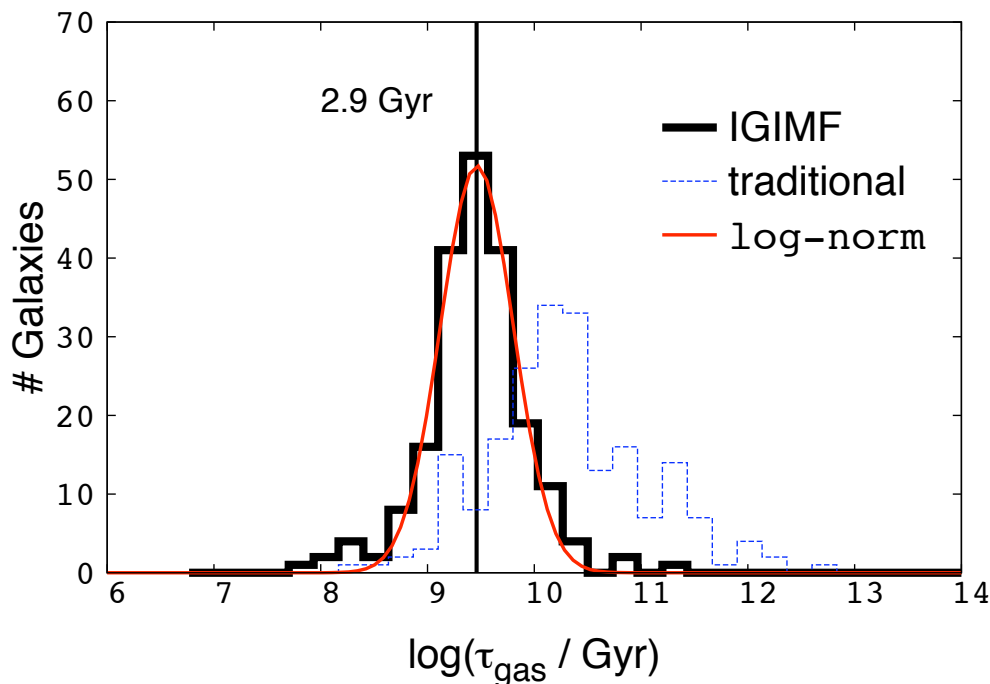
Pavel Kroupa: University of Bonn / Charles University

IGIMF theory



constant
gas depletion
time-scale !

for galaxies with
 $10^5 \lesssim \frac{M_{\text{gas}}}{M_{\odot}} \lesssim 10^{11}$



Pflamm-Altenburg,
Weidner &
Kroupa
(2007);
Pflamm-A. &
Kroupa
(2009)

for all late-type galaxies $10^6 \lesssim \frac{M_{\text{gas}}}{M_{\odot}} \lesssim 10^{11}$ 2.9 Gyr ?

Pavel Kroupa: *ifU*, University of Bonn

Dwarf galaxies have the same star-formation efficiencies
as L* galaxies

$$SFR = \frac{1}{3 \text{ Gyr}} M_{\text{neutral gas}}$$

i.e. every 10 Myr a galaxy transforms
0.3 % of its neutral gas mass
into stellar mass.

Pavel Kroupa: *Alfa*, University of Bonn

Conclusions

What we call the IMF is a mathematical "hilfskonstrukt"

This "hilfskonstrukt" is
an optimally sampled distribution function
(star formation is not probabilistic but
highly self-regulated)

This "hilfskonstrukt" varies with physical conditions

Star-formation "hilfskonstrukt" $\neq$ galaxy-wide "hilfskonstrukt"

Important implication for galaxy evolution (& cosmology)