How Turbulence Enables the Explosion of Core-Collapse Supernova

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1-D simulations do not explode, yet multi-D do

Simulations suggested, but did not prove, that turbulence is the dominant factor that aids explosion

We develop a self-consistent turbulence model and show that it reduces the critical conditions for explosion

The semi-analytic nature also allows us to explain *how* it enables explosion

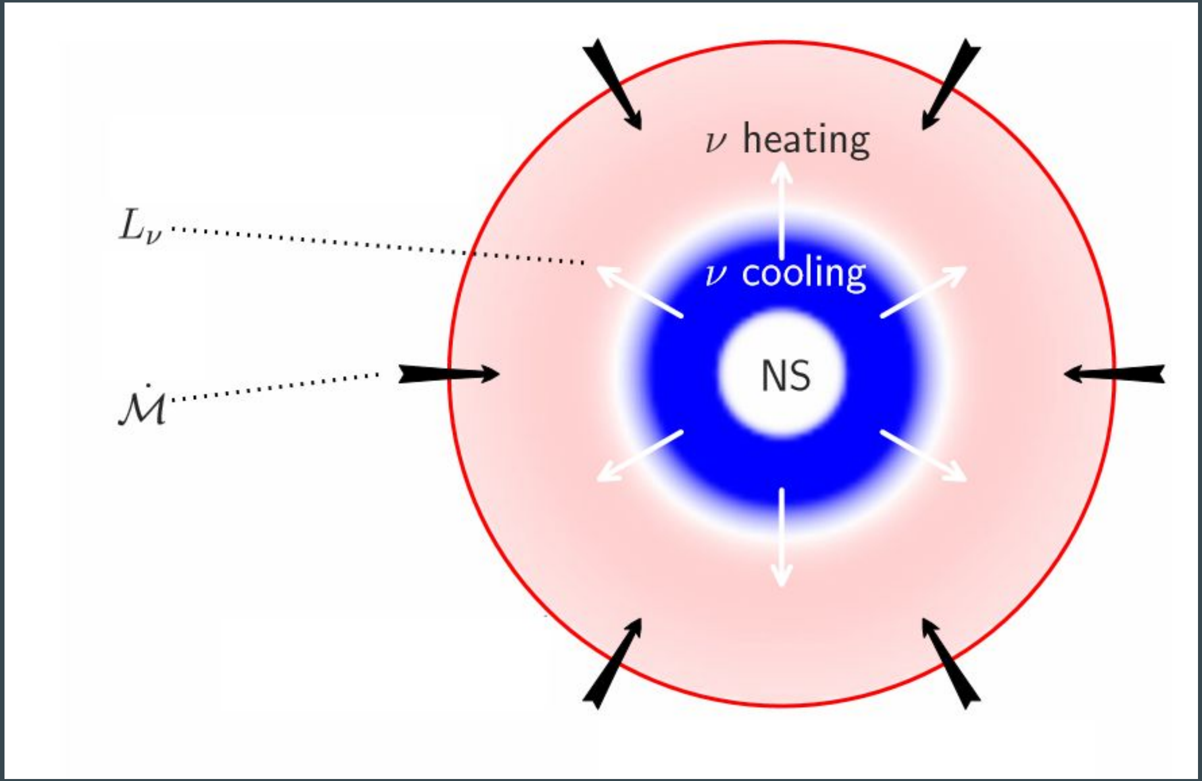
Outline

- Background
  - ➢ Why turbulence is important
  - ➢ “Critical Curve”
- Methodology
  - ➢ Reynolds decomposition
  - ➢ Turbulence Models
  - ➢ Isolating Turbulent Terms
  - ➢ Source of Energy for Turbulent Dissipation
- Results and Future Work

The fundamental question is: how does the stalled shock revive into an blastwave?

Reviving the Shock

- Bethe and Wilson (1985) hypothesized that neutrino heating alone revitalized the shock



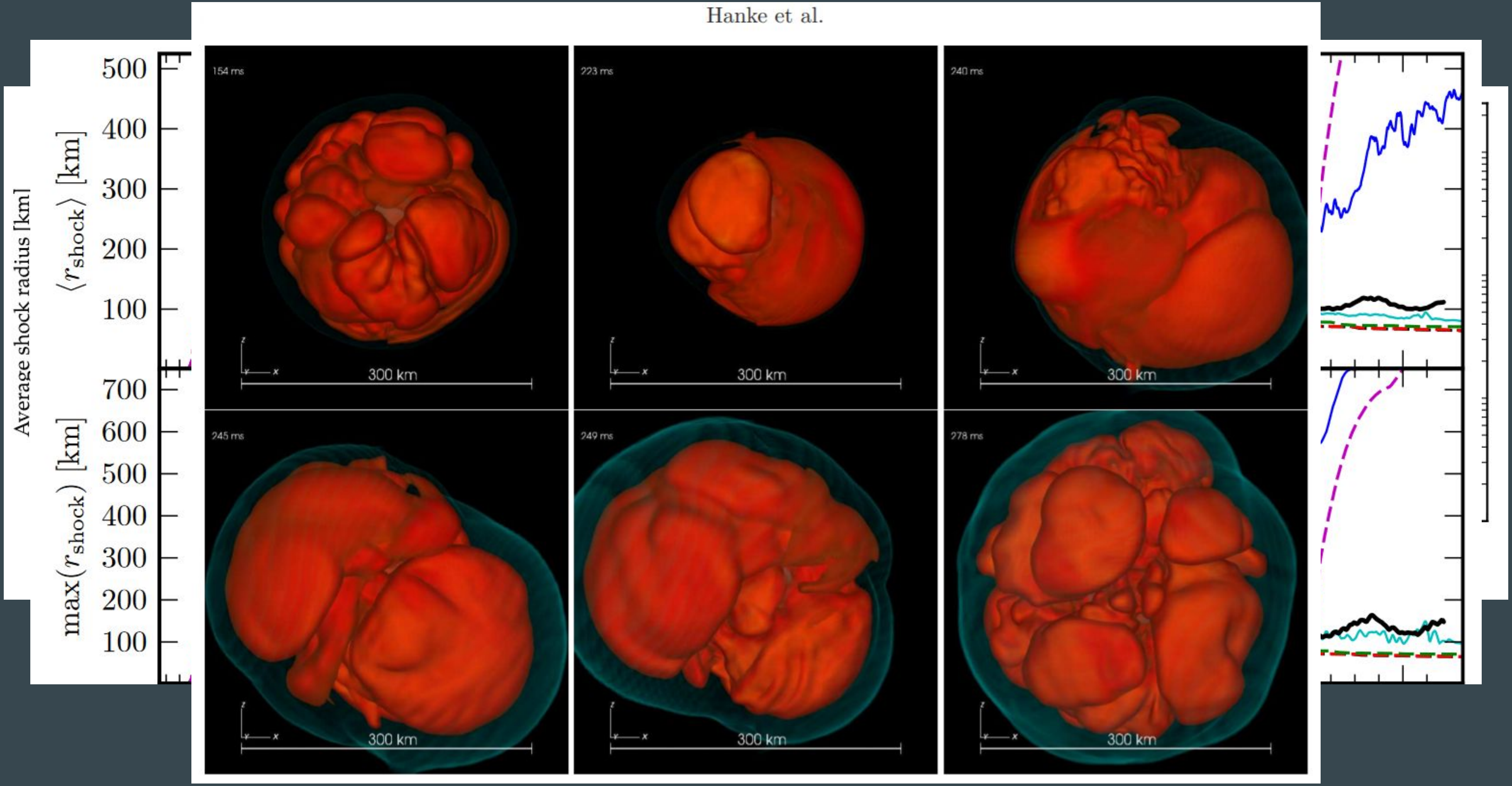
- As the core heats up, a few processes are involved
  - Nuclear Dissociation
  - Electron Capture
  - Positron Capture

- Energy is converted to neutrinos
  - $p + e^- \rightarrow n + \nu_e$
  - $n + e^+ \rightarrow p + \bar{\nu}_e$

- Lightbulb Model

1-Dimensional hydrodynamic and neutrino transport simulations have been unsuccessful in creating realistic supernovae...

Multi-D succeeds where 1-D fails



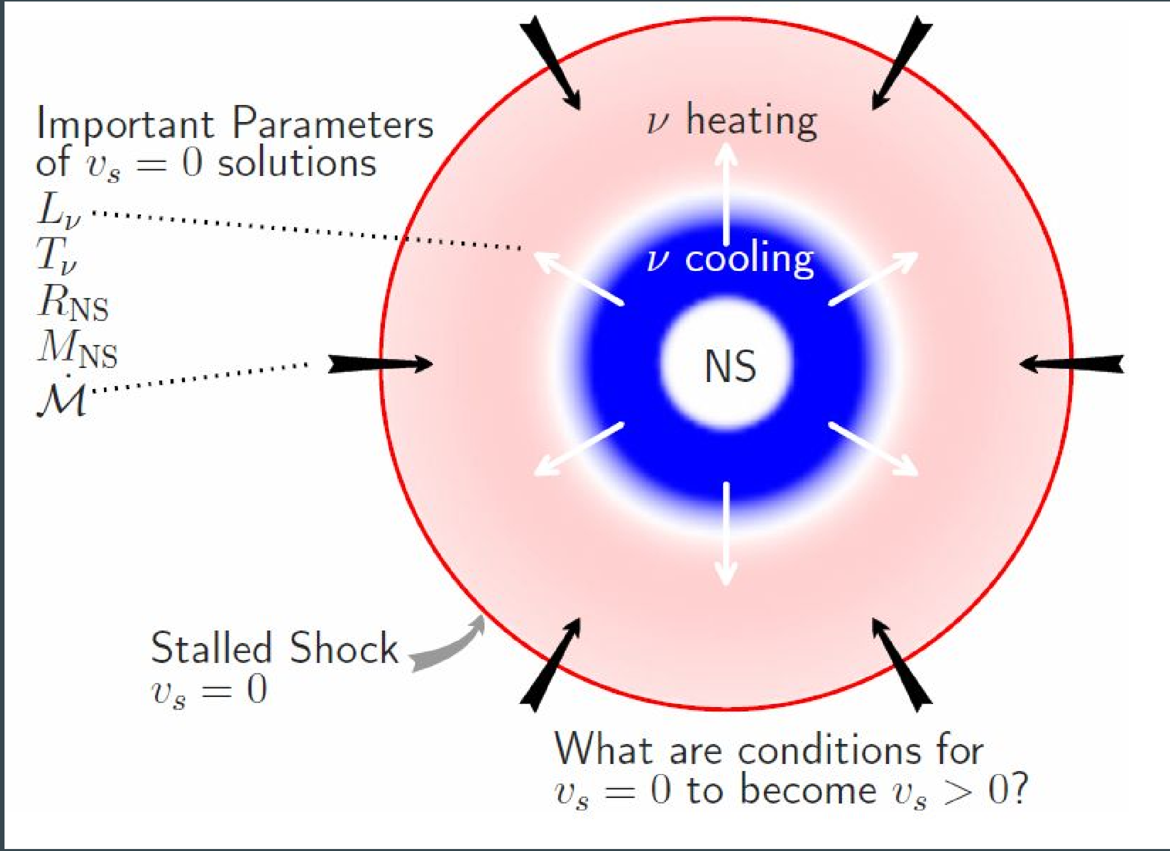
How does turbulence aid explosion?

- People suspect that turbulence is a key contributor to successful supernova explosions (Murphy & Burrows 2008; Marek & Janka 2009; Murphy & Meakin 2011; Hanke+ 2012; Murphy+ 2013; Couch & Ott 2015; Melson+ 2015; Radice+ 2016; etc.)
- Though there is some progress, these are guesses and suppositions. No one had yet shown *how* turbulence aids explosion

Simulations are important, but they’re often complex and computationally expensive, making it difficult to thoroughly investigate the various effects

We use another approach, which is to build upon and develop a semi-analytic theory, which enables a deeper understanding of the mechanisms at play

The Big Picture



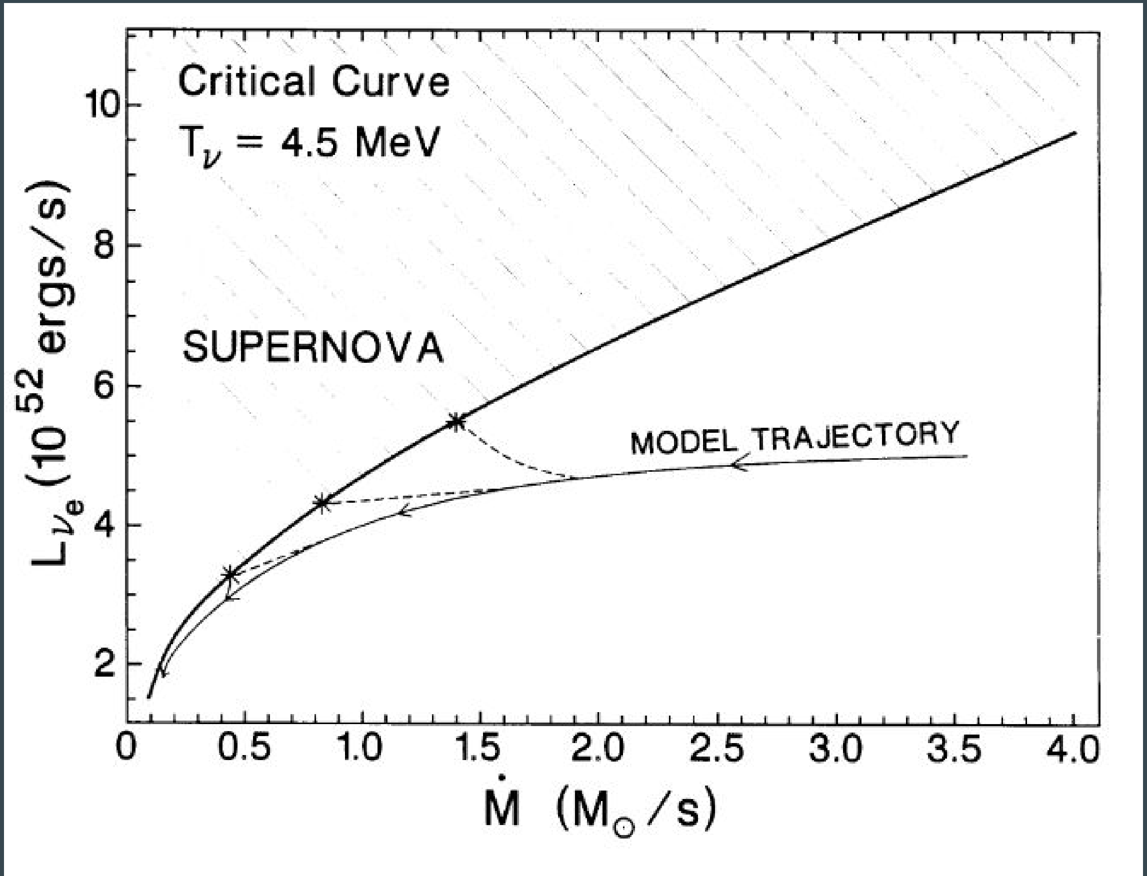
Murphy & Dolence (2017)

My Parameters

- M_{NS}: 1.5 $M_\odot$
- R_{NS}: 65 km
- T_ν: 4 MeV
- Let $\dot{M}$ vary

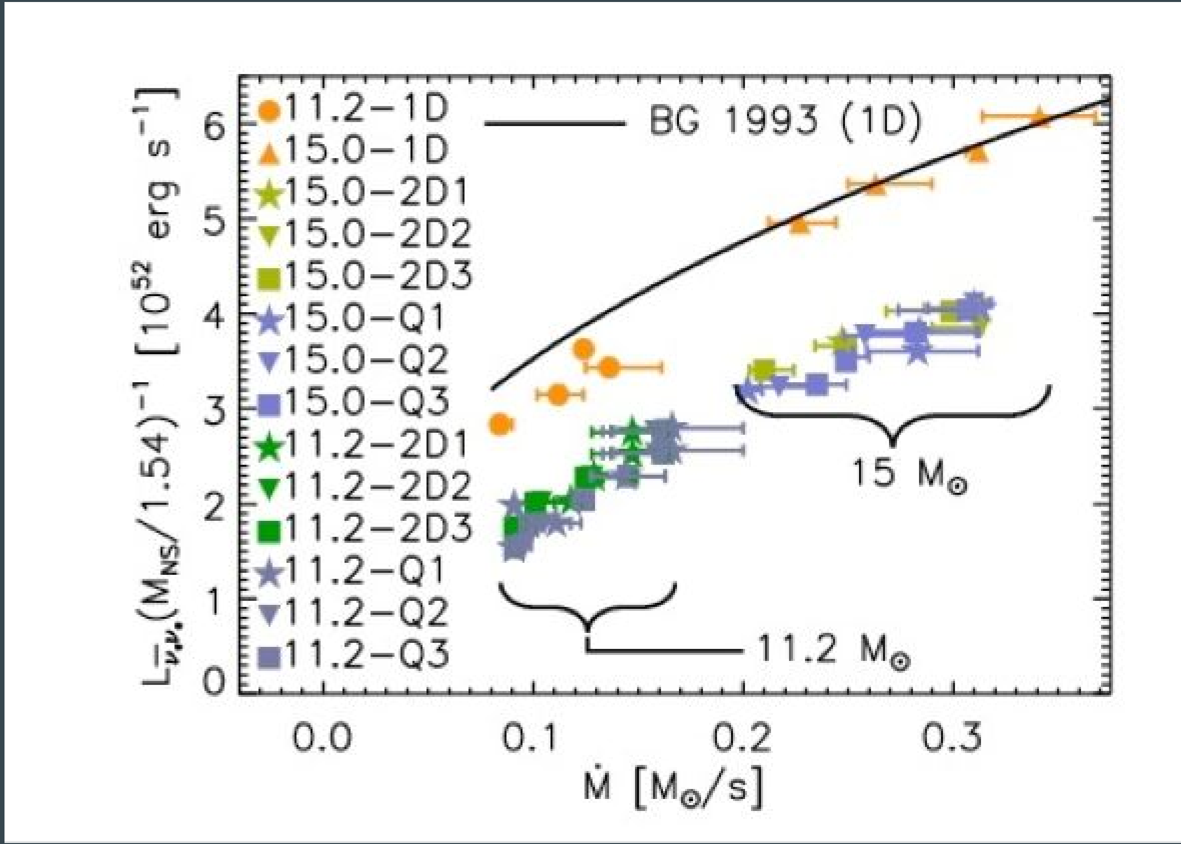
- Find critical L_ν

Original Critical Curve



Burrows & Goshy (1993)

Some Inspiration



Murphy & Burrows (2008)

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Let's augment this critical condition analysis to include turbulence in a self-consistent way

With Reynolds decomposition, we can explore how turbulence reduces the critical curve

Start With Continuity Equations

$$\frac{d\rho}{dt} = -\rho \nabla \cdot \boldsymbol{v},$$

$$\rho \frac{d\boldsymbol{v}}{dt} = -\rho \nabla \Phi - \nabla P,$$

$$\rho \frac{d\varepsilon}{dt} = -P \nabla \cdot \boldsymbol{v} + \mathcal{H} - \mathcal{C},$$

Equations With Turbulence

$$\nabla \cdot (\rho_0 \vec{u}_0 + \langle \rho' \vec{u}' \rangle) = 0$$

$$\langle \rho \vec{u} \rangle \cdot \nabla \vec{u}_0 = -\nabla P_0 + \rho_0 \vec{g} - \nabla \cdot \langle \rho \mathbf{R} \rangle$$

$$\langle \rho \vec{u} \rangle \cdot \nabla h = \langle \rho q \rangle + \rho_0 \epsilon_k - \nabla \cdot \langle \vec{L}_e \rangle$$

- By Reynolds decomposing our equations we can peer into the various components involved in creating a successful supernova explosion

$$L_{\mathbf{e}} = 4\pi r^2 \rho \langle e'v' \rangle$$

$$R = {v'_{rr}}^2 + {v'_{\theta\theta}}^2 + {v'_{\phi\phi}}^2$$

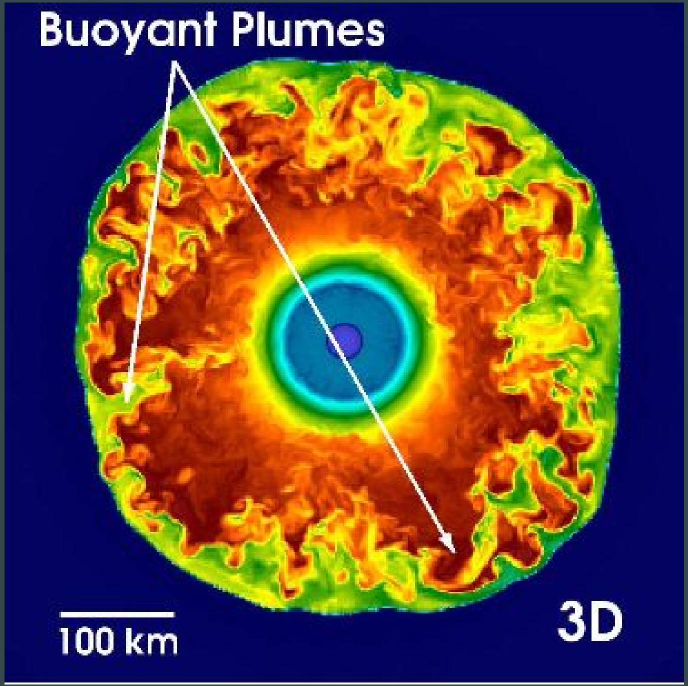
$$\varepsilon = \nu \left[\nabla^2 \langle \mathbf{R} \rangle - 2(\nabla v') \cdot (\nabla v') \right]$$

By introducing these new terms, we now have more unknowns than equations

Must develop a model for the turbulent dissipation (ε_k), the Reynolds stress (**R**), and turbulent luminosity (L_e)

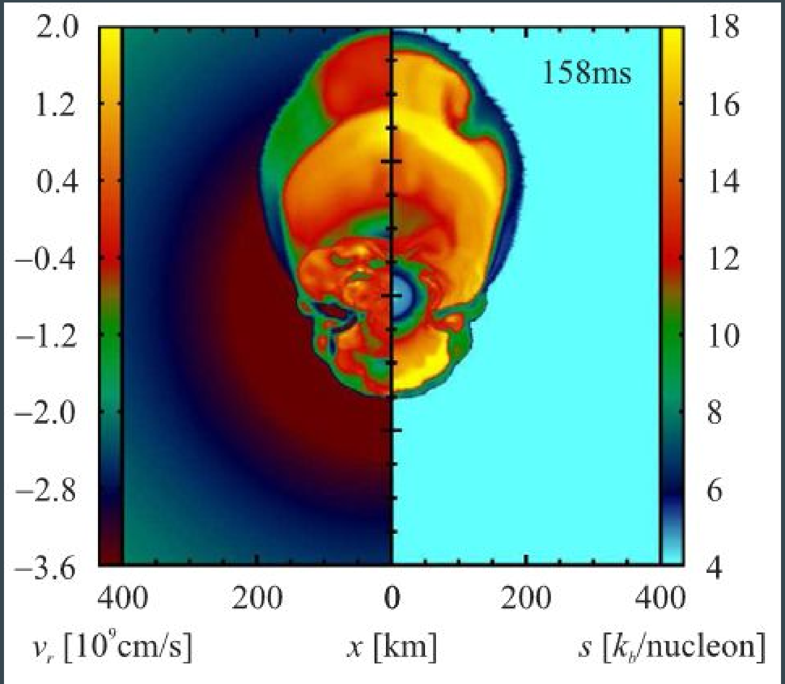
Two Sources of Turbulence

Neutrino Driven Convection



Murphy+ (2013)

Standing Accretion Shock Instability (SASI)



Müller+ (2012)

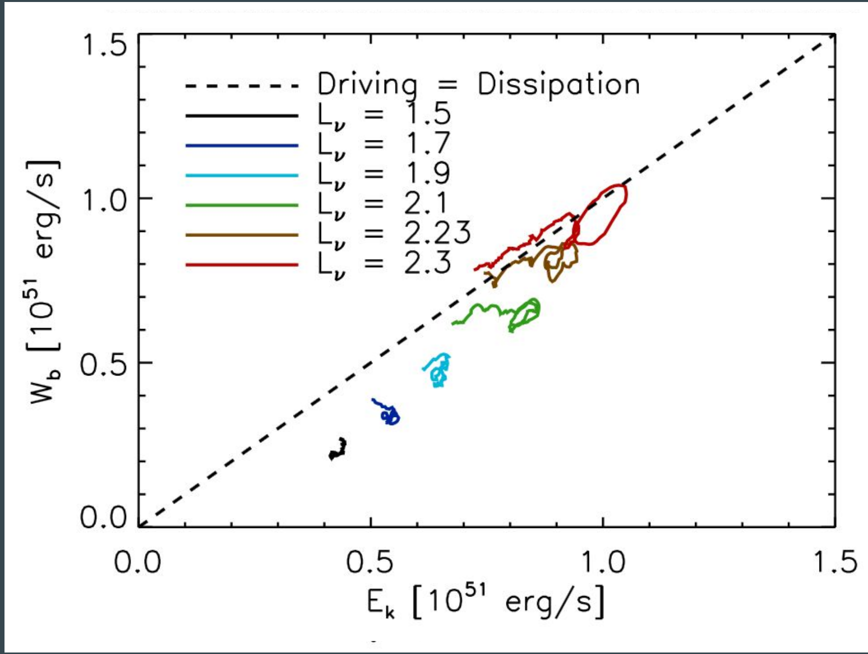
To correctly incorporate these effects we need
a non-linear model for turbulence

A non-linear model exists for convection but not yet for the SASI
(work in progress by Erica Bloor)

For now, Neutrino Driven Convection is the only model
we have to probe for the intricacies turbulent dynamics

Making some constraints...

- From Murphy & Dolence (2013), we can impose a few conditions on the turbulent kinetic energy and turbulent luminosity. Namely:
  - ➢ $L_\nu \tau = E_K + L_e$
  - ➢ $W_B = E_k$



The radial Reynolds stress is in rough equipartition with both of the tangential components

$$R_{\phi\phi} + R_{\theta\theta} \sim R_{rr}$$

The transverse components are roughly balanced

$$R_{\phi\phi} \sim R_{\theta\theta}$$

Following Kolmogorov's hypothesis, we can also approximate the turbulent dissipation to be:

$$\varepsilon_k = v'^3 \mathcal{L}^{-1} = R^{3/2} \mathcal{L}^{-1}$$

We need some sort of local description

Based upon simulations, we have
a profile for these terms

We use the global conditions
to set the scales

Some Assumptions...

- $E_k \sim \varepsilon_k M_g$
- We take the lengthscale of convection to be the entire length of the gain region
- Use a hypertangent prescription of the Turbulent Luminosity

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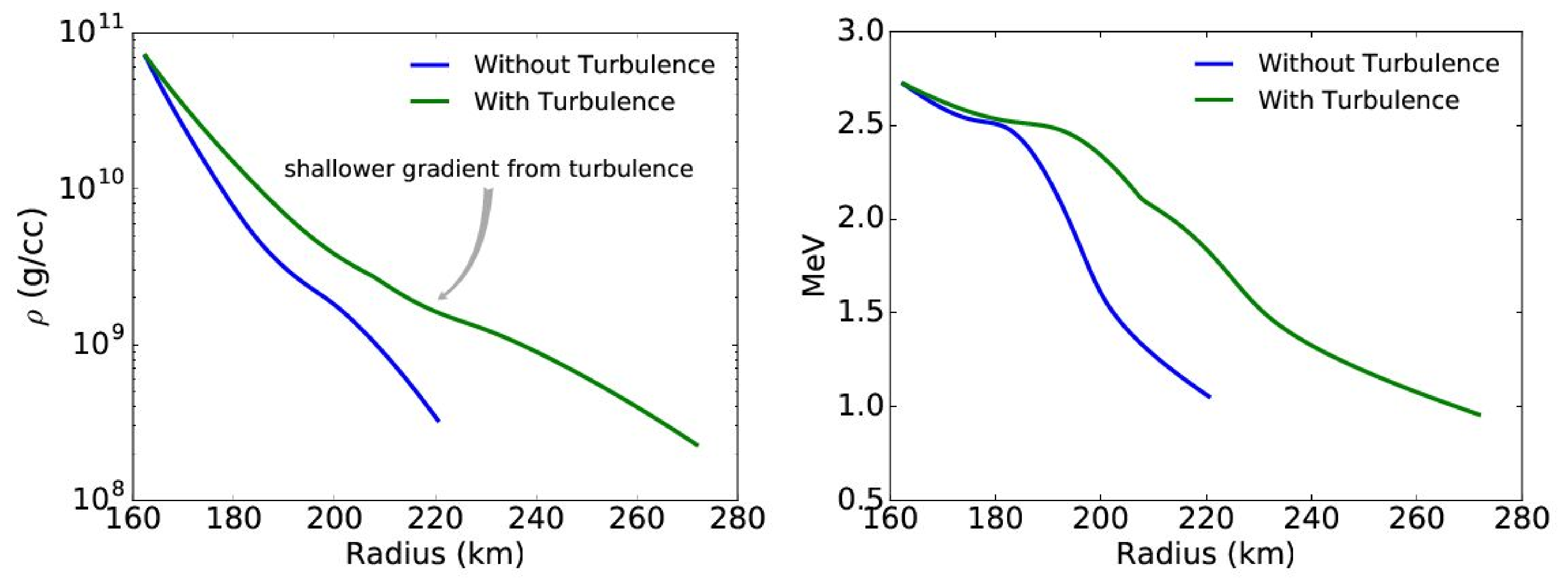
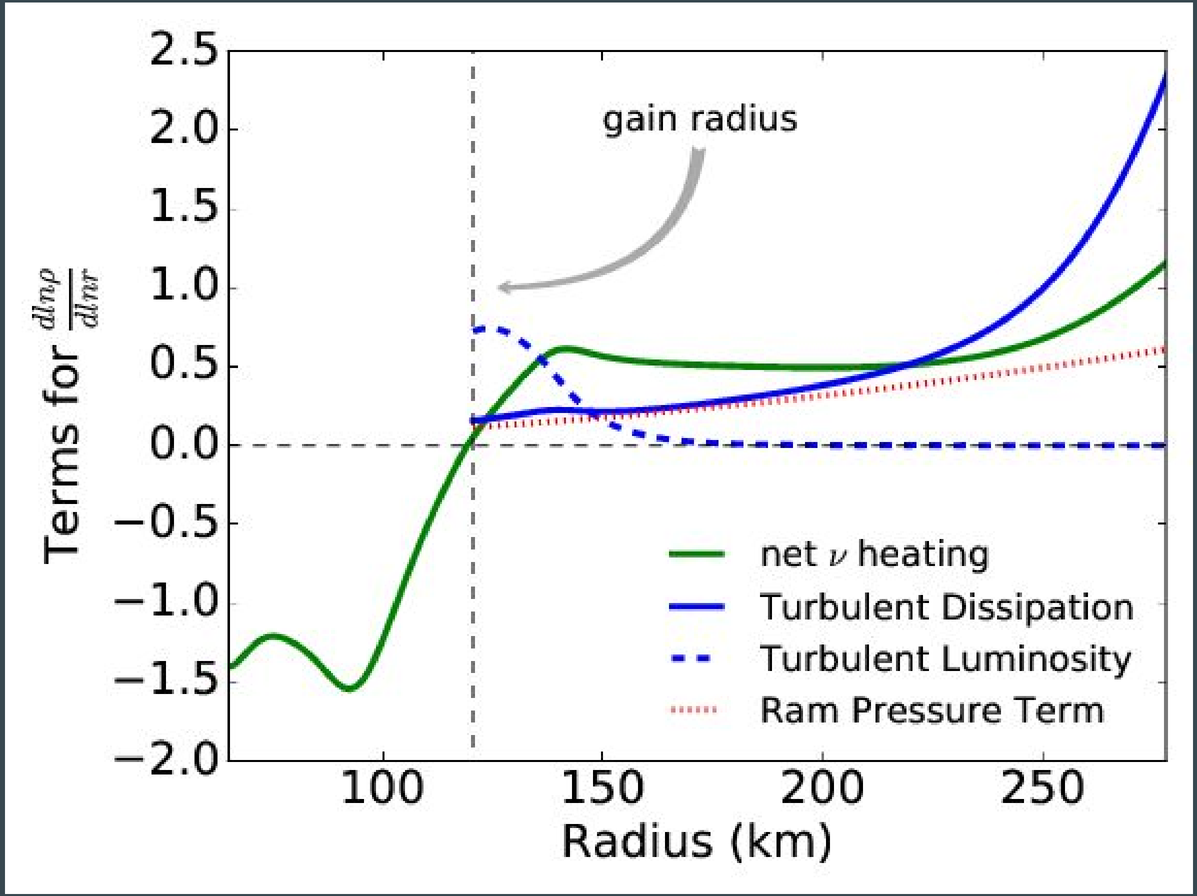
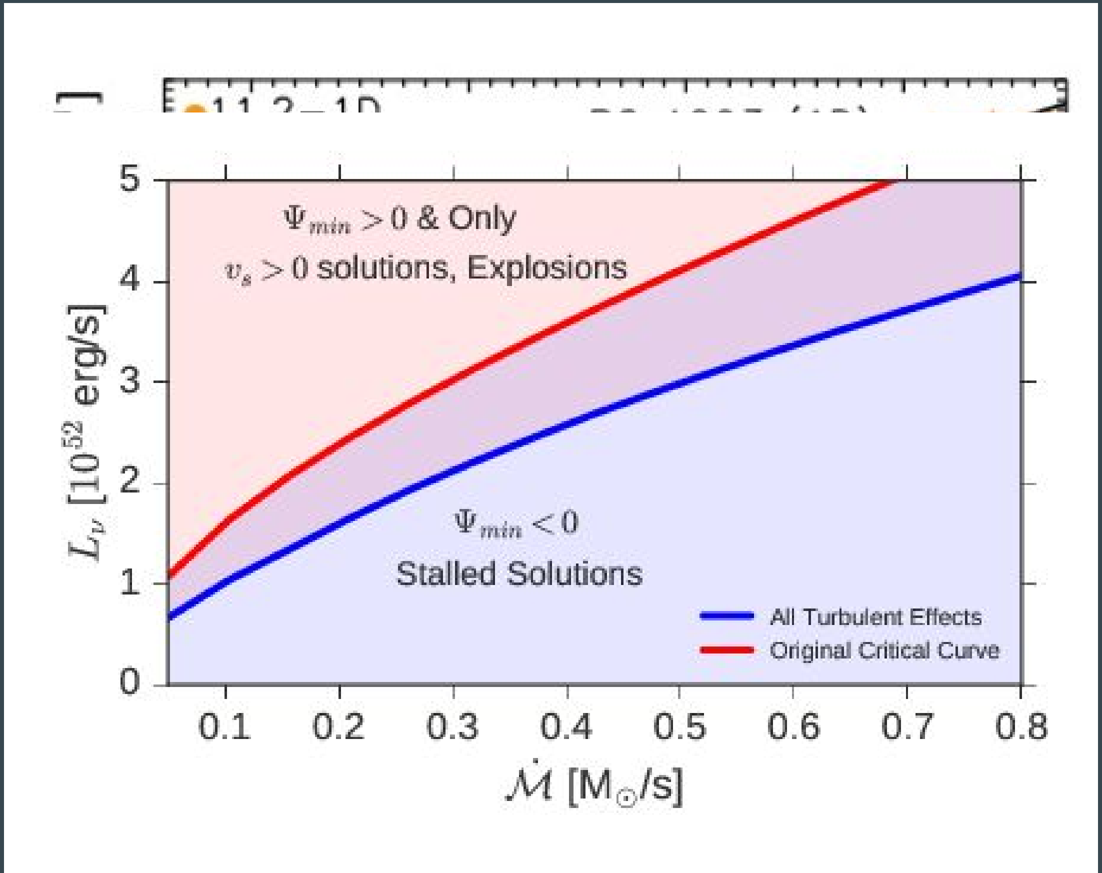
Altered Density and Temperature Profiles

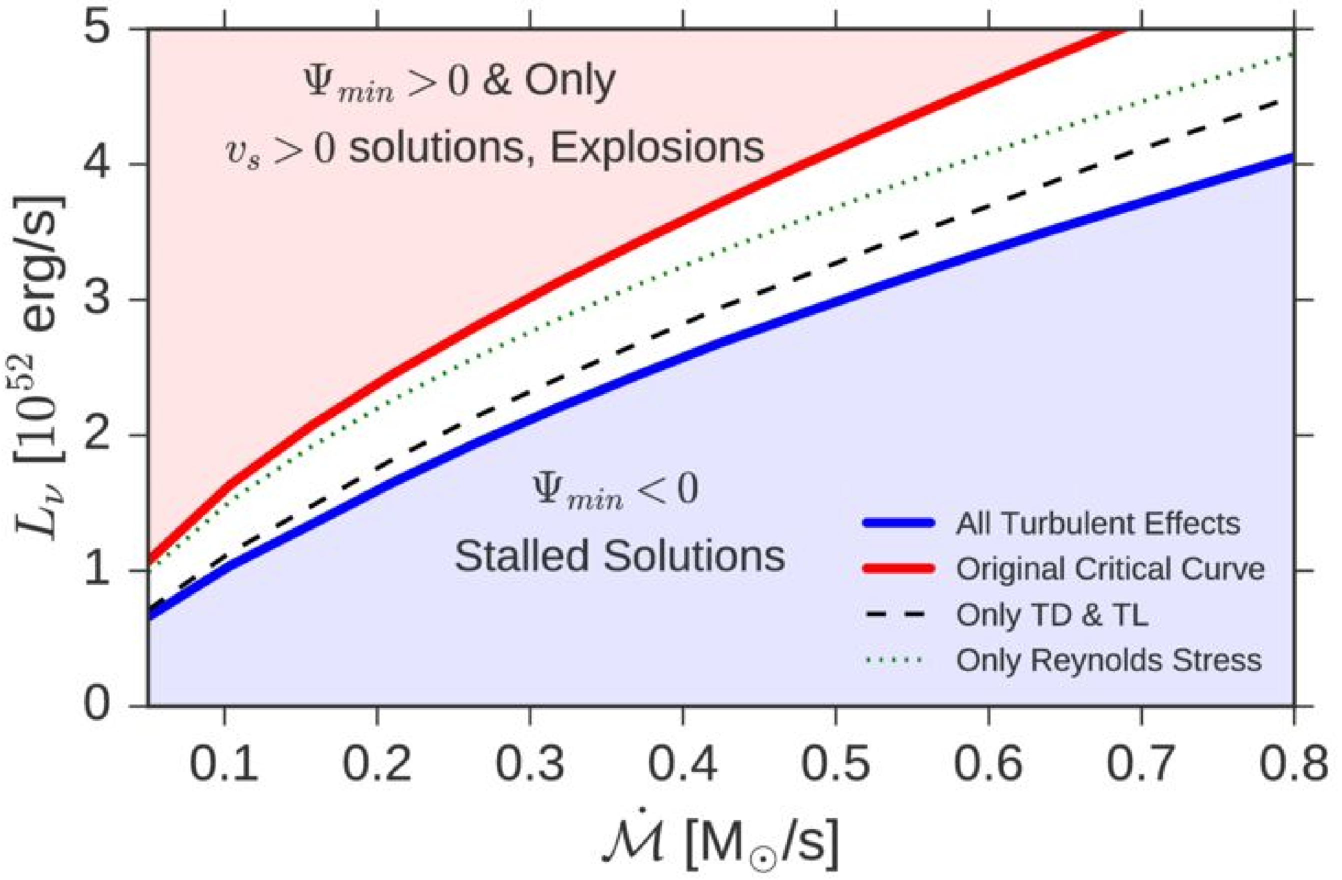


How Turbulence Affects the Profile



How Turbulence Affects the Critical Curve



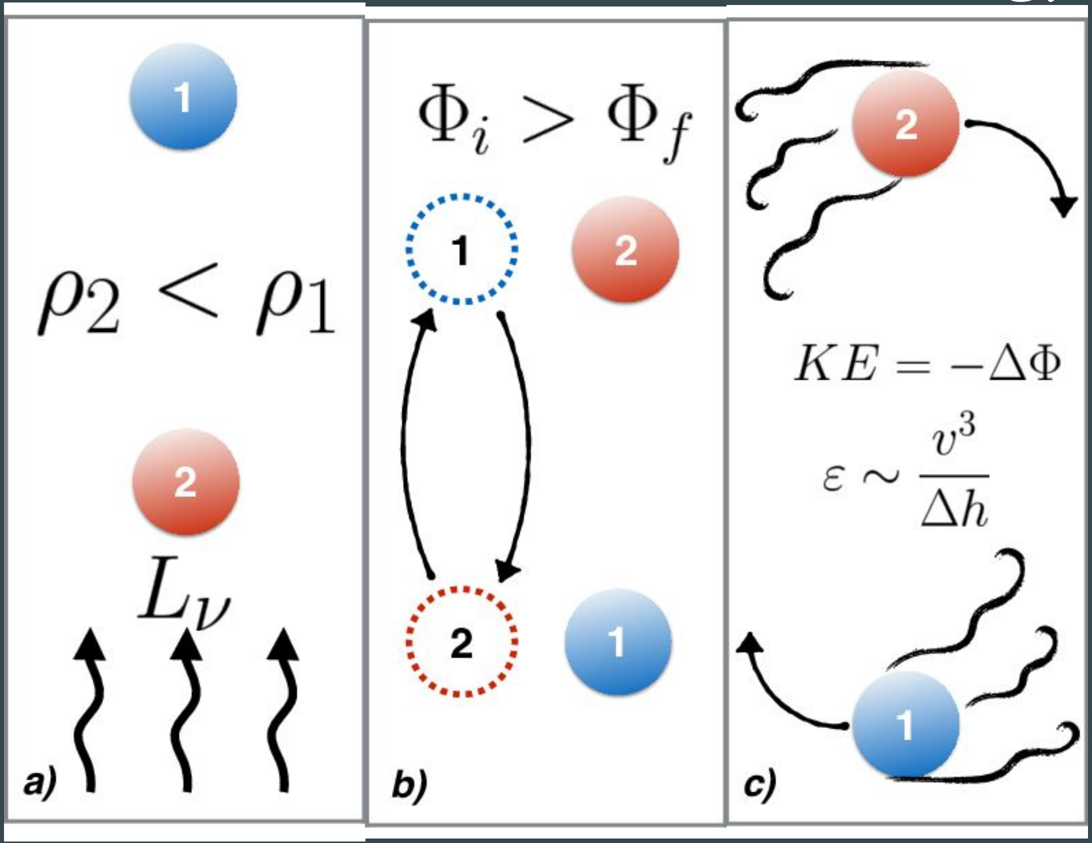


$\Psi_{min} > 0$ & Only
$v_s > 0$ solutions, Explosions
$\Psi_{min} < 0$
Stalled Solutions
All Turbulent Effects
Original Critical Curve
Only TD & TL
Only Reynolds Stress
L_ν [10^{52} erg/s]
$\dot{\mathcal{M}}$ [$M_\odot$/s]
0
1
2
3
4
5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

So what is the source of the energy
for turbulent dissipation?

Is there double counting of the neutrino energy?

Conversion of Convective Kinetic Energy to Heat



Summary

- Simulations suggest that multidimensional effects ease explosion
- Burrows & Goshy (1993) constructed a critical curve which divided the boundary between which steady-state solutions do or do not exist
- We include turbulence self-consistently in a critical conditions analysis (Reynolds decomposition)
- Investigate how turbulence reduces the critical condition
- All turbulent effects play a role in this reduction, but we find turbulent dissipation to be the dominant effect
- We implore simulators check our assumptions and check the values of R, ε_k, and L_e

Looking Forward

- Implement a more rigorous neutrino transport model (two-moment closure, ray-by-ray, Boltzmann transport)
- Verify and validate our model with 3-D simulations
- Incorporate SASI..?
- Probe how sensitive the critical curve is to other detailed physics (e.g. General Relativity, Equation of State, absorption cross-section, etc.)

Thanks!