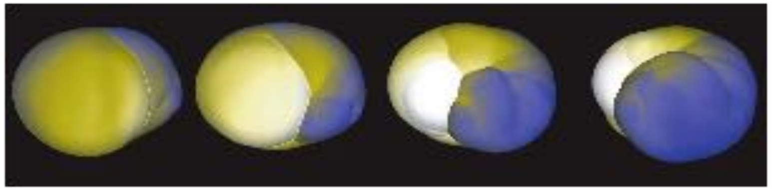
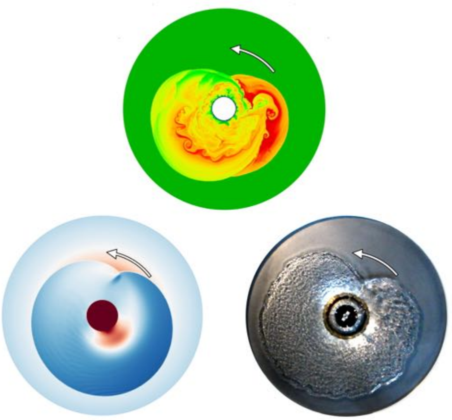
The impact of modest stellar rotation on the asymmetric explosion of massive stars



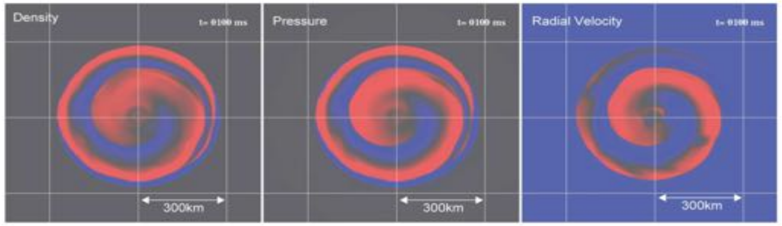
Blondin & Mezzacappa 07

Why is the prograde mode of SASI destabilized by rotation?



How much rotation can make a difference on the explosion threshold and NS birth?

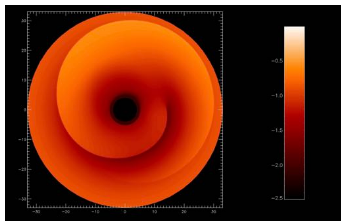
Should we expect different GW signatures from these two instabilities?

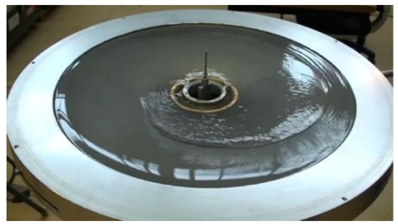


Takiwaki+16

What is the interplay between SASI and the corotation 'low T/|W|' instability?

Is the corotation instability similar to isolated NS with differential rotation ?

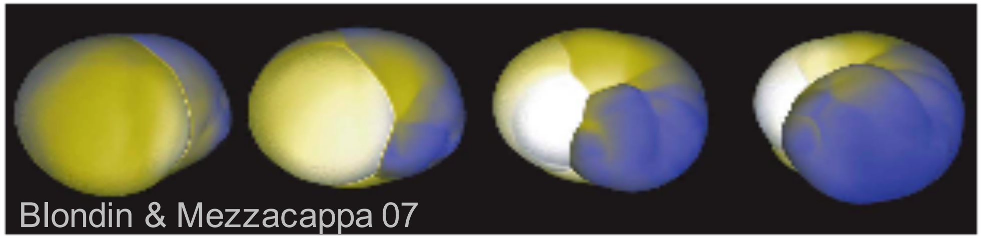




Thierry Foglizzo, Rémi Kazeroni*, Jérôme Guilet, Matthias Gonzalez, Gilles Durand, Bruno Pagani
CEA Saclay, *MPA Garching

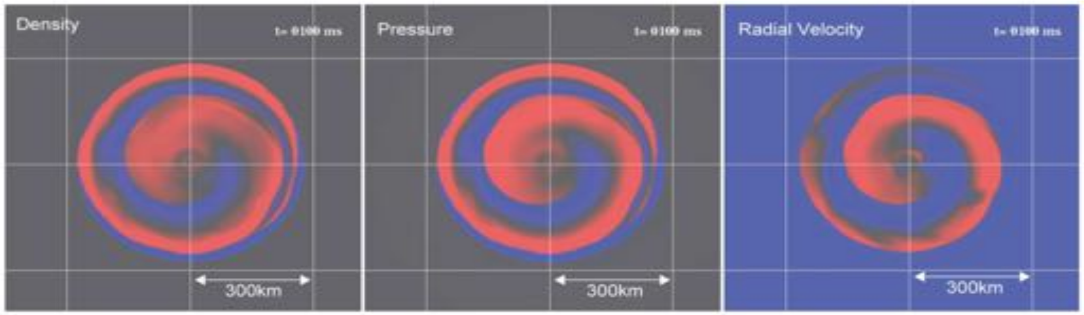
How much rotation is needed to affect the explosion?

slow rotation ($j = 10^{15}$ cm^2/s): spiral SASI



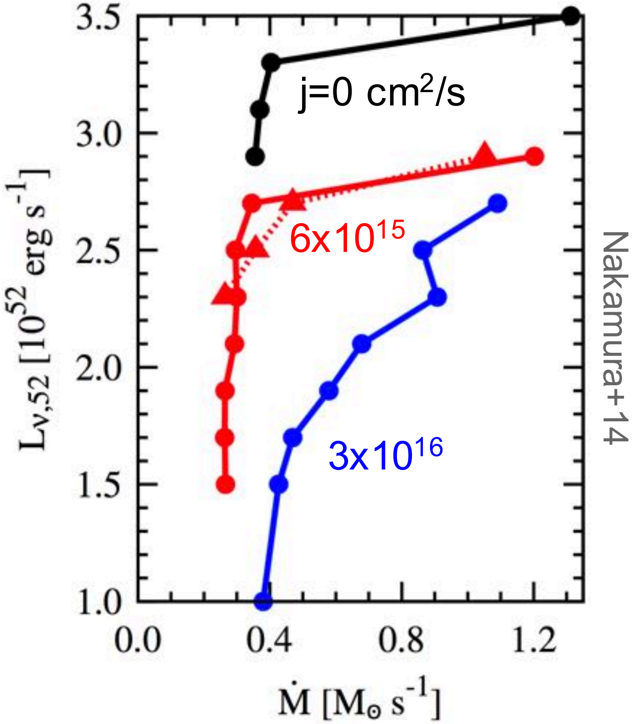
retrograde pulsar spin is possible at birth

Very fast rotation (Ω_0=2rad/s, j = 4×10^{16} cm^2/s): low-T/|W| instability



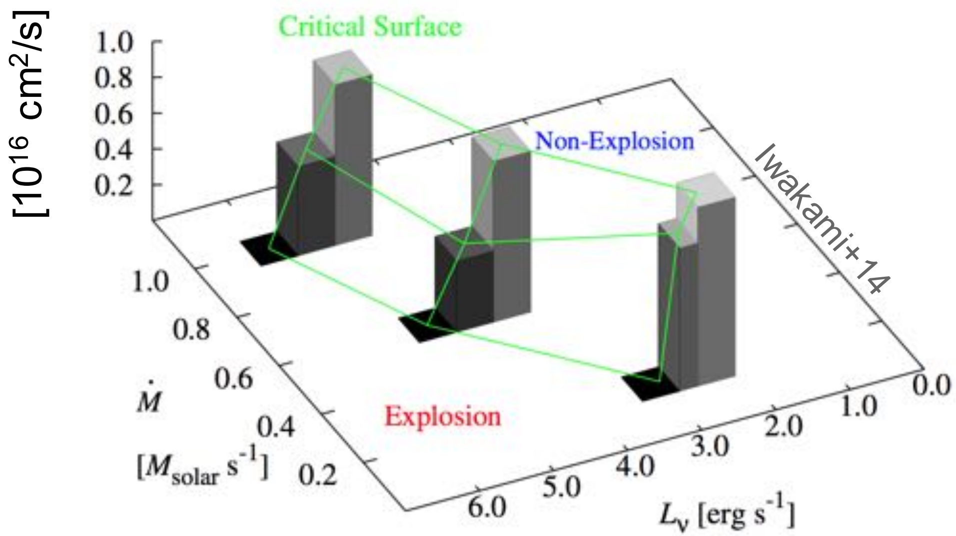
Takiwaki+16

but no effect for Ω_0=1rad/s, j = 2×10^{16} cm^2/s



$\Delta L_\nu/L_\nu$~10%

for j=5×10^{15}cm^2/s
(~ms pulsar)
modest effect compared to the rotational kinetic energy involved

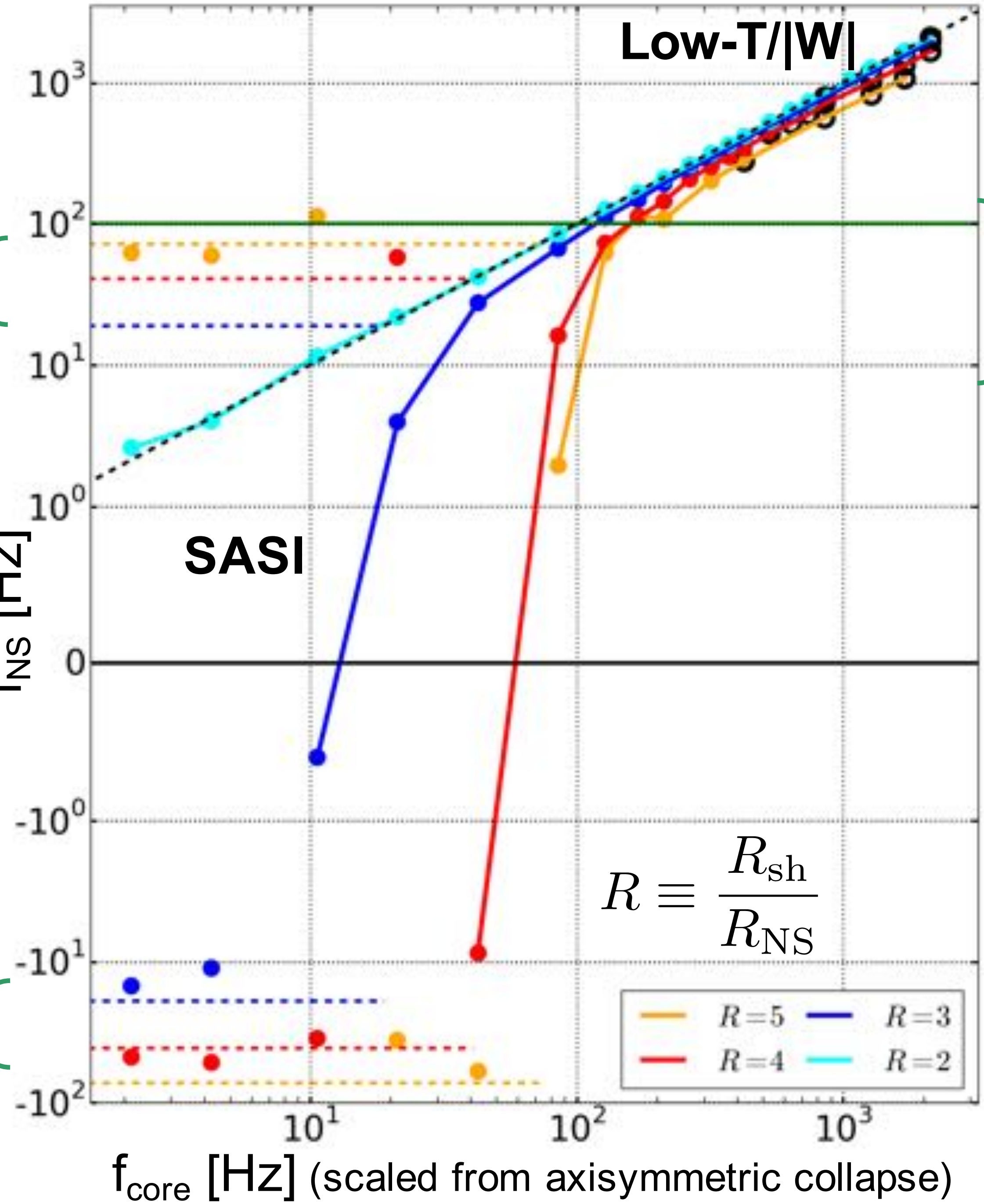


$$E_{\rm rot} \sim 1.5\times10^{52}{\rm erg}\left(\frac{M_{\rm ns}}{1.5M_{\rm sol}}\right)\left(\frac{10{\rm km}}{R_{\rm ns}}\right)^2\left(\frac{j}{5\times10^{15}{\rm cm^2s^{-1}}}\right)^2$$

Spin-up or spin-down of the neutron star?

(Kazeroni+17)

SASI alone



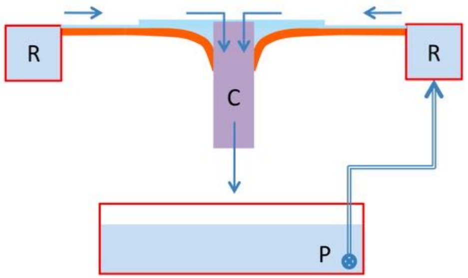
Neutron Star rotation frequency

SASI alone

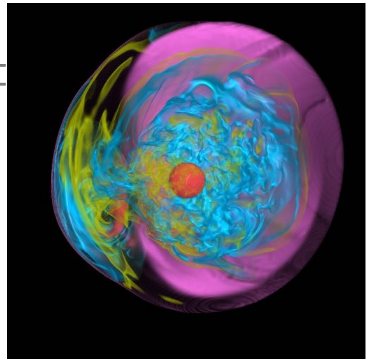
range of NS spin at birth

For a strong rotation rate, the corotation instability decelerates the neutron star by less than 30%.

Physical insight from an experimental analogue of SASI



Blondin & Mezzacappa 07



adiabatic gas

$$c_s^2 \equiv \frac{\gamma P}{\rho}$$

$$\Phi \equiv -\frac{GM_{ns}}{r}$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho v) = 0$$

$$\frac{\partial v}{\partial t} + (\nabla\times v)\times v + \nabla\left(\frac{v^2}{2} + \frac{c_s^2}{\gamma-1} + \Phi\right) = \frac{c_s^2}{\gamma}\nabla S$$

Inviscid shallow water is analogue to an isentropic gas γ=2

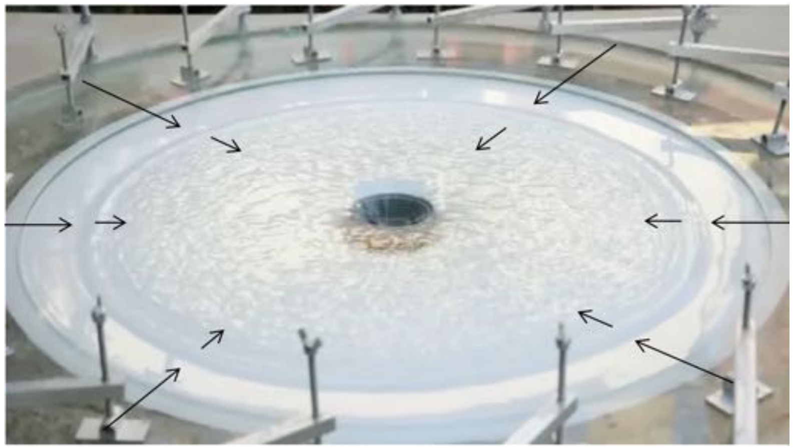
St Venant

$$c_{sw}^2 \equiv gH$$

$$\Phi \equiv gH_\Phi$$

$$\frac{\partial H}{\partial t} + \nabla\cdot(Hv) = 0$$

$$\frac{\partial v}{\partial t} + (\nabla\times v)\times v + \nabla\left(\frac{v^2}{2} + c_{sw}^2 + \Phi\right) = 0$$



acoustic waves ↔ surface waves
shock wave ↔ hydraulic jump
pressure ↔ depth

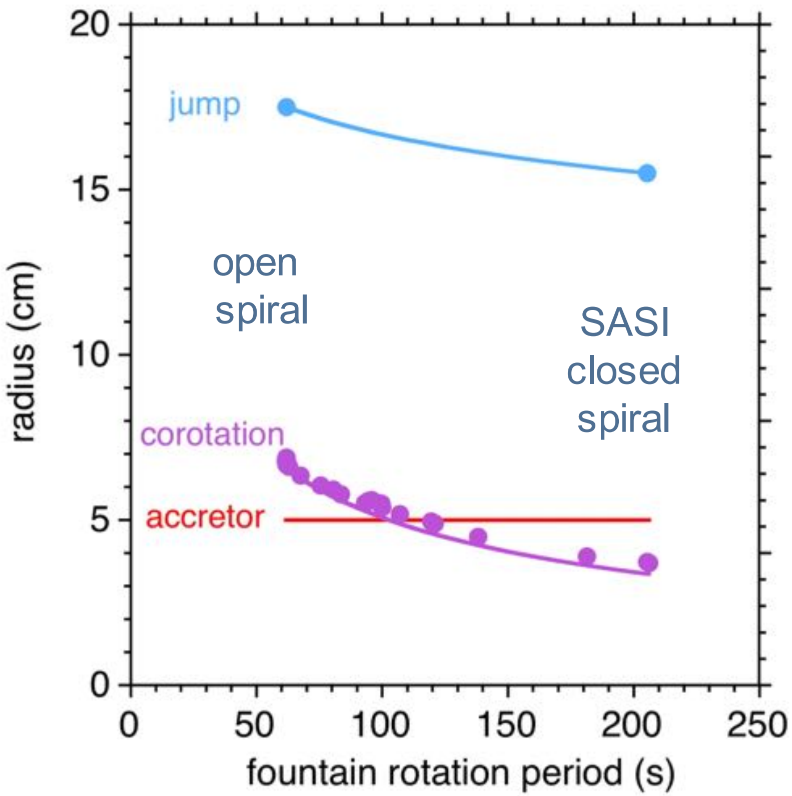
expected scaling

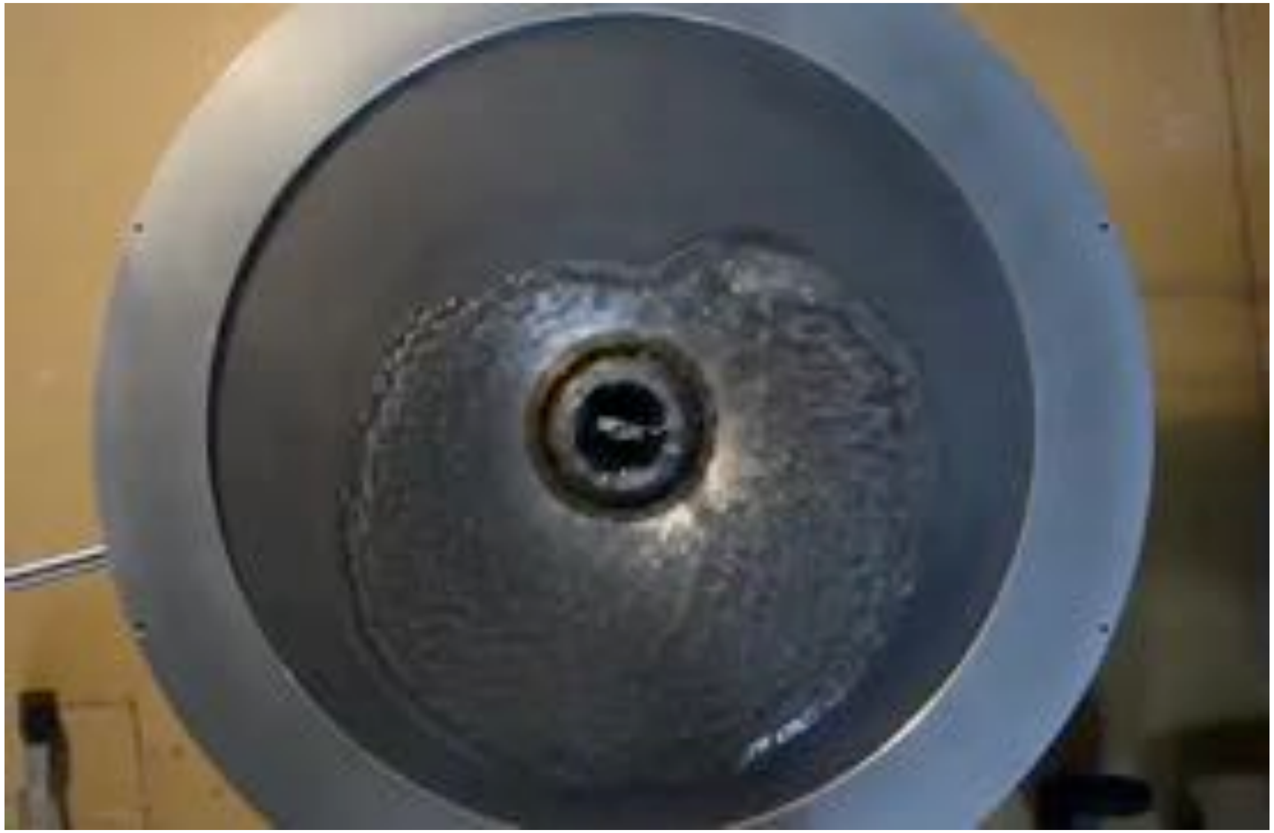
$$\frac{t_{ff}^{sh}}{t_{ff}^{jp}} \equiv \left(\frac{r_{sh}}{r_{jp}}\right)\left(\frac{r_{sh}gH_{jp}}{GM_{NS}}\right)^{\frac{1}{2}} \sim 10^{-2}$$

shock radius $\times 10^{-6}$ 200 km → 20 cm

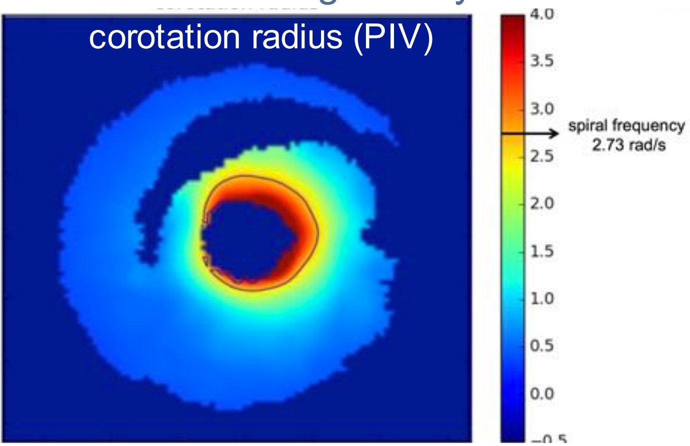
oscillation period $\times 10^{2}$ 30 ms → 3 s

Increasing the rotation rate: continuous transition from SASI to the corotation instability

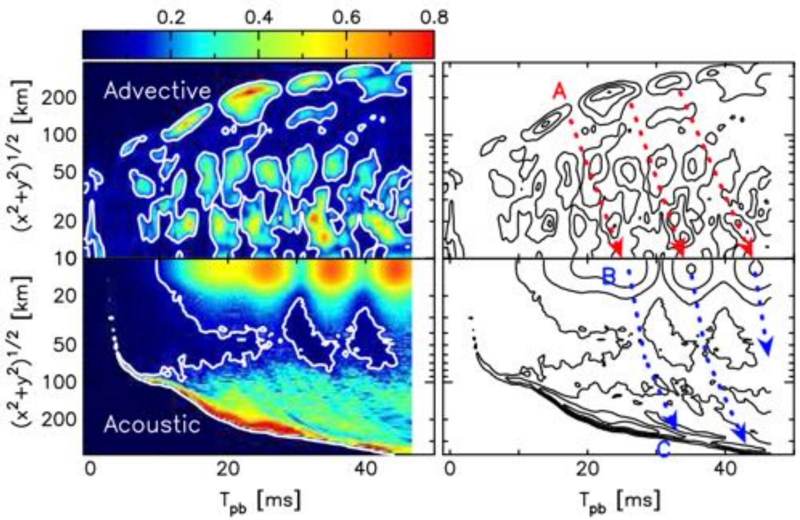




the rotation period is gradually decreased (205s → 62s)
the flow rate is gradually decreased (1.1 L/s → 0.59 L/s)



→the gravitational wave signature of the low T/|W| instability may be hard to disentangle from the SASI oscillations (Kuroda+14)



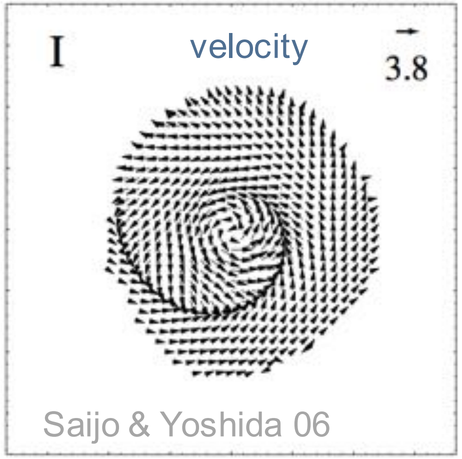
Unexpectedly robust spiral shock driven at the corotation radius when the inner rotation rate reaches 20% Kepler (low T/|W|=0.02)

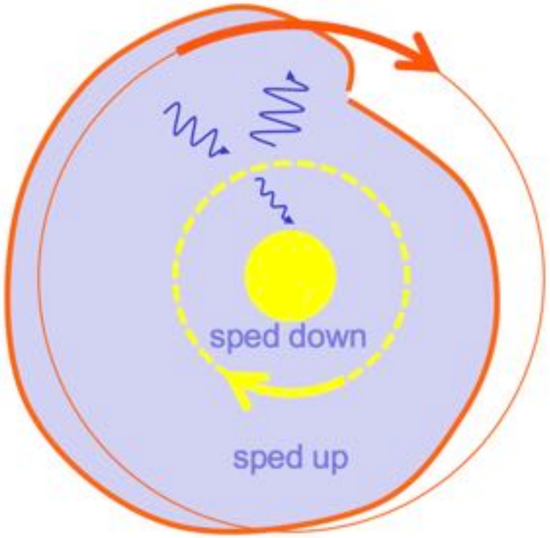
Spiral instability with a weak shock

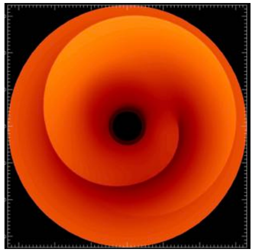
Radial accretion enforces differential rotation

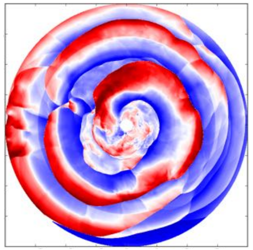
$$\frac{\Omega}{\Omega_{NS}} \propto \left(\frac{R_{NS}}{R}\right)^2$$

Analogue to the "low T/|W| instability" of a neutron star rotating differentially (Shibata+02,03, Saijo+03,06, Watts+05, Corvino+10, Passamonti & Andersson 15, Yoshida & Saijo 17)

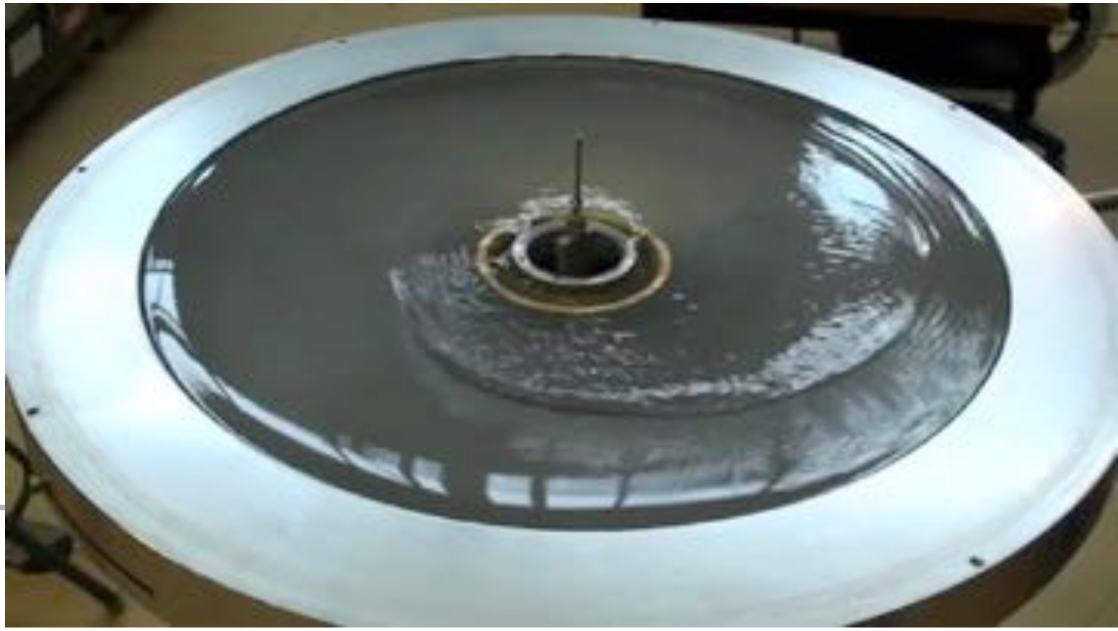






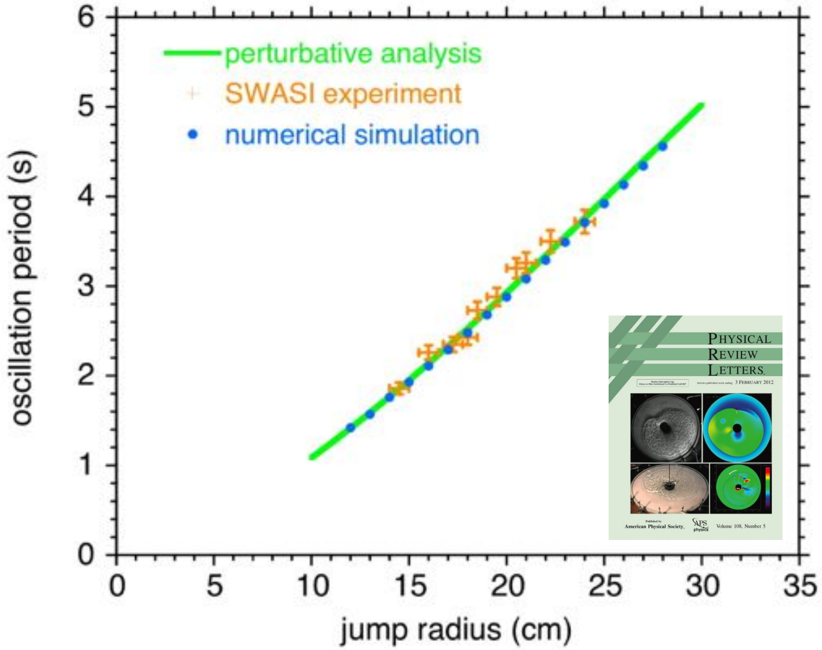


Spiral instability with subsonic accretion

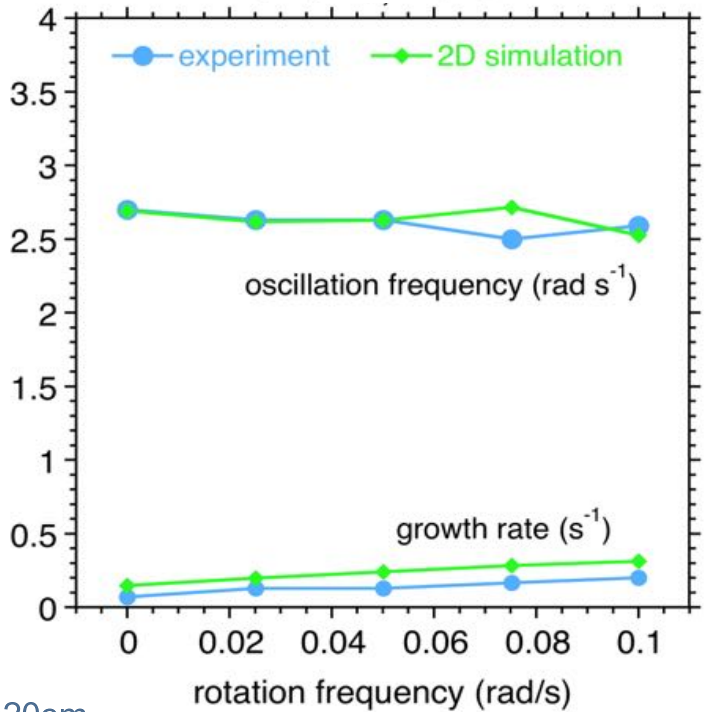


Instability mechanism: interaction of a corotation radius with acoustic waves (Papaloizou & Pringle 84, Goldreich & Narayan 85)

Experimental growth rate and oscillation period compared to shallow water modelling: a hint for an advective mechanism



Foglizzo+12



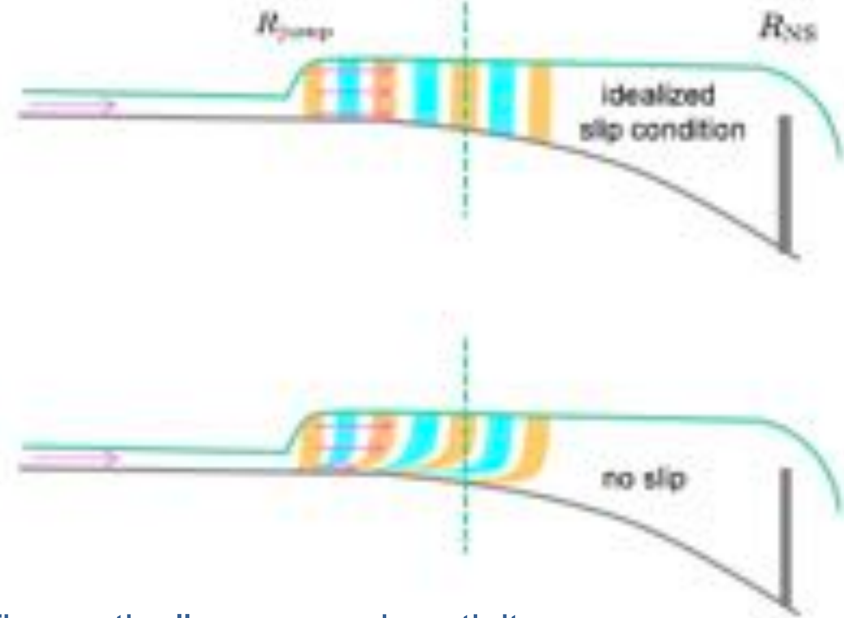
R_{jp}~20cm

-excellent modelling of the oscillation frequency

limited by the measured radial width of the hydraulic jump

-systematic offset of the experimental growth rate

expected phase mixing of the dragged vorticity



The vertically averaged vorticity is damped by a factor Q

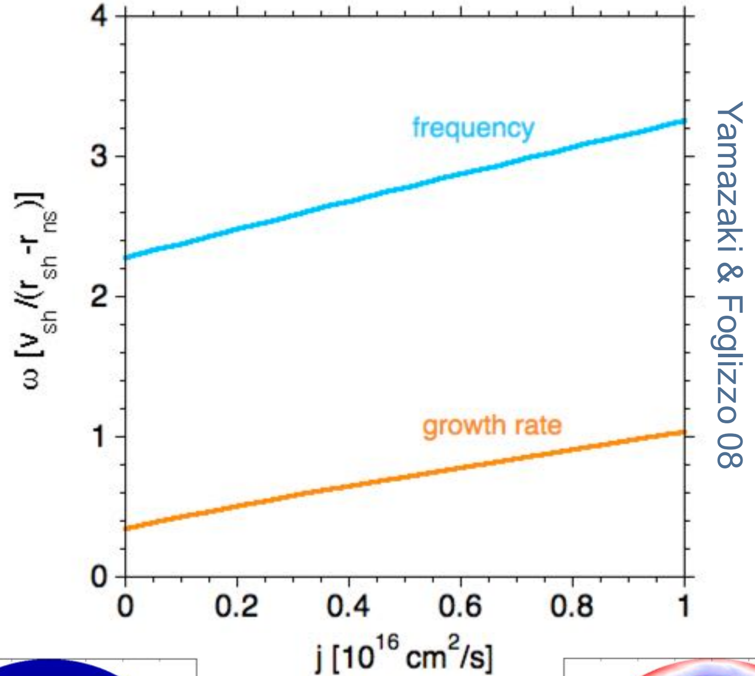
$$Q \sim \int_0^H \frac{\mathrm{d}z}{H} \cos\left[\frac{\omega_{\rm SASI}\Delta R}{v(z)}\right] \sim 0.27 \text{ (laminar)}$$

$$\sim 0.52 \text{ (turbulent)}$$

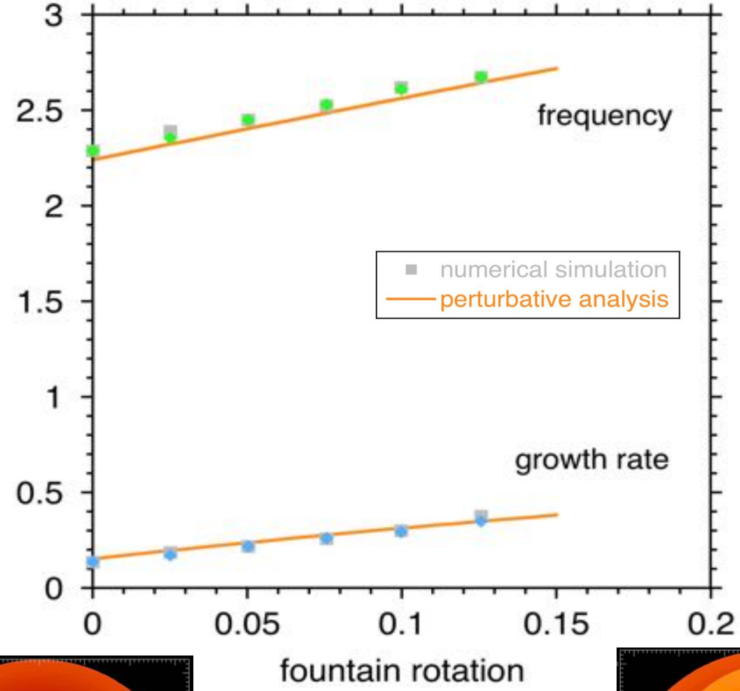
→ at odds with the idea of a transition to an acoustic corotation instability?

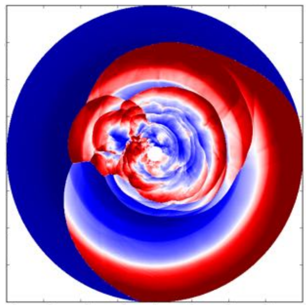
Rotation effect in shallow water equations compared to gas dynamics

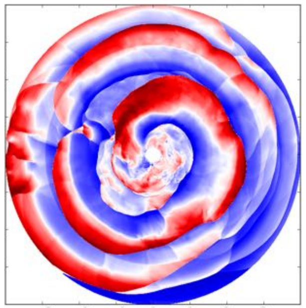
shocked gas dynamics

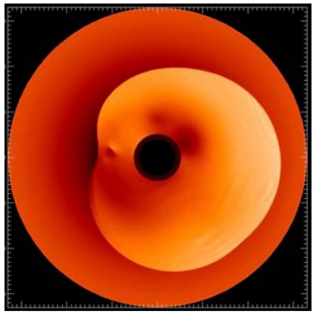


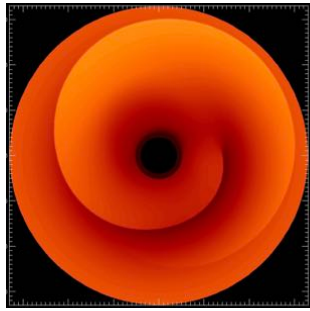
shallow water equations











Why is the prograde mode of SASI destabilized by rotation?
Why is the transition to the corotation instability so smooth?

The St Venant system of shallow water equations is a simpler framework to understand the coupling of SASI with rotation and the transition to the low T/|W| instability:
- -adiabatic equations (no neutrino cooling)
- -no buoyancy effects

Compact formulation of the perturbative problem with rotation

Differential system for the linearized perturbations

$$\begin{cases} \dfrac{\partial H}{\partial t} + \nabla \cdot (H\boldsymbol{v}) = 0, & \text{mass conservation} \\ \dfrac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{w} \times \boldsymbol{v} + \nabla \left(\dfrac{|\boldsymbol{v}|^2}{2} + gH + \Delta\Phi \right) = 0. & \text{Euler equation} \end{cases}$$

Doppler shifted frequency

$$\omega' \equiv \omega - \frac{mL}{r^2},$$

same as in a cylindrical flow (Yamasaki & Foglizzo 08)

$$\delta w_z = \frac{r_{sh} v_{sh}}{r v_r} (\delta w_z)_{sh} e^{\int_{sh} i\omega' \frac{dr}{v_r}},$$

conserved specific vorticity

same changes of variables as in a plane parallel flow (Foglizzo 09)

$$\begin{cases} \dfrac{dX}{dr} \equiv \dfrac{v_r}{1 - Fr^2}, & \text{new variable } X \\ \delta\tilde{v}_\theta \equiv e^{\int_{sh} i\omega' \frac{dX}{c^2}} \delta v_\theta, & \text{phase shifted velocity} \end{cases}$$

$$\left\{ \frac{d^2}{dX^2} + \frac{\omega'^2 - \frac{m^2}{r^2}(c^2 - v_r^2)}{v_r^2 c^2} \right\} r\delta\tilde{v}_\theta = e^{\int_{sh} i\omega' \frac{dX}{c^2}} \frac{d}{dX} \frac{r\delta w_z}{v_r}.$$

acoustic equation with a source term similar to Foglizzo 09 without rotation

+ boundary conditions from conservation equations

-at the shock/jump

$$\begin{cases} (\delta w_z)_{sh} = -\dfrac{im}{r_{sh} v_{sh}} \left(1 - \dfrac{v_{sh}}{v_1}\right) \Delta\zeta \left[-i\omega' v_1 \left(1 - \dfrac{v_{sh}}{v_1}\right) + \dfrac{\partial\Phi}{\partial r} - \dfrac{v_1 v_{sh}}{r_{sh}} - \dfrac{L^2}{r_{sh}^3} \right], \\ (r\delta v_\theta)_{sh} = im \left(1 - \dfrac{v_{sh}}{v_1}\right) v_1 \Delta\zeta, \\ \dfrac{d}{dX}(r\delta\tilde{v}_\theta)_{sh} = -\dfrac{im}{v_{sh}^2} \left(1 - \dfrac{v_{sh}}{v_1}\right) \Delta\zeta \left[-i\omega' v_1 \left(1 - 2\dfrac{v_{sh}}{v_1}\right) + \dfrac{\partial\Phi}{\partial r} - \dfrac{v_1 v_{sh}}{r_{sh}} - \dfrac{L^2}{r_{sh}^3} \right], \end{cases}$$

-at the inner boundary

$$v_*^2 \frac{d}{dX} \delta(r\delta\tilde{v}_\theta) = i\omega' Fr_*^{\frac{4}{3}} (r\delta\tilde{v}_\theta)_* + \left(1 - Fr_*^{\frac{4}{3}}\right) (r v_r \delta\tilde{w})_*$$

Wronskien resolution: convolution of the acoustic solution with the source term

acoustic solution satisfying the inner boundary condition

$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}X^2}+\frac{\omega'^2-\frac{m^2}{r^2}(c^2-v_r^2)}{v_r^2c^2}\right\}r\delta\tilde{v}_\theta^0=0,$$

inner boundary condition

$$v_*^2\frac{\mathrm{d}}{\mathrm{d}X}\delta(r\delta\tilde{v}_\theta^0)=i\omega'\mathrm{Fr}_*^{\frac{4}{3}}(r\delta\tilde{v}_\theta^0)_*,$$

definition of the perturbed mass flux

$$h^0\equiv\frac{\delta v_r}{v_r}+\frac{\delta H}{H}=\frac{1-\mathrm{Fr}^2}{imv_r}\mathrm{e}^{\int_{\mathrm{sh}}-i\omega'\frac{v_r}{1-\mathrm{Fr}^2}\frac{\mathrm{d}r}{c^2}}\frac{\mathrm{d}}{\mathrm{d}r}\left(\mathrm{e}^{\int_{\mathrm{sh}}i\omega'\frac{v_r}{1-\mathrm{Fr}^2}\frac{\mathrm{d}r}{c^2}}r\delta v_\theta^0\right).$$

acoustic solution | advected phase of the source term

$$\int_*^{\mathrm{sh}}\left(h^0+\frac{\omega'}{mc^2}r\delta v_\theta^0\right)\frac{\mathrm{e}^{\int_* i\omega'\frac{1+\mathrm{Fr}^2}{1-\mathrm{Fr}^2}\frac{\mathrm{d}r}{v_r}}}{1-\mathrm{Fr}^2}\frac{\mathrm{d}r}{v_r}=-\frac{\frac{\omega'}{mv_{\mathrm{sh}}}(r\delta v_\theta)_{\mathrm{sh}}^0+v_1h_{\mathrm{sh}}^0}{\frac{\partial\Phi}{\partial r}-\frac{v_1v_{\mathrm{sh}}}{r_{\mathrm{sh}}}-\frac{L^2}{r_{\mathrm{sh}}^3}-i\omega'v_1\left(1-\frac{H_1}{H_{\mathrm{sh}}}\right)}\mathrm{e}^{\int_*^{\mathrm{sh}}i\omega'\frac{1+\mathrm{Fr}^2}{1-\mathrm{Fr}^2}\frac{\mathrm{d}r}{v_r}}-\frac{h_*^0}{i\omega'_*}.$$

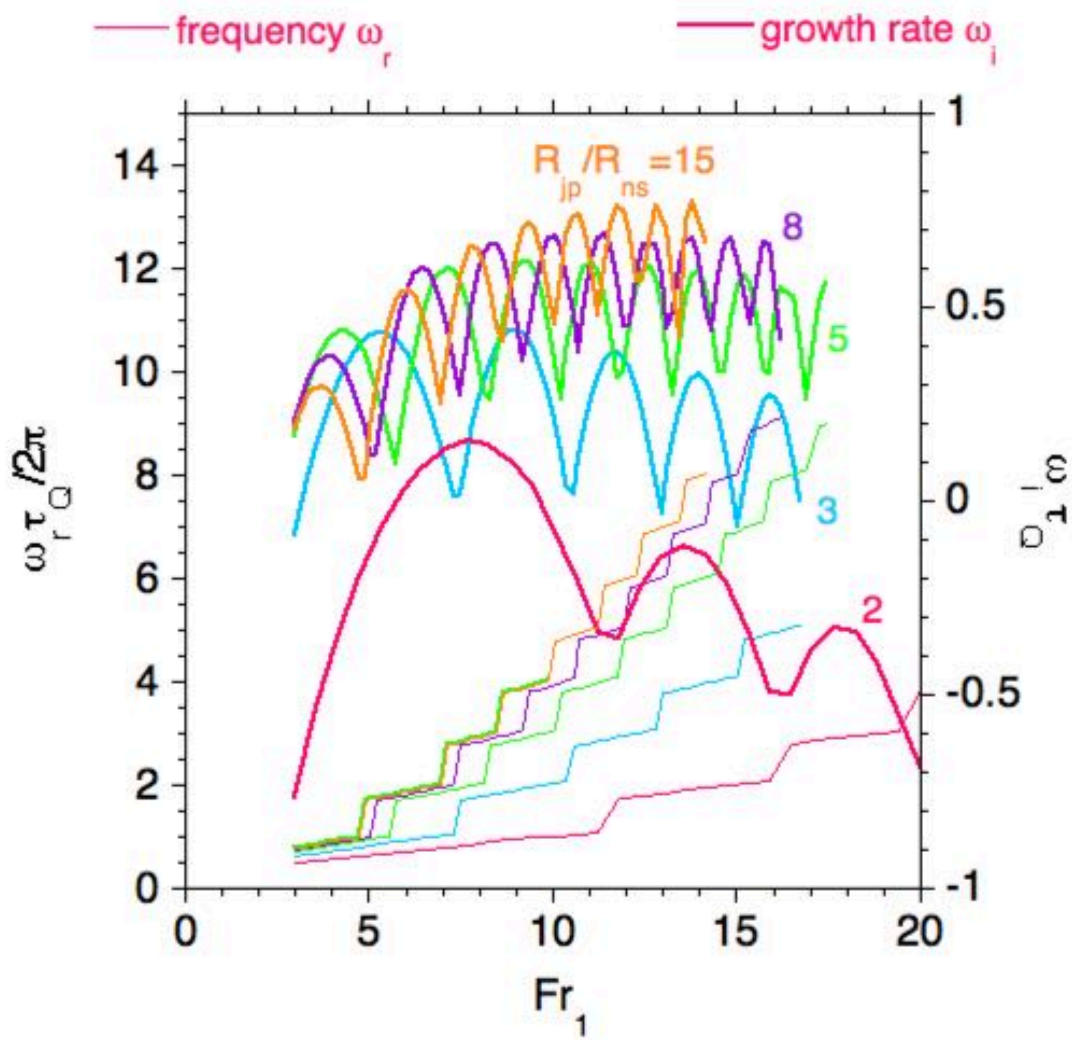
advective-acoustic coupling | shock boundary condition | inner boundary condition

The Doppler shifted frequency $\omega'\equiv\omega-\frac{mL}{r^2}$ affects the phase mixing between the source and the acoustic wave

The frequency of the prograde mode is locally decreased by the doppler shift: the decrease of $\omega'\tau_\nabla$ is favourable to the advective-acoustic coupling as in Sheck+08 and Foglizzo 09 without rotation.

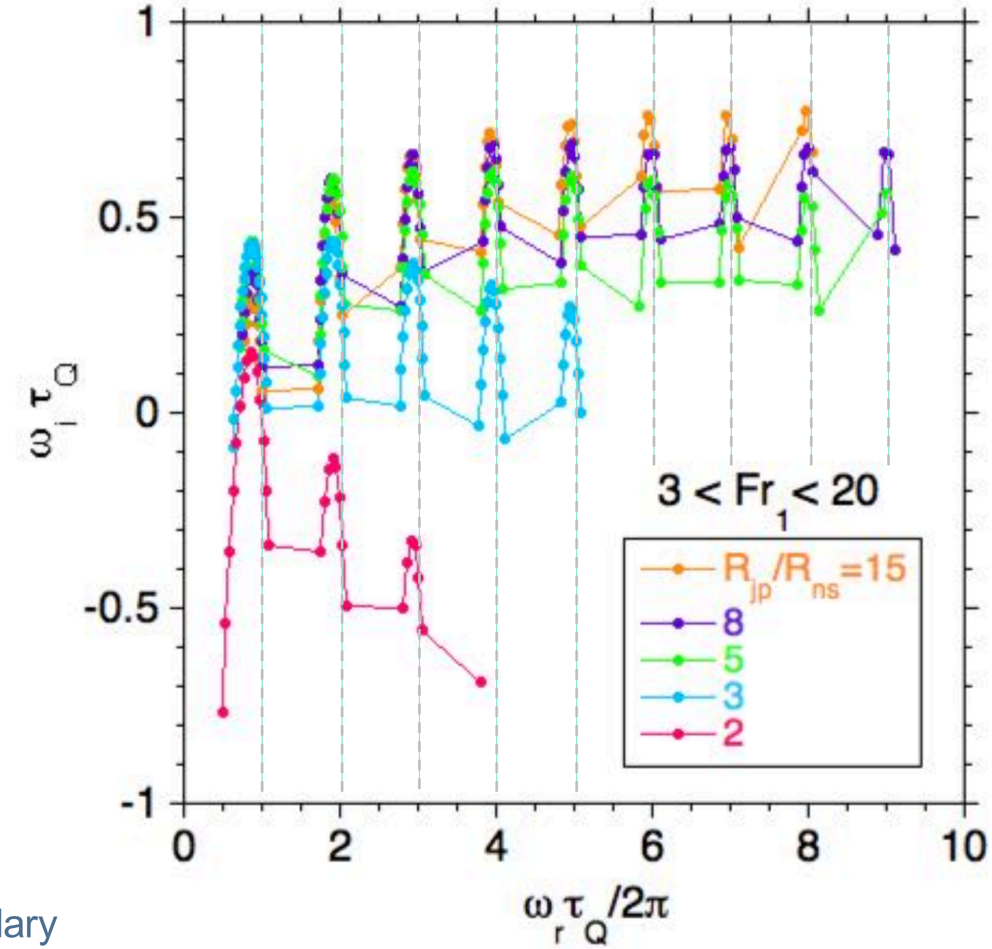
The corotation condition $\omega'=0$ favours the advective-acoustic coupling: the stationary phase prevents phase mixing

Analogue of SASI modes without rotation: Fr_1, R_{jp}/R_{ns}



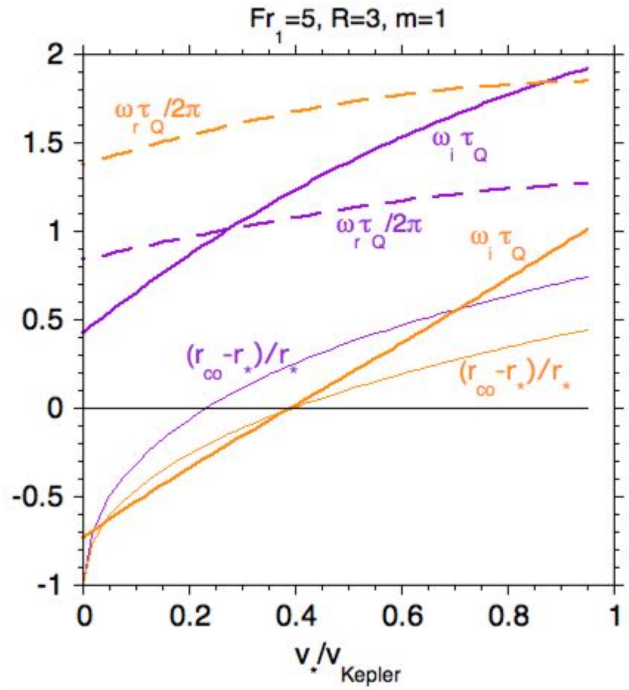
radial advective-acoustic time

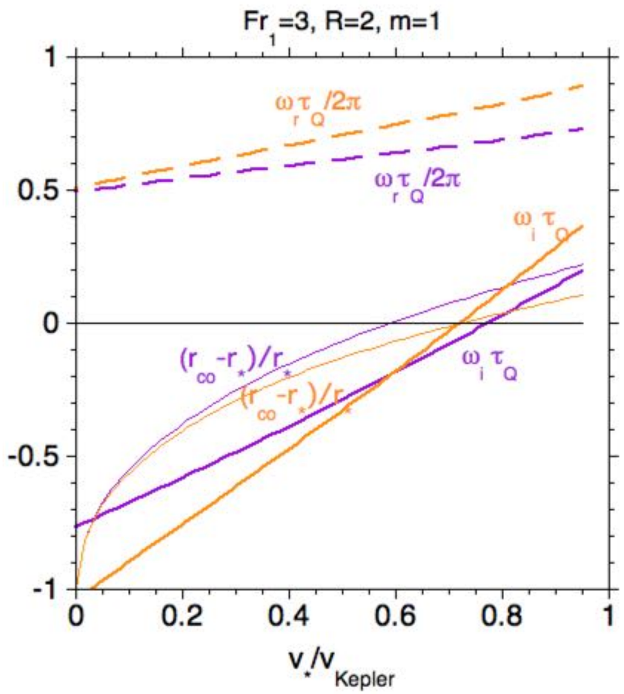
$$\tau_{\rm Q} \equiv \int_*^{\rm sh} \frac{{\rm d}r}{|v_r|} + \int_*^{\rm sh} \frac{{\rm d}r}{c-|v_r|}$$



eigenfrequencies ~multiples of $2\pi/\tau_Q$ suggest that the advective-acoustic coupling is dominated by the lower boundary

Comparison of a shocked rotating flow and a trapped acoustic mode





normal shock condition: vorticity + pressure perturbation

- as in Yamasaki & Foglizzo 08, the growth rate of the prograde mode increases with the rotation rate

- a corotation radius can exist for rotation rates as low as 3% v_{Kepler} at r_* (T/W~0.05%)

- a corotation radius is not a sufficient condition for instability (e.g. Fr_1=3, R=2)

- the transition from SASI to an instability with a corotation is very smooth

ad-hoc shock condition: total acoustic reflexion, no vorticity

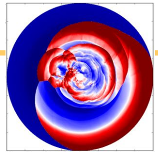
- when acoustic reflexion at the shock is total, the existence of the corotation radius is a sufficient condition for instability: similar to differentialy rotating NS (Watts+05, Passamonti & Andersson 15, Yoshida & Saijo 17)

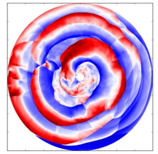
- a corotation radius can exist for rotation rates as low as 6% v_{Kepler} at r_* (T/W~0.02%)

- however, the growth rate of this corotation instability seems loosely correlated with the growth rate of the shocked flow.

- estimated growth rate: $$\frac{\omega_i}{\Omega_{\rm sh}} \propto (2.2 \pm 0.4) \left(\frac{\Omega_{\rm sh}}{\Omega_*}\right)^{\frac{1}{2}} \left[1 - \frac{\Omega_{\rm corot}}{\Omega_*}\right]$$

Conclusions



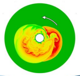


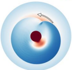
2D Cylindrical gas dynamics (Kazeroni+17) suggests that

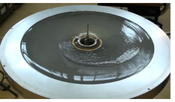
-SASI can account for pulsar rotation periods down to ~50ms

-for rotation rates >100Hz the 'corotation instability' decreases the pulsar spin by <30%

Both instabilities are captured in the supernova fountain experiment



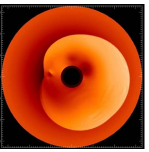


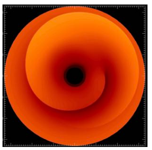


-as the injected angular momentum increases, the prograde spiral mode of SASI seems to connect smoothly to the 'low T/|W|' instability

-the offset growth rate in the experiment suggests advection may play a dominant role even when a corotation is present

The shallow water model offers a simple analytical framework to study the interplay of SASI & 'low T/W'





-equations are both simple and connected to a real experiment

-the rotational destabilization of the prograde mode of SASI can be explained by its lower doppler shifted frequency which benefits to the advective-acoustic coupling

-a classical corotation instability is recovered as a purely acoustic process, despite radial advection, if the shock is artificially replaced by a total acoustic reflection and no advected vorticity

-the existence of a corotation radius is not a sufficient condition for instability in a shocked flow

-the prograde mode of SASI can be more unstable than an acoustic corotation instability: the stationary phase at the corotation radius favours the advective-acoustic coupling

→A sharp transition between SASI and the 'low T/|W|' instability in a shocked flow is not expected